

**AN APPLIED OPTIMIZATION MODEL FOR  
FREIGHT DELIVERY IN A PHYSICAL INTERNET  
SUPPLY CHAIN**

by

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Submitted in partial fulfilment of the requirements  
for the degree of Master of Applied Science

at

Dalhousie University

Halifax, Nova Scotia

December 2019

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# ABSTRACT

The Physical Internet (PI) is a recent infrastructure in the supply chain, which allows transformation of the current logistics system into a universally interconnected system. It aims at tackling the issues of economic, environmental, and social sustainability in conventional logistics, allowing improvement of unsustainable freight transportation. In analogy to Digital Internet (DI), the key concept of PI is an open interconnected logistics system with a collaborative distribution network of PI hubs. PI offers a common operating framework for different companies in which physical products can be transported seamlessly in standard modularised  $\pi$ -containers similar to data packets in a DI.

This thesis is a study of the PI and the conventional logistic system. The main objective of the thesis is to compare the 3 different models, PI system (P), standard system (S), and hybrid (H). From a system-wide cost perspective, it is shown that Model H performs better than the others.

## **LIST OF ABBREVIATIONS USED**

4IR: Fourth Industrial Revolution

BLP: Binary Linear Programming

CO: Conventional logistics network

CVRP: Capacitated Vehicle Routing Problem

DC: Distribution Centers

DI : Digital Internet

GDP: Gross Domestic Product

GHG: Green House Gasses

GLW: Global Logistics Web

IoT: Internet of Things

KPIs: Key Performance Indicators

MBLP: Mixed Binary Linear Programming

MILP: Mixed Integer Linear Programming

OEMs: Original Equipment Manufacturers

OLI: Open Logistics Interconnection

OSI: Open System Interconnection

OSW: Open Supply Web

P2P: point-to-point

PI: Physical Internet

PSN: Private Supply Networks

RFID: Radio Frequency Identification

SSW: Shared Supply Webs



VRP: Vehicle Routing Problem

WEF: World Economic Forum

$\pi$ -container: Physical Internet containers

$\pi$ -hubs : Physical Internet containers

$\pi$ -movers : Physical Internet movers

$\pi$ -nodes : Physical Internet nodes

## **ACKNOWLEDGMENT**

Firstly, I would like to express my sincere gratitude to my supervisors Dr. M. Ali Ülkü and Dr. Uday Venkatadri, and how deeply indebted I am for their continuous support for my master's study and research. I thank them for their patience, motivation, enthusiasm, and immense knowledge. Their guidance facilitated me in all the time of research and writing of this thesis.

Besides my advisors, I would like to thank the rest of my thesis committee: Dr. Claver Diallo and Dr. Ahsan Habib, for offering their time and knowledge in evaluating my thesis. Moreover, I am immensely grateful to all my teachers for their academic guidance in my graduate courses. Thanks are also due to Ms. Ana Mora Sanchez who helped with the data configuration of the Mexican Automotive network.

I am incredibly thankful to my parents for their love, prayers, caring, and sacrifices for educating and preparing me for my future. Last but not least, a very special thanks to my friends for their continuous encouragement throughout this thesis.

# CHAPTER 1 INTRODUCTION

A supply chain is a set of facilities, suppliers, customers, products, and methods of controlling inventory, purchasing, and distribution. The chain links suppliers and customers, beginning with the production of raw material by a supplier, and ending with the consumption of a product by the customer. Logistics is the flow of goods or materials between suppliers and customers passing through several echelons in a supply chain, and each echelon may consist of many facilities. Logistics activities typically include inbound and outbound transportation management, fleet management, warehousing, materials handling, order fulfillment, logistics network design, inventory management, supply-and-demand planning, and management of third-party logistics services providers.

Generally, research and practice in supply chain logistics focus mainly on key factors of maintaining relations and coordination between suppliers and customers to achieve a more profitable outcome. The main emphasis is on customer response, inventory planning and management, supply, transportation, and warehousing. Even though these attributes are substantially important value addition, the current logistics system in implementation throughout the world is not sustainable economically, environmentally, and socially (Ballot et al., 2012). In Montreuil, B. (2011), thirteen unsustainability symptoms (Figure 1) were identified, providing the evidence to the previous assertion. From an economic perspective, the contribution of transportation to the economy is a measure of its contribution to Gross Domestic Product (GDP), which includes investments made, and transportation goods and services consumed. As per the statistics from the 2015 Department of Transportation reports U.S. Department of transportation (2017),

transportation accounts for 9% of GDP (roughly \$1,477.9 billion) and employs over 13.0 million people in a variety of roles, from driving buses to manufacturing cars to building and maintaining ports and railroads.

	<b>Unsustainability symptoms</b>	<b>Economical</b>	<b>Environmental</b>	<b>Societal</b>
1	We are shipping air and packaging	X	X	
2	Empty travel is the norm rather than the exception.	X	X	
3	Truckers have become the modern cowboys.	X		X
4	Products mostly sit idle, stored where unneeded, yet so often unavailable fast where needed.	X		X
5	Production and storage facilities are poorly used.	X	X	
6	So many products are never sold, never used.	X	X	X
7	Products do not reach those who need them the most.	X	X	X
8	Fast and reliable intermodal transport is still a dream or a joke.	X	X	
9	Getting products in, through, and out of cities is a nightmare	X	X	X
10	Products unnecessarily move, crisscrossing the world.	X	X	X
11	Networks are neither secure nor robust.	X		X
12	Smart automation and technology are hard to justify.	X		X
13	Innovation is strangled.	X	X	X

Figure 1 Unsustainability symptoms, source: Montreuil (2011)

From an environment perspective, Study by transport Canada Transport Canada (2019), shows that out of overall transport mediums (aviation, railways, etc.), road transport maintains its high among other mediums since 2005 to 2015 (Figure 2). Despite fuel efficiency improvements, emissions from road transportation, which represents 21% of total Canadian greenhouse gas emissions in 2016, have increased by 12% from 2005 to 2016.

From a social perspective, the U.S. Department of Transportation (2017) reported that from 2000 to 2015, road transportation (via trucks) had the highest percentage of fatalities, which raises the very critical issue of social sustainability (Figure 3). Another social sustainability concern is the number of hours drivers have to spend away from home, even if they have sufficient rest time during their trips.

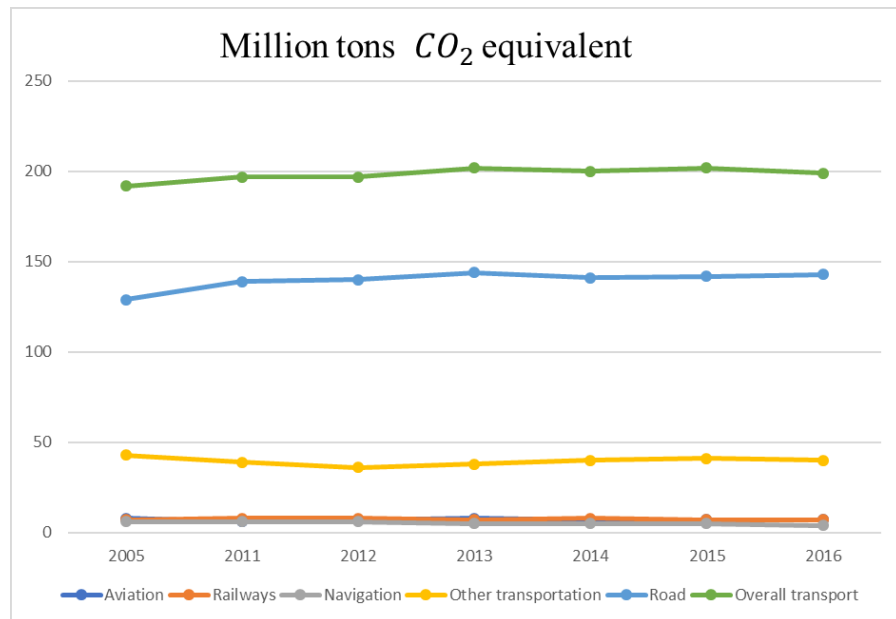


Figure 2 Million tons of CO<sub>2</sub> equivalent, source : U.S. Department of Transportation (2017)

As per Transport Canada (2019), the transportation sector grew by 3.2% in 2017, which is almost 1.4 times the growth rate of all other industries. In 2018, 920,800 employees (including self-employed people) were employed in the transportation and warehousing sector, demonstrating the growth of 2.7% from 2017. These growing trends in the transportation sector indicate that the demand for resources in the future will be substantially high. This also raises the concern for environmental and social sustainability.

**Table 6-1 Fatalities by Freight Transportation Mode: 2000, 2010, and 2013–2015**

	2000	2010	2013	2014	2015
<b>Total transportation fatalities</b>	<b>44,276</b>	<b>35,039</b>	<b>34,685</b>	<b>34,638</b>	<b>36,973</b>
<b>Total highway fatalities</b>	<b>41,945</b>	<b>32,999</b>	<b>32,894</b>	<b>32,744</b>	<b>35,092</b>
<b>Total freight transportation fatalities</b>	<b>6,079</b>	<b>4,287</b>	<b>4,558</b>	<b>4,523</b>	<b>4,631</b>
<b>Freight as a share of total fatalities</b>	<b>13.7%</b>	<b>12.2%</b>	<b>13.1%</b>	<b>13.1%</b>	<b>12.5%</b>
<b>Highway<sup>1</sup></b>	<b>5,282</b>	<b>3,686</b>	<b>3,981</b>	<b>3,902</b>	<b>4,067</b>
Large-truck occupants	754	530	695	656	667
Others killed in crashes involving large trucks	4,528	3,156	3,286	3,246	3,400
<b>Railroad</b>	<b>717</b>	<b>520</b>	<b>505</b>	<b>552</b>	<b>502</b>
Train accidents	8	4	6	2	1
Highway-rail grade crossing <sup>2</sup>	353	187	157	203	155
Trespassers	328	310	317	325	302
Other incidents	28	19	25	22	44
<b>Waterborne<sup>3</sup></b>	<b>42</b>	<b>62</b>	<b>64</b>	<b>50</b>	<b>52</b>
Freight	NA	22	19	18	40
Industrial/other	NA	40	45	32	12
<b>Pipeline</b>	<b>38</b>	<b>19</b>	<b>8</b>	<b>19</b>	<b>10</b>
Hazardous liquid pipeline	1	1	1	0	1
Gas pipeline	37	18	7	19	9

Figure 3 Fatalities by various freight transportation modes from 2000 to 2016, source:

U.S. Department of Transportation (2017)

Various new innovative paradigms in logistics and supply chains can be observed in the literature, which tends to solve these sustainability issues by the integration of novel technological advancements. Organizations such as Airbnb and Uber are the prime example of the Fourth Industrial Revolution (4IR), which has developed a notion of shared economy and asset sharing (Barber, 2018). 4IR represents a series of significant shifts in values related to the emergence of new technologies that encompasses the digital, physical, and biological worlds (Philbeck & Davis, 2019). The World Economic Forum (WEF) has provided five themes related to 4IR for enhancement of the supply chain logistics and making them more sustainable. Themes include information services, logistics services, delivery capabilities, circular economy, and sustainability. The concept of 4IR is often confused with Industry 4.0. Industry 4.0 conceals the technological developments in manufacturing and production systems by focussing on relationships between digitization,

organizational transformation, and productivity enhancement. It focuses on the vision of modular and efficient manufacturing systems for future production. In Lasi et al. (2014), authors also list the fundamental concept of smart factory, cyber-physical systems, self organizations, new systems in distribution and procurement, new systems in the development of products and services, adaptation to human needs, Corporate Social Responsibility. Another innovative area of research is Internet of Things (IoT). Sun (2012) defines IoT as “The radio frequency identification (RFID), infrared sensors, global positioning systems, laser scanners and other information sensing device, according to the agreed protocol, to any article connected to the Internet up to information exchange and communication, in order to achieve intelligent identify, locate, track, monitor and manage a network”. IoT facilitates in managing a company’s logistic architecture and helps in supervising the circulation in supply chain and share information.

Recently, there has been an intense wave of innovative change in business models generated by a new infrastructure called Physical Internet (PI) or “ $\pi$ ”. The PI is a novel concept in supply chain logistics with the potential of modernizing material handling, logistics, and facilities design aiming to enhance economic, environmental, and societal efficiency. PI is a vision for moving physical objects via a set of processes, procedures, systems, and mechanisms from an origin point to the desired destination in a way like how digital Internet moves packets of information from a host computer to another computer. PI thus emphasizes on following (1) Digital Internet exploitation, (2) seamless interconnectivity of logistics services, and (3) the magnitude expected for changes that are required (Montreuil et al., 2013). The framework of PI is based on standard and smart modular PI containers or  $\pi$ -containers that can be transported by all means (e.g., planes,

trucks, barges, drones, and private cars) with ease (Crainic & Montreuil, 2015). PI containers are modularly sized from small parcels to large maritime containers that are moved through multimodal, distributed transportation networks in which the transit sites consolidate containers from various origins to optimize the loading on the upcoming level. PI is an open interconnected network that includes open logistics facilities such as open semi-trailer transit centers, open cross-docking hubs, and open warehouses, which enables a Global Logistics Web (GLW) (Montreuil et al., 2013).

