A Dynamic Asset Allocation Strategy with Macroeconomic Indicators

by

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

This thesis proposes a novel approach to strategic asset allocation (SAA) that uses macro-econometric factors and a regime-switching regression model to capture systemic risk, which is crucial in setting target allocations for various asset classes and measuring associated risk premiums. The proposed model employs a Hidden Markov Model (HMM) to filter the four macro-econometric factors, which are the main source of market risk, and uses a mean-variance-based dynamic selection model to increase the portfolio's risk-adjusted return. The empirical analysis utilizes global bond and equity ETFs, as well as sector ETFs, to test the asset pricing model and evaluate the performance of the portfolio. Weekly financial data from January 01, 2002, to December 31, 2022, are used to construct the four macro-econometric factors, and the four weekly macro-econometric factors from September 06, 2016, to January 1, 2019, are employed to estimate the HMM and asset return parameters. The results show that the proposed model consistently outperforms the benchmark MSCI ACWI Index in the out-of-sample period from January 1, 2019, to December 31, 2022, and outperforms the selected ETFs and the mean-variance models in terms of the Sharpe Ratio. The proposed approach to SAA provides a promising avenue for investors to achieve superior risk-adjusted returns in the current global financial landscape.

Glossary

Strategic Asset Allocation (SAA) - A long-term investment strategy that involves setting target allocations for various asset classes, based on an investor's risk tolerance, investment objectives, and time horizon.

Dynamic Asset Allocation (DAA) - A portfolio management strategy that involves adjusting the asset allocation mix over time based on changing market conditions or economic outlook.

Macroeconomic Factor (Indicators) - A broad economic variable or metric that has a significant impact on the overall economy, industries, sectors, and financial markets, such as interest rates, inflation, GDP growth.

Financial Risk Premium - The excess return that investors demand for holding a risky asset over a risk-free asset, which reflects the compensation for bearing the risk of a particular asset.

Macro-econometric Factor - Factors that are typically derived from macroeconomic data using statistical methods such as regression analysis, principal component analysis, or factor analysis. These factors are systematically related to asset returns and are used as risk factors in asset pricing models.

CAGR - Cumulative Annualized Growth Rate is a measure used to calculate the annual growth rate of an investment over a specific period. CAGR = (Ending value / Beginning value) ^ (1 / Number of years) – 1.

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Chapter 1 Introduction

Strategic asset allocation (SAA) is a pivotal strategy for institutional investors, such as pensions, endowments, or sovereign wealth funds, to set target allocations in various asset classes, and it forms the basis for factor investing. The initial study of asset allocation was done by Markowitz (1952), who developed the seminal modern portfolio theory (MPT). Subsequently, Sharpe (1964), Treynor (1962), Lintner (1965a, b), and Mossin (1966) introduced the capital asset pricing model (CAPM) and the market factor. Empirically, Fama and French (1993, 2005) proposed additional factors such as size, value, profitability, and investment factor. Carhart (1997) proposed the momentum factor, which refers to the tendency of assets that have recently performed well or poorly to continue that trend in the short term. Carhart's momentum factor has since been widely studied and applied in finance, with numerous empirical studies demonstrating its ability to explain cross-sectional returns in asset pricing models. In addition, researchers are studying numerous other factors, including liquidity, volatility, and quality, among others, to better understand their impact on asset pricing and investment performance. Since the Global Financial Crisis (GFC, 2008–2009), institutional investors have increasingly focused on factor investing strategies to target specific drivers of securities or asset classes such as size or value factors.

Factor investing is a strategy that involves selecting securities or assets based on specific characteristics or factors that are expected to contribute to higher returns and improve portfolio performance, instead of relying solely on traditional asset class diversification. This approach involves identifying factors such as value, growth, momentum, and quality that have historically been associated with higher risk-adjusted returns. Factor investing can be implemented using various investment vehicles, including index funds and exchange-traded funds (ETFs), which provide investors with a cost-effective and transparent way to gain exposure to specific factors or combinations of factors. By tracking certain factor indices, investors can achieve more targeted exposure to the factors and diversify their portfolio across multiple securities and asset classes.

The factors commonly used to explain fundamentals-based return premia, also known as style factors, are primarily constructed based on equity portfolios. However, it's not sufficient for managing macroeconomic risks, such as interest rate or credit risk, particularly when making broad asset allocation decisions. Therefore, there is a significant need for a macroeconomic factor model that can effectively account for and manage these types of risks. The macroeconomic factors are significantly different from the style factors in that they are largely related to macroeconomic conditions such as GDP Growth or Inflation versus the style factors based on the company's internal performance or share-related variables such as Value or Size. Macroeconomic conditions can reveal the overall state of the economy and can impact investor sentiment and market expectations, leading to changes in the equity risk premium (ERP). The ERP is the excess return that investors expect to receive from investing in stocks compared to risk-free government bonds, which is an important indicator of the performance of the stock market and reflects the expectations of future stock market returns (Duarte and Rosa, 2015). The historical ERP is typically calculated as the difference between the average annual return on a broad stock market index, such as the S&P 500, and the yield on a risk-free government bond, such as a 10-year Treasury note. While historical ERP cannot guarantee future ERP outcomes, it can provide valuable insights. According to Buncic and Tischhauser (2017), "any financial and/or macroeconomic variables that help to predict the state of the economy should in theory also help to predict the equity premium" (p. 621). For example, when the economy is growing rapidly, and unemployment rates are low, investors are more likely to be optimistic about the future of the stock market and expect higher returns, leading to an increase in the equity premium. Conversely, during an economic recession or a period of high inflation, for instance, from mid-2022 till now, investors may be more cautious and expect lower returns, leading to a decrease in the equity premium. This suggests that macroeconomic factors play a fundamental role in predicting stock market returns and can be used to design factor-based investment strategies.

The macroeconomic factor model is a widely used investment strategy that employs various macroeconomic factors, such as GDP (Growth), Consumption, Consumer confidence, LIBOR, Unemployment rate, Credit spread, and Yield spread. However, one significant challenge for this model is managing the increasing correlations between the underlying assets during downturns or adverse market conditions. During times of economic stress or market turbulence, asset classes became more closely correlated, and the macroeconomic factors overlapped, making it difficult for the macroeconomic factor model to accurately predict stock market returns, which lead to increased volatility, ineffective asset allocation and a higher risk of losses. Longin and Solnik (2001) and Ang and Bekaert (2003) have demonstrated that international equity returns tend to show higher correlations during extreme times than during normal times. To address this challenge, researchers have proposed various approaches to enhance the macroeconomic factor model's performance, such as incorporating non-linear relationships between variables to adjust for correlation changes during adverse times. One such approach is the Markov switching model introduced by Hamilton (1989), which is used to model time series data and allows for alterations in the underlying economic regime over time. This approach has become widely used in macroeconomic analysis. It is noted that the macroeconomic factor model offers an advantage in that certain sectors may not experience contraction during times of economic recession. This is because the return and risk profiles can vary greatly across different business sectors. For instance, Edirisinghe and Zhao (2021) found that investors who do or do not track the benchmark index can benefit from the use of smart beta strategies when investing in utilities, which is a non-cyclical sector that tends to outperform cyclical sectors (such as the technology sector) during economic downturns. Thus, a successful macroeconomic factor investing strategy should utilize the varying performance of different business sectors in order to optimize portfolio performance.

Institutional investors have unique investment objectives and constraints that require a comprehensive approach to portfolio management. Relying solely on style factors, such as value or momentum, without considering macroeconomic factors, could result in suboptimal portfolio performance. Similarly, implementing a strategy that does not account for the dynamic nature of the macroeconomic conditions could lead to an inadequate response to the market. Therefore, it is essential to incorporate macroeconomic factors and consider time-varying regimes in the development of an investment strategy that can effectively meet the needs of institutional investors. In my thesis, I have developed a global asset allocation strategy that integrates both macroeconomic factors and time-varying regimes to explain the variation of global portfolios. This approach can help institutional investors improve their long-term portfolio performance while effectively managing macroeconomic exposures.

In Chapter 2, a comprehensive review of the relevant literature regarding macroeconomic factor investing and financial market states will be presented. Chapter 3 will explicate the methodological approach employed in this thesis, including the utilization of the Markov-switching vector autoregressive (MS-VAR) technique to filter relevant factors, as well as the regime-switching regression model and the investment model. Chapter 4 will address the details of data collection and software used in this research. In Chapter 5, empirical results will be presented and discussed in depth. Finally, Chapter 6 will provide a conclusive summary of the findings and their implications for future research.

Chapter 2 Literature Review

2.1 Macroeconomic Factors

Asset allocation and risk management using factor investing models have been well-researched in various studies. Among these models, The MPT and CAPM are two well-known approaches that rely on mean-variance analysis. Another model, the arbitrage pricing theory (APT), was introduced by Ross (1976) as an alternative to the CAPM. The APT differs from the CAPM in that it relies on three assumptions: (1) security returns can be described using a linear factor model; (2) there are sufficient securities to diversify away the asset's idiosyncratic risk; and (3) security markets do not allow arbitrage opportunities to last, and pure arbitrage profits are impossible. With these assumptions, asset returns can be characterized as a linear combination of factor-mimic portfolios which are based on systematic (or "pervasive") factors.

The APT has been widely used as a fundamental framework for developing financial models that attempt to explain asset returns or excess returns using macroeconomic factors. Typically, excess returns of portfolios are commonly used to test APT, and the portfolios can vary in composition, ranging from those with different asset classes, industries, or regions to those with different investment styles. Additionally, the ERP is often used as a benchmark for evaluating the expected return on individual stocks or portfolios. It should be noted that various studies have employed different methods when utilizing the ERP in financial models. Some studies have employed the ERP as an independent variable, using it as a factor of asset returns, while others have treated it as a dependent variable, attempting to model and estimate its value based on other factors. Chen et al. (1986) examined the relationship between the historical ERP and several macroeconomic variables, such as inflation surprise, output growth gap, and the term structure of interest rates. They found that the historical ERP was significantly related to these macroeconomic factors and could be used as a proxy for a common factor that influences stock returns. On the other hand, Goyal and Welch (2008), Neely et al. (2014), and Buncic and Tischhauser (2017) have made significant strides in modelling ERP using both financial and macroeconomic variables. These models integrate a variety of macro-econometric indicators, as well as technical indicators, to capture the complex dynamics of ERP. Rapach et al. (2013) found that lagged U.S. equity returns were more effective in predicting asset returns than non-U.S. countries' economic variables such as nominal interest rates or dividend yields. Cochrane (2011) discusses the importance of understanding the ERP in asset pricing models. He emphasizes that understanding the ERP is important not only for theoretical reasons but also for practical asset allocations. For example, if an investor believes that the ERP is lower than what is priced in the market, they may conclude that equities are undervalued and increase their allocation to stocks. Conversely, if an investor believes that the ERP is higher than what is priced, they may conclude that equities are overvalued and reduce their allocation to stocks. However, Fama and French (2004) investigated whether the historical ERP could proxy for a factor that was not included in a standard asset pricing model, and they found that the historical ERP did not add any explanatory power to the model beyond what was already captured by other factors, such as market risk, size, book-to-market equity. They conclude that the historical ERP is only “bad estimates of expected return in applications” (p. 43). It’s worth noting that the historical ERP in their analysis was calculated as the difference between the estimated market risk premium (MRP,[1] by regressing the excess returns of the market on the

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[1] The equity risk premium (ERP) looks more narrowly than the market risk premium (MRP), which is broader and more diversified. Ref: https://www.investopedia.com/terms/m/marketriskpremium.asp

Treasury bill rate) and the average excess returns on a portfolio of Treasury bills over the same rolling window period. Although several studies have examined the possibility of using the historical ERP as a proxy for a macroeconomic factor, the evidence remains mixed. Further research is needed to determine the validity of using historical ERP as a proxy for a macroeconomic factor in different asset pricing models. This is particularly important when considering more complex models that may incorporate regime-switching conditions or other non-linear relationships between asset returns and macroeconomic factors.

Moreover, in addition to the equity risk premium, some researchers have proposed using other risk premia such as the interest rate risk premium (IRP) or credit risk premium (CRP) as proxies for macroeconomic factors. These factors have been shown to have significant explanatory power for asset returns and are increasingly being incorporated into asset pricing models. Rapach et al. (2013) conducted Granger causality tests to determine the predictive power of different macroeconomic variables in explaining asset returns across countries and found that interest rates were a better predictive variable than dividend yields, as interest rates exhibited stronger causal relationships with asset returns in multiple countries. Hou et al. (2015) propose an investment approach that incorporates multiple risk factors, including IRP, and found that the IRP has significant explanatory power for asset returns. In a recent study, Cerniglia and Fabozzi (2018) identified several risks associated with factor investing, including those related to interest rates and beta. They found that these risks can vary over time and may be difficult to control, particularly if a fixed portfolio re-balancing horizon is used. Thus, alternative approaches may be needed. It's observed that there was a positive correlation between equity markets and short-term default-free bonds, meaning that when equity markets were performing well, short-term default-free bonds tended to also perform well. However, there is a negative correlation between equity markets and long-term risky bonds, meaning that when equity markets were performing well, long-term risky bonds tended to perform poorly. This suggests that the risk associated with interest rates may be more complex and need to be carefully considered when developing factor-based investment strategies. Asvanunt and Richardson (2016) conducted a study using 80 years of U.S. data and nearly 20 years of Europe data and found strong evidence of a credit risk premium over the duration risk. They concluded that this premium is not spanned by other known risk premia, which has important implications for institutional investors and academia in terms of asset pricing.

