



SECOND-ORDER ANALYSIS OF SLENDER GFRP REINFORCED CONCRETE COLUMNS USING ARTIFICIAL NEURAL NETWORK

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Abstract: Analysis of slender concrete columns reinforced with glass fiber-reinforced polymer (GFRP) bars is required for design and optimization purposes. Finite element, finite difference, and analytical-numerical tools were developed in literature to accurately predict the response of slender columns reinforced with GFRP bars. The primary challenge with using these tools is the high computational cost required to accurately predict the response of the slender columns due to the material and geometric nonlinearities. Design codes and standards provide a simplified second-order analysis method, called the moment magnification method, to avoid the high computational cost associated with conducting complex nonlinear second-order analyses. The accuracy of the moment magnification method is often compromised when used in analyzing slender elements. Therefore, there is a need for developing efficient and accurate methods of analyzing slender columns, particularly when a large number of analyses is required (e.g., reliability analysis and optimization applications). The objective of this study is to propose an efficient regression-based method to predict the second-order effects on GFRP reinforced concrete (RC) columns by utilizing the artificial neural network (ANN) approach. Nonlinear finite-difference analysis of slender GFRP-RC columns was utilized to generate a training dataset including multiple eccentricity ratios, slenderness ratios, reinforcement ratios, section aspect ratios, and material properties to train the ANN. Preliminary results indicate an average error of less than 1 kN for the second-order analysis based on the trained ANN model. Preliminary reliability analysis of slender GFRP-RC columns indicates that using the developed ANN method yields a significant reduction in the computational cost as compared with existing finite difference methods.

1 INTRODUCTION

Extensive research was conducted to examine the behavior of short and slender concrete columns reinforced with glass fiber-reinforced polymer (GFRP) bars (Afifi et al. 2014, Maranan et al. 2016, Khorrarnian and Sadeghian 2017a, 2017b, 2020). Slender columns (i.e., columns with secondary moment effects) can be analyzed using simplified methods such as the moment magnification method specified in design codes (ACI 318-19 2019, CSA S806-12 2012) or using advanced analytical methods such as finite element method (FEM), finite difference method (FDM), or other iterative methods.

Advanced FEM and FDM are typically used to analyze slender columns for design optimization and reliability analysis as opposed to the simplified moment magnification method since the latter involves simplifying assumptions that hinder the accuracy of the solution. The use of FEM and FDM is challenged by the high computational cost required to conduct the analyses. Surrogate models using artificial neural

network (ANN) were developed to improve the analysis efficiency of FEM and FDM models by providing a more efficient analysis model with almost the same degree of precision. ANN was used in the literature to predict the strength of FRP-confined concrete columns (Cascardi et al. 2017), assess the bond between FRP bars and concrete (Yan et al. 2017), evaluate the shear capacity of FRP-reinforced concrete beams (Naderpour et al. 2018), evaluate the reliability of structures (Malakzadeh and Daei 2020), and in other applications related to structural engineering. Recently, Raza et al. (2020) developed an ANN model to predict the axial load-carrying capacity of GFRP-RC columns. The study considered an experimental database of 279 specimens and introduced the optimum ANN model from 18 candidate ANN models. However, the study was limited and did not cover the effect of eccentricity and slenderness.

In this paper, an ANN model is developed using a computer-based database of approximately 3 million GFRP-RC columns. The database considered multiple eccentricity ratios, end-moment ratios, GFRP mechanical properties, reinforcement ratio, reinforcement layout, concrete strength, and slenderness ratios. The developed ANN model is an efficient replacement of conventional FEM and FDM with material and geometric nonlinearities. The development and performance of the ANN model are presented in the subsequent sections.

2 ARTIFICIAL NEURAL NETWORK (ANN) MODEL

2.1 Artificial Neural Network Concepts

The principles of neural network modeling including definitions, activation functions, cost function, training, and performance are briefly presented in this section. The ANN developed in this study is used to perform nonlinear regression over a set of known data, where each set consists of multiple inputs and outputs.