<b>Logistics Web</b>	
<b>1. Mobility Web</b> 1.1 Unimodal transport 1.2 Multimodal transport 1.3 Transport management 1.4 Innovation	<b>2 Distribution Web</b> 2.1 Distribution Center 2.2 Warehouse 2.3 Inventory control 2.4 Innovation
<b>3 Realization Web</b> 3.1 Production center (open fabs) 3.2 Production module 3.3 Demand management 3.4 Innovation	<b>4 Service Web</b> 4.1 Legal framework 4.2 Access rules & structure 4.3 Security

Figure 4 Logistics web attributes, source: Fergani et al. (2019).

In Fergani et al. (2019), a clustering strategy is provided to classify the literature dedicated to PI paradigm. This paper reviews scientific articles presented in the field of PI and categorise them on the basis of three factors: logistics web, organization, and resources. Figure 4 presents the attributes of the logistics web addressed in the literature.



As observed from the survey conducted by Fergani et al. (2019), there are a very limited number of publications on the transport management concerning route planning and cost determination in PI, specifically multimodal transport is more addressed in the literature compared to the unimodal one. This gap provides supports the motivation for this thesis.

The problem statement in this thesis consists of optimizing the transportation cost for unimodal freight delivery in a supply chain consisting of a multiple suppliers and multiple buyers/customers in what we term as a “ $\pi$ -Supply Chain” ( $\pi$ -SC). Figure 5 depicts the conceptual model of  $\pi$ -SC.

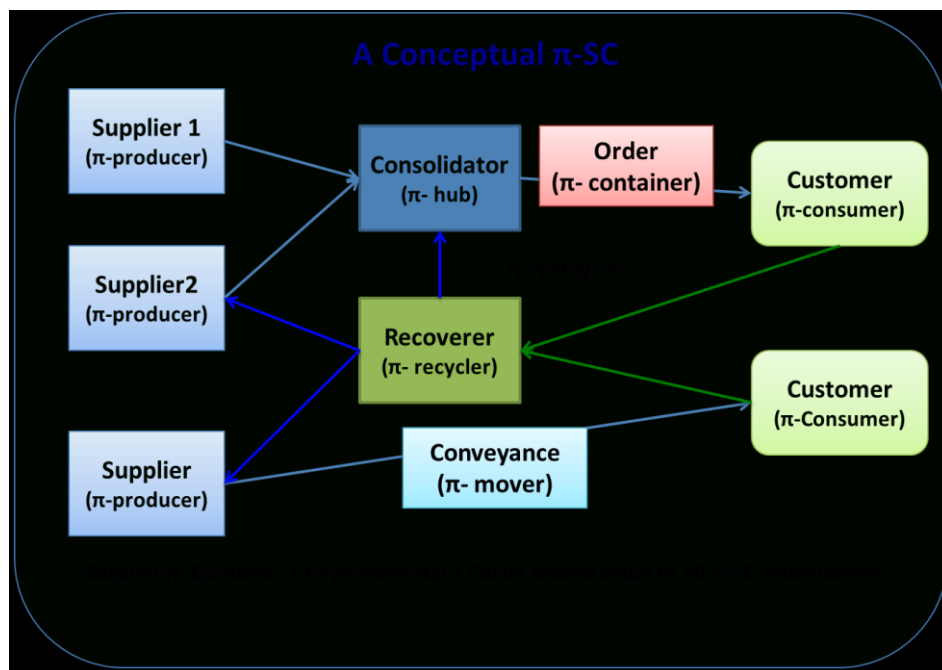


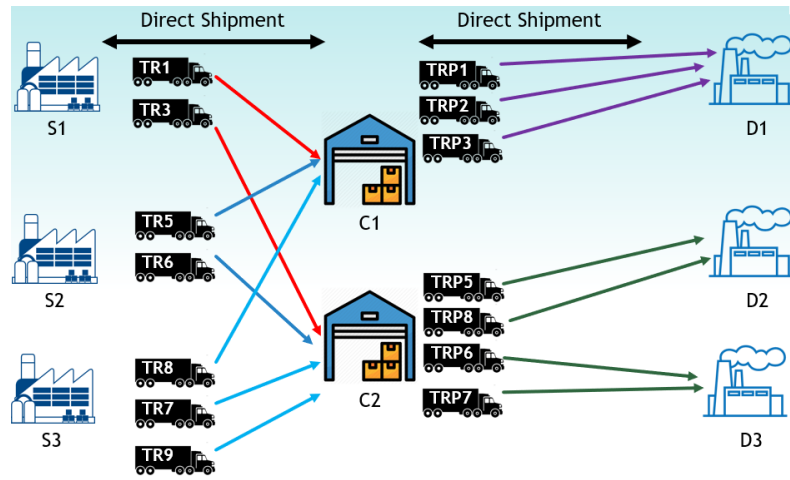
Figure 5 A conceptual  $\pi$ -Supply Chain, source: Ülkü, M.A. (2019)

Ülkü, M.A. (2019) states that “A Physical Internet Supply Chain (PI- or  $\pi$ -SC) is a collective set of suppliers, customers, and value-recovery (e.g., reverse logistics) companies with the common goal of achieving sustainable production, delivery and consumption, by maximizing economic, environmental, and social shared-value of their

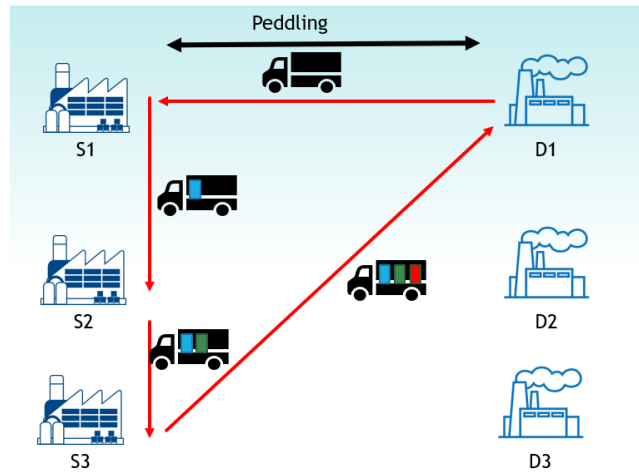
business eco-system on a global scale, and by collaboratively devising and utilizing smart technologies, modular resources (e.g.,  $\pi$ -containers,  $\pi$ -movers) and infrastructures (e.g.,  $\pi$ -nodes) on a  $\pi$ -network.”  $\pi$ -SC management, poised to a paradigm shift, probes the concept of business-as usual, brings about implementable disruptive innovations to SC infrastructure, operations, and leadership.

This research presents a comparison analysis based on freight transportation cost between the three models shown in Figure 6. The first model is a Binary Linear Programming (BLP) model for the PI supply chain configuration with size constraint of  $\pi$ -containers, with the configuration name Model P. The  $\pi$ -containers with varying sizes as per the orders (combination of products manufactured/assembled at a supplier) are delivered to the buyers as per their demands from specific suppliers through a network of uncapacitated  $\pi$ -hubs. In contrast to the PI system, which includes a multiple  $\pi$ -hub network, this configuration consists of 2-echelons. The first leg of the network comprises of set of suppliers delivering the  $\pi$ -containers on a set trucks owned by the supplier to the selected  $\pi$ -hubs as direct shipments, this selection is based on the minimum transportation cost and size constraints of  $\pi$ -containers. The second leg of the network comprises of a set of  $\pi$ -hubs, similar to the one in first leg and a set of buyers. Here, the fleet of trucks are owned by the  $\pi$ -hubs and they deliver to the buyers as per their demands on the routes as direct shipments, selected by the mathematical model based on minimum transportation cost.

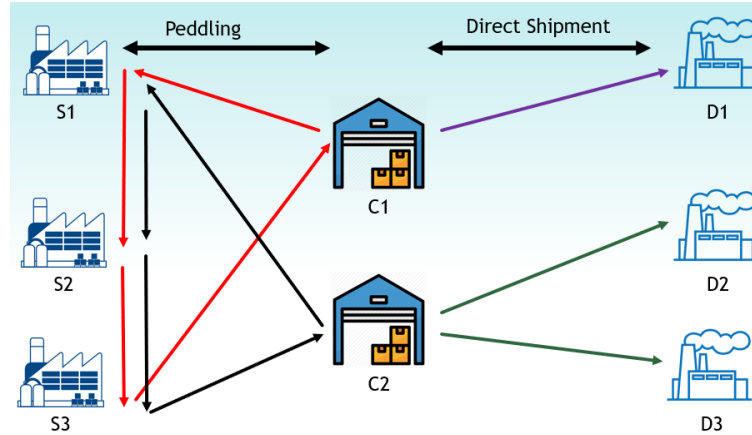
The second model is a standard transportation model; we call it Model S. As observed in literature, Model S is analogous to peddling, which a transportation configuration in which fleet of trucks is owned by the buyers, pick up products at one or



(a)



(b)



(c)

Figure 6 (a) Model P (b) Model S (c) Model H

several suppliers and deliver them to one or several plants. Peddling does not involve any hubs or cross-docking centres, which increases the distance travelled by a single truck, as it has to travel larger distance without any consolidation point/center. For our problem, this configuration falls under the category of Capacitated Vehicle Routing Problem (CVRP), which is a subcase of Vehicle Routing Problem (VRP). A CVRP consists of a fleet of identical vehicles located at a depot (buyers in our case) with a known capacity. There are buyers have a known demand to be fulfilled by the suppliers and the cost of transportation between any pair of suppliers, or between any supplier and the buyer, is also known. The objective is to find the minimum transportation cost for the freight delivery routes, each route starting and ending at the buyer. Mathematical model of CVRP ensures that each supplier is visited by only one truck and no truck exceeds its capacity. The third model is a hybrid model, so the name Model H. This configuration integrates the concepts of Model P and Model S in the transportation network. Model P has greater influence on this configuration as it follows majorly the PI framework with a network of  $\pi$ -hubs in contrast to Model S. The supply chain network here is 2-echelon as well, but with the integration of peddling on the first leg, in which the fleet of trucks owned by the  $\pi$ -hubs pick up  $\pi$ -containers from one or several suppliers and deliver them back to the respective  $\pi$ -hub. For the second leg, the fleet of trucks owned by the  $\pi$ -hubs deliver to the buyers as per their demands in as direct shipment. Further, we compare these three models based on their system wide optimization of minimum total transportation cost and study the results based on a case study of Mexican automotive supply chain consisting of tier 1 suppliers and Original Equipment Manufacturers (OEMs).

Through this research, we contribute to the literature by conducting a performance analysis based on systemwide transportation cost between the Models P, S, and H on the basis of the following KPIs: fuel, labor, and truck cost. Our hybrid model (H) aims to bring together the logistical benefits of both peddling (via Model S) and PI (via Model P). As an illustrative example, we test our models based on the logistics data obtained from an automotive supply chain in Mexico. The optimality results showcased that Model H may outperform Models S and P, total systemwide transportation cost, which is a function of vehicle capacity utilization, total distance travelled (loaded or empty), among others.

Remainder of the thesis is arranged as follows. In chapter 2, a literature review related to PI paradigm is conducted, which includes the research conducted in introduction, design and implementation of PI infrastructure with the application of novel technologies that enable the PI. The proposed methodologies for all three configurations are described in chapter 3. In this chapter we present the mathematical formulations and heuristics used to develop the three models (Model P, Model S, and Model H). Chapter 4 focuses on the computational experimentation and analysis between the proposed systems based on a case study of Mexican automotive industry. This chapter discusses the approach adopted for data collection and selection of parameters, with the static optimization results obtained by implementation of the proposed models in chapter 3. In addition, we also define the Key Performance Indicators (KPIs) and based on these KPIs we compare the three supply chain networks. Finally, conclusions and future research directions are discussed in chapter 5.

## CHAPTER 2 LITERATURE REVIEW

In this chapter we focus on the literature review on Physical Internet. The Term Physical Internet (PI) was first introduced as a big headline on the front page of The Economist in June 2006. The issue consisted of a logistics survey, with supply chain and logistics articles. Apart from the headline, the term PI was not emphasized until Montreuil (2011). Its capabilities, key features, comparison with Digital Internet have led to the main question: “Why would the world need a Physical Internet?” The answer to the latter question became vivid after the thirteen bold unsustainability symptoms presented by Montreuil (2011). The rest of the curiosities led to the definition of PI vision through 13 characteristics, briefly shown in Figure 7.

S.No	Thirteen characteristics defining the Physical Internet vision
1	Encapsulation merchandises in world standard smart green modular containers
2	Aiming towards universal interconnectivity
3	Evolve from material to $\pi$ -containers handling and storage systems
4	Exploit smart networked containers embedding smart objects
5	Evolve from point to point hub and spoke transport to distributed multi-segment intermodal transport
6	Embrace a unified multi-tier conceptual framework
7	Activate and exploit an Open Global Supply Web
8	Design products fitting containers with minimal space waste
9	Minimize physical moves and storages by digitally transmitting knowledge and materializing objects as locally as possible
10	Deploy open performance monitoring and capability certifications
11	Prioritize webbed reliability and resilience of networks
12	Stimulate business model innovation
13	Enable open infrastructural innovation

Figure 7 Thirteen characteristics defining the Physical Internet vision,

source: Montreuil (2011)

An overview of key elements of PI, which are the foundation of PI paradigm, is provided in Montreuil et al. (2010). These elements are classified into three main categories: containers, movers and nodes.

In Ballot et al. (2012), design approach for interconnected logistic services using open hub network is introduced through encapsulation of materials/goods in PI container or  $\pi$ -containers to tackle the sustainability issues in fragmented supply chains. This paper explains that PI enables users to contemplate and act in terms of open global mobility web and supply web, which helps in transforming logistics towards seamless and efficient interconnections of all logistics network.