Lastly, currency risk premium (CURP) has been proposed as a macroeconomic factor for factor investing, but its application has been limited. Covered interest rate parity (CIP) and purchasing power parity (PPP) theories suggest a relationship between interest rates, foreign exchange rates, and inflation, indicating that currency risk relative to the local currency should be considered in global asset allocation. However, the literature has not fully incorporated CURP as a proxy for macroeconomic factors in factor investing, despite its potential significance for institutional investors and academia in asset pricing. The existence of CURP implies that investors can earn additional returns by investing in assets denominated in foreign currencies, beyond what they would earn from investing in domestic assets. By accounting for the risk of currency fluctuations, investors can make a more informed investment though there is still debate among academics about its true nature and magnitude for it can be difficult to capture and measure.

While macroeconomic factors such as GDP growth, inflation, and the unemployment rate are widely acknowledged to have an indirect impact on the financial markets, it's important to note that these factors can have a systematic influence on financial risk premiums. Previously, the equity risk premium (ERP) has been commonly used as a macroeconomic factor to explain asset returns. However, more recent studies indicate that considering additional risk premiums, such as the interest rate risk premium (IRP), credit risk premium (CRP), and currency risk premium (CURP), could improve the predictability of asset returns. As such, a comprehensive approach that considers multiple macroeconomic factors may be necessary to identify the relationship between macroeconomic factors and asset classes, which can lead to the development of more effective investment strategies. The magnitude of the macroeconomic risk premium can vary over time due to changes in factors such as expected economic growth, GDP, or LIBOR. These changing risk premiums can be estimated by analyzing the equity, bond, or currency markets, where asset prices are dynamically updated based on market conditions. The realized return is typically calculated as the change in asset price over time, including any dividends or coupons paid out.

Using realized return as a proxy for future risk premium can be problematic, particularly during times of economic recession. In such situations, investors may demand a higher risk premium on financial assets, particularly those considered risky, as they become less willing to take on additional credit or default risk. This can result in a realized return that deviates significantly from a normal distribution. To address this issue, some argue for the use of a rolling return method to capture the dynamic nature of financial markets. However, such an approach risks ignoring the trend of the financial market. To overcome these challenges, this thesis proposes a filtrated method for estimating macroeconomic risk premiums that can dynamically update the expected risk premium for the next period based on the financial market while also accounting for market trends. By incorporating both market trends and risk premiums into the estimation process, this method provides a more comprehensive and accurate way to assess macroeconomic risk premiums.

The information presented in Table 1 includes a comprehensive enumeration of the macroeconomic variables that are relevant to the analysis, along with the corresponding financial risk premiums.

Table 1. Literature of financial risk premiums relative macroeconomic factors.

Researchers	Dependent Variables	Independent Variables	Main Method	Results	Influential Factor or Methods
Chen et al. (1986)	Individual stock return (20 portfolios)	Macro variables such as industrial production, inflation, risk premium (bond), etc.; and market index (return of NYSE-listed stocks)	Fama–MacBeth (1973)	Macroeconomic variables are more significant in explaining cross-sectional expected stock returns than market index; however, market index is significant in explaining timeseries expected stock returns	"The spread between long and short interest rates, expected and unexpected inflation, industrial production, and the spread between high- and low-grade bonds (PP 383.)".
Goyal and Welch (2008)	ERP (S&P 500)	14 Goyal and Welch (2008) predictors[2]	Bivariate predictive regressions (OLS)	"The best monthly U.S. equity premium forecast" (Rapach et al., 2013, p. 344)	14 macroeconomic variables
Neely et al. (2014)	ERP (S&P 500)	14 popular technical indicators added to the 14 Goyal and Welch (2008) predictors	Bivariate predictive regressions (OLS); Principal Components Analysis (PCA)	Statistically significant technical indicators for the monthly equity risk premium	Economy and technical principal components
Rapach et al. (2013)	Industrialized countries' excess stock returns	Lagged U.S. market returns and non-U.S. countries' economic variables	Granger causality tests[3]	Lagged U.S. equity returns were more effective in predicting asset returns; interest rates are powerful than dividend yields in explaining stock returns.	Lagged U.S. market returns (historical ERP); Interest rate risk premium.
Cochrane (2011)	U.S. Stock excess returns; Dividend yield	Dividend yield and U.S. Stock excess returns	VAR	Long-term forecasts contain variables can lead to different price implications compared to forecasts based solely on one-period returns.	VAR model for multivariate analysis

[2] The data are available from Amit Goyal's webpage at http:// www.hec.unil.ch/agoyal/

[3] The standard tool for studying lead-lag relationships by Clive Granger (1969).

Hou et al. (2015)	NYSE, Amex, and NASDAQ small and big portfolio	The investment channel: project-level discount rates equal firm-level expected returns	GRS test[4]	Significant IRP in explaining asset returns	Interest rate risk premium
Asvanunt and Richardson (2016)	Excess corporate bond return (US and Europe), such as Barclays U.S. Corporate Investment Grade Index, U.S. Corporate High Yield Index	Markit CDX and Markit iTraxx credit default swap (CDS) indexes	Rolling OLS	Existence of credit risk premium	Credit risk premium

[4] On the null that the alphas are jointly zero across a given set of deciles.

2.2 Financial Market States

To achieve diversification in the global market, it is essential to identify the state of financial markets with respect to macroeconomic indicators that have a significant impact on financial risk premiums, as risks may overlap. For instance, changes in interest rates can alter the value or risk premium of financial assets like bonds, stocks, or currencies, which in turn affect other economic indicators like inflation, unemployment, and GDP growth, as well as the future financial risk premiums. This makes it critical for investors to keep a close eye on both these macroeconomic indicators and the financial assets when diversifying their portfolios in the global market.

The relationship between asset returns and the state of the market has been extensively studied in academic literature. Schwert (1989) argues that asset returns may be characterized by high or low volatility that changes over time and that a single-regime model may not be adequate to explain the fluctuations in aggregate stock volatility during historical events such as the Great Depression. Turner et al. (1989) utilized a regime-switching model and demonstrated that a mixture of normal densities with different means and variances can effectively model excess returns. Similarly, Ang and Bekaert (2002) developed an international asset allocation model that incorporates regime-switching to examine correlations between international market returns and volatility. Kim and Kim (2015) applied the regime-switching ARMA model to US business cycle data and find that this approach provides a better fit than traditional ARMA models, suggesting that incorporating changes in economic regimes is important for modelling the US business cycle. In addition, Guidolin and Timmermann (2006, 2008) conducted research on the impact of different market regimes on investments and utilized an econometric framework that allowed for switching between different states of the economy. They optimized asset allocation based on the state of the economy, accounting for macroeconomic factors and their impact on asset returns. Liu et al. (2011) employed a regime-switching model to extract time-varying risk premiums in analyzing sector ETFs. Their proposed model led to significant improvements in explaining the variations of the funds' returns within time-varying risk premiums of sector ETFs. Most recently, MacLean et al. (2022) introduced an entropy-based dynamic portfolio selection model that relaxes the constraints of return distribution and outperformed mean-variance models under both the empirical Sharpe and return to entropy ratios.

To effectively navigate financial markets, it is therefore critical to identify the relationships between macroeconomic factors themselves, as well as the impact of those factors on the market states. Macroeconomic factors such as inflation, interest rates, and government policies can all have a significant impact on the performance of financial markets. However, the impact of these factors can vary depending on the regime and market states. For example, during a recessionary period, interest rate cuts by the central bank may have a more significant impact on the market than during a period of economic growth. Similarly, the relationship between inflation and the stock market may be different during periods of high inflation compared to periods of low inflation. Duarte and Rosa (2015) argue that the ERP is primarily driven by bond yields (IRP), rather than expected stock returns. This implies that traditional indicators of the ERP, such as simple valuation ratios, may not be as reliable in predicting future excess returns as they have been in the past. In addition, Chen et al. (1986) found that macroeconomic variables can have varying relationships between different asset classes. For instance, the performance of fixed income and equity can show opposing reactions to the same inflation surprise. Thus, a factor-interaction model incorporating regime-switching would be a suitable approach to modelling the relationship between macroeconomic variables and asset returns.

Incorporating regime-switching into factor-interaction models can help capture the nonlinear and time-varying relationship between macroeconomic factors and asset returns, thereby improving the accuracy of forecasts and supporting better-informed investment decisions, particularly in the face of economic uncertainty or market instability. Kim et al. (2001) used a Markov-switching vector autoregressive (MS-VAR) model to study the impact of monetary policy on stock prices and their relationship with macroeconomic variables such as interest rates, inflation, and output growth. With an MS-VAR model, Hammerschmid and Lohre (2018) discussed the usefulness of identifying economic regimes. Their resulting regime factor is shown to be relevant in forecasting the ERP and leads to significant utility gains in a mean-variance portfolio strategy. It's worth noting that the MS-VAR model often employs Bayesian estimation, which enables the integration of prior knowledge or assumptions regarding the parameters, leading to enhanced estimation accuracy. The thesis proposes a foresight strategy for asset allocation that relies on the frequent use of Bayesian estimation in the MS-VAR model. '

Chapter 3 Methodology

3.1 Filtrated Factors

The current macroeconomic models used in academia and industry to predict the return on assets rely on economic or financial factors, which assume a linear relationship between expected return and the associated systematic risk. However, this linear assumption does not always hold true in the real world, as non-linear relationships can more accurately reflect market observations. Additionally, dynamic risk premiums can vary significantly across different market regimes. For example, during times of economic prosperity, investors tend to prefer stocks over bonds, while during economic downturns, investors favour bonds over stocks. These fluctuations in investor preference demonstrate that the relationship between risk and return is not always linear and might be better modelled with a non-linear model, such as a hidden Markov model (HMM). Financial markets can exhibit varying regimes, including Bull, Bear, and Transition. As these regimes can exist in a financial market, it's reasonable to assume that the dynamics of these regimes follow a stochastic process. The transitions between these regimes are often dominated by fundamental financial and economic risk premiums, also known as macroeconomic indicators. Given that economic conditions periodically change, we can use economic factors to model the probabilities of these economic conditions using the HMM framework. This allows us to predict the likelihood of a financial market transitioning between different regimes and to adjust investment strategies accordingly.

Assuming that (1) these risk premiums follow the financial market regimes; (2) the regimes follow a Markov chain with a finite number of regimes, S; and (3) there is an independent and constant regime transition probability matrix P that governs the transitions.

$$P = \begin{bmatrix} P_{11}\ P_{12} \dots P_{1S} \\ P_{21}\ P_{22} \dots P_{2S} \\ \dots \\ P_{S1}\ P_{S2} \dots P_{SS} \end{bmatrix},$$

where $P_{ij} = \Pr\{M_t = j | M_{t-1} = i\}$ is the transition probability from regime i at time $t-1$ to regime j at time t. I assume that Markov chain dynamics are embedded in a set of macroeconomic indicators. Therefore, the parameters of hidden Markov chains can be estimated based on macroeconomic indicators. This allows me to estimate the prior and posterior probabilities of the regimes under the Bayesian framework.

Assuming that the macroeconomic indicators follow an S-regime autoregressive regime change process and that the regime change is governed by a first-order Markov chain, the Markov-switching vector autoregressive (MS-VAR) process can be expressed as:

$$f_t = A_{M_t} + f_{t-1} B_{M_t} + C_{M_t} \varepsilon_t \ , \tag{1}$$

where M_t is the hidden regime at time t, f_t is the macroeconomic indicator observed at time t. A_{M_t}, B_{M_t}, and C_{M_t}, are regime-dependent regression coefficients of Equation (1); ε_t follows a multivariate normal (MVN) distribution with zero mean and identity covariance matrix. It is important to note that Equation (1) varies from a standard HMM, as the macroeconomic risk premium is not independent but rather dependent on its previous value. This approach has been successfully used by MacLean et al. (2022) to make portfolio constructions based on this regime change process.

Determining the optimal number of regimes (or market states) involves the trade-off between fitting in-sample data and predicting out-of-sample data. There are various statistical methods that can help determine the optimal number of regimes, but three commonly used criteria are the maximum log-likelihood (Log-Likelihood), Akaike information criterion (AIC) and Bayes information criterion (BIC). These criteria evaluate the goodness of fit of the model to the training data while adjusting for the number of parameters used in the model. Let Y_t be the sequence of unobservable (hidden) state at time t, and all unknown parameters of Equation (1) to be in a vector Θ, Log-likelihood of an HMM with S regimes can be expressed as a mathematical function that considers the observed macroeconomic data and the estimated values of the parameters:

$$L(S) = \max_{\Theta} \left\{ ln \sum_{Y \in \mathcal{Y}} Pr(f_t, Y; \Theta) \right\}. \quad (2)$$

$L(S)$ is the maximized Log-Likelihood with S regimes. Maximizing the Log-likelihood can help us find the most likely values of the unknown parameters and the hidden states of the system at each time t. Increasing the number of regimes S in a hidden Markov model can improve the accuracy of the model's fit to the data. However, a model that is too complex can result in overfitting. To determine the optimal number of regimes, the AIC or BIC can be used. Either balances the goodness of fit of the model to the data with the complexity of the model, penalizing models with a large number of parameters. AIC and BIC can be expressed as:

$$AIC(S) = -2L(S) + \rho(S) * 2, \quad (3)$$

$$BIC(S) = -2L(S) + \rho(S) ln(T), \quad (4)$$

respectively, where $\rho(S)$ is the total number of free parameters function[5] and T is the length of in-sample. The BIC penalizes models with more parameters more heavily than the AIC because the BIC involves multiplying the number of parameters by the natural logarithm of the sample size,

[5] $\rho(S)$ = 3/2 N (N + 1) S + S^2− 1, where N is the number of dimensions of observed data and S is the number of regimes.

which is larger than the linear penalty term in the AIC. To determine the optimal number of states for Equation (1), this thesis employs the BIC selection criterion.