Figure 1 presents the neural network procedure and definitions. Figure 1(a) presents an ANN with three layers, including the input layer, output layer, and one hidden layer. Each layer consists of several neurons: 5, 10, and 4 neurons for input, hidden, and output layers, respectively. Except for the input layers where each neuron is a certain value corresponding to an input parameter, all neurons are evaluated based on functions that are related to the neurons in the previous layer adjusted by weights. To calculate the value of each neuron, the sum of all weights times neuron values from the previous layer plus a constant number, named bias, is considered as the argument for the activation function as shown in Figure 1(b). Sigmoid and rectified linear unit (ReLU) activation functions were considered in this study, except for the output layer where a linear function was used. The prescribed procedure of calculating the values of each neuron is called feedforward in which the values of each neuron are calculated from the previous layer. Once all output layers are calculated, the output data-driven from the feedforward procedure is compared to the given output. The sum of squared error for each data set, called the cost for that set, is also calculated. The total cost of the network is the sum of the calculated costs for all data sets, as illustrated in Figure 1(c). The cost function is a function of all weights and bias values. The goal of the network is to optimize the weights and bias values for the network using mathematical algorithms such as gradient descent, which minimizes the cost function by applying the negative of the gradient of the cost function to the cost function and propagate backward to update the weights and bias values (Backpropagation). In this study, Levenberg-Marquardt backpropagation (LMB) and Bayesian regularization backpropagation (BRB) were considered.

In this study, the Deep Learning Toolbox from MATLAB software was used for the analysis. The analysis output was the axial capacity of GFRP bars loaded under eccentric compressive loads. Several second-order training sets (i.e., input and outputs) were required to train the network. Since the number of experimental data set for slender columns in the literature is limited, FDM was used to train the proposed ANN model.

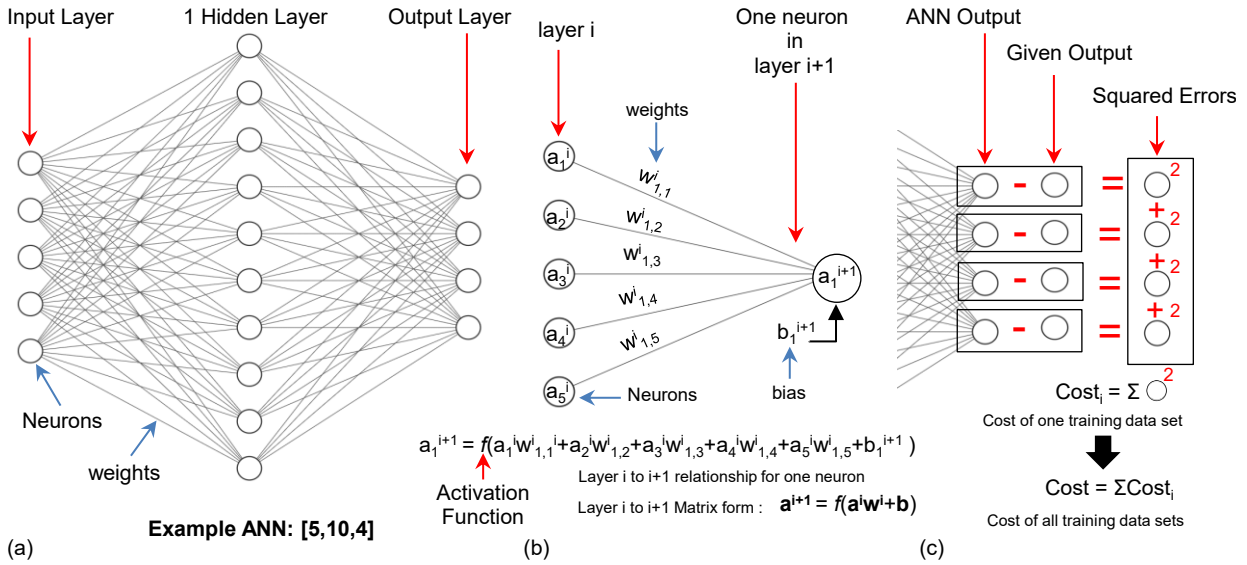


Figure 1: Neural Network Procedure: (a) One-Hidden-Layer neural network; (b) activation function; (c) cost function for one training data.

2.2 Second-Order Reference Analysis using FDM

The FDM by Khorramian and Sadeghian (2019) and Khorramian (2020) was utilized as the reference second-order analysis for training the ANN model. The FDM was used to analyze a simply supported column loaded axially under arbitrary independent end eccentricities, while considering the material and geometric nonlinearities.