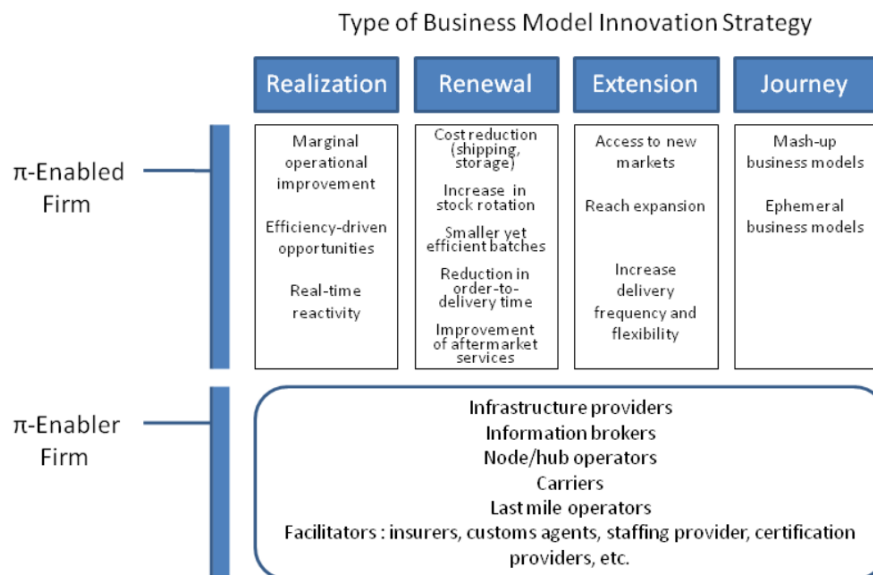


Figure 8 Implications of different types of business model innovation strategies for  $\pi$ -Enablers and  $\pi$ -Enabled firms, source: Montreuil et al. (2012)

Potential impacts of PI on business model innovation were the main focus of Montreuil et al. (2012). This paper classifies firms into two categories:  $\pi$ -enablers and  $\pi$ -

enabled. The  $\pi$ -enablers are those firms that provide the infrastructural tools for the implementation purpose and  $\pi$ -enabled firms are the ones that exploit those tools for value creation. Figure 8 shows the relationship between  $\pi$ -enablers and  $\pi$ -enabled firms.

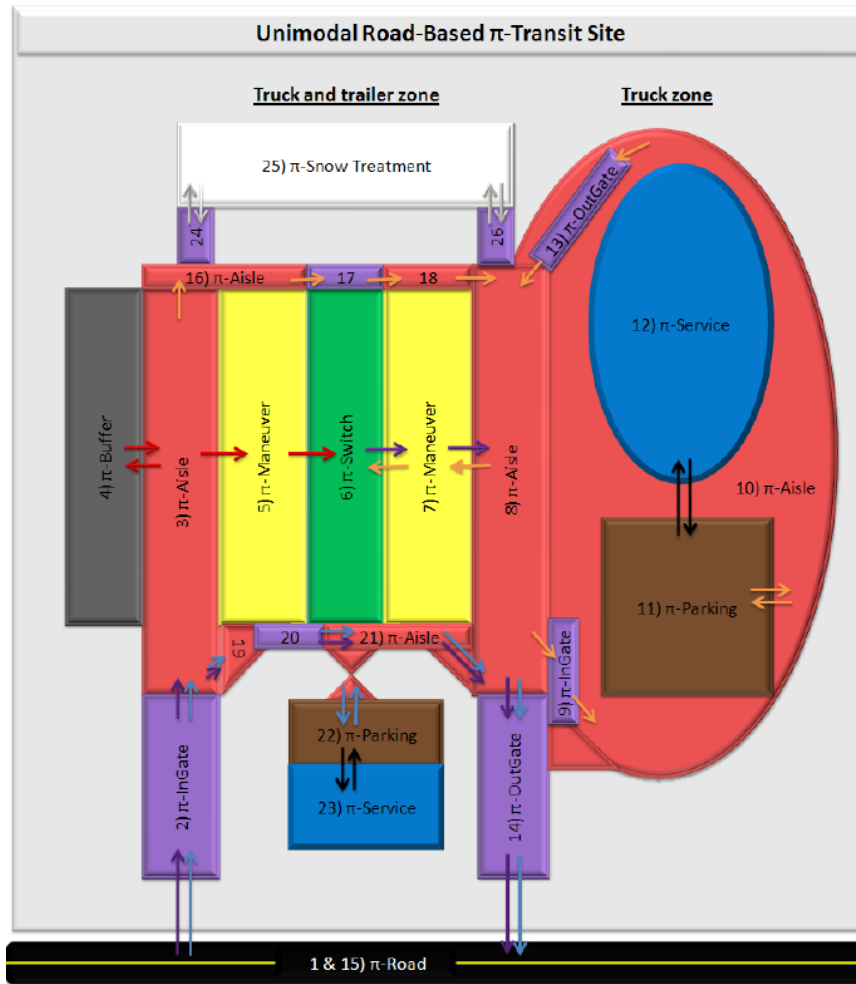


Figure 9 Block Layout for the Proposed Functional Design, source: Meller et al. (2012).

In Meller et al. (2012), functional design of road-based PI facilities needed for the operation of PI is provided as shown in Figure 9. Essentially, PI facilities are  $\pi$ -transit centres that facilitate the transfer of  $\pi$ -carriers from inbound to outbound destinations. It provides the method of transferring  $\pi$ -trailers from one truck to other but with a notion of



considering the uncertainties involved due to arrival times of driver-trailer pairs. This paper also discusses two KPIs with the perspective of customers of the  $\pi$ -transit centre and the operators.

In Sohrabi et al. (2012), scenario-based distribution network design model is introduced which involves rigorous assessment of economic performance potentials of PI. This paper adopts an optimization-based approach to develop a generic distribution design network and adapts the model for existing closed and collaborative distribution systems, as well as for open distribution web, a key constituent of logistics web in PI (Montreuil, 2011). Research conducted in Sohrabi et al. (2012) also characterises three key drivers of distribution network design, which are available capacity, market-demand, and network cost. Also, it describes the future shaping variables that depend on time, location of distribution network resources, and product type for the future business environments.

Table 1 Layers of OSI, Internet and OLI models, source: Fontane et al. (2012)

<b>Layer</b>	<b>OSI model: Digital Internet</b>	<b>TCP/IP model: Digital Internet</b>	<b>OLI model: Physical Internet</b>
1	<i>Physical</i>	<i>Physical</i>	<i>Physical</i>
2	<i>Data Link</i>	<i>Data Link</i>	<i>Link</i>
3	<i>Network</i>	<i>Network</i>	<i>Network</i>
4	<i>Transport</i>	<i>Transport</i>	<i>Routing</i>
5	<i>Session</i>	<i>Application</i>	<i>Shipping</i>
6	<i>Presentation</i>		<i>Encapsulation</i>
7	<i>Application</i>		<i>Logistics Web</i>

A seven layered Open Logistics Interconnection (OLI) model and five layered TCP/IP model (Digital Internet) is describes and illustrated in Fontane et al. (2012). Similar to Open System Interconnection (OSI) model and Digital Internet (DI), which has structured layers that facilitates interconnections between digital counterparts, an OLI

model is proposed in contrast to enable the interconnections of logistic services globally in PI (Table 1).

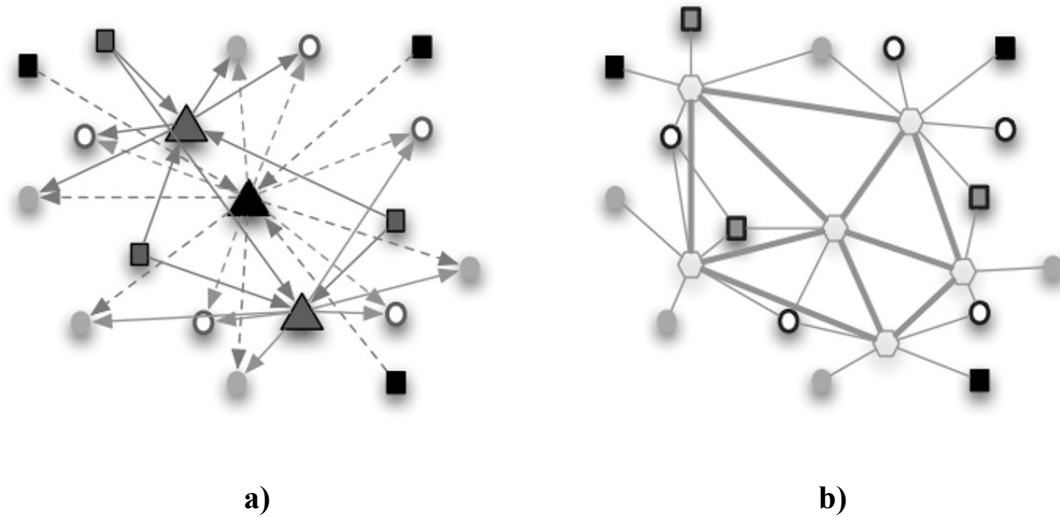


Figure 10 a) Overlapping but disconnected logistics network;  
b) Interconnected logistics network, source: Fontane et al. (2012)

Figure 10 depicts two different logistic network topologies, overlapping yet disconnected and interconnected to justify the radical impacts on logistics structuring, operations and performance as well as on the business models of Physical Internet users (retailers, distributors, manufacturers, etc.) proposed by Montreuil (2011).

Meller et al. (2012) introduced a first formal definition of PI as “An open global logistics system founded on physical, digital and operational interconnectivity through encapsulation, interfaces and protocols.” This paper provides an insight to the foundations of PI introduced by Montreuil (2011) and explains eight foundations of PI, shown in Figure 11. The design and development methodology of a mobility web simulator is proposed in Montreuil et al. (2013) to study and quantify the effect of evolving from current logistics

system to PI on economical, environmental, and social efficiencies with a case study in France. This paper adapts the three-level approach defined in Montreuil et al. (2010) to provide a general architecture for developing a mobility web simulator. The approach is to capture the complexity and dynamicity of logistics contexts by mapping software agents with real world decision-making actors or systems.

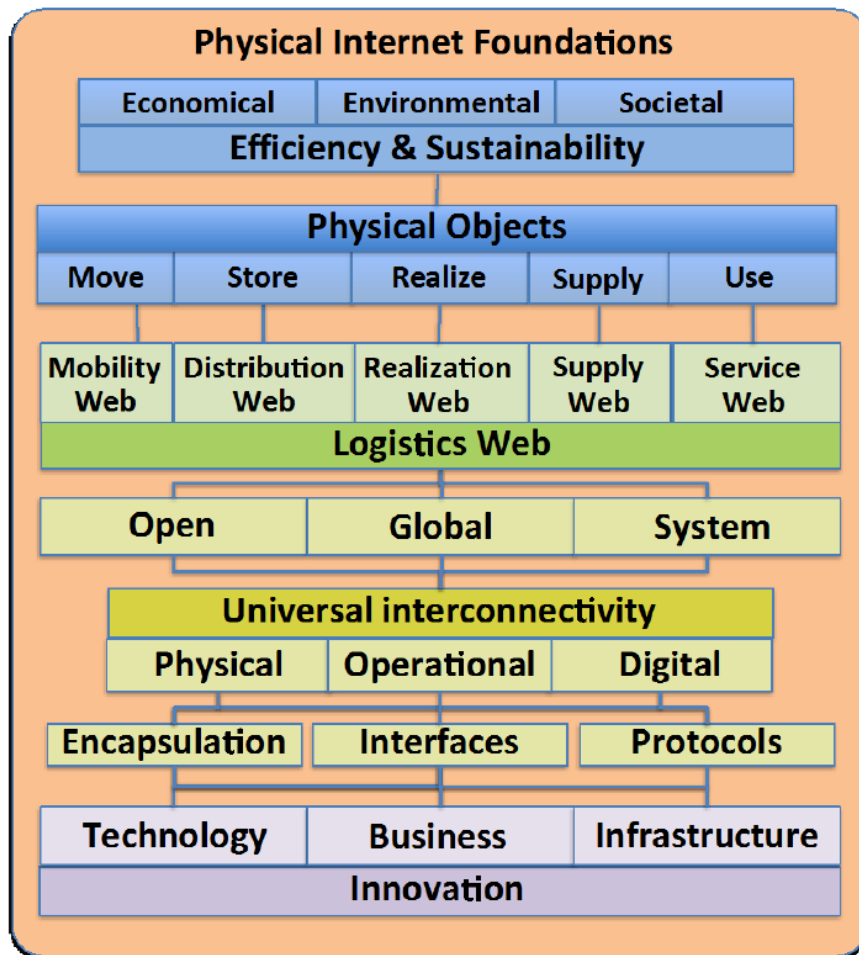


Figure 11 Physical Internet foundations framework, source: Montreuil et al. (2013)

In Sohrabi & Montreuil (2014), an interconnected distribution-planning framework is proposed exploiting the open distribution web in PI. The framework is structured through four layers: network, distribution, deployment, and delivery, which facilitate businesses to

distribute its products across global markets, utilizing the currently open Distribution Centers (DC) that are operated by other businesses. In addition, this paper also proposes the distribution policy for a mid term planning horizon (e.g. season, month) and parameterize the policy for each product at each open DC in regard to the market served.