During the estimation process, for each time t, the transition matrix P of the Markov model can be determined, which enables the inference of the financial market regime Y_t. In the end, the regime-dependent coefficients A_{M_t}, B_{M_t} and C_{M_t} can be estimated too. These coefficients can reveal valuable insights into the underlying factors that drive macroeconomic risk premia. Once the parameters of the HMM have been estimated, I can use the model to dynamically forecast the underlying asset return and risk profile. Conditional on the previous macroeconomic risk premiums f_{t-1} and the expected current regime M_t, the one-period regime-dependent expected macro-econometric factor vector can be forecasted as,

$$\overline{f_t} = A_{M_t} + f_{t-1} B_{M_t}. \tag{5}$$

Additionally, the corresponding covariance matrix can be calculated, which is $C_{M_t} {C_{M_t}}^T$. Table 5 would present the parameter estimation for the HMM and Figure 2 would depict the filtrated macro-econometric factors $\overline{f_t}$ with the hidden regimes.

3.2 Regime-Switching Regression Model

To model asset returns according to different regimes, a regime-switching regression model is applied. With the filtrated one-period regime-dependent macroeconomic factors, the return of an asset (such as a fund or a portfolio) i can be written into the following traditional time-series framework. Specifically, the return of asset *i* at time t can be expressed as follows:

$$r_{t,i} - RF_t = \alpha_{M_t} + \overline{f_t}\beta_{M_t} + \varepsilon_{t,i} , \tag{6}$$

where RF_t is the risk-free rate at time t; α_{M_t}, β_{M_t} are regime-dependent parameters. Specifically, α_{M_t} is the intercept of one regime and β_{M_t} is the coefficient (risk exposure) to the filtrated macro-econometric factor $\overline{f_t}$, which is the expected risk premium at time t. $\varepsilon_{t,i}$ is an error term with a zero mean that represents the portion of the return to asset i not explained by the macroeconomic factor model.

In practice, it's common to represent Equation (6) in a matrix format for all the n risky assets:

$$r_t - RF_t = \alpha_{M_t} + \overline{f_t}\beta_{M_t} + \gamma_{M_t}\,\varepsilon_t \ , \tag{7}$$

where r_t represents the return vector of risky assets at time t; γ_{M_t} is a diagonal matrix of size n*n, indicating that the idiosyncratic risks of the risky securities are uncorrelated. Suppose there are S regimes, ε_t is a standard normal random vector with size S*n, indicating the idiosyncratic risks of the n risky securities under different regimes, which is serially independent and uncorrelated with f_t, given f_{t-1}. Hence, the n asset returns jointly follow a multivariate normal (MVN) distribution that is conditioned on the regime at time t and the previous macroeconomic factor f_{t-1},

$$(r_t - RF_t |\, M_t = s, f_{t-1}) \sim MVN(\mu_s, \sigma_s)\ , \tag{8}$$

where r_t represents the return vector of risky assets at time t; MVN denotes the multivariate normal distribution; the regime-dependent mean of the excess returns of the risky securities is:

$$\mu_s = \mu_{s,t} = \alpha_{M_t} + \overline{f_t}\beta_{M_t} = \alpha_{M_t} + (A_{M_t} + f_{t-1}B_{M_t})\beta_{M_t}, \tag{9}$$

and the regime-dependent covariance matrix is:

$$\sigma_s{}^2 = \sigma_{s,t}{}^2 = \beta_{M_t}{}^T C_{M_t} C_{M_t}{}^T \beta_{M_t} + \gamma_{M_t}{}^2. \tag{10}$$

The implication of the above equation is that the excess returns of the risky securities are time-varying and depend on the information on the previous macroeconomic factors f_{t-1}. However, the regime-dependent covariance matrix is constant over time.

Let $M_t = s$ and the prior probabilities for regimes in Equation (1) be $p_{s,t}$ at time t, the unconditional returns of the n risky assets follow a mixture of normal distributions with a mean vector,

$$\mu_t = \sum_{s=1}^{S} (p_{s,t}\, \mu_{s,t}), \tag{11}$$

and a covariance matrix,

$$\sigma_t{}^2 = \sum_{s=1}^{S} p_{s,t} \left(\sigma_{s,t}{}^2 + (\mu_{s,t} - \mu_t)(\mu_{s,t} - \mu_t)^T\right). \tag{12}$$

The calculated unconditional mean vector μ_t and covariance matrix $\sigma_t{}^2$ will be the inputs of a subsequent mean-variance model. Table 11 would present the estimates of the return parameters α_{M_t}, β_{M_t} and γ_{M_t} of a group of assets. Table 6 would report the conditional mean vectors μ_s or $\mu_{s,t}$, and standard deviation σ_s of a group of assets, with regimes at time t.

3.3 The Investment Model

Given that the regime-switching regression model has demonstrated the ability to accurately capture the regime-dependent mean returns of the risky securities, it is justifiable to consider replicating the assets utilizing the proposed macroeconomic factors and incorporating the model into investment decision-making processes, as well as optimizing the portfolio's risk-return profile. The primary challenge lies in determining the appropriate weights to assign to the underlying assets, which requires careful consideration of various factors such as risk tolerance, return expectations, and market conditions.

With the same settings as in Equation (6), the act of investing involves the allocation of capital to the risky assets. The expected excess return of a portfolio can be calculated as follows:

$$E(r_{t,p} - RF_t) = \Sigma_{i=1}^{n} \omega_{t,i}(r_{t,i} - RF_t)\,, \tag{13}$$

where $r_{t,p}$ is the total return of a portfolio at time *t;* $r_{t,i}$ represents the return of a risky asset i at time *t;* Furthermore, $\omega_{t,i}$ denotes the proportion of available capital that is allocated to asset i, where i is a value between 1 and n, from the previous time point t-1 until time t. Notably, the proportion of capital allocated to the risk-free asset is: $1 - \Sigma_{i=1}^{n} \omega_{t,i}$.

It is assumed that there are S regimes, and at any given time t, the regime $M_t = s$ determines the returns on the n risky assets, which are assumed to be multivariate normally distributed. The mean vector of these returns is denoted by $\mu_{s,t}$ and the covariance matrix is denoted by $\sigma_{s,t}$. These values are derived from Equation (9) and Equation (10), respectively. Therefore, if the regime at time t (i.e., $M_t = s$) and asset p are known, the return of the asset p is normally distributed with a mean equal to the corresponding element of $\mu_{s,t}$, weighted by the weight vector W_t. This can be expressed mathematically as follows:

$$\mu_{s,W_t} = RF_t + \left(\mu_{s,t} - RF_t\right)^T W_t \,. \tag{14}$$

Here, $W_t = \{\omega_{t,1}, \omega_{t,2}, \dots, \omega_{t,n}\}$ represents the weight vector for the portfolio at time t, RF_t represents the risk-free rate at time t, while the superscript T denotes the transpose of the vector. Likewise, the return of asset p is also assumed to follow a normal distribution, with the variance of the distribution equal to the corresponding element of the covariance matrix $\sigma_{s,t}$, weighted by the weight vector W_t. This can be expressed mathematically as follows:

$$\sigma_{s,W_t}{}^2 = W_t{}^T \sigma_{s,t} W_t \,. \tag{15}$$

Given the assumption that the prevailing regime at time t is $M_t = s$, and that the prior probabilities of the regimes in Equation (1) are denoted by $p_{s,t}$, the multivariate normal density function with mean vector $\mu_{s,t}$ and covariance matrix $\sigma_{s,t}$ is represented by $\emptyset_s(x)$; please refer to

Equation (8). Using this notation, the density function of the one-period portfolio return can be expressed as follows:

$$\mathrm{P(x)} = p_{1,t}\emptyset_1(x; W_t) + \cdots + p_{s,t}\emptyset_s(x; W_t), \tag{16}$$

where the symbol x represents the random variable of the portfolio return, $p_{s,t}$ represents the prior probabilities for regimes; and $\emptyset_s(x; W_t)$ is the normal density function with mean μ_{s,W_t} and variance ${\sigma_{s,W_t}}^2$ for portfolio weight vector W_t; please refer to Equation (14) and Equation (15). It is evident from this equation that the portfolio return density function is a weighted sum of the regime-specific normal density functions, each weighted by the corresponding prior probability.

The distribution of the one-period portfolio return is a mixture of normal distributions because the one-period portfolio return is a linear combination of the returns of the individual assets in the portfolio, which are assumed to follow normal distributions (Markowitz, 1952; Bodie et al., 2017). The Markowitz mean-variance model is a widely used framework for portfolio optimization. In this model, the portfolio weights of the risky assets are determined based on their expected returns and covariances. Specifically, let $\bar{\mu}$ denote the mean and $\bar{V}$ denote the covariance matrix of the one-period portfolio return P(x) in Equation 15, and set the target return level to be τ, for the Markowitz mean-variance model, the optimal portfolio weights in the risky assets are:

$$\mathrm{W} = \frac{(\tau - RF)\bar{V}^{-1}(\bar{\mu} - RF)}{I^{\top}\bar{V}^{-1}(\bar{\mu} - RF)}, \tag{17}$$

where RF is the risk-free rate and *I* denotes the vector of ones whose size equals the size of the risky assets, and $\top$ denotes the transpose of a vector.

Chapter 4 Data Collection

4.1 Data Source for Risk Premiums

As elaborated in Section 2.1, risk premiums can be utilized as a means of measuring the expected return. These risk premiums can serve as proxies for macroeconomic factors, given their ability to predict expected returns or be predicted by macroeconomic factors. When constructing a portfolio with global investments, it is crucial to consider the various sources of risk, including interest rate risk, credit risk, equity risk, and currency risk. To accurately capture these sources of risk, it is necessary to account for the four types of risk premiums.

In this thesis, weekly index data were collected from January 1, 2002, to December 31, 2022, inclusive. All the data for macroeconomic risk premiums are from Refinitiv Eikon.

Interest Rate Data

For investment-grade (IG) bonds, the risk of default is typically limited. To calculate the interest rate risk associated with IG bonds, the yield of long-term government bonds (typically 10-year) is subtracted from the short-term (1-month or 3-month) Treasury bill rate. In this thesis, ICE BofA 7-10 Year US Treasury Index is used to proxy for long-term government bonds.

Credit Risk Data

For a high-yield (HY) bond, the default risk or credit risk is not limited. To estimate credit risk, the return on long-term (usually 10-year) corporate bonds is subtracted from the return on long-term (usually the same duration as the corporate bonds) government bonds. In this study, the ICE Bank of America United States High Yield Index was used as a proxy for risky bonds, in line with industry practice.

Equity Risk Data

This study estimates equity risk premium using the MSCI All Country World Index, which is a widely recognized and respected benchmark for global equity markets. The MSCI ACWI includes large and mid-cap equities from 23 developed markets and 24 emerging markets, covering approximately 85% of the global investable equity universe[6] with over 2,933 stocks. Fund managers often use the MSCI ACWI as a guide for global asset allocation and as a benchmark for the performance of global equity funds due to its broad coverage of global equities.

To select the most appropriate risk premiums, a combination of industry experience and statistical methods was employed. A principal component analysis (PCA) method was employed in this study, as expounded in Appendix A1, to select appropriate risk premiums for global investments. The analysis revealed that the MSCI ACWI effectively captures the significant variances of major markets worldwide, making it a suitable index to estimate equity risk premiums.

Currency Risk Data

In investment practice, several mutual funds and ETFs are formulated to mitigate currency risk through hedging methods such as forex, options, or futures. In this study, the focus is primarily on the currency exchange rate between CAD and USD, which is crucial in assessing the currency risk exposure of a Canadian institution. To this end, Refinitiv Eikon is utilized to collect weekly data on the MSCI Canadian Dollar to 1 United States Dollar exchange rate data.[7]

Risk-Free Rate Data

In this thesis, the risk-free rate uses the US 3-month Treasury bill rate to proxy the risk-free rate for the US market, as well as the global markets. This rate is commonly used as a

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[6] https://www.msci.com/our-solutions/indexes/acwi

[7] MSCI CAD TO 1 USD has been used (Symbol: MSERCAD)

benchmark for short-term interest rates and is generally regarded as a good indicator of the risk-free rate. The 3-month Treasury bill rate is also considered to have negligible default risk due to its short maturity. Refinitiv Eikon is used for US 3-month Treasury bill rate data.[8]

4.2 Global Portfolios Data Source

This thesis analyzes a global portfolio consisting of 21 assets from bond and equity ETFs. The portfolio is evaluated over the period September 06, 2016,[9] to December 31, 2022.

The bond ETFs selected for this study include TIP (iShares TIPS Bond ETF), TERM20 (iShares 20+ Year Treasury Bond ETF), IG (iShares iBoxx Investment Grade Corporate Bond ETF), HY (SPDR Bloomberg High Yield Bond ETF), and HY-Emerging (iShares J.P. Morgan USD Emerging Markets Bond ETF). These bond ETFs exhibit varying degrees of risk, such as interest rate risk and credit risk.

On the other hand, the equity ETFs selected for this study include global equity ETFs and US sector ETFs. The global equity ETFs include ACWX (iShares MSCI ACWI ex U.S. ETF), AAXJ (iShares MSCI All Country Asia ex Japan ETF), IWB (iShares Russell 1000 ETF), EFG (iShares MSCI EAFE Growth ETF), and SCZ (iShares MSCI EAFE Small-Cap ETF). These ETFs are mainly exposed to equity risk, interest rate risk, currency risk, and credit risk. The global sector ETFs, which include M1AFCD (Consumer Discretionary), M1AFCS (Consumer Staples), M1AFE1 (Energy), M1AFFN (Financial), M1AFHC (Health Care), M1AFID (Industrial), M1AFM1 (Materials), M1AFIT (Technology), M1AFU1 (Utilities), M1AFR1 (Real Estate), and

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[8] Canada, Interest Rate : 3 Month Treasury Bills (End Month) has been used (Symbol: CNGBILL3)

[9] The maximize of the start date of the 21 ETFs is September 06, 2016.

M1AFT1 (Communication), mainly expose investors to equity risk, interest rate risk, and credit risk.

These ETFs are utilized to perform regime-switching regression models and assist with investment allocation.