Figure 2 illustrates the procedure of the FDM analysis. The column is divided into a finite number of nodes. The differential equation for modeling the column behavior is based on the central expansion of the differential equation of a beam. The load is applied in increments. For a load increment, the displacement at node 1 is considered as zero, while an arbitrary value for the second displacement is assumed. The displacements of all nodes are calculated based on the differential equation. An iterative algorithm is used such that the boundary condition is satisfied by varying the displacement of the second node and building the displacement profile for the column. The procedure requires obtaining the moment-curvature diagram of the GFRP-RC column at each considered load level. More detail of the analysis can be found in literature (Khorramian 2020).

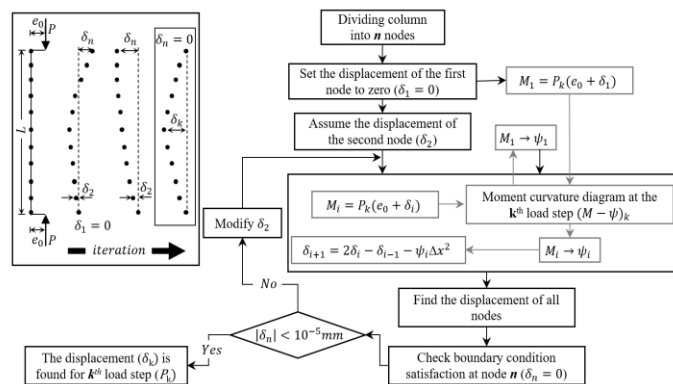


Figure 2: Iterative procedure for finite difference second-order analysis (Khorramian 2020)

2.3 Training of ANN

The FDM analysis was used to generate 2,916,000 cases by varying design parameters including end moment ratios, rebar layout, the shape of the cross-section, concrete strength, reinforcement depth ratio, strength of GFRP bars in compression and tension, reinforcement ratio of GFRP bars, modulus of elasticity of GFRP bars, eccentricity ratio, and slenderness ratio as included in Table 1. The column width for all cases was set to 254 mm, while an ultimate concrete strain of 0.0035 mm/mm was considered.

A total of 11 input data are required for the ANN model to calculate the axial capacity. The input data were normalized to present the values for neurons on a scale ranging from zero to unity, except for the output layer. The datasets were divided into training, validation, and test datasets. The data was divided into multiple sets to avoid overfitting.

Table 1: Training datasets for B=254 mm

Parameter	Value	No. of cases
End moment/eccentricity ratio (e_1/e_2)	-1, -0.5, 0, +0.5, +1	5
Rebar layout (RL)	Two or Four-sided	2
Shape of Cross section (SC)	Rectangular (h/b = 0.75, 1, 1.25)	3
concrete strength (f'_c)	20, 35, 50, 65	4
Reinforcement depth ratio (γ)	0.6, 0.75, 0.9	3
Strength of GFRP bars in tension (f_{ftu})	500, 700, 900	3
Strength of GFRP bars in compression (f_{fcu}/f_{ftu})	0.5, 0.75, 1	3
Reinforcement ratio (ρ)	1, 2, 4, 6, 8%	5
Modulus of elasticity of GFRP (E_f)	40, 50, 60 GPa	3
Eccentricity ratio (e/h)	0.1, 0.2, 0.3, 0.4, 0.5, 1	6
Slenderness ratio (λ)	14, 17, 20, 22, 24, 27, 30, 33, 37, 40	10
Total cases		2,916,000

2.4 Optimum Configuration

The optimum configuration of the ANN developed in this research is presented in this section. Figure 3 presents the concept of optimization of the neural network, in which each node in Figure 3(a) to 3(c) represents a schematical input (containing 11 inputs in each node) in the horizontal axis and one output in the vertical axis. The network can be under-trained, over-trained, or reach an optimum training based on the characteristics of the network such as number of epochs (number of times training is completed for the whole dataset), activation function, number of layers, number of neurons per each layer, and other characteristics. If the network is under-trained, the root mean squared error (RMSE) for the network will be high. If the network is over-trained, the RMSE of the training dataset will be very low while the RMSE of the validation and/or test dataset would be high, which leads to a high overall RMSE. For optimally trained network, RMSE is low, while the training and validation RMSE values are approximately similar as shown in Figure 3(d).