Table 2 Analogy between computing network and physical network, source: Sarraj et al. (2014)

Network	Internet	Physical Internet	Interconnection function
Flow	Datagram	$\pi$ -Container	Encapsulation of merchandise
Node	Router	Hub	Place of orientation (sorting), change of mode, service provider
	Host (unique address)	Supplier or consumer	Place of containerisation and de-containerisation
Arc	Wire or wave connection	Transport services	Punctual or regular transport between two hubs

Sarraj et al. (2014) presents analogy between DI and novel logistics service networks based on PI, due to the strong similarities in both the networks (Table 2). Even though the type of object being transported differs. The proposed analogy is based on three vital characteristics: the definition of interconnection, the structure of the networks and the routing of objects through these networks.

In Tremblay et al. (2015), a synthesis of transformation of goods encapsulation by implementation of  $\pi$ -containers is proposed with introduction of three-tier structural characterization of  $\pi$ -containers. Figure 12 depicts the general characteristics of  $\pi$ -containers, presented in the paper. The three tiers modular design of  $\pi$ -containers consist of transport containers (T-containers), handling containers (H-containers) and packaging containers (P-containers).

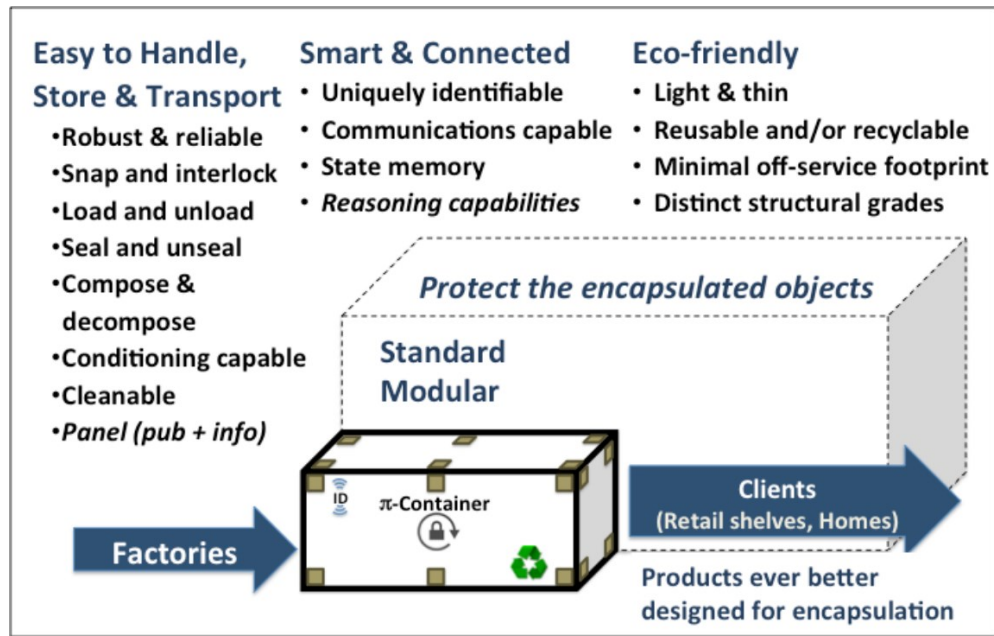


Figure 12 General characteristics of  $\pi$ -containers, source: Tremblay et al. (2015)

Table 3 Contrasting characteristics of existing private supply networks and shared supply networks, source: Sohrabi & Montreuil (2011).

Supply Network design	Decision making level	Facility utilization	Geographical extension	Customer service level
PSN	Strategic	Poor as facilities are utilized by a single company through long-term contracts	Constrained to the single company's network	Compromised by location and capacity of single company's facilities
SSW	Strategic	Improved relative to PSN but still constrained to partnering companies and long-term commitments	Constrained to the shared web of partnering companies	Compromised by location and capacity of partnering companies' facilities
OSW	Tactical, Operational	Improved by opening available space to other companies within short-term contracts	Globally extended	Fast and reliable by exploiting globally dispersed open facilities

In Sohrabi & Montreuil (2011), an exploratory study is conducted to assess the potential benefits of evolving from current supply network design, typically Private Supply Networks (PSN) and Shared Supply Webs (SSW) to PI enabled Open Supply Web (OSW). This research provides the contrasting characteristics of existing PSN and SSW with the proposed OSW on the basis of facility utilization, geographical extension, and customer service level as depicted in Table 3.

In Peng et al. (2019), a multi-objective mixed integer linear programming model is proposed to explore the sustainability performance of PI in an integrated production-inventory-distribution system. This paper adopted the approach of augmented  $\varepsilon$ -constraint method to solve the model and compared the sustainability performance of PI system with that of traditional and horizontal collaboration networks covering all the three dimensions of sustainability: economic (total cost), environmental (Green House Gasses or GHG emissions) and social (accident risks).

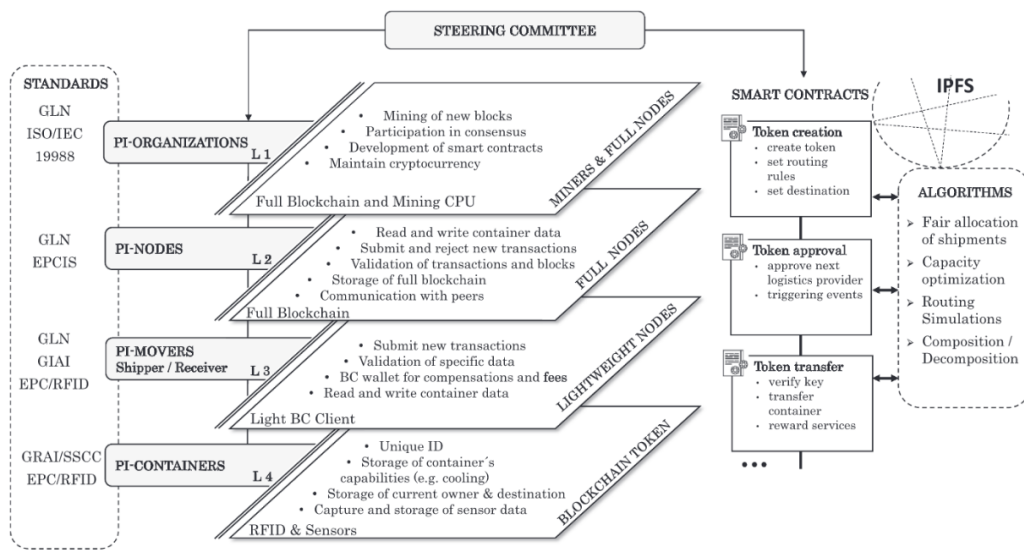


Figure 13 conceptual framework of Blockchain design architecture in PI, source:

Treiblmaier (2019)

In Marino et al. (2019), the solutions for tackling the challenges on implementation of Internet of Things (IoT) in PI are discussed. Challenges of efficient implementation of IoT in PI are due to the modularity and dynamicity of PI systems. This paper explores the role of IoT in designing of hyper-connected and interoperable  $\pi$ -containers to achieve the goal of EU H2020 ICONET project, which is to extend the research around the PI concept by designing a new networked architecture for interconnected logistics hubs and by developing a cloud-based PI framework and platform. In Treiblmaier (2019), this research provides a decentralised approach based on Blockchain technology, which offers tackles the barriers concerning the exchange of value and physical assets in logistics networks under the PI paradigm. This paper presents a conceptual framework of Blockchain design architecture, which comprises a four-layered framework as shown in Figure 13 and illustrates its application through an industry case study.

In Venkatadri et al. (2016), a MILP model for one-way optimal dispatch between a pair of nodes is developed and is extended for the two-way point-to-point (P2P) optimal dispatch between a pair of nodes. This model is used to characterise the performance of both the networks on the basis of the following logistics KPIs: total cost, inventory cost, transportation cost, number of truck trips or sector trips, average number of transfers, and average delivery time. This paper also presents the direction of implementing the P2P model to traditional and PI based logistics network.

In Fazili et al. (2017), a Monte-Carlo based comparison is presented within a sequential three-phase optimisation framework to quantify the advantages and disadvantages of PI logistics system over Conventional (CO) logistics network. This paper provides the logistics and routing optimization framework of three systems, PI, CO and

intermediate hybrid (HY) and compares them on the basis of a three-phased optimisation framework on following KPIs: Number of container packing and unpacking instances, total hours of container routing, number of trucks in service, average hours worked per truck, percentage drivers back home at the end of the day, and total systems cost including the costs operations, social impact, material handling and fixed installation.

The scope of PI and its application is diverse. Collaboration of PI concept with different sectors such as humanitarian logistics is the future of making supply chains sustainable by interconnecting them globally. In Abdoukadre et al. (2014), an interconnected humanitarian logistics system based on PI paradigm is proposed. This paper provides literature on the main humanitarian logistics and the issues encountered in current practices with a proposition of repositioning it through a PI conceptual framework. Ülkü et al. (2015) also provides the insights on humanitarian logistics and challenges faced due to inefficiencies in supply chains. Abdoukadre et al. (2014) states that “the aim is to enable efficient interconnectivity of individuals, donors, contractors, suppliers, NGOs, international institutions, government and beneficiaries.”

As observed from the literature, the PI has gathered a lot of research interest; however, there are a number of questions which remain unaddressed. This thesis aims to fill the gap in literature by comparing the logistics performances between peddling in conventional logistics, PI logistics, and a combination of these both systems, which we refer to as a hybrid model. Peddling generally improves vehicle capacity utilization, but also involves longer travel distance. PI tries to improve vehicle capacity utilization through consolidation and also distance travelled by locating hubs strategically. The thesis analyzes the costs of both configurations in addition to proposing a hybrid model to potentially



realize the benefits offered by each. Additionally, we emphasize the potential enabling capacities to PI of those new (technological) paradigms such as IoT, Big Data, Industry 4.0 etc.

## CHAPTER 3 PROBLEM FORMULATION

In this chapter, we will provide the mathematical formulation for our freight delivery problem with the view of optimizing the system wide transportation cost for the three supply chain networks PI (Model P), Standard model (Model S) and Hybrid model (Model H). In the following subsection, we have discussed the three settings:

### MODEL P – PHYSICAL INTERNET

In this model there are a set of multiple suppliers  $s$ , multiple demand points/buyers  $d$  and uncapacitated consolidation  $\pi$ -hubs  $c$  in a 2-echelon framework. Each Supplier own a homogenous fleet of trucks  $t_1$  for direct shipment to  $\pi$ -hubs and each  $\pi$ -hub owns a homogenous fleet of trucks  $t_2$  for direct shipment to buyers. There is a set of  $\pi$ -containers  $i$ , which are assigned to suppliers for encapsulation of the products manufactured or assembled at the supplier's facility. The  $\pi$ -containers have different sizes  $q_i$ , which interconnect and form a composite cluster that is loaded in the truck with capacity constraint. Each buyer has a known demand of  $\pi$ -containers from specific suppliers and supplier fulfils the demand by shipping it through a  $\pi$ -hub. We present a binary linear program adapted from Küçüköğlü & Öztürk (2017) in which they present a two-stage Mixed Integer Linear Programming (MILP) model for the transportation problem of cross-docking network design integrated with truck-door assignments to minimize total transportation costs from suppliers to customers. For our research, we have focused on the first stage of their model, which formulates the transportation problem of the network with two-dimensional truck-loading constraints. Their problem consisted of suppliers, customers (or destinations) and cross-docking centers. The products flow from suppliers to

customers through cross-docking centers according to customer demand. Each product from the suppliers is loaded into incoming and outgoing trucks according to the two-dimensional truck-loading constraints. We eliminated the two-dimensional truck-loading constraints, but we introduce the concept of  $\pi$ - containers with size constraint. The objective of this model is to minimize the overall logistics cost. Table 4 below gives the set of parameters and decision variables considered. We used Binary Linear Programming (BLP) to solve this model, as the objective function is linear, and the variables are binary. The linear program implemented in GUSEK for Model P is presented in Appendix I.

Table 4 Parameters and decision variables for Model P

<b>Parameters</b>	
$s$	Index for supplier $s \in S$ , $ S  = n$ .
$c$	Index for consolidation $\pi$ -hub $c \in C$ , $ C  = h$ .
$d$	Index for demand point (buyer/customer) $d \in D$ , $ D  = m$ .
$i$	Index for an order consolidated in a $\pi$ -container $i \in I$ , $ I  = o$ .
$t_1$	Index for a truck between suppliers and $\pi$ -hubs $t_1 \in T_1$ , $ T_1  = \bar{t}_1$ .
$t_2$	Index for a truck between $\pi$ -hubs and demand points $t_2 \in T_2$ , $ T_2  = \bar{t}_2$ .
$sl_i$	Supplier label of $\pi$ -container $i$ ; $i \in I$ .
$dl_i$	Demand point label of $\pi$ -container $i$ ; $i \in I$ .
$st_{t_1}$	Supplier label of truck $t_1$ between suppliers and $\pi$ -hubs, $t_1 \in T_1$ .
$ct_{t_2}$	PI hub label of truck $t_2$ ; $t_2 \in T_2$ .
$T_s$	Trucks available at supplier $s$ ; $s \in S$ .
$T_c$	Trucks available at $\pi$ -hubs $c$ ; $c \in C$ .