All the data above are from Refinitiv Eikon.

4.3 Software Packages

To conduct rigorous statistical analysis of financial econometric models, this study utilizes standard econometric and statistical packages available in both Python and MATLAB software. These software packages are well-known for their comprehensive libraries and tools for time series and cross-sectional asset pricing tests, which allow for advanced data management and robust statistical analysis.

Chapter 5 Empirical Results and Discussions

5.1 Estimation of Risk Premiums

Risk premiums are key factors in determining the expected excess returns of an asset. To calculate these premiums, historical data analysis can be used to measure the performance of various asset classes over time. In this study, I adopted weekly returns as the basis for analysis. Specifically, I employ a one-period simple net return to measure the interest rate factor on a weekly basis, which is consistent with the Refinitiv Eikon net return calculation methodology. Let P_t be the price or index value of an asset at week t, then the weekly return R_t is calculated with the following formula:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}. \tag{18}$$

To determine the equity risk premium, I rely on the historical return of ACWI as a basis for estimating the expected excess return of equities over the risk-free rate. To estimate the interest rate risk premium, I calculate the historical return of the ICE BofA 7-10 T-Bond Index over the risk-free rate. This is an additional yield that investors expect to receive for taking on interest rate risk. The credit risk premium can be measured as the credit spread. To calculate the credit risk premium, I subtract the corresponding historical return of ICE BofA 7-10 T-Bond Index from the return of the ICE BofA United States High Yield Index. To calculate the currency risk premium, I use the weekly returns of the MSCI Canadian Dollar to 1 United States Dollar as a proxy. However, it's important to note that this approach only captures changes in the exchange rate between the Canadian and U.S. dollar, and there may be other sources of currency risk when investing globally, such as changes in foreign interest rates, geopolitical events, and currency depreciation of other currencies against the local currency.

Table 2 presents a descriptive summary of the global index data, providing information on the average returns and distributions of the global market. Panel A indicates that MSCI ACWI has the highest mean return (0.1555%) among the indices, followed by MSCI CAD/USD (0.1311%) and ICE BofA HY (0.0866%). Interestingly, the data suggest that the spot foreign exchange rate may provide relatively higher returns than previously assumed. Thus, incorporating Currency risk premium in the investment model is crucial. Panel B reports the correlations among the index returns. The correlation coefficient between MSCI ACWI and ICE BofA 7-10 TB is negative (-0.2889), suggesting that the two indices may move in opposite directions, which is consistent with the traditional finance observation that stocks and bonds have a negative correlation. The correlation between ICE BofA HY and MSCI ACWI is positive (0.0835), indicating that there is a slight tendency for high-yield corporate bonds to move in the same direction as global equity markets. Lastly, there is a positive correlation between MSCI CAD/USD and ICE BofA HY (0.5195), implying that these two indices may be affected by similar economic factors or market conditions.

Table 3 presents a summary of the key statistics for the four financial risk premiums, namely Equity RP, Interest rate RP, Credit RP, and Currency RP. One important feature of these risk premiums is that their distributions are positively skewed, indicating that lower values are more frequent than higher values. Credit RP is particularly skewed, with a skewness value of 8.1510. Additionally, the distributions display negative excess kurtosis, which suggests that they have flatter tails than a normal distribution. Credit RP also has the lowest ex-kurtosis value of -0.8146, indicating that it has the flattest tail.

Another noteworthy aspect is the correlation between the different risk premiums. Equity RP has a positive correlation with Credit RP (0.6318) and Currency RP (-0.5143), implying that these risk premiums may be influenced by similar economic factors or market conditions. Moreover, Interest rate RP and Credit RP have a high negative correlation of -0.6428, indicating a close opposite side relationship between interest rates and credit spreads.

Furthermore, the average mean return for Equity, Currency, and Interest rate risk premiums is higher than that of Credit RP. This suggests that these three risk premiums play a more foundational role in generating asset returns.

All in all, the positively skewed and negatively kurtotic distributions suggest that these risk premiums may be affected by extreme events and non-normal market conditions. The significant correlations observed among the risk premiums suggest that they may be driven by shared risk factors or economic conditions, which is consistent with the underlying assumptions of a MS-VAR model.

Table 2. Summary statistics for macroeconomic factor-proxy data, from January 01, 2002, to December 31, 2022. Panel A reports average weekly returns, standard deviation, kurtosis, and skewness. Panel B reports correlations among the index returns. All returns and Std. Dev are expressed in percentages.

Panel A	**MSCI ACWI**	**ICE BofA 7-10 TB**	**ICE BofA HY**	**MSCI CAD/USD**	**3-MONTH T-bill [10]**
Weeks	1105	1105	1105	1105	1105
Return (Mean)	0.1555	0.0752	0.0866	0.1311	0.0243
Std. Dev.	2.2052	0.8704	0.7765	1.0830	0.0285
Return (Min)	-12.3540	-4.0406	-6.6530	-8.3275	-0.0002
Return (Median)	0.4250	0.0897	0.1075	0.1965	0.0152
Return (Max)	12.1700	3.2791	5.5456	8.2312	0.0963
Skewness	4.0351	1.7227	11.3306	14.6983	0.3613
Ex Kurtosis	-0.5897	-0.1182	-1.0014	-1.0186	1.2095
Panel B: Correlations					
MSCI ACWI	1.0000				
ICE BofA 7-10 TB	-0.2889	1.0000			
ICE BofA HY	0.0835	0.7305	1.0000		
MSCI CAD/USD	0.5891	-0.0304	0.5195	1.0000	

[10] Adjusted yearly return to weekly by dividing by 52.

Table 3. Summary statistics for macroeconomic risk premiums after risk-free rate adjustment, from January 01, 2002, to December 31, 2022. Panel A reports average weekly returns, standard deviation, kurtosis, skewness, and T-mean (ratio of the mean to its standard error) of each macroeconomic factor. Panel B reports correlations among the macroeconomic risk premiums. All returns and Std. Dev are expressed in percentages.

Panel A	**Equity RP**	**Interest rate RP**	**Credit RP**	**Currency RP**
Weeks	1096	1096	1096	1096
Return (Mean)	0.1287	0.0521	0.0547	-0.0073
Std. Dev.	2.2108	0.8697	1.4135	1.2756
Return (Min)	-12.3561	-4.0725	-8.9943	-10.7922
Return (Median)	0.3968	0.0683	0.0964	-0.0454
Return (Max)	12.1698	3.2341	8.6748	6.7941
Skewness	4.0096	1.7436	8.1510	5.8933
Ex Kurtosis	-0.5799	-0.1078	-0.8146	-0.1454
T-mean	0.0582	0.0600	0.0387	-0.0057
Panel B: Correlations				
Equity RP	1.0000			
Interest rate RP	-0.2936	1.0000		
Credit RP	0.6318	-0.6428	1.0000	
Currency RP	-0.5143	0.1145	-0.4093	1.0000

5.2 Filtrated Factors Results

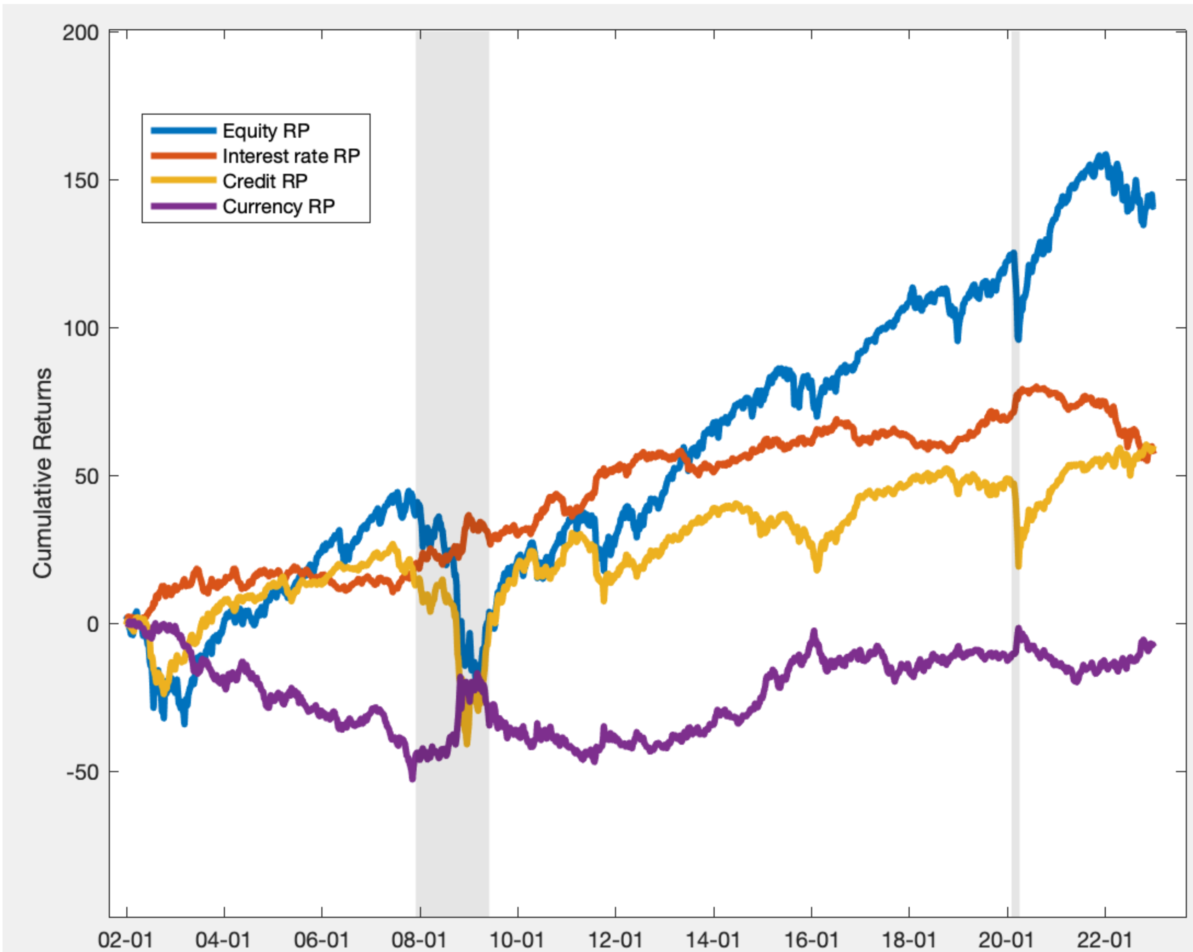
The following section examines the implications of macroeconomic risk premium trends and the estimation of HMM. By analyzing trends in macroeconomic risk premiums, we can better understand their impact on financial markets.

5.2.1 Risk Premium Trend Implication

Financial markets are inherently affected by numerous macroeconomic factors, which are reflected in financial risk premiums. Figure 1 illustrates that these macroeconomic risk premiums can fluctuate significantly over time, resulting in varying cumulative returns in financial markets. The shaded grey areas indicate the US recession periods. These grey periods represent a contraction in the US economy, which can have a significant impact on the global behaviour of macroeconomic risk premiums and financial markets as a whole. The thesis focuses on three significant global market events that have occurred in the past two decades. The first event was the global financial crisis (GFC) of 2008–2009, which stemmed from a subprime mortgage crisis. The second event was the European sovereign debt crisis, which lasted from 2011 to 2013. The third event that this thesis covers is the COVID-19 pandemic, which had a profound impact on the global financial markets. In March 2020, the pandemic caused a significant dip in the global stock market. These three major events have had a notable influence on financial markets. For instance, they have caused the interest rate risk premium to increase, while the equity and credit risk premiums have decreased notably. For the European sovereign debt crisis event, it’s observed that the cumulative credit risk premium dipped from 2011 to 2013.

The cumulative returns of the four macroeconomic risk premiums suggest that certain hidden regimes may exist within financial markets. These regimes may not be immediately apparent from examining the trends in risk premiums over time.

Figure 1. The cumulative returns of Financial Risk Premiums in percentage. The GFC and the COVID-19 pandemic at the beginning of March 2020 significantly affect Interest rate risk premium (upward) and Equity risk premium and Credit risk premium (downward). The shaded grey areas indicate the US recession periods.



Notes: The US recession periods data are from the National Bureau of Economic Research (NBER), [Business Cycle Expansions and Contractions](#).

5.2.2 Estimation of the HMM

To conduct an HMM-based analysis of economic regimes, this thesis utilizes in-sample data from September 06, 2016, to December 31, 2018. The out-of-sample data period chosen for this analysis is January 1, 2019, to December 31, 2022.

To estimate these hidden regimes, an HMM framework is applied. Table 4 presents the number of regimes with Criteria such as Maximum Loglikelihood (Log-likelihood), AIC, and BIC. While the model fits well with a higher number of hidden regimes, it may result in poor predictive performance due to overfitting the in-sample data. Therefore, the optimal number of financial market regimes is identified as S=3, based on the lowest BIC criterion. This decision ensures a balance between the goodness of fit and predictive performance, providing a more accurate and robust analysis of market behaviour. The initial regime distribution is estimated as $\pi = [1,0,0]$ and the transition probability matrix P is estimated as $\begin{bmatrix} .9096 & .0647 & .0257 \\ .0695 & .9305 & \mathbf{.0000} \\ .1417 & \mathbf{.0000} & .8583 \end{bmatrix}$. The diagonally dominant transition matrix indicates that there is a higher probability that the regime will remain in a specific state once the process enters that regime. In addition, the presence of a 0 element in the transition probability matrix signifies that certain states of the system would never transition to certain other states. For instance, in this transition probability matrix, regime 2 is incapable of transitioning directly into regime 3, and regime 3 is incapable of transitioning directly into regime 2.

Figure 2 illustrates the four macro-econometric factors and the identified hidden regimes in the financial markets. The bear time of the financial markets in the last two decades has been identified correctly, such as the bear times for the GFC period, and the COVID-19 pandemic period. The shaded grey areas in Figure 2 indicate the US recession periods.