Preliminary analysis showed that three hidden layers provide the best performance of the ANN for the analysis of GFRP-RC columns, by comparing one, two, and three hidden layers with a different number of neurons. The sigmoid activation function was found to yield better performance as compared with ReLU. Also, Levenberg-Marquardt backpropagation (LMB) was found to yield better performance as compared with Bayesian regularization backpropagation (BRB). Therefore, the following ANN configuration was used

to search for the optimum number of neurons in each hidden layer: three hidden layers, sigmoid function for hidden layers, and LBM algorithm for training.

The search for the optimum number of neurons was based on a reduced dataset (5% of the original dataset included in Table 1) for the preliminary phase of the study. The optimum configuration was obtained by varying the number of neurons in each layer as shown in Figure 4. For this phase, 70% of the studied datasets were used for training, 15% for validation, and 15% for the testing of ANN.

The RMSE of the considered networks showed that networks with 35 neurons in the first layer have the best performance. As a result, for the second phase of the study, the number of neurons in the first hidden layer was set to 35, while the second and third layers were changed as shown in Figure 5. This phase of study consists of 50% of the original dataset in Table 1. The performance of the considered networks suggests that 30 neurons yield an optimum network configuration for the second layer. For the third hidden layer, 6 or 12 neurons results in the best performance of the network. However, 12 neurons were selected for the rest of study since the network with 12 neurons in the last hidden layer showed more consistent RMSE values for training and test datasets.

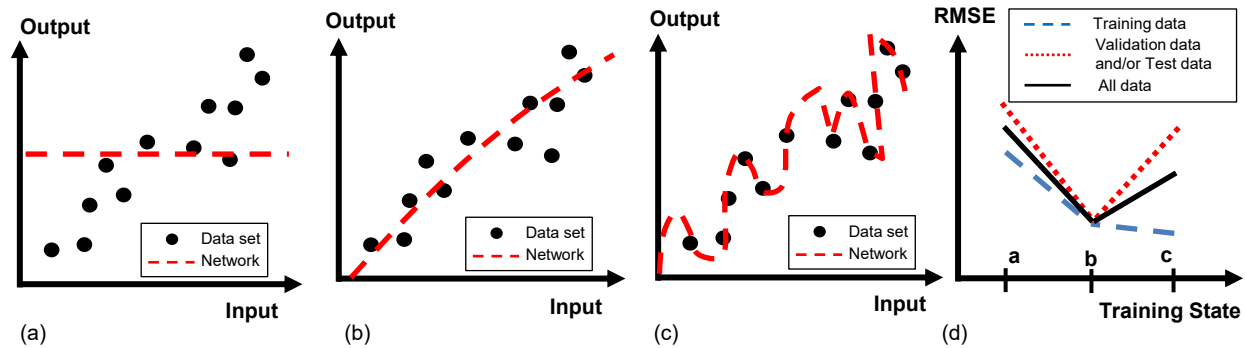


Figure 3: Training states: (a) Under-trained; (b) Optimum training; (c) over-trained; and (d) RMSE comparison