$I_d$	Set of $\pi$ -containers destined to $d$ , $d \in D$ .
$q_i$	Size of $\pi$ -container $i$ ; $i \in I$ .
$Q$	Maximum capacity of a truck.
$td_{sc}$	Minimum travel distance from supplier $s$ to $\pi$ -hub $c$ , $s \in S, c \in C$ .
$td_{cd}$	Minimum travel distance from $\pi$ -hub $c$ to demand point $d$ ; $c \in C, d \in D$ .
<b>Surrogate sets</b>	
$\varepsilon_i$	$\{t_1 \in T_1: st_{t_1} = sl_i\}$ ; If the trucks belong to first leg (i.e. between suppliers and $\pi$ -hubs), this is the subset of trucks which have same supplier label as of $\pi$ -containers.
$\rho_{t_1}$	$\{i \in I: sl_i = st_{t_1}\}$ ; For all the $\pi$ -containers, this is the subset of $\pi$ -containers with same supplier label on the trucks belonging to first leg.
$\pi_i$	$\{i \in I: st_{t_1} = sl_i\}$ ; For all the $\pi$ -containers, this is the subset of supplier trucks with same supplier label on the $\pi$ -containers.
<b>Decision variables</b>	
$z_{it_1c}$	$\left\{ \begin{array}{l} 1, \text{ if } \pi\text{-container } i \text{ is transported to } \pi\text{-hub } c \text{ on a truck } t_1 \\ 0, \text{ otherwise} \end{array} \right\}$
$z^o_{it_2d}$	$\left\{ \begin{array}{l} 1, \text{ if } \pi\text{-container } i \text{ is transported to demand point } d \text{ on a truck } t_2 \\ 0, \text{ otherwise} \end{array} \right\}$
$v_{t_1c}$	$\left\{ \begin{array}{l} 1, \text{ if truck } t_1 \text{ is assigned to } \pi\text{-hub } c \\ 0, \text{ otherwise} \end{array} \right\}$
$v^o_{t_2d}$	$\left\{ \begin{array}{l} 1, \text{ if truck } t_2 \text{ is assigned to demand point } d \\ 0, \text{ otherwise} \end{array} \right\}$

## Objective function for Model P:

$$\begin{aligned} & \text{Min } Z_p \tag{P.0} \\ & z_{it_1c}, z^0_{it_2d}, v_{t_1c}, v^0_{t_2d} \forall i, t_1, t_2, c, d \\ \text{where } Z_p = & \sum_{s \in S} \sum_{c \in C} \sum_{t_1 \in T_s} td_{sc} v_{t_1c} + \sum_{c \in C} \sum_{d \in D} \sum_{t_2 \in T_c} td_{cd} v^0_{t_2d} \end{aligned}$$

Subject to:

$$\sum_{t_1 \in \varepsilon_i} \sum_{c \in C} z_{it_1c} = 1 \quad \forall i \in I \tag{P.1}$$

$$\sum_{c \in C} v_{t_1c} \leq 1 \quad \forall t_1 \in T_1 \tag{P.2}$$

$$\sum_{i \in \rho_{t_1}} z_{it_1c} \leq M \cdot v_{t_1c} \quad \forall t_1 \in T_1, c \in C \tag{P.3}$$

$$\sum_{i \in \pi_i} \sum_{c \in C} z_{it_1c} q_i \leq Q \quad \forall t_1 \in T_1 \tag{P.4}$$

$$\sum_{t_2 \in T_2} \sum_{d \in dl_i} z^0_{it_2d} = 1 \quad \forall i \in I \tag{P.5}$$

$$\sum_{d \in D} v^0_{t_2d} \leq 1 \quad \forall t_2 \in T_2 \tag{P.6}$$

$$\sum_{i \in I_d} z^0_{it_2d} \leq M \cdot v^0_{t_2d} \quad \forall t_2 \in T_2, d \in D \tag{P.7}$$

$$\sum_{i \in I} \sum_{d \in dl_i} z^0_{it_2d} q_i \leq Q \quad \forall t_2 \in T_2 \tag{P.8}$$

$$\sum_{t_1 \in \varepsilon_i} z_{it_1c} = \sum_{t_2 \in T_c} \sum_{d \in dl_i} z^0_{it_2d} \quad \forall i \in I, c \in C \tag{P.9}$$

$$z_{it_1c}, z^0_{it_2d}, v_{t_1c}, v^0_{t_2d} \in \text{Binary} \tag{P.10}$$

The objective function (P.0) minimizes the total transportation cost of the incoming and outgoing trucks from  $\pi$ -hubs in the PI network. Constraints (P.1) ensure that each  $\pi$ -container can be transported to only one  $\pi$ -hub by only one of the supplier trucks. Similarly, constraint (P.5) ensures that each  $\pi$ -container can be delivered to its destination by only one of the vehicles available at  $\pi$ -hubs. Constraints (P.2) and (P.6) show that each of the available trucks at the supplier and  $\pi$ -hub is used at most once and assigned to one location. Constraint (P.3) ensures that a truck at supplier can only be used for transportation if it is loaded with at least one  $\pi$ -container. Similarly, constraint (P.7) ensures that a truck at  $\pi$ -hub can only be used for transportation if it is loaded with at least one  $\pi$ -container. Constraint (P.4) and (P.8) ensures that only those  $\pi$ -containers will be transported in a truck in which the sum of their sizes is less than or equal to the truck capacity. Constraint (P.9) is a flow balance constraint that maintains the product continuity at  $\pi$ -hubs, i.e., if a  $\pi$ -container is dropped at a  $\pi$ -hub, and then it must be delivered from that same hub to its destination. Finally, constraints (P.10) impose the bounds on the decision variables.

## **MODEL S - PEDDLING**

This configuration consists of multiple suppliers  $s$  and multiple demand points  $d$ . Every demand point owns a homogenous fleet of trucks  $T_d$ , performing the freight delivery in peddling fashion. We adapted this model from Sungur et al. (2008). Each of the truck has a maximum capacity of  $Q$ , which limits the number of suppliers it can visit before returning to the demand point. Each route must start at a demand point, visit a subset of suppliers and then return to the demand point. All suppliers must be visited exactly once. We represent the problem using a graph  $G(S^o, \partial)$ , in which  $\partial \in S \cup T_d$  is a set of nodes associated to

suppliers  $s$  and the homogenous fleet of trucks  $t_3$  at a demand point. Set  $\partial$  contains the arcs  $(x, y)$  for each pair of nodes  $x, y \in S^o$ . The cost of crossing an arc  $(x, y) \in \partial$  is  $td_{xy}$ . Each supplier has a requirement to fulfil the demand  $d_x > 0$  for each  $x \in S$  and  $d_r = 0$  from the demand point. For each demand point, the model is implemented separately as it works with a fleet of truck at a single demand point. The objective is to minimize the total logistics cost of the network, which is obtained by addition of all the minimum transportation costs acquired from implementation of Model S for each demand point. We used Mixed Binary Linear Programming (MBLP) to solve this model, due to linear objective function and mixed variables (binary and continuous). The linear programming mathematical model is presented as follows:

<b>Parameters</b>	
$s$	Index for supplier $s \in S$ , $ S  = n$ .
$T_d$	Index for homogenous trucks at demand point $t_3 \in T_d$ , $ T_d  = r$ .
$Q$	Maximum capacity of a truck at demand point $d$ .
$s^o$	Index for supplier and homogenous trucks at demand point $s^o \in S^o$ , $ S^o  = n \cup r$ .
$td_{xy}$	Minimum travel distance between two nodes $x$ and $y$ ; $x, y \in S^o$ .
$d_x$	Index for size of $\pi$ -containers required from each supplier $s$ ; $d_x > 0$ , $d_r = 0$ , $x \in S$
<b>Decision variables</b>	
$z_{xy}$	$\begin{cases} 1, & \text{if } \pi\text{-container } i \text{ is transported supplier } x \text{ to supplier } y \\ 0, & \text{otherwise} \end{cases}; x, y \in S^o$
$a_x$	Cumulated $\pi$ -containers from supplier $s$ on the route; $x \in S^o$ , $x \geq 0$

## Objective function for Model S:

$$\text{Min } Z_S = \sum_{i \in S^o} \sum_{j \in S^o} t d_{xy} z_{xy} \quad (\text{S.0})$$

Subject to:

$$\sum_{i \in S} z_{xy} = 1 \quad \forall y \in S^o \quad (\text{S.1})$$

$$\sum_{j \in S} z_{xy} = 1 \quad \forall x \in S^o \quad (\text{S.2})$$

$$a_x \geq 0 \quad \forall x \in S^o \quad (\text{S.3})$$

$$d_x \leq a_x \leq Q \quad \forall x \in S^o \quad (\text{S.4})$$

$$a_y \geq a_x + d_y z_{xy} - Q(1 - z_{xy}) \quad \forall x, y \in S^o \quad (\text{S.5})$$

$$z_{xy} \in \text{Binary} \quad (\text{S.6})$$

The Objective function (S.0) imposes that the total travel cost of the route in between supplier  $s$  and demand point  $d$  is minimised. Constraints (S.1) and (S.2) ensure that all suppliers are visited exactly once. Constraints (S.4) and (S.5) ensure together that the vehicle capacity is not exceeded. Constraints (S.5) also avoid subtours in the solution. Constraint (S.3) imposes that the  $\pi$ -containers picked up from a supplier  $s$  is a positive. Finally, constraint (S.6) introduces the boundary limitation on the decision variable  $z_{xy}$ .



## MODEL H - HYBRID

This configuration is an integrated version of Model P and Model S, so it is a hybrid model. As Model P, it is a 2-echelon model with multiple suppliers and multiple demand points. Each demand point has a specific requirement of  $\pi$ -containers  $i$  with different sizes from multiple suppliers. Supplier must fulfil the requirement by delivering the freight to demand point as per requirement through a  $\pi$ -hub  $c$ . Each  $\pi$ -hub owns a homogenous fleet of trucks  $t_2$  for collecting  $\pi$ -containers from suppliers in peddling fashion and for direct shipment to demand point. There is a set of  $\pi$ -containers  $i$  that are assigned to suppliers for encapsulation of the products at the supplier's facility. The  $\pi$ -containers have different sizes that are loaded in the truck with capacity constraint. We implemented Model S in the first leg of the network, which follows the peddling logistics in between suppliers and  $\pi$ -hubs. Each truck owned by  $\pi$ -hubs collects the  $\pi$ -containers from multiple suppliers and return to the same  $\pi$ -hub for consolidation. For the second leg, the consolidated shipment at the  $\pi$ -hub is then shipped directly to the demand point.

The heuristics of the integration process is as follows:

1. The solutions obtained from solving Model P, provides us with the data of  $\pi$ -containers being received at the  $\pi$ -hubs for consolidation and for further direct shipment on the second leg. Utilization of same parameters ensures that no matter what the route is the quantity of  $\pi$ -containers received at the  $\pi$ -hubs remains the same. This obtained data of quantity of  $\pi$ -containers serve as the demand parameter for the first leg of the model.
2. Demand parameters (quantity of  $\pi$ -containers at  $\pi$ -hubs) obtained in the first step is used to perform peddling on the first leg, which lead us to implement the Model S.

3. After implementing the Model S for each  $\pi$ -hub receiving  $\pi$ -containers from suppliers in peddling fashion, we add the minimum transportation costs from all the routes and obtain the minimum total logistics cost for the first leg. As for the second leg, previously implemented Model P gave us the total minimum transportation cost for the direct shipments from  $\pi$ -hubs to demand points.

$$\text{Total Cost} = \underbrace{\text{Transportation cost (First leg)}}_{\text{Model S}} + \underbrace{\text{Transportation cost (Second leg)}}_{\text{Model P}}$$

## **CHAPTER 4 NUMERICAL ANALYSIS**

In this chapter, we will discuss the implementation of all the three models (Model P, Model S, and Model H) proposed in chapter 3 on a case study of Mexican automotive industry and the results obtained by running the proposed linear programming mathematical formulation. In addition, we will compare the three configurations based on collected data and selected parameters. GUSEK (GLPK Under Scite Extended Kit, GLPSOL: GLPK LP/MIP Solver, v4.65) was used to implement the mathematical models and solved by GUROBI 8.0.1 platform. The codes for the models including data sets are provided in Appendix 1. All the software was operated in 64-bit personal computer with Intel(R) Core (TM) i7-7500U CPU, 2.7GHz.

### **DATA COLLECTION**

We started with a set of data consisting of coordinate locations of 25 Original Equipment Manufacturers (OEM) and 149 tier 1 suppliers all over Mexico. For our study, we chose five OEMs and 30 suppliers as depicted in the Figure 14.

The first step was to utilize these coordinates (Table. 5 and 6) and figure out the routing distances between the nodes. To accomplish this task, we used Geopy 1.20.0 library of python to calculate the geodesic distance between two points using the geodesic distance or the great-circle distance, with a default of the geodesic distance available as the function `geopy.distance.distance` (Python Software Foundation, 2019).