During a recession market (Regime), the corresponding factors are falling or expected to fall, and investor sentiment is negative (the interest rate factor and the currency factor jumps during the recession period inception). In contrast, a bull market, as seen during periods of economic growth and expansion, is characterized by high returns and positive investor sentiment. Additionally, a transit market may occur during a period of transition or uncertainty, with no clear trend in either direction. Thus, it can transfer into either of the other two market regimes.

Table 4. Determination of the optimal number of regimes. The minimum BIC indicates that the optimal number of financial market regimes is 3.

K	**1**	**2**	**3**	**4**	**5**	**6**
Parameter Numbers	30	63	98	135	174	215
Log-likelihood	-6666.0	-6023.4	-5857.3	-5764.8	-5720.0	-5656.0
AIC	13392.0	12172.8	11910.6	11799.7	11787.9	11742.0
BIC	13542.2	12488.3	12400.4	12475.7	12659.3	12818.6

Note: when the sample size is large, the BIC imposes a greater penalty on complex models than the AIC.

Figure 2. Macro-econometric Factors and Regimes. The shaded grey areas indicate the US recession periods. Returns (left axis) are expressed in percentages.

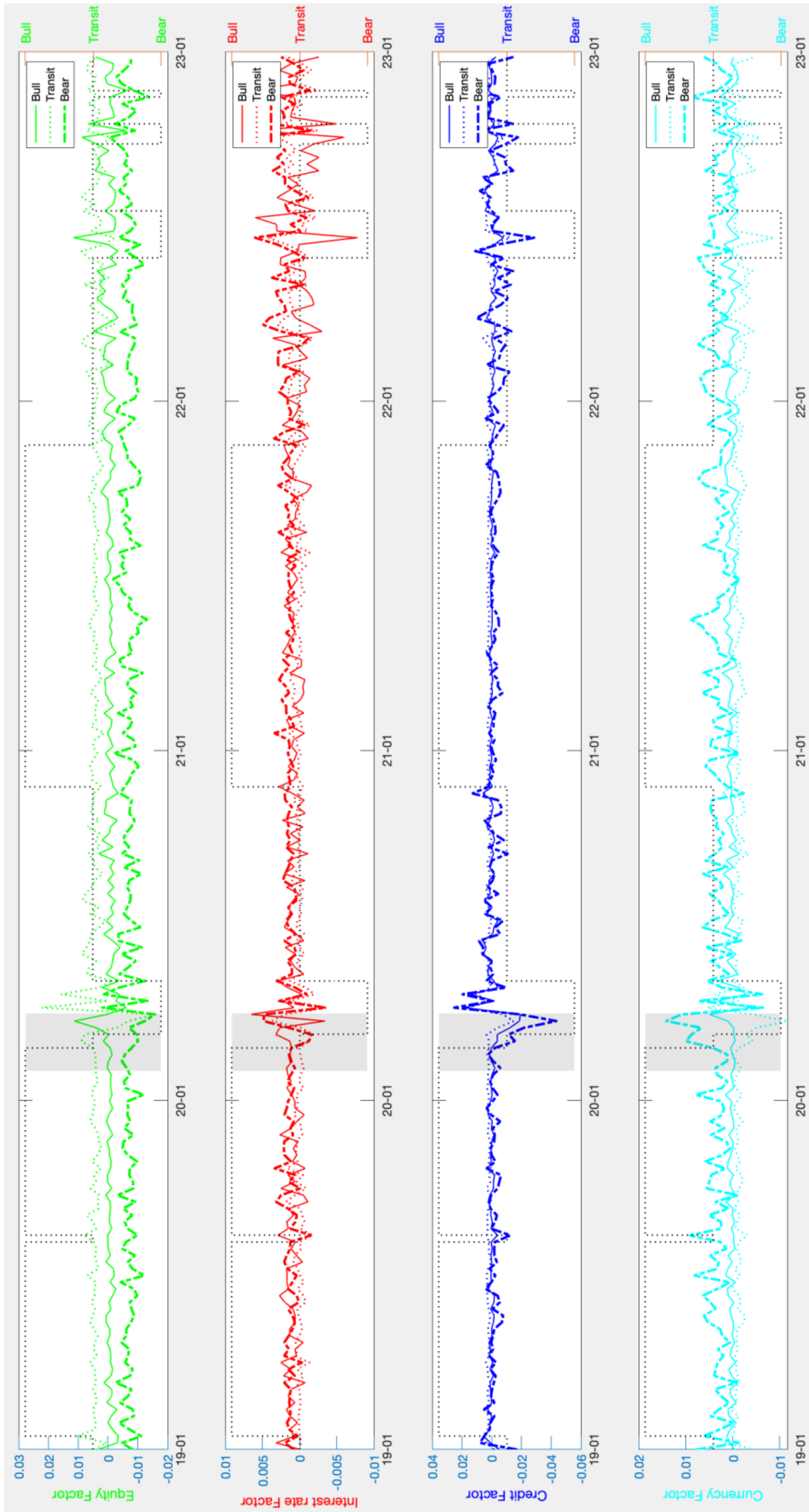


Table 5 presents the parameter estimation for the HMM, as seen in Equation (1). With the information about the HMM dynamics and the coefficients of the macroeconomic risk premiums, I can now separate the financial market into three regimes.

In terms of parameter A_{M_t}, the equity risk premium in Regime 2 exhibits a significantly higher positive mean compared to the other three risk premiums, which have relatively lower means. In terms of Residual Risk, all four macroeconomic risk premiums in Regime 2 have lower standard deviations. The impulse response function (IRF) in Figure 6 provides information on how an exogenous shock or impulse in one variable affects other variables in a multivariate time series model over the subsequent periods. In particular, the IRF shows that the impulse of all four factors decays very quickly (within two periods) in Regime 2, indicating an active action of this regime. Thus, Regime 2 can be characterized as an (Equity) bull market.

During Regime 3, the equity risk premium exhibited a negative mean that was relatively lower compared to the other regimes, while the interest rate risk premium had a higher mean due to the flight to quality resulting in higher yields. Additionally, all four macroeconomic risk premiums had relatively higher Residual Risk during this period. Specifically, the IRF in Figure 7 shows that the impulse of all four factors decays slightly slowly (within four periods) in Regime 3, indicating a numb action of this regime. Therefore, it is more likely that Regime 3 can be characterized as an (Equity) bear market.

During Regime 1, the parameter A_{M_t} and Residual Risk are found to be intermediate between those of Regime 2 and Regime 3. It is likely that Regime 1 represents a transitional market. To highlight, the parameter B_{M_t} reflects the dynamics of the macroeconomic risk premiums. A high value of the parameter for the equity risk premium to credit risk premium (0.2775 in Regime 2, row 3) suggests a stronger positive correlation between credit RP and equity RP. This could be because investors are willing to take on more credit risk (which is relatively low in a bull market) to earn higher returns in the equity markets and increase the equity risk premium. Conversely, a negative value of the parameter for the equity risk premium to itself (-0.1133 in Regime 2, row 1) indicates that in a favourable market environment, investors would require relatively less equity premium due to the decreasing market risk, such as credit risk. This suggests that investors may be more willing to invest in equity markets when the overall market risk is low, and as a result, the equity risk premium may decrease. The IRF in Figure 5 shows that the impulse of all four factors at regime 1.

Table 5. Coefficient estimates for the macroeconomic RPs (06 September 2016–31 December 2018). All returns and Std. Dev are expressed in percentages.

Full Market		Single Regime		
Parameter	**Equity RP**	**Interest Rate RP**	**Credit RP**	**Currency RP**
$\boldsymbol{A_{M_t}}$	0.1237	0.0538	0.0176	0.0006
	-0.0547	0.0249	0.0661	0.0198
	0.0735	-0.0404	0.2885	-0.0766
$\boldsymbol{B_{M_t}}$	0.1460	-0.0635	0.2582	-0.1271
	0.0953	-0.0174	0.0116	-0.0990
Res. Risk	2.2042	0.8675	1.3629	1.2677
	4.8583	-0.5558	1.9744	-1.4279
Res. Cov	-0.5558	0.7526	-0.7850	0.1200
	1.9744	-0.7850	1.8576	-0.7235
	-1.4279	0.1200	-0.7235	1.6072
Market Label		Transit Market Regime 1		
Parameter	**Equity RP**	**Interest Rate RP**	**Credit RP**	**Currency RP**
$\boldsymbol{A_{M_t}}$	-0.0228	0.0688	-0.0083	0.0253
	-0.0613	0.0458	0.0021	0.0272
	-0.1092	0.1025	0.1670	-0.0736
$\boldsymbol{B_{M_t}}$	0.0027	-0.0605	0.2443	-0.0895
	0.0776	-0.0281	0.0264	-0.1000
Res. Risk	2.2258	0.8449	1.2438	1.1871
	4.9544	-0.6928	1.9115	-1.1560
Res. Cov	-0.6928	0.7139	-0.7673	0.1051
	1.9115	-0.7673	1.5470	-0.5386
	-1.1560	0.1051	-0.5386	1.4092
Market Label		Bull Market Regime 2		
Parameter	**Equity RP**	**Interest Rate RP**	**Credit RP**	**Currency RP**
$\boldsymbol{A_{M_t}}$	0.4401	0.0193	0.1649	-0.1401
	-0.1133	0.0068	0.0154	0.0547
	0.3124	-0.1244	0.2834	0.0890
$\boldsymbol{B_{M_t}}$	**0.2775**	-0.0454	0.1432	0.0482
	-0.0529	-0.0232	-0.0095	0.0405
Res. Risk	1.0444	0.6365	0.7051	0.9155

Res. Cov	1.0908	-0.2492	0.4289	-0.2268
	-0.2492	0.4051	-0.3967	0.0771
	0.4289	-0.3967	0.4972	-0.1574
	-0.2268	0.0771	-0.1574	0.8381
Market Label		Bear Market		
		Regime 3		
Parameter	**Equity RP**	**Interest Rate RP**	**Credit RP**	**Currency RP**
$\boldsymbol{A_{M_t}}$	-0.7193	0.1394	-0.2615	0.3548
$\boldsymbol{B_{M_t}}$	0.0084	0.0285	0.2021	-0.0702
	0.1054	-0.1528	0.2503	-0.0584
	0.1885	-0.0611	0.2181	-0.1524
	0.3395	0.0134	0.0905	-0.3069
Res. Risk	4.6719	1.6147	3.1581	2.4907
Res. Cov	21.8270	-1.1907	9.6167	-8.3508
	-1.1907	2.6073	-2.7428	0.4174
	9.6167	-2.7428	9.9735	-4.3188
	-8.3508	0.4174	-4.3188	6.2035

Note: $\boldsymbol{B_{M_t}}$ are the coefficients with 1 unit. $\boldsymbol{A_{M_t}}$ and Res. Risk are in percentages. Res. Risk is the standard deviation of the macroeconomic RPs, which equal the square root of the diagonal entries of the covariance matrix Res. Cov ($C_{M_t} C_{M_t}{}^T$) of each regime. Res. Cov is in 10 ^ (-4) unit given the percentage Res. Risk.

5.3 Regime-Switching Regression Model Results

5.3.1 Estimation of Underlying Asset Return Parameters

The weekly asset risk/return profile of the financial markets is estimated using the weekly returns of the 21 ETFs with the in-sample data. Based on the discussion of the regime-switching model, the asset return/risk profile can be explicitly expressed using the macro-econometric factor regression model as shown below,

$$r_t = RF_t + \alpha_{M_t} + f_{t,E}\beta_{E,M_t} + f_{t,I}\beta_{I,M_t} + f_{t,C}\beta_{C,M_t} + f_{t,X}\beta_{X,M_t} + \gamma_{M_t}\varepsilon_t, \tag{19}$$

where r_t is a vector of weekly returns on the risky assets. RF_t is the risk-free rate at time t. $f_{t,E}$, $f_{t,I}$, $f_{t,C}$, and $f_{t,X}$ are the filtered equity, interest, credit, and currency macro-econometric factors, respectively. α_{M_t}, β_{E,M_t}, β_{I,M_t}, β_{C,M_t}, β_{X,M_t}, and γ_{M_t} are regime-dependent parameters. ε_t is a multivariate standard normal random vector.

To estimate the regime-dependent parameters, I used a weighted least squares (WLS) method for each regime based on the estimated hidden Markov process and the posterior probabilities of regimes over time. The posterior probabilities of each regime were used as weights in the WLS method to account for the uncertainty of the regime assignment. This approach can capture the dynamics of the market across different regimes and obtain more accurate estimates of the regime-specific parameters. Table 11 offers insightful information on return parameters, highlighting their significant variations across diverse asset classes. Notably, there are discernible differences in the return parameters of bond assets and equity assets. Furthermore, the prevailing market conditions play a crucial role in shaping the return parameters, as evidenced by the contrasting equity exposures of ACWX in bull and bear regimes. For instance, ACWX has greater equity exposure (1.2185) in bull regimes while it has a slower equity exposure (0.5776) in bear regimes. These varying equity risk exposures indicate a better optimization to gain a risk-adjusted return. Lastly, the return parameters α_{M_t}, β_{E,M_t}, β_{I,M_t}, β_{C,M_t}, β_{X,M_t} can help for risk budgeting. By leveraging these parameters, portfolio managers can efficiently gauge the collective cumulated exposures of diverse investments within a portfolio. This, in turn, can enable portfolio managers to make informed decisions regarding portfolio diversification, hedging strategies, and risk mitigation, thus enhancing overall portfolio performance.

5.3.2 Conditional and Unconditional Portfolio Returns

Based on the estimated parameters in Table 11 in Appendix A3, we can now calculate the conditional mean vectors, standard deviations, and correlation coefficients of security returns over time. It is worth noting that the conditional covariance matrix remains steady within a specific regime. To demonstrate this, Table 6 exhibits the conditional mean returns and standard deviations of the 21 ETFs for the first week of the out-of-sample period in January 2019, given a regime.