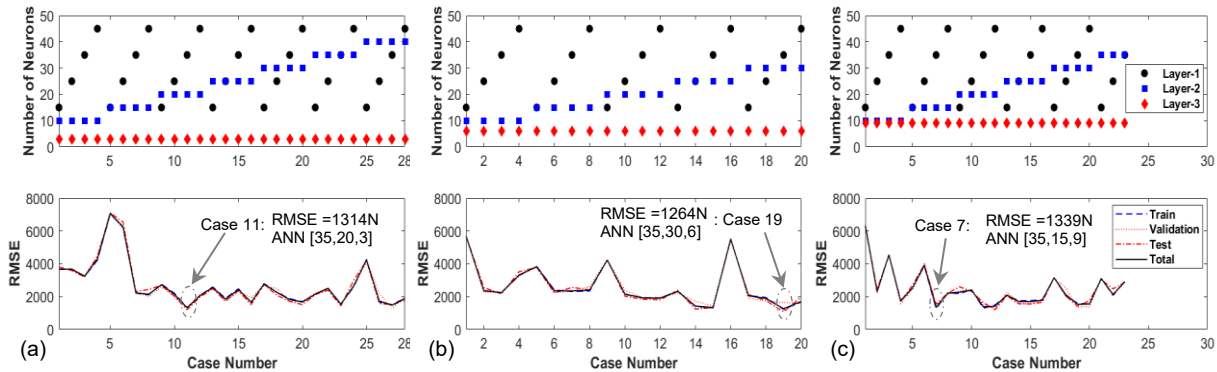


Figure 4: Optimum configuration search for the first hidden layer: (a) with 3 neurons in hidden layer-3, (b) with 6 neurons in hidden layer-3, and (c) with 9 neurons in hidden layer-3

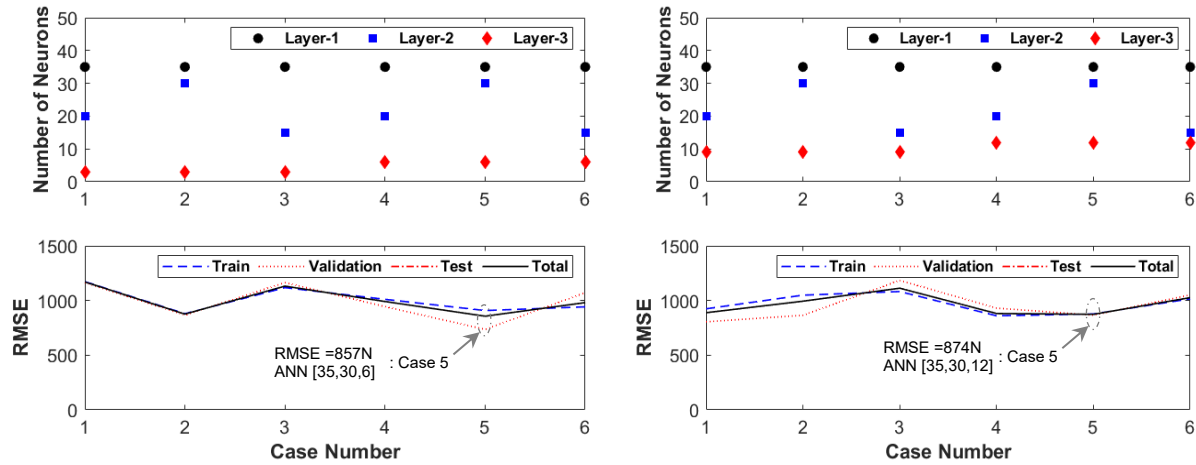


Figure 5: Optimum configuration search for the second hidden Layer

3 PERFORMANCE OF ANN MODEL

3.1 ANN Model Validation

The model validation (with 100% of the dataset in Table 1) was conducted using two ANN configurations: optimum configuration recommended in Section 2.4, and a configuration of 35, 30, and 15 for the first, second, and third layers, respectively. The reason for examining the latter configuration relates to observing an improving trend with increasing the number of neurons in the third layer. The results showed improved performance for the network with 15 neurons in the third hidden layer as shown in Figure 6. The coefficient of determination (R^2) and RMSE for the validated model were 1, and 1 kN, respectively. The final configuration of the ANN is shown in Figure 7.