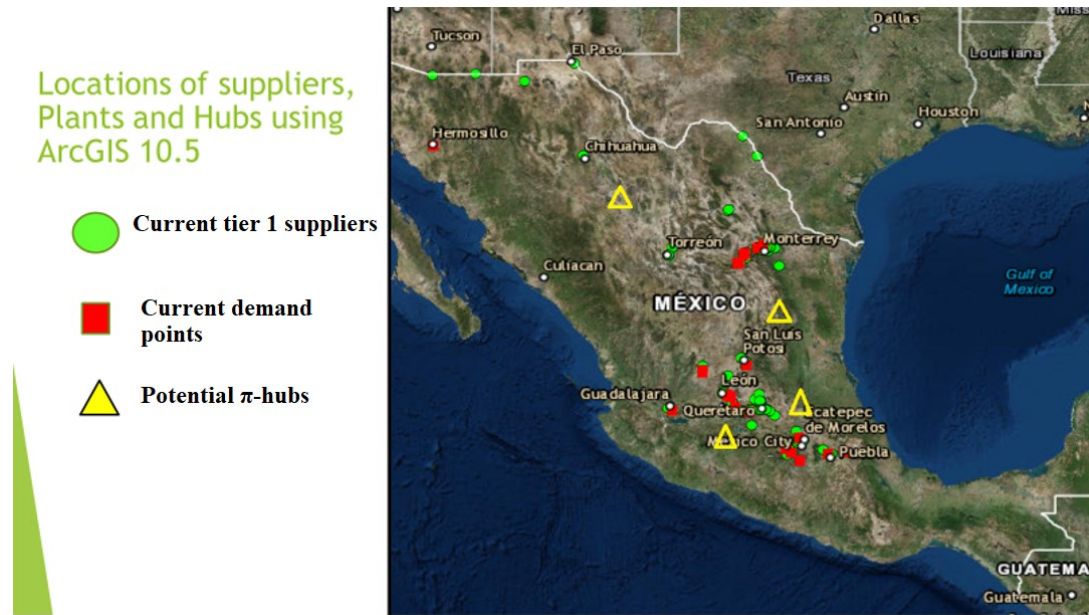


Figure 14 Locations of current tier-1 suppliers, plants & potential  $\pi$ -hubs using ArcGIS

Here's an example application of the geodesic distance:

```
>>> from geopy.distance import geodesic

>>> newport_ri = (41.49008, -71.312796)

>>> cleveland_oh = (41.499498, -81.695391)

>>> print(geodesic(newport_ri, cleveland_oh).miles)

538.390445368
```

Without loss of generality, the geodesic distance is used in our analysis. Actual road distances may also be used. In the case of the Mexican Automotive industry, the geodesic distance is not a bad approximation given the general North West/South East orientation of highways.

Table 5 Coordinates of suppliers.

Suppliers	X	Y
Brose	20.555	-100.26434
Brose	20.628	-100.44201
Brose	19.115	-98.258
Industrias Norm	19.113	-98.25806
Rassini Suspensiones	28.689	-100.52078
Rassini Suspensiones	19.514	-99.086
Rassini Frenos	19.263	-98.41653
Rassini BYPASA	20.368	-99.96755
Unicar Plastics	25.394	-100.93449
Unicar Plastics	19.102	-98.20617
Accuride International	32.586	-115.3815
Accuride de Mexico	25.946	-100.22842
Cemm Mex	25.664	-100.15756
Cifunsa	25.458	-100.99398
Cifunsa	22.106	-100.90932
Cifunsa	20.699	-101.29425
CSA - Castellón México	25.781	-100.13881
DBG	25.744	-100.21524
Denso	25.771	-100.16737
Denso - Air Systems De Mexico	26.932	-101.47303
Denso - Hamaden México	25.915	-100.29368
Denso - Asmo Manufacturing	20.78	-101.31986
Denso	25.645	-100.18306
Denso	21.01	-101.47352
Ficosa	25.856	-100.29158
Ficosa	25.808	-100.35587
Frisa	25.679	-100.43329
Frisa	25.755	-100.5399
Port 1	25.738	-99.98302
Port 2	25.776	-100.14799

Table 6 Coordinates of demand points.

Plants	X	Y
Volkswagen	19.117	-98.25169
Audi	19.206	-97.74914
Navistar International	21.01	-101.4726
Caterpillar	25.733	-100.5222
Daimler (Truck Manufacturing Plant)	25.241	-101.1589

Once we gathered the distances between the nodes, the next step was to locate the potential  $\pi$ -hubs, and for this, we opted the K-means clustering algorithm (Keen, 2017).

We used python to solve the clustering algorithm, which consists of three steps:

- Initialization – K initial “means” (centroids) are generated at random
- Assignment – K clusters are created by associating each observation with the nearest centroid
- Update – The centroid of the clusters becomes the new mean

The application of this algorithm on our chosen data set gave us the outcome depicted in Figure 15.

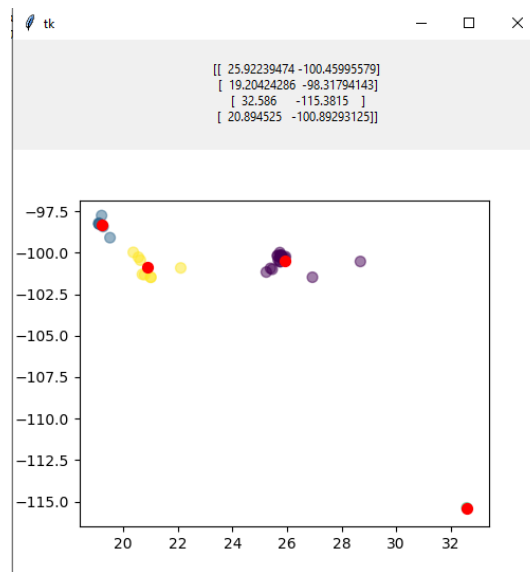


Figure 15 Location of potential  $\pi$ -hubs using K-means clustering

After gathering the potential  $\pi$ -hubs location, we again used the Geopy library to get the routing distances, including the recently added nodes (i.e.,  $\pi$ -hubs). These computations provided us with our significant parameters to our transportation problem. But a critical parameter was still missing, the sizes of the  $\pi$ - containers. For this parameter, we settled our search to the random generation of the sizes as a ratio of truck volume.

Finally, after gathering all the required parameters and data sets, the subsequent task was to model this into a PI system.

## **SELECTION OF PARAMETERS**

As stated in the beginning of this chapter, our primary problem of the PI system consists of a data set of 30 suppliers, four PI hubs, and five destinations/OEMs. One hundred products were selected for our network optimization, which was randomly divided among the suppliers. Demands at the destinations were also generated randomly. Considering the number of products, we selected 45 trucks for the first leg and 35 trucks for the second leg. Due to a lack of standardization in dimensions of PI containers at the current stage of research, the products were assumed to be a PI container with a volume ratio of the capacity of trucks with a maximum capacity of 50% volume of a truck's capacity. Depending on the size ratios of the PI containers at the supplier's end, a minimum of one vehicle to a maximum of three trucks was assigned to each supplier. Similarly, a minimum of eight trucks was attached to each hub. We utilized distances obtained using Geopy 1.20.0 library of python as a metric for our cost parameter, which was used to calculate labor cost, truck cost, and fuel cost.

The distance being the base metric for our calculations, all the costs (fuel, labour, truck) depend on it. As per the Mexican statistics, we assumed fuel cost to be 3.3 km/litre, labor cost to be 4.6 \$/hr. The cost of a new truck of \$175,000 was taken with an average of 140,000 km/yr., which runs at 83.04 km/hr. Using this data collected, we converted all the costs to a dollar per kilometer standards. This resulted in fuel cost to be 0.303 \$/KM, labour cost to be 0.06 \$/km, and truck cost to be 0.1643 \$/km.





### Model S

#### Demand Point 1

$ S $	$ T_d $	$ S^o $
16	6	22

#### $td_{xy}$

	S2	S7	S8	S9	S10	S11	S12	S13	S14	S15	S19	S20	S22	S26	S28	S30
S2	10000.00	260.49	57.25	532.35	288.89	1989.34	591.74	560.73	540.03	171.32	572.56	708.77	92.86	576.06	570.18	573.22
S7	260.49	10000.00	203.52	729.21	28.44	2246.51	766.00	733.87	738.01	408.85	745.63	908.13	347.06	754.49	754.06	745.70
S8	57.25	203.52	10000.00	567.57	231.95	2044.75	620.82	589.21	575.65	216.51	601.13	745.79	148.05	606.20	601.86	601.63
S9	532.35	729.21	567.57	10000.00	753.80	1614.48	93.67	83.54	9.29	365.62	87.62	179.26	514.56	74.07	56.37	89.59
S10	288.89	28.44	231.95	753.80	10000.00	2274.81	788.81	756.70	762.66	436.70	768.42	932.50	375.14	777.65	777.63	768.44
S11	1989.34	2246.51	2044.75	1614.48	2274.81	10000.00	1642.55	1664.64	1605.60	1840.81	1657.69	1480.79	1913.38	1639.37	1626.63	1659.07
S12	591.74	766.00	620.82	93.67	788.81	1642.55	10000.00	32.15	93.96	432.55	20.40	165.46	585.13	19.95	37.72	20.54
S13	560.73	733.87	589.21	83.54	756.70	1664.64	32.15	10000.00	86.97	402.94	11.94	192.55	555.90	25.51	39.62	12.49
S14	540.03	738.01	575.65	9.29	762.66	1605.60	93.96	86.97	10000.00	372.83	89.89	170.73	521.24	74.88	56.25	91.90
S15	171.32	408.85	216.51	365.62	436.70	1840.81	432.55	402.94	372.83	10000.00	414.44	539.65	153.44	415.47	407.48	415.35
S19	572.56	745.63	601.13	87.62	768.42	1657.69	20.40	11.94	89.89	414.44	10000.00	183.28	567.31	19.32	37.35	2.02
S20	708.77	908.13	745.79	179.26	932.50	1480.79	165.46	192.55	170.73	539.65	183.28	10000.00	684.25	167.35	160.54	184.26
S22	92.86	347.06	148.05	514.56	375.14	1913.38	585.13	555.90	521.24	153.44	567.31	684.25	10000.00	567.68	558.90	568.27
S26	576.06	754.49	606.20	74.07	777.65	1639.37	19.95	25.51	74.88	415.47	19.32	167.35	567.68	10000.00	19.35	21.11
S28	570.18	754.06	601.86	56.37	777.63	1626.63	37.72	39.62	56.25	407.48	37.35	160.54	558.90	19.35	10000.00	39.32
S30	573.22	745.70	601.63	89.59	768.44	1659.07	20.54	12.49	91.90	415.35	2.02	184.26	568.27	21.11	39.32	10000.00

#### Demand Point 2

$ S $	$ T_d $	$ S^o $
13	6	19

#### $td_{xy}$

	S3	S6	S7	S10	S11	S16	S18	S19	S22	S23	S24	S25	S29
S3	10000.00	97.56	23.41	5.63	2269.92	363.02	764.03	765.64	369.71	752.56	396.46	778.09	757.45
S6	97.56	10000.00	75.56	103.07	2177.06	265.57	702.36	704.53	272.38	690.97	299.49	715.95	698.17
S7	23.41	75.56	10000.00	28.44	2246.51	340.48	743.93	745.63	347.06	732.46	373.60	757.93	737.73
S10	5.63	103.07	28.44	10000.00	2274.81	368.47	766.84	768.42	375.14	755.37	401.83	780.93	760.12
S11	2269.92	2177.06	2246.51	2274.81	10000.00	1921.73	1655.11	1657.69	1913.38	1663.54	1883.88	1642.17	1675.44
S16	363.02	265.57	340.48	368.47	1921.73	10000.00	571.70	575.60	9.39	561.57	39.28	582.50	576.10
S18	764.03	702.36	743.93	766.84	1655.11	571.70	10000.00	5.66	563.38	11.47	541.82	14.61	23.27
S19	765.64	704.53	745.63	768.42	1657.69	575.60	5.66	10000.00	567.31	14.10	545.91	15.62	18.82
S22	369.71	272.38	347.06	375.14	1913.38	9.39	563.38	567.31	10000.00	553.29	30.15	574.10	567.95
S23	752.56	690.97	732.46	755.37	1663.54	561.57	11.47	14.10	553.29	10000.00	531.95	25.86	22.55
S24	396.46	299.49	373.60	401.83	1883.88	39.28	541.82	545.91	30.15	531.95	10000.00	552.17	547.28
S25	778.09	715.95	757.93	780.93	1642.17	582.50	14.61	15.62	574.10	25.86	552.17	10000.00	33.56
S29	757.45	698.17	737.73	760.12	1675.44	576.10	23.27	18.82	567.95	22.55	547.28	33.56	10000.00

### Demand Point 3

$ S $	$ T_d $	$ S^o $
14	6	20

#### $td_{xy}$

	S1	S4	S6	S7	S9	S12	S13	S14	S17	S20	S21	S23	S25	S26
S1	10000.00	264.09	168.97	240.75	542.43	599.46	568.20	550.28	581.25	719.67	596.01	566.04	589.45	584.18
S4	264.09	10000.00	97.66	23.56	750.69	786.24	754.12	759.53	766.19	929.46	784.67	752.77	778.30	774.98
S6	168.97	97.66	10000.00	75.56	680.83	724.73	692.63	689.36	705.17	860.08	722.44	690.97	715.95	711.87
S7	240.75	23.56	75.56	10000.00	729.21	766.00	733.87	738.01	746.01	908.13	764.32	732.46	757.93	754.49
S9	542.43	750.69	680.83	729.21	10000.00	93.67	83.54	9.29	90.66	179.26	86.50	80.40	82.42	74.07
S12	599.46	786.24	724.73	766.00	93.67	10000.00	32.15	93.96	20.42	165.46	7.38	33.78	11.83	19.95
S13	568.20	754.12	692.63	733.87	83.54	32.15	10000.00	86.97	13.14	192.55	31.06	3.32	25.22	25.51
S14	550.28	759.53	689.36	738.01	9.29	93.96	86.97	10000.00	92.96	170.73	86.64	83.97	83.16	74.88
S17	581.25	766.19	705.17	746.01	90.66	20.42	13.14	92.96	10000.00	184.53	21.50	15.76	17.42	21.94
S20	719.67	929.46	860.08	908.13	179.26	165.46	192.55	170.73	184.53	10000.00	163.03	192.40	167.82	167.35
S21	596.01	784.67	722.44	764.32	86.50	7.38	31.06	86.64	21.50	163.03	10000.00	32.00	6.56	13.43
S23	566.04	752.77	690.97	732.46	80.40	33.78	3.32	83.97	15.76	192.40	32.00	10000.00	25.86	25.06
S25	589.45	778.30	715.95	757.93	82.42	11.83	25.22	83.16	17.42	167.82	6.56	25.86	10000.00	8.36
S26	584.18	774.98	711.87	754.49	74.07	19.95	25.51	74.88	21.94	167.35	13.43	25.06	8.36	10000.00