When there is only one market regime, the mean returns are relatively small and have mixed signs, while the standard deviations are quite large for a single-regime market as returns are averaged out. However, in multiple regime markets, the security's conditional mean return varies. Notably, bonds tend to gain significant positive return premiums during bear markets, while experiencing significant negative premiums during bull markets, which is consistent with actual observations. Regarding equity, staple equities with low Equity factor beta perform well during bear markets but are slightly weaker during bull markets, such as MSCI ACWI Cons staples (M1AFCS). Conversely, equities with high Equity factor beta perform well in bull markets but are slightly weaker in bear markets, such as MSCI ACWI Financials (M1AFFN) or MSCI ACWI IT (M1AFIT). The 21 ETFs exhibit a diverse range of unconditional returns, which can be modelled as a mixture of normal distributions. Notably, the spreads in the returns across different asset classes are substantial, suggesting that there are ample opportunities for investors to diversify their portfolios and potentially mitigate risk.

Table 6. Conditional means and standard deviations of security returns for the first week of the out-of-sample period in January 2019.

ETF Name	TIP	TERM20	IG	HY	HYEmerging	ACWX	AAXJ	IWB	EFG	SCZ	M1AFCD
Market						Mean (in %)					
Single Regime	-0.0014	-0.0007	-0.0016	-0.0029	-0.0024	-0.6384	-0.6891	-0.5247	-0.6503	-0.9055	-0.6424
Transit	-0.0012	-0.0034	-0.0066	-0.0085	-0.0099	0.5318	1.2860	-0.2137	0.0996	-0.0315	-0.0742
Bull	-0.0003	0.0012	0.0006	-0.0036	0.0021	0.5314	0.2608	0.2748	0.4974	-0.0068	0.1593
Bear	-0.0010	-0.0011	-0.0127	-0.0266	-0.0185	-0.7705	0.0944	-2.2544	-1.1008	-1.8832	-2.7095
						Standard Deviation (in %)					
Single Regime	0.4228	1.1759	0.4715	0.8841	0.8852	2.0045	2.4858	2.2382	1.9752	2.2096	2.3244
Transit	0.6224	2.0018	0.7515	1.2103	1.0172	2.3281	3.0086	2.7332	2.1438	2.6555	2.9779
Bull	0.4665	1.3826	0.5314	0.5002	0.9430	1.7871	2.1741	1.1814	1.5771	1.7976	1.3936
Bear	1.1846	3.8385	1.9289	4.5539	2.4819	4.2490	4.9357	6.6433	3.5816	4.9675	6.9974
ETF Name	**M1AFE1**	**M1AFM1**	**M1AFID**	**M1AFCS**	**M1AFHC**	**M1AFFN**	**M1AFIT**	**M1AFT1**	**M1AFU1**	**M1AFR1**	
Market						Mean (in %)					
Single Regime	-0.8139	-0.7058	-0.7005	-0.2565	-0.1784	-0.6272	-0.7251	-0.3101	-0.0397	-0.3337	
Transit	0.6586	0.7491	0.7332	0.0187	0.4320	1.6901	-0.9780	0.2340	0.4159	0.6711	
Bull	1.0035	0.5349	0.2128	0.0515	0.6306	0.5066	0.6691	0.3359	-0.1136	-0.0646	
Bear	0.0849	-0.1566	-0.4713	-0.5851	-1.0144	1.4546	-4.0824	-1.0707	0.2174	0.4509	
						Standard Deviation (in %)					
Single Regime	2.7315	2.5912	2.2601	1.3739	1.9396	2.0430	2.7639	1.7612	1.0511	1.5853	
Transit	3.9967	3.6143	2.9460	1.6353	2.3841	3.2037	3.1647	1.9365	1.2466	2.0641	
Bull	1.9603	2.2350	1.5095	1.0127	1.0366	1.6200	2.1747	1.4428	1.0935	1.3275	
Bear	7.2805	6.1384	6.2571	3.1678	5.1809	6.3142	8.2009	5.1280	2.6676	4.4769	

5.4 The Investment Model Results

To invest, the dynamic risk exposure and regime-dependent weekly asset returns on 21 ETFs between January 1, 2019, and December 31, 2022, were estimated based on the financial market's regime, which was determined by estimating parameters derived from weekly observations of financial risk premiums and the four macro-econometric factors, which were calculated based on the regimes and previous financial risk premiums.

At the beginning of each out-of-sample week t, investment decisions are made in the 21 ETFs: $W = \{\omega_1, \ldots, \omega_{21}\}$. The information available to the decision-maker is the prior probabilities of the three regimes and the regime-dependent weekly asset returns at week t-1. The prior probabilities would be dynamically updated each week with new information at the end of week t.

5.4.1 Foresight Regime Strategy

It is worth noting that the 21 risky underlying assets have varying sensitivity to the macro-econometric factors with different magnitudes and signs, which create opportunities for constructing an optimal portfolio with combinations of long and short positions in the risky assets. This is consistent with one of the hedge fund investment styles, which has become popular among hedge fund portfolio managers. To make portfolio returns meaningful (without a large holding in some of the individual funds), we will set a target return equal to

$$\tau = RF + required\ weekly\ risk\ premium, \tag{20}$$

where RF represents the weekly risk-free rate.

The underlying assumption for an HMM is that asset returns follow a multivariate normal distribution, depending on the regime outcome. Each regime is depicted by a unique probability distribution for asset returns, which is expected to reduce the overall uncertainty of asset returns.

Using the prior probabilities of the regimes and the transition matrix, the Markov chain can provide "foresight" for predicting the future regime of the economy. Let s be the most likely regime to occur, i.e., s=argmax {Prior Probabilities of Regimes}. With the forecast regimes in asset allocation, this strategy is called Foresight Regime Strategy. Thus, Equation (15) can be written as:

$$\mathrm{P(x)} = \emptyset_s(x; W_t), \tag{21}$$

where s= argmax $\{p_{s,t}\}$. If the density function of one-period portfolio returns changes, the expected returns and risks of the assets in the portfolio have changed. Thus, recalculating the optimal portfolio weights in response to changes in the density function of one-period portfolio return can improve portfolio performance by incorporating foresight and adapting to new information.

5.4.2 Alternative Portfolio Strategies

Portfolio managers pay great attention to risk-adjusted returns. The most popular method is to minimize portfolio risk subject to constraints on the portfolio return.

The alternative models with which to compare the dynamic portfolio are the equally weighted model, the mean-variance model with a single regime, and the mean-variance model with multiple regimes, as MacLean et al. (2022) showed in their asset allocation strategies. The equally weighted model is a simple asset allocation strategy where the amount invested in each asset is equal, regardless of the individual asset's risk or expected return. The equally weighted model assumes that each asset has an equal contribution to the overall portfolio performance and does not factor in the individual asset's risk-return characteristics. On the other hand, the mean-variance model is a widely used optimization framework in portfolio management. This model aims to find the portfolio allocation that minimizes portfolio risk subject to a given level of portfolio return. In the mean-variance model with a single regime, the optimization is performed under the assumption that the market has a single regime, which remains constant over time. In the mean-variance model with multiple regimes, the optimization is performed under the assumption that the market has multiple risk regimes, and the portfolio allocation is dynamically adjusted based on the market regimes. The MSCI ACSI serves as the baseline for the models. All the models are listed in Table 7 for a summary view.

Table 7. Portfolio Strategies. This thesis proposes a foresight regime portfolio to compare with the other three traditional portfolio strategies and the benchmark ACWI.

Strategy				**Base Line**
Equally Weighted	**Single Regime MV**	**Multiple Regimes MV**	**Foresight Regime**	**ACWI Index**
$\omega_i = \frac{1}{21} * W$	$\min_W W^T \sigma_A^2 W \ s.t.$ $(\mu_A - RF)^T W \geq \tau - RF$	$\min_W W^T \sigma_B^2 W \ s.t.$ $(\mu_B - RF)^T W \geq \tau - RF$	$\min_W W^T \sigma_M^2 W \ s.t.$ $(\mu_M - RF)^T W \geq \tau - RF$	

Note: The subscript A is the case with all data from one regime; B and M are the cases of blended mean and variance estimates.

5.4.3 Portfolio Cumulative Returns and Net Weighting

Based on the predetermined settings for each strategy, Figure 3 depicts that the foresight regime strategy and the mean-variance model with multiple regimes outperform the mean-variance model with a single regime and the equally weighted portfolios in terms of cumulative portfolio returns, indicating a financial benefit to incorporate the macro-econometric factors and the dynamics of the financial markets. Furthermore, Figure 3 shows that the foresight regime strategy has a clear advantage in that it consistently improves cumulative returns with minimal drawdown, a feature that is highly valued by institutional investors, as well as individual investors. Other strategies exhibit much larger drawdowns than the foresight regime strategy, which is less desirable for institutional investors seeking consistent portfolio performance.

Effective portfolio risk management involves monitoring and controlling the turnover of portfolio weights, which refers to the frequency and extent of changes in the allocation of assets within the portfolio. It is important to balance the benefits of high turnover (such as maintaining optimal asset allocation in changing market conditions) with the costs associated with it, such as increased transaction costs, tax implications, and the impact of bid-ask spreads and liquidity costs on portfolio performance. Conversely, low portfolio turnover may provide a more stable asset allocation but could result in deviation from optimal allocation over time. Figure 4 depicts the net weights of the risky assets for the various investment strategies. It is evident from the figure that the foresight regime strategy would result in the lowest turnover compared to other strategies.

Figure 3. Cumulative portfolio returns in the risky assets for the various investment strategies. The proposed Foresight Regime Strategy performs best. All cumulative returns are expressed in percentages.

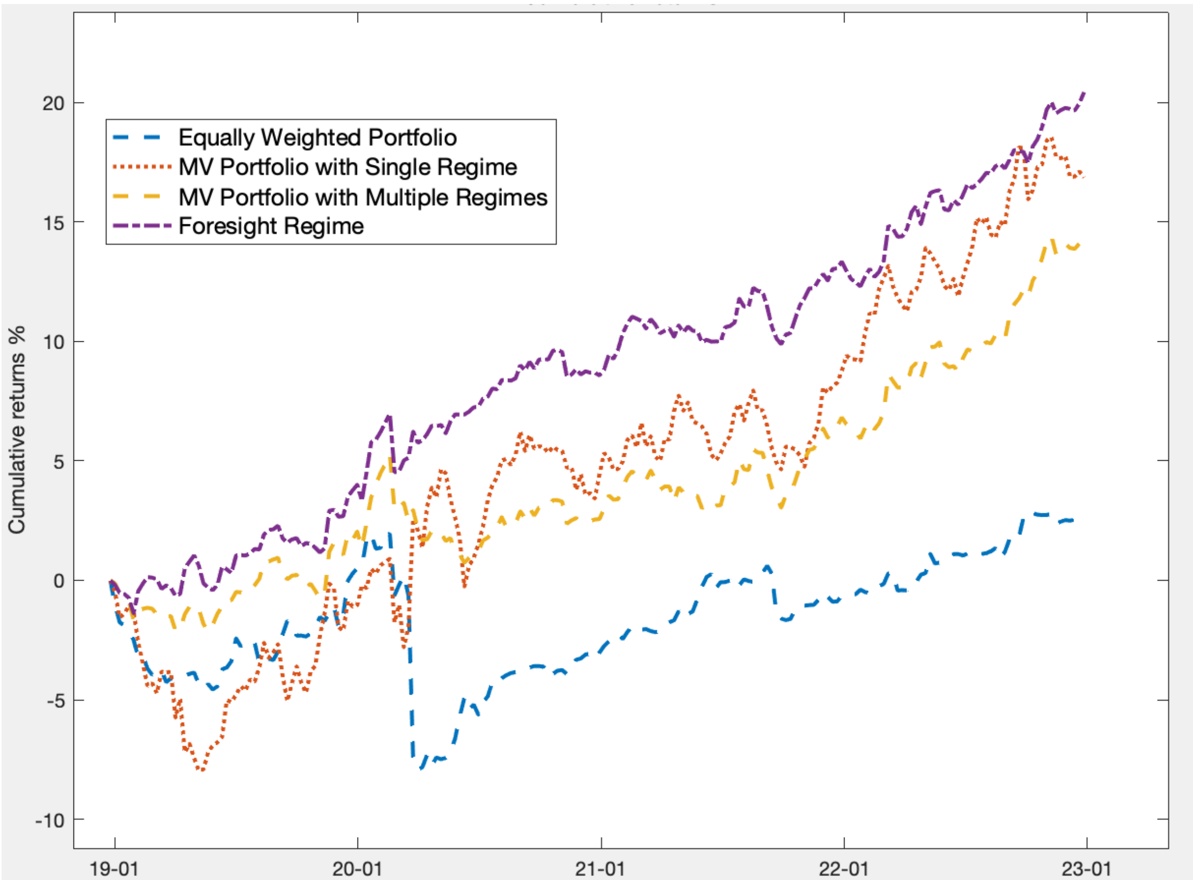
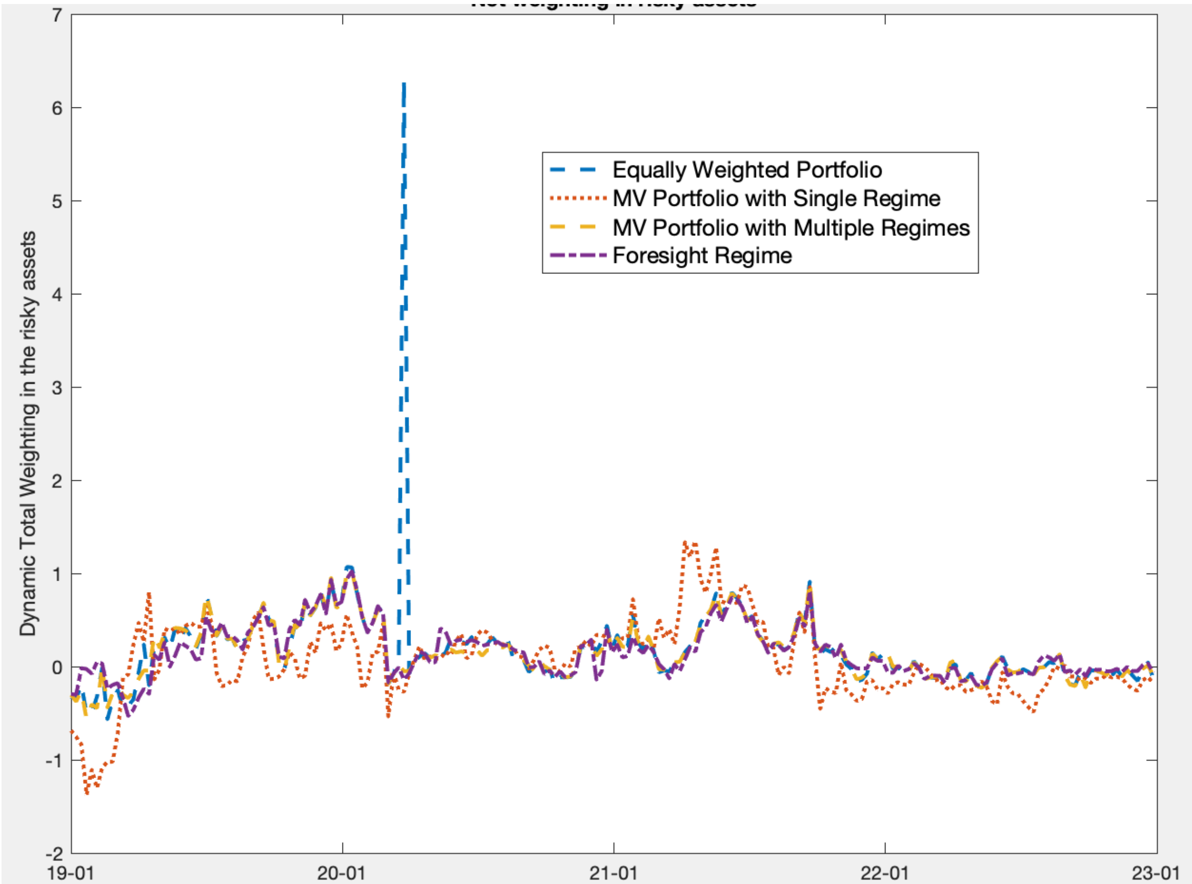