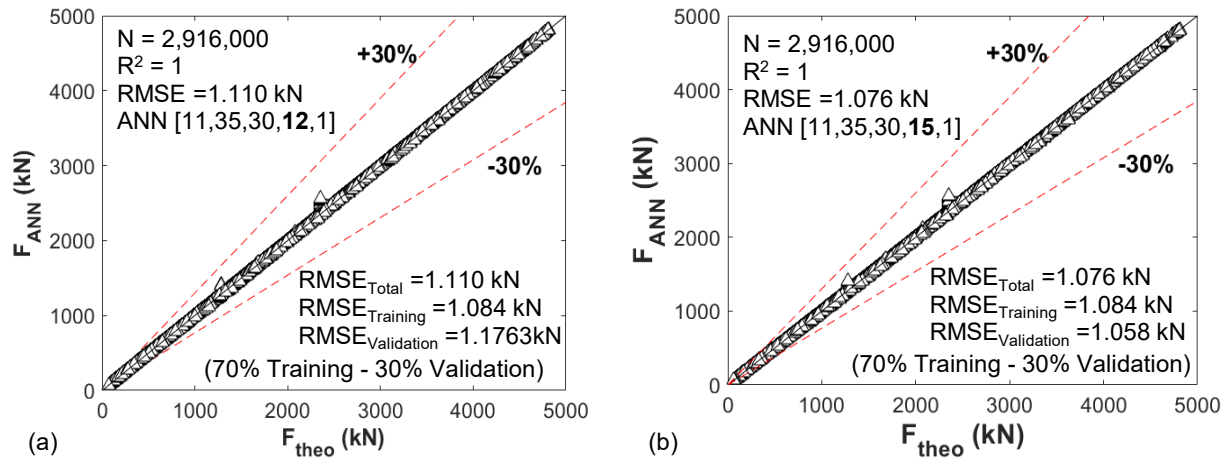


Figure 6: Performance of ANN model: (a) 12 neurons in Layer-3; and (b) 15 neurons in Layer-3

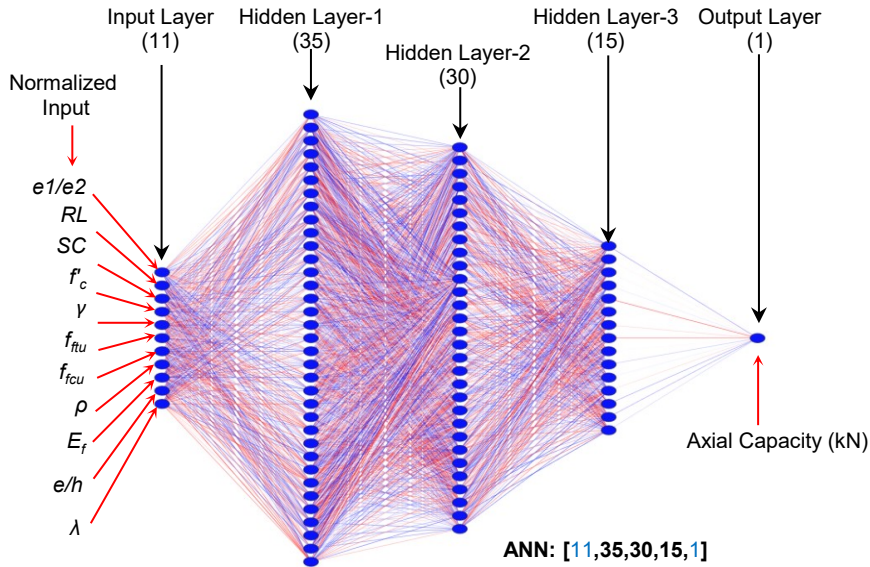


Figure 7: Proposed ANN configuration for second-order analysis of GFRP-RC Columns

3.2 Time Efficiency of ANN

The efficiency of using the developed ANN in computationally demanding engineering applications such as reliability analysis is examined in this section. Reliability analysis using Monte Carlo Simulation (MCS) involves conducting millions of simulations of random phenomena. Developing efficient tools like the ANN considered in this study can significantly reduce the computational cost for conducting reliability analysis of slender columns. To compare the time-efficiency of the ANN versus the FDM method, MCS was performed on a square section with a width of 254 mm with single curvature, slenderness ratio of 22, three layers of reinforcement, 2.5% reinforcement ratio, and 30% eccentricity ratio. The bar tensile strength, compressive strength, and modulus of elasticity were 700 MPa, 500 MPa, and 50 GPa, respectively. A dead-to-live load ratio of 4.0 was considered. The means of dead and live loads were calculated using a first-order analysis. The load and material statistics and simulation procedure can be found in the literature (Nowak and Szerszen 2003, Oudah et al. 2019, Khorramian 2020). Only the ultimate limit state (ULS) was considered in the analysis ($Y = R - P_D - P_L$, where Y , R , P_D , and P_L are performance function, axial resistance, axial dead load, and axial live load, respectively).