### Demand Point 4

$ S $	$ T_d $	$ S^o $
17	7	23

#### $td_{xy}$

	S2	S3	S4	S9	S10	S11	S15	S16	S17	S18	S20	S21	S23	S27	S28	S29	S30
S2	10000.00	283.66	283.79	532.35	288.89	1989.34	171.32	89.02	573.82	569.35	708.77	588.08	558.49	561.65	570.18	570.14	573.22
S3	283.66	10000.00	0.22	750.49	5.63	2269.92	432.12	363.02	765.97	764.03	929.26	784.46	752.56	763.32	774.63	757.45	765.67
S4	283.79	0.22	10000.00	750.69	5.59	2270.08	432.29	363.12	766.19	764.24	929.46	784.67	752.77	763.53	774.84	757.67	765.89
S9	532.35	750.49	750.69	10000.00	753.80	1614.48	365.62	523.36	90.66	81.97	179.26	86.50	80.40	59.44	56.37	102.82	89.59
S10	288.89	5.63	5.59	753.80	10000.00	2274.81	436.70	368.47	768.73	766.84	932.50	787.28	755.37	766.28	777.63	760.12	768.44
S11	1989.34	2269.92	2270.08	1614.48	2274.81	10000.00	1840.81	1921.73	1659.58	1655.11	1480.79	1638.66	1663.54	1640.15	1626.63	1675.44	1659.07
S15	171.32	432.12	432.29	365.62	436.70	1840.81	10000.00	161.45	416.07	410.63	539.65	428.13	400.38	400.24	407.48	414.68	415.35
S16	89.02	363.02	363.12	523.36	368.47	1921.73	161.45	10000.00	577.28	571.70	693.32	588.92	561.57	560.69	567.46	576.10	576.55
S17	573.82	765.97	766.19	90.66	768.73	1659.58	416.07	577.28	10000.00	8.69	184.53	21.50	15.76	31.60	40.27	16.32	1.07
S18	569.35	764.03	764.24	81.97	766.84	1655.11	410.63	571.70	8.69	10000.00	182.10	20.57	11.47	23.01	32.54	23.27	7.62
S20	708.77	929.26	929.46	179.26	932.50	1480.79	539.65	693.32	184.53	182.10	10000.00	163.03	192.40	173.65	160.54	199.18	184.26
S21	588.08	784.46	784.67	86.50	787.28	1638.66	428.13	588.92	21.50	20.57	163.03	10000.00	32.00	29.73	30.39	36.80	21.25
S23	558.49	752.56	752.77	80.40	755.37	1663.54	400.38	561.57	15.76	11.47	192.40	32.00	10000.00	25.36	37.79	22.55	14.98
S27	561.65	763.32	763.53	59.44	766.28	1640.15	400.24	560.69	31.60	23.01	173.65	29.73	25.36	10000.00	13.62	45.59	30.55
S28	570.18	774.63	774.84	56.37	777.63	1626.63	407.48	567.46	40.27	32.54	160.54	30.39	37.79	13.62	10000.00	55.81	39.32
S29	570.14	757.45	757.67	102.82	760.12	1675.44	414.68	576.10	16.32	23.27	199.18	36.80	22.55	45.59	55.81	10000.00	17.05

<b>Demand Point 5</b>																		
<b> S </b>						<b> T<sub>d</sub> </b>						<b> S<sup>o</sup> </b>						
18						6						24						
<b><i>td<sub>xy</sub></i></b>																		
	S1	S2	S5	S6	S8	S9	S14	S15	S16	S19	S22	S24	S25	S26	S27	S28	S29	S30
S1	10000.00	20.20	904.83	168.97	37.26	542.43	550.28	184.95	108.37	580.08	112.63	135.51	589.45	584.18	570.03	578.90	577.04	580.67
S2	20.20	10000.00	896.38	188.15	57.25	532.35	540.03	171.32	89.02	572.56	92.86	115.32	581.53	576.06	561.65	570.18	570.14	573.22
S5	904.83	896.38	10000.00	1030.52	926.94	368.67	362.31	733.04	891.87	326.34	883.13	859.25	315.83	320.77	334.81	326.25	332.42	326.00
S6	168.97	188.15	1030.52	10000.00	132.32	680.83	689.36	344.93	265.57	704.53	272.38	299.49	715.95	711.87	699.31	709.81	698.17	704.77
S8	37.26	57.25	926.94	132.32	10000.00	567.57	575.65	216.51	142.97	601.13	148.05	172.15	611.14	606.20	592.48	601.86	597.12	601.63
S9	542.43	532.35	368.67	680.83	567.57	10000.00	9.29	365.62	523.36	87.62	514.56	490.58	82.42	74.07	59.44	56.37	102.82	89.59
S14	550.28	540.03	362.31	689.36	575.65	9.29	10000.00	372.83	530.07	89.89	521.24	497.01	83.16	74.88	61.37	56.25	106.05	91.90
S15	184.95	171.32	733.04	344.93	216.51	365.62	372.83	10000.00	161.45	414.44	153.44	135.12	421.68	415.47	400.24	407.48	414.68	415.35
S16	108.37	89.02	891.87	265.57	142.97	523.36	530.07	161.45	10000.00	575.60	9.39	39.28	582.50	576.12	560.69	567.46	576.10	576.55
S19	580.08	572.56	326.34	704.53	601.13	87.62	89.89	414.44	575.60	10000.00	567.31	545.91	15.62	19.32	28.54	37.35	18.82	2.02
S22	112.63	92.86	883.13	272.38	148.05	514.56	521.24	153.44	9.39	567.31	10000.00	30.15	574.10	567.68	552.22	558.90	567.95	568.27
S24	135.51	115.32	859.25	299.49	172.15	490.58	497.01	135.12	39.28	545.91	30.15	10000.00	552.17	545.56	529.91	536.15	547.28	546.93
S25	589.45	581.53	315.83	715.95	611.14	82.42	83.16	421.68	582.50	15.62	574.10	552.17	10000.00	8.36	24.26	27.28	33.56	16.90
S26	584.18	576.06	320.77	711.87	606.20	74.07	74.88	415.47	576.12	19.32	567.68	545.56	8.36	10000.00	16.31	19.35	38.14	21.11
S27	570.03	561.65	334.81	699.31	592.48	59.44	61.37	400.24	560.69	28.54	552.22	529.91	24.26	16.31	10000.00	13.62	45.59	30.55
S28	578.90	570.18	326.25	709.81	601.86	56.37	56.25	407.48	567.46	37.35	558.90	536.15	27.28	19.35	13.62	10000.00	55.81	39.32
S29	577.04	570.14	332.42	698.17	597.12	102.82	106.05	414.68	576.10	18.82	567.95	547.28	33.56	38.14	45.59	55.81	10000.00	17.05
S30	580.67	573.22	326.00	704.77	601.63	89.59	91.90	415.35	576.55	2.02	568.27	546.93	16.90	21.11	30.55	39.32	17.05	10000.00

<b>Model H.1 (implementation of Model S on first leg)</b>																		
<b>II-hub 1</b>																		
<b> S </b>						<b> T<sub>c</sub> </b>						<b> S<sup>o</sup> </b>						
15						9						26						

**$td_{xy}$**

	S5	S9	S13	S14	S17	S18	S19	S20	S21	S23	S25	S27	S28	S29	S30
S5	10000.00	368.67	1089.06	1089.28	325.55	328.86	326.34	216.65	309.27	340.12	315.83	334.81	326.25	332.42	326.00
S9	368.67	10000.00	750.49	750.69	90.66	81.97	87.62	179.26	86.50	80.40	82.42	59.44	56.37	102.82	89.59
S13	1089.06	750.49	10000.00	0.22	765.97	764.03	765.64	929.26	784.46	752.56	778.09	763.32	774.63	757.45	765.67
S14	1089.28	750.69	0.22	10000.00	766.19	764.24	765.86	929.46	784.67	752.77	778.30	763.53	774.84	757.67	765.89
S17	325.55	90.66	765.97	766.19	10000.00	8.69	3.07	184.53	21.50	15.76	17.42	31.60	40.27	16.32	1.07
S18	328.86	81.97	764.03	764.24	8.69	10000.00	5.66	182.10	20.57	11.47	14.61	23.01	32.54	23.27	7.62
S19	326.34	87.62	765.64	765.86	3.07	5.66	10000.00	183.28	20.40	14.10	15.62	28.54	37.35	18.82	2.02
S20	216.65	179.26	929.26	929.46	184.53	182.10	183.28	10000.00	163.03	192.40	167.82	173.65	160.54	199.18	184.26
S21	309.27	86.50	784.46	784.67	21.50	20.57	20.40	163.03	10000.00	32.00	6.56	29.73	30.39	36.80	21.25
S23	340.12	80.40	752.56	752.77	15.76	11.47	14.10	192.40	32.00	10000.00	25.86	25.36	37.79	22.55	14.98
S25	315.83	82.42	778.09	778.30	17.42	14.61	15.62	167.82	6.56	25.86	10000.00	24.26	27.28	33.56	16.90
S27	334.81	59.44	763.32	763.53	31.60	23.01	28.54	173.65	29.73	25.36	24.26	10000.00	13.62	45.59	30.55
S28	326.25	56.37	774.63	774.84	40.27	32.54	37.35	160.54	30.39	37.79	27.28	13.62	10000.00	55.81	39.32
S29	332.42	102.82	757.45	757.67	16.32	23.27	18.82	199.18	36.80	22.55	33.56	45.59	55.81	10000.00	17.05
S30	326.00	89.59	765.67	765.89	1.07	7.62	2.02	184.26	21.25	14.98	16.90	30.55	39.32	17.05	10000.00

**II-hub 2**

<b> S </b>	<b> T<sub>c</sub> </b>	<b> S° </b>
8	7	15

**$td_{xy}$**

	S3	S4	S6	S7	S10	S11	S14	S30
S3	10000.00	0.22	97.56	23.41	5.63	2269.92	759.33	765.67
S4	0.22	10000.00	97.66	23.56	5.59	2270.08	759.53	765.89
S6	97.56	97.66	10000.00	75.56	103.07	2177.06	689.36	704.77
S7	23.41	23.56	75.56	10000.00	28.44	2246.51	738.01	745.70
S10	5.63	5.59	103.07	28.44	10000.00	2274.81	762.66	768.44
S11	2269.92	2270.08	2177.06	2246.51	2274.81	10000.00	1605.60	1659.07
S14	759.33	759.53	689.36	738.01	762.66	1605.60	10000.00	91.90
S30	765.67	765.89	704.77	745.70	768.44	1659.07	91.90	10000.00

<b>II-hub 3</b>										
<b> S </b>			<b> T<sub>c</sub> </b>				<b> S<sup>o</sup> </b>			
10			8				18			
<b><i>td<sub>xy</sub></i></b>										
	S1	S2	S8	S9	S12	S15	S16	S22	S24	S26
S1	10000.00	20.20	37.26	542.43	599.46	184.95	108.37	112.63	135.51	584.18
S2	20.20	10000.00	57.25	532.35	591.74	171.32	89.02	92.86	115.32	576.06
S8	37.26	57.25	10000.00	567.57	620.82	216.51	142.97	148.05	172.15	606.20
S9	542.43	532.35	567.57	10000.00	93.67	365.62	523.36	514.56	490.58	74.07
S12	599.46	591.74	620.82	93.67	10000.00	432.55	593.49	585.13	563.34	19.95
S15	184.95	171.32	216.51	365.62	432.55	10000.00	161.45	153.44	135.12	415.47
S16	108.37	89.02	142.97	523.36	593.49	161.45	10000.00	9.39	39.28	576.12
S22	112.63	92.86	148.05	514.56	585.13	153.44	9.39	10000.00	30.15	567.68
S24	135.51	115.32	172.15	490.58	563.34	135.12	39.28	30.15	10000.00	545.56
S26	584.18	576.06	606.20	74.07	19.95	415.47	576.12	567.68	545.56	10000.00

As observed in the solutions obtained from implementation of Model P, in Model H.1 the first leg of our hybrid configuration has only three  $\pi$ -hub as compared to Model P, which has four. This is due to the fact that post optimization of Model P, the results show that only three out of four  $\pi$ -hubs are consumed in the network. This limits our implementation of Model S on the first leg of Model H (i.e. H.1) to only three  $\pi$ -hubs.

## STATIC OPTIMAL RESULTS

This section focuses on the mathematical results obtained from implementation of the models on the Mexican automotive case study using GUSEK solver and using the results to compare the performance of the three different configurations. As the distance being the base metric for our calculations, all the costs including fuel, labour, truck relies

on it. Using the collected data, we converted all the costs to a dollar per kilometer standards shown in Table 8.

Table 8 Cost parameters

Fuel cost	0.303 \$/km
Labour cost	0.06 \$/km
Truck cost	0.1643 \$/km

Findings are as follows:

### **MODEL P**

The results obtained from implementing the Model P in GUROBI are shown in the Figure 16. The optimal solution obtained is 16173.7 in 22.3 minutes. This is a distance metric and does not include the backhaul of trucks. For the total transportation cost of the network, we must double the distance and then multiply the cost parameters as shown below.