Figure 4. Net weights in the risky assets for the various investment strategies. The single-regime model has the highest portfolio turnover for the out-of-sample period.



5.4.4 Performance Measurement

In addition to the period-by-period cumulative portfolio excess returns presented in Figure 3, Table 8 presents the overall results for the alternative portfolio strategies during the out-of-sample period.

It is shown that the baseline ACWI index exhibited the highest Cumulative Annualized Growth Rate (CAGR) of 8.7729%, but also had the highest standard deviation of 16.7790% and the highest maximum drawdown (MaxDD) of -26.5014% during the period, which indicates that the overall global equity market is volatile in comparison with the alternative strategies.

Portfolio performance is often evaluated by examining its historical returns. The Sharpe ratio, which is the average excess return over the risk-free rate divided by its standard deviation, is the most commonly used criterion, assuming a normal distribution of portfolio returns. Among the portfolio strategies examined, the equally weighted portfolio has the lowest growth rate with a CAGR of 0.6225% and a substantial amount of risk with a standard deviation of 4.4093% and a MaxDD of -9.3995%. Moreover, the equally weighted portfolio underperforms the risk-free rate, resulting in a negative Sharpe ratio.

The foresight regime portfolio presented in this study has demonstrated a compelling performance compared to other alternative strategies. The portfolio has achieved a CAGR of 4.7570%, with slightly lower volatility of 3.1436% and the lowest MaxDD of -2.4227% during the out-of-sample period. These results suggest that the foresight regime strategy has a superior risk and return trade-off compared to the ACWI index and other alternative strategies. Additionally, the foresight regime portfolio has the largest Sharpe ratio, indicating that it provides the highest return per unit of risk, and therefore has strong potential applications in investment. Finally, it is worth noting that the highest T-mean is observed for the foresight regime strategy, indicating that this strategy has the highest ratio of mean excess return to its standard error compared to the other strategies, implying that it has a more favorable Sharpe ratio.

Table 8. Investment portfolio performances for the out-of-sample period. The out-of-sample period is from 1 January 2019 to 31 December 2022.

Performance Measure	**Strategy**				**ACWI Index**	**RF**
	Equally Weighted	**Single Regime MV**	**Multiple Regimes MV**	**Foresight Regime**		
CAGR (%)	0.6225	3.9727	3.4120	4.7570	8.7729	1.1395
Std. Dev. (%)	4.4093	5.8789	3.2074	3.1436	16.7790	-
MaxDD (%)	-9.3995	-7.6600	-4.4428	-2.4227	-26.5014	
T-mean	0.0090	0.0058	1.0638	1.5132	0.5228	
Sharpe Ratio	-0.1173	0.4819	0.7085	1.1508	0.4549	

Notes: RF is the average risk-free rate of the out-of-sample period. Std. Dev is annualized. MaxDD is calculated throughout the out-of-sample period.

Chapter 6 Conclusion

The role of macroeconomic indicators in driving asset returns has become a topic of intense interest among portfolio managers seeking to enhance their portfolio performance through factor exposure analysis. In this context, a MS-VAR model has been applied to model the excess asset returns (financial risk premiums) to identify financial market regimes and filter financial risk premiums that are assumed to be affected by macroeconomic indicators. Subsequently, a regime-switching regression model has been applied to model the conditional asset returns with the previous generated macro-econometric factors. Lastly, an optimal portfolio performance has been proposed through a foresight regime strategy with dynamic adjustments of portfolio weights based on the current regime and regime-dependent macro-econometric factors to capture the associated systemic risk. The out-of-sample test showed that the proposed asset allocation strategy outperformed other strategies and the baseline index in terms of the Sharpe ratio. These findings indicate that portfolio managers can benefit from incorporating filtered macro-econometric factors into their investment decisions, leading to improved risk-adjusted returns.

This study contributes significantly to the literature on macroeconomic factor investments by proposing a novel investment model that enriches the existing theoretical frameworks. By integrating the four macro-econometric factors, namely equity, interest rate, credit, and currency factors, with market regimes, this research presents a comprehensive framework that dynamically explains the variation in cross-sectional and time-series asset returns. The macro-econometric factors proposed in this thesis can serve as a pole risk premium for global portfolios, thereby enhancing the effectiveness of asset exposure analysis. Furthermore, this study provides valuable insights into global asset allocation by demonstrating the superiority of the proposed dynamic strategy in achieving a risk-adjusted return that outperforms other commonly used strategies. The proposed investment model and dynamic asset allocation strategy offer a more comprehensive understanding of financial risk premiums and provide a foundation for further research in this field.

While this study offers comprehensive analysis of the proposed macro-econometric factors and investment models, there remain some concerns about their practical application. For example, the overall variability of macro-econometric factors is a crucial consideration when implementing these models in real-world investment decisions.

Furthermore, it should be noted that the in-sample and out-of-sample data used in this study were constructed during a period that included quantitative easing (QE) or occurred after QE had ended. During this period, global interest rates remained at a low level. However, there has been a recent increase in interest rates, which may impact the effectiveness of the proposed models and strategies.

These limitations should be considered when applying the proposed models and strategies to real-world investment decisions. Further research is needed to address these concerns and validate the effectiveness of the proposed models under different macroeconomic conditions.

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Appendices

A1. Principal Component 1 of the major markets' index return and its correlation with the major markets.

Table 9 PC 1 of the major markets' index return and its correlation with the major markets. Data include weekly returns from January 01, 2002, to December 31, 2022. ACWI LCL is chosen for historical weekly ERP calculation due to its high correlation with PC1.

Variables	S&P 500	DJI	RUSSELL 2000	Nasdaq Composite	TSX	FTSE100	STXE600
S&P 500	1.000						
DJI	0.966	1.000					
RUSSELL 2000	0.897	0.856	1.000				
Nasdaq Composite	0.935	0.864	0.883	1.000			
TSX	0.812	0.774	0.803	0.753	1.000		
FTSE100	0.748	0.746	0.705	0.671	0.721	1.000	
STXE600	0.801	0.792	0.761	0.746	0.748	0.923	1.000
NIKKEI 225							
SHENZHEN Composite							
ACWI USD							
ACWI LCL							
Principal Component 1							

Principal Component 1	ACWI LCL	ACWI USD	SHENZHEN Composite	NIKKEI 225
0.957	0.953	0.937	0.188	0.559
0.931	0.923	0.904	0.178	0.552
0.913	0.884	0.871	0.174	0.581
0.909	0.901	0.879	0.184	0.548
0.873	0.852	0.856	0.192	0.570
0.862	0.860	0.838	0.183	0.593
0.907	0.911	0.881	0.182	0.645
0.702	0.711	0.690	0.196	1.000
0.241	0.227	0.231	1.000	
0.980	0.984	1.000		
0.994	1.000			
1.000				

Notes 1: PCA mainly summarizes the macroeconomic variable’s dominant variation into parsimonious common components (Neely et al., 2014; Aït-Sahalia and Xiu, 2019).

A2. The underlying ETFs

Table 10 Summary statistics for global portfolios. Data range from September 06, 2016, to December 31, 2022. Return (mean and median) and Std. Dev denotes the annualized returns and annualized standard deviation, respectively.

Global Portfolios Weekly Return Data Describe							
Asset Class	**Variable** [11]	**Name**	**Mean Return**	**Median Return**	**Std. Dev**	**Skewness**	**Ex Kurtosis**
	ACWX	iShares MSCI ACWI ex U.S. ETF	0.003	0.376	0.133	-0.991	5.025
Global Equity	AAXJ	iShares MSCI All Country Asia ex Japan ETF	0.008	0.408	0.143	-0.454	1.902
	IWB	iShares Russell 1000 ETF	0.090	0.354	0.216	-1.162	4.967

[11] ETF Start date:
iShares MSCI ACWI ex U.S. ETF: March 26, 2008
iShares MSCI All Country Asia ex Japan ETF: August 19, 2008
iShares Russell 1000 ETF: January 01, 2002
iShares MSCI Growth ETF: August 09, 2005
iShares MSCI EAFE Small-Cap ETF: December 14, 2007
MSCI ACWI Cons Discretionary: October 14, 1994
MSCI ACWI Energy: October 14, 1994
MSCI ACWI Materials: October 14, 1994
MSCI ACWI Industrials: October 14, 1994
MSCI ACWI Cons staples: October 14, 1994
MSCI ACWI Health Care: October 14, 1994
MSCI ACWI Financials: October 14, 1994
MSCI ACWI IT: October 14, 1994
MSCI ACWI Communication: October 14, 1994
MSCI ACWI Utilities: October 14, 1994
MSCI ACWI Real Estate: September 6, 2016
iShares TIPS Bond ETF: December 4, 2003
iShares 20+ Year Treasury Bond ETF: July 30, 2002
iShares iBoxx Investment Grade Corporate Bond ETF: July 30, 2002
SPDR Bloomberg High Yield Bond ETF: December 4, 2007
iShares J.P. Morgan USD Emerging Markets Bond ETF: December 25, 2007

	EFG	iShares MSCI EAFE Growth ETF	0.027	0.373	0.155	-0.936	4.298
	SCZ	iShares MSCI EAFE Small-Cap ETF	0.024	0.399	0.198	-1.795	12.508
Global Sector Equity	M1AFCD	MSCI ACWI Cons Discretionary	0.069	0.409	0.217	-1.088	6.217
	M1AFE1	MSCI ACWI Energy	-0.017	0.551	0.099	-1.146	8.362
	M1AFM1	MSCI ACWI Materials	0.001	0.445	0.161	-0.760	3.216
	M1AFID	MSCI ACWI Industrials	0.049	0.378	0.161	-1.182	6.829
	M1AFCS	MSCI ACWI Cons staples	0.054	0.262	0.121	-0.746	3.252
	M1AFHC	MSCI ACWI Health Care	0.097	0.311	0.208	-0.800	3.953

	M1AFFN	MSCI ACWI Financials	0.027	0.410	0.135	-0.926	4.636
	M1AFIT	MSCI ACWI IT	0.120	0.416	0.269	-0.883	2.807
	M1AFT1	MSCI ACWI Communication	-0.002	0.342	0.065	-0.773	3.265
	M1AFU1	MSCI ACWI Utilities	0.020	0.311	0.073	-1.114	6.596
	M1AFR1	MSCI ACWI Real Estate	-0.018	0.391	0.054	-2.136	15.322
Global Bond	TIP	iShares TIPS Bond ETF	-0.002	0.133	0.019	-0.111	13.584
	TERM20	iShares 20+ Year Treasury Bond ETF	0.006	0.301	0.033	-0.022	0.681
	IG	iShares iBoxx Investment Grade Corporate Bond ETF	-0.003	0.162	0.022	-1.564	16.738

HY	SPDR Bloomberg High Yield Bond ETF	-0.025	0.199	0.040	-0.548	10.896
HY-Emerging	iShares J.P. Morgan USD Emerging Markets Bond ETF	-0.019	0.210	0.028	-2.294	17.259

A3. The conditional factor exposure (coefficients) of the underlying ETFs

Table 11. Regime-dependent weekly asset return parameters. Parameters are the first week of the out-of-sample period in January 2019.