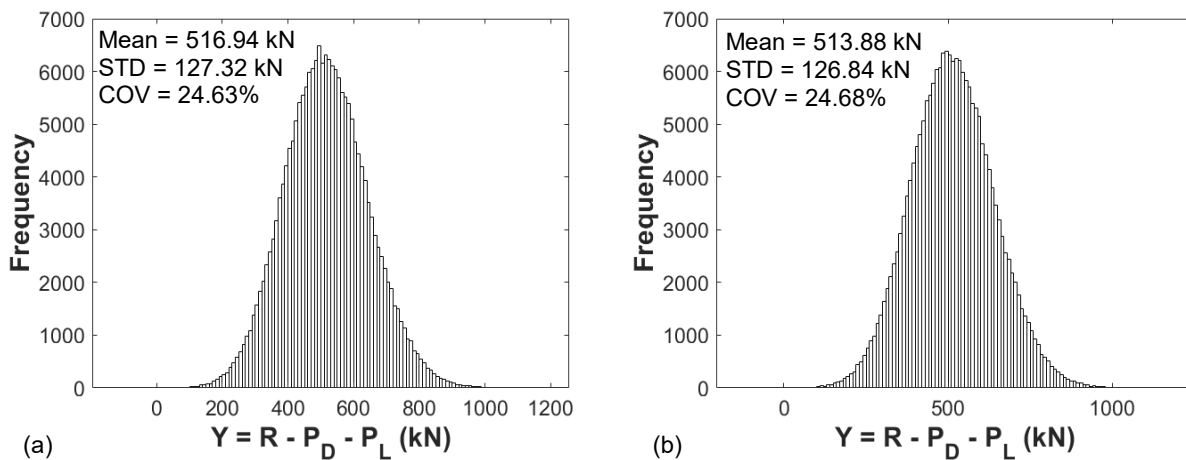


Figure 7: MCS comparison for 200,000 trials: (a) FDM analysis; and (b) ANN analysis

The distribution of the performance function obtained for the MCS using FDM and ANN is shown in Figure 7, while the efficiency of the analysis using ANN compared to FDM is compared in Table 2. The coefficient of variation (COV) and the standard deviation (STD) are also reported in Figure 7 and Table 2. Analysis results indicate similar reliability indexes for the considered column cases, while the time it took to run the ANN model was only 0.1% of the time required to conduct the analysis using the FDM model. Therefore, the developed model is considered efficient for reliability applications.

Table 2: Efficiency of ANN versus FDM

No.	MCS Analysis	Failed Trials	Reliability Index	Limit State Function Distribution (Y)			Time Efficiency
				Mean (kN)	STD	COV	
1	FDM	1/200,000	4.4172	516.92	127.32	24.63%	100%
2	ANN	1/200,000	4.4172	513.88	126.84	24.68%	0.1%
3	ANN/FDM ratio	1	1	1.006	0.996	1.002	0.001

4 CONCLUSION

In this study, an artificial neural network (ANN) was developed to analyze slender glass fiber-reinforced polymer (GFRP) reinforced concrete (RC) columns. The developed ANN was trained against a nonlinear finite-difference model (FDM) found in the literature. Multiple analysis parameters such as slenderness ratio, load eccentricity, end moment ratio, the shape of the cross-section, bar reinforcement ratio, bar layout, reinforcement depth ratio, concrete strength, bar modulus of elasticity, compressive and tensile strength were considered to develop a dataset used to train the ANN model (approximately 3 million cases). It should be emphasized that the developed ANN model is valid in the range of the trained parameters. The considered network had three hidden layers, sigmoid function for hidden layers, and Levenberg-Marquardt backpropagation (LBM) algorithm for training. Multiple configurations were investigated to optimize the number of neurons. The optimum configuration consisted of 35, 30, and 15 neurons in the first, second, and third hidden layers, respectively. The ANN model showed a coefficient of determination of 1 and a root mean squared error (RMSE) of 1 kN. The efficiency of the developed ANN in engineering applications was examined by conducting a Monte Carlo Simulation (MCS) reliability analysis of axially loaded slender GFRP columns at the ultimate limit state. The efficiency of the developed ANN was compared with the MCS of the original FDM model used in training the ANN model. Analysis results indicated similar reliability indexes for the considered column cases using ANN and FDM, while the time it took to run the ANN model was only 0.1% of the time required to conduct the analysis using the FDM model.

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