Market	Factor Exposure	TIP	TERM20	IG	HY	HY Emerging	ACWX	AAXJ	IWB	EFG	SCZ	M1AFCD
Single Regime	α_{M_t}	-0.0013	-0.0009	-0.0013	-0.0014	-0.0015	-0.0019	-0.0017	-0.0002	-0.0019	-0.0043	-0.0010
	β_{E,M_t}	0.0358	0.0795	-0.0425	0.0038	-0.0667	**0.9151**	1.2742	1.0248	0.9935	0.8891	1.0660
	β_{I,M_t}	0.7293	2.1690	0.8979	1.1816	0.9045	-1.3601	-1.0197	0.5975	-0.6773	-0.8458	0.9676
	β_{C,M_t}	0.0250	-0.0379	0.4002	1.3002	0.5385	-0.7986	-1.2507	0.8172	-0.4490	-0.0246	0.8382
	β_{X,M_t}	-0.0752	0.0243	-0.0716	0.0337	-0.4672	-0.7576	-0.6395	0.3871	-0.3976	-0.3877	0.1982
	γ_{M_t}	0.0023	0.0038	0.0035	0.0035	0.0068	0.0076	0.0146	0.0048	0.0073	0.0089	0.0054
Transit	α_{M_t}	-0.0016	-0.0021	-0.0022	-0.0012	-0.0025	0.0007	0.0060	-0.0012	-0.0024	-0.0029	0.0025
	β_{E,M_t}	0.0283	0.0224	-0.0946	0.0163	-0.2159	**0.7994**	1.1072	1.0335	0.9278	0.8414	0.9377
	β_{I,M_t}	0.6275	2.5081	1.1865	1.2484	1.7326	-0.6518	0.0864	0.4616	0.1660	-0.4728	1.0864
	β_{C,M_t}	-0.0486	0.1976	0.7214	1.3492	1.2224	-0.3006	-0.5243	0.7847	-0.0414	0.0978	1.2072

	β_{X,M_t}	-0.1964	0.2262	0.2550	0.1289	0.0623	-0.6642	-0.8103	0.4604	-0.0108	-0.5107	-0.0640
	γ_{M_t}	0.0023	0.0038	0.0034	0.0045	0.0034	0.0076	0.0152	0.0058	0.0080	0.0098	0.0055
Bull	α_{M_t}	-0.0008	-0.0005	-0.0005	-0.0016	-0.0018	-0.0047	-0.0086	0.0008	-0.0034	-0.0061	-0.0022
	β_{E,M_t}	0.0306	0.2141	0.0508	-0.0119	0.2833	**1.2185**	1.7426	0.9788	1.2564	1.0478	1.2398
	β_{I,M_t}	0.5856	1.7782	0.6909	1.2314	0.4545	-1.7867	-1.6545	0.7439	-1.1541	-1.3231	0.5036
	β_{C,M_t}	-0.0822	-0.4434	-0.0441	1.3337	-0.0885	-0.9102	-0.9879	0.7438	-0.6412	-0.0484	0.4384
	β_{X,M_t}	-0.0362	0.0309	-0.1561	-0.0039	-0.4952	-0.6502	-0.3684	0.3265	-0.3451	-0.2372	0.2862
	γ_{M_t}	0.0022	0.0034	0.0027	0.0024	0.0074	0.0056	0.0088	0.0035	0.0054	0.0072	0.0044
Bear	α_{M_t}	-0.0015	-0.0032	-0.0023	0.0008	-0.0040	-0.0001	0.0052	0.0004	-0.0038	-0.0040	0.0027
	β_{E,M_t}	0.0548	-0.0742	-0.1174	0.0659	-0.2440	**0.5776**	0.6857	1.0975	0.7064	0.5501	0.7589
	β_{I,M_t}	0.5749	2.3798	1.3304	1.5427	1.7091	-0.7354	-0.3605	0.6997	0.1320	-0.5537	0.9789
	β_{C,M_t}	-0.1209	0.0951	0.8505	1.6585	1.1190	-0.2465	-0.6790	1.0201	0.0379	0.2387	1.2728
	β_{X,M_t}	-0.2019	0.1743	0.2874	0.0742	0.1856	-0.8396	-1.2799	0.4119	-0.1017	-0.7494	-0.3083

	γ_{M_t}	0.0015	0.0033	0.0025	0.0044	0.0025	0.0053	0.0108	0.0053	0.0057	0.0068	0.0036
Market		M1AFE1	M1AFM1	M1AFID	M1AFCS	M1AFHC	M1AFFN	M1AFIT	M1AFT1	M1AFU1	M1AFR1	
Single Regime	α_{M_t}	-0.0026	-0.0019	-0.0017	0.0001	0.0029	-0.0019	-0.0012	0.0002	0.0006	-0.0005	
	β_{E,M_t}	0.9538	0.7885	0.9415	0.4471	0.8969	1.3088	1.2361	0.5334	0.1693	0.7382	
	β_{I,M_t}	-1.4427	-0.9312	-0.3753	0.3822	0.8079	-2.4048	1.3151	-0.3764	0.7293	-0.5036	
	β_{C,M_t}	-0.5515	-0.5374	-0.0696	0.3404	0.4451	-1.7256	1.0402	-0.1056	0.3294	-0.6473	
	β_{X,M_t}	-1.1538	-1.4650	-0.7121	-0.1612	-0.1267	-0.2593	0.4147	-0.6554	0.0199	-0.1800	
	γ_{M_t}	0.0131	0.0131	0.0073	0.0086	0.0061	0.0077	0.0103	0.0109	0.0094	0.0115	
Transit	α_{M_t}	-0.0031	0.0017	-0.0008	-0.0010	0.0014	0.0011	-0.0007	-0.0009	0.0026	0.0005	
	β_{E,M_t}	0.9135	0.6340	1.1699	0.6839	0.9704	1.2946	0.7626	0.8217	0.5145	1.0515	
	β_{I,M_t}	-2.2741	-0.5553	-0.8540	0.4943	0.4533	-2.9870	2.3025	0.1567	0.2704	0.4895	
	β_{C,M_t}	-1.0417	-0.4910	-0.7094	0.1641	0.0742	-2.0342	2.1450	-0.1330	-0.0149	-0.5857	
	β_{X,M_t}	-2.0477	-2.2429	-0.8638	0.5635	-0.2105	-0.9422	0.1624	0.4686	0.5206	0.9324	
		0.0140	0.0147	0.0066	0.0089	0.0085	0.0079	0.0073	0.0119	0.0078	0.0103	

	γ_{M_t}										
Bull	α_{M_t}	0.0001	-0.0055	-0.0027	-0.0006	0.0033	-0.0031	-0.0007	-0.0013	0.0007	-0.0043
	β_{E,M_t}	0.8641	1.1662	0.5443	0.1054	0.7714	1.2656	2.0821	0.1771	-0.5360	0.4253
	β_{I,M_t}	-2.1857	-1.9072	0.3410	1.4267	1.3825	-2.0550	-0.9066	0.6648	1.7054	0.0124
	β_{C,M_t}	-1.1195	-0.5182	0.9504	1.2306	0.9337	-1.2491	-1.0428	0.8986	1.1000	0.4580
	β_{X,M_t}	-0.9253	-0.9929	-0.7735	-0.4845	-0.1356	-0.1067	0.7956	-1.0685	-0.4489	-0.5005
	γ_{M_t}	0.0111	0.0079	0.0069	0.0070	0.0029	0.0065	0.0093	0.0079	0.0074	0.0096
Bear	α_{M_t}	-0.0044	-0.0002	-0.0001	-0.0003	0.0018	0.0013	-0.0002	0.0043	0.0045	0.0039
	β_{E,M_t}	1.2500	0.6312	1.2131	0.7659	0.9101	1.1775	0.7041	0.9245	0.7146	1.2768
	β_{I,M_t}	-2.4587	-1.1102	-0.8371	0.5949	0.3656	-3.1833	2.3275	0.8196	0.4679	1.0075
	β_{C,M_t}	-1.7625	-1.2387	-0.7553	0.2832	0.0879	-2.0002	2.1487	0.5774	0.0834	-0.1382
	β_{X,M_t}	-1.6641	-2.1447	-0.8965	0.6726	-0.4014	-1.1614	0.0601	0.2498	0.6097	1.0492
	γ_{M_t}	0.0099	0.0109	0.0051	0.0067	0.0070	0.0060	0.0043	0.0112	0.0060	0.0085

Note: α_{M_t} and γ_{M_t} are in percentages. Others are the regression coefficients with 1 unit.

A4. impulse response function (IRF) graph of MS-VAR model

Figure 5. IRF graph of MS-VAR model in Regime 1

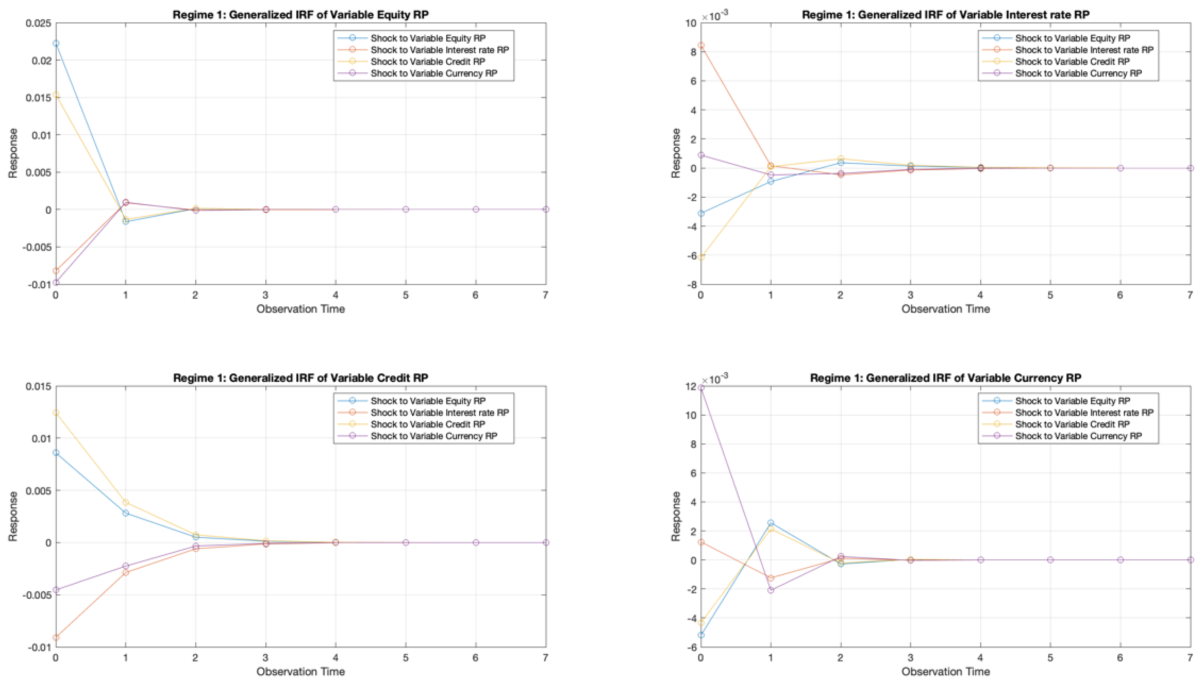

Figure 6. IRF graph of MS-VAR model in Regime 2

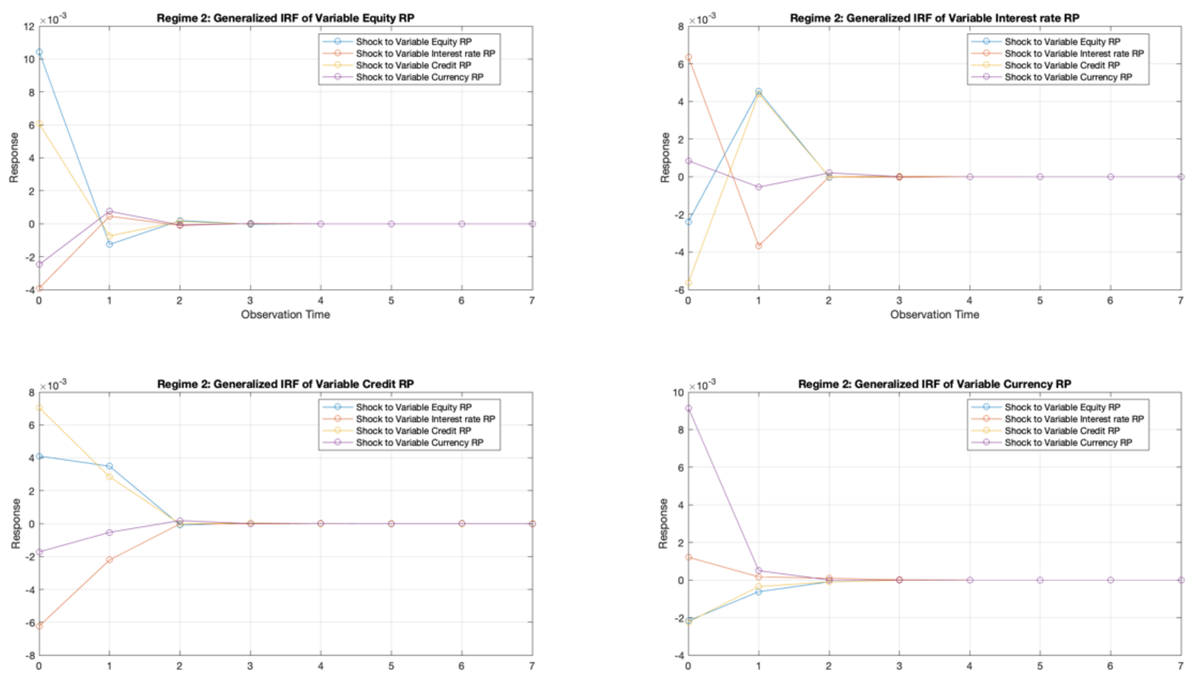

Figure 7. IRF graph of MS-VAR model in Regime 3