<b>Model P - PI</b>	Distance	Fuel cost	Labour cost	Truck cost	Total cost
Network objective	16173.87	\$1,788,927.73	\$327,022.19	\$ 970,187.98	\$3,086,137.90
Total Costs (2XNetwork objective)	32347.73	\$3,577,855.46	\$654,044.39	\$1,940,375.95	
				<b>Total</b>	<b>\$6,172,275.80</b>

After performing these calculations, the total fuel cost is \$3,577,855.46, labour cost is \$654,044.39, and truck cost is \$1,940,375.95. The system wide transportation cost is the sum of total fuel, labour and truck cost, which is \$6,172,275.80.

---

190809	6776	cutoff	72		16684.7147	16584.3779	0.60%	195	1300s
192653	6139	16592.0620	84	93	16684.7147	16592.0620	0.56%	193	1305s
194367	5674	cutoff	76		16173.7147	16597.7721	0.52%	192	1310s
196079	5003	infeasible	83		16173.7147	16605.0963	0.48%	191	1315s
197788	4331	16640.9703	88	158	16173.7147	16613.5007	0.43%	190	1320s
199530	3509	cutoff	78		16173.7147	16623.5680	0.37%	189	1325s
201230	2816	cutoff	84		16173.7147	16628.9632	0.33%	188	1330s
203601	1622	16648.2494	90	43	16173.7147	16648.2494	0.22%	186	1335s
205931	154	infeasible	94		16173.7147	16677.5076	0.04%	184	1340s

Cutting planes:  
 Gomory: 136  
 Cover: 1744  
 Implied bound: 21  
 Clique: 74  
 MIR: 296  
 StrongCG: 112  
 Flow cover: 493  
 GUB cover: 3  
 Inf proof: 19  
 Zero half: 94

Explored 206201 nodes (37994211 simplex iterations) in 1340.51 seconds  
 Thread count was 6 (of 6 available processors)

Solution count 10: 16173.7 16173.7 16173.7 ... 16173.7

Optimal solution found (tolerance 1.00e-04)  
 Best objective 1.61738671604099e+04, best bound 1.6172548764e+04, gap 0.0050%

---

Figure 16 GUROBI solution - Model P

## MODEL S

The mathematical formulation is implemented on each demand point separately and the results obtained are discussed as follows:

### Demand Point 1

As shown in the Figure 17, the optimal solution obtained is 10395.59 in 91.41 minutes or 1.51 hours. After performing the calculations, the fuel cost is \$1,149,815.51, labour cost is \$210,190.26, and truck cost is \$623,578.68. The total transportation cost for Demand point 1 is \$1,983,584.45.

```

15693750 155500 cutoff 62 10395.5884 10279.5083 1.12% 29.1 5445s
15712862 141848 cutoff 84 10395.5884 10286.3364 1.05% 29.1 5450s
15731375 128517 infeasible 74 10395.5884 10293.4411 0.98% 29.1 5455s
15751228 113979 cutoff 68 10395.5884 10301.4986 0.91% 29.0 5460s
15772675 97840 cutoff 80 10395.5884 10311.0643 0.81% 29.0 5465s
15795200 80432 cutoff 68 10395.5884 10322.4052 0.70% 29.0 5470s
15818568 61723 infeasible 65 10395.5884 10335.8665 0.57% 29.0 5475s
15842119 41648 cutoff 76 10395.5884 10352.2401 0.42% 29.0 5480s
15870926 14824 cutoff 86 10395.5884 10378.2536 0.17% 28.9 5485s

Cutting planes:
  Gomory: 7
  Cover: 1
  Implied bound: 43
  Clique: 1
  MIR: 214
  StrongCG: 9
  Flow cover: 332
  GUB cover: 7
  Inf proof: 51
  Zero half: 26

Explored 15885211 nodes (459010664 simplex iterations) in 5487.98 seconds
Thread count was 6 (of 6 available processors)

Solution count 10: 10395.6 10395.6 10395.6 ... 10395.6

Optimal solution found (tolerance 1.00e-04)
Best objective 1.039559231066e+04, best bound 1.039456507456e+04, gap 0.0099%

```

Figure 17 GUROBI solution - Demand point 1

```

58186 12132 5475.10759 39 25 6914.14660 5069.07863 26.7% 27.9 20s
84941 17361 cutoff 42 6914.14660 5296.66949 23.4% 28.2 25s
114755 20626 6511.80868 42 26 6914.14660 5529.94444 20.0% 28.4 30s
147808 21451 cutoff 63 6914.14660 5767.71589 16.6% 28.1 35s
*175778 20001 67 6910.9794404 5962.92166 13.7% 27.9 39s
180239 19436 infeasible 41 6910.97944 5999.23942 13.2% 27.9 40s
H193421 17471 6910.9793670 6100.60945 11.7% 27.8 42s
H195064 17193 6910.9791739 6118.01200 11.5% 27.8 42s
208322 14236 6440.35880 49 33 6910.97917 6237.86686 9.74% 27.6 45s

Cutting planes:
  Learned: 2
  Gomory: 17
  Implied bound: 33
  Clique: 1
  MIR: 74
  StrongCG: 3
  Flow cover: 157
  GUB cover: 1
  Inf proof: 28
  Zero half: 21

Explored 238068 nodes (6454136 simplex iterations) in 49.94 seconds
Thread count was 6 (of 6 available processors)

Solution count 10: 6910.98 6910.98 6910.98 ... 6914.15

Optimal solution found (tolerance 1.00e-04)
Best objective 6.910979440388e+03, best bound 6.910979173889e+03, gap 0.0000%

```

Figure 18 GUROBI solution - Demand point 2



## **Demand Point 2**

Figure 18 displays the optimal solution as 6910.97 in 45 seconds. After performing the calculations, the fuel cost is \$764,396.21, labour cost is \$139,734.28, and truck cost is \$414,554.48. The total transportation cost for Demand point 2 is \$1,318,684.97.

## **Demand Point 3**

Displayed in Figure 19, the optimal solution obtained is 5529.55 in 5.67 minutes. After performing the calculations, the fuel cost is \$611,602.84, labour cost is \$111,803.12, and truck cost is \$331,690.16. The total transportation cost for Demand point 3 is \$1,055,096.11.

## **Demand Point 4**

As shown in the Figure 20, the optimal solution is found to be 7360.98 in one second. After performing the calculations, the fuel cost is \$814,169.53, labour cost is \$148,833.01, and truck cost is \$441,548.02. The total transportation cost for Demand point 4 is \$1,404,550.56.

## **Demand Point 5**

Figure 21 displays the optimal solution that is 5633.87, in a runtime of 91.4 minutes or 1.51 hours. After performing the calculations, the fuel cost is \$623,140.20, labour cost is \$113,912.19, and truck cost is \$337,947.21. The total transportation cost for Demand point 4 is \$1,074,999.60.

1228387	87781	5390.33105	52	24	5529.55722	5258.09210	4.91%	33.5	290s
1253060	85141	cutoff	64		5529.55722	5300.49987	4.14%	33.4	295s
1278692	82812	5428.02535	80	30	5529.55722	5329.73559	3.61%	33.2	300s
1299073	80329	5493.66152	55	35	5529.55722	5347.61610	3.29%	33.1	305s
1321353	76561	5480.35354	58	30	5529.55722	5365.14697	2.97%	32.9	310s
1345892	71333	cutoff	67		5529.55722	5381.40499	2.68%	32.8	315s
1371091	64268	cutoff	66		5529.55722	5397.23189	2.39%	32.6	320s
1395223	55970	5435.47847	64	39	5529.55722	5411.88945	2.13%	32.4	325s
1421951	44680	5511.93741	82	26	5529.55722	5428.89353	1.82%	32.2	330s
1447101	32317	5460.31672	70	40	5529.55722	5446.46905	1.50%	32.0	335s
1476159	15477	cutoff	78		5529.55722	5474.38626	1.00%	31.7	340s

Cutting planes:  
 Gomory: 3  
 Implied bound: 28  
 MIR: 164  
 StrongCG: 10  
 Flow cover: 268  
 GUB cover: 1  
 Inf proof: 14  
 Zero half: 14

Explored 1496137 nodes (47103709 simplex iterations) in 342.86 seconds  
 Thread count was 6 (of 6 available processors)

Solution count 10: 5529.56 5529.56 5529.56 ... 5529.56

Optimal solution found (tolerance 1.00e-04)  
 Best objective 5.529559892744e+03, best bound 5.529557218879e+03, gap 0.0000%

Figure 19 GUROBI solution – Demand point 3

0	0	5772.60277	0	63	7400.42392	5772.60277	22.0%	-	0s
0	0	5774.06625	0	63	7400.42392	5774.06625	22.0%	-	0s
0	0	5774.06625	0	63	7400.42392	5774.06625	22.0%	-	0s
0	0	5775.69325	0	57	7400.42392	5775.69325	22.0%	-	0s
0	0	5776.22450	0	59	7400.42392	5776.22450	21.9%	-	0s
0	0	5776.23065	0	60	7400.42392	5776.23065	21.9%	-	0s
0	0	5776.44372	0	61	7400.42392	5776.44372	21.9%	-	0s
0	0	5776.44372	0	55	7400.42392	5776.44372	21.9%	-	0s
0	2	5776.44372	0	52	7400.42392	5776.44372	21.9%	-	0s
H	894	502			7400.4239051	6488.16133	12.3%	17.0	0s
H	907	479			7370.8493383	6488.16133	12.0%	17.1	0s
H	2084	616			7370.8489432	6853.37519	7.02%	23.2	0s
H	2090	619			7370.8489423	6853.37519	7.02%	23.2	0s
*	3046	635	40		7370.6876101	7204.99358	2.25%	23.9	0s
H	3311	678			7370.6876092	7215.41673	2.11%	23.3	0s
H	3335	690			7370.6876060	7219.07320	2.06%	23.2	0s
H	4106	967			7360.9847962	7260.85217	1.36%	22.0	0s
H	6451	1400			7360.9847944	7280.17613	1.10%	22.3	1s

Cutting planes:  
 Learned: 1

Explored 30553 nodes (506457 simplex iterations) in 4.89 seconds  
 Thread count was 6 (of 6 available processors)

Solution count 10: 7360.98 7360.98 7370.69 ... 7400.42

Optimal solution found (tolerance 1.00e-04)  
 Best objective 7.360984794353e+03, best bound 7.360984794353e+03, gap 0.0000%

Figure 20 GUROBI solution - Demand point 4

```

3702032 1206420 cutoff 62 5633.86884 5402.19054 1.12% 29.1 5445s
3712517 1208117 cutoff 84 5633.86884 5402.39017 1.05% 29.1 5450s
3724875 1210277 infeasible 59 5633.86884 5402.64216 0.98% 29.1 5455s
3736165 1212194 cutoff 70 34 5633.86884 5402.86840 0.91% 29.0 5460s
3746301 1213996 cutoff 73 5633.86884 5403.07228 0.81% 29.0 5465s
3757961 1215916 cutoff 65 5633.86884 5403.31103 0.70% 29.0 5470s
3769395 1217754 cutoff 55 17 5633.86884 5403.55205 0.57% 29.0 5475s
3780386 1219715 infeasible 60 42 5633.86884 5403.79023 0.42% 29.0 5480s
3791812 1221603 cutoff 58 36 5633.86884 5404.03760 0.17% 28.9 5485s
3803205 1223503 cutoff 77 37 5633.86884 5404.26947 0.17% 28.9 5485s

Cutting planes:
Gomory: 7
Cover: 1
Implied bound: 43
Clique: 1
MIR: 214
StrongCG: 9
Flow cover: 332
GUB cover: 7
Inf proof: 51
Zero half: 26

Explored 15885211 nodes (459010664 simplex iterations) in 5487.98 seconds
Thread count was 6 (of 6 available processors)

Solution count 10: 5633.8 5633.8 5633.8 ... 5633.8

Optimal solution found (tolerance 1.00e-04)
Best objective 5.63387654126e+03, best bound 5.6326253621e+03, gap 0.0030%

```

Figure 21 GUROBI solution - Demand point 5

For the system wide transportation cost of Model S, we sum all the results obtained from individual Demand point model implementation.

<p>Total transportation cost for Model S =</p>	<p>Transportation cost of Demand point 1 + Transportation cost of Demand point 2 + Transportation cost of Demand point 3 + Transportation cost of Demand point 4 + Transportation cost of Demand point 5</p>
--	--

The system wide transportation cost achieved for Model S is \$6,836,915.70.

## MODEL H

As guided by the heuristics for the integration process of Model S on the first leg and Model P on the second of the model, the second step is to implement Model S on each of the  $\pi$ -hubs (Model H.1). The results obtained from the implementation of Model H.1 were achieved in less than one second and are discussed as follows:

### II – Hub 1

As shown in the Figure 22, the optimal solution is 3027.98. After performing the calculations, the fuel cost is \$334,912.86, labour cost is \$61,223.23, and truck cost is \$181,633.06. The total transportation cost for  $\pi$ -hub 1 is \$1,404,550.56.

	0	0	2854.07500	0	26	-	2854.07500	-	-	0s
	0	0	2854.07500	0	26	-	2854.07500	-	-	0s
	0	0	2854.07500	0	24	-	2854.07500	-	-	0s
H	0	0				4731.6200000	2854.07500	39.7%	-	0s
	0	0	2854.07500	0	18	4731.62000	2854.07500	39.7%	-	0s
H	0	0				4669.6300000	2854.07500	38.9%	-	0s
	0	2	2854.07500	0	18	4669.63000	2854.07500	38.9%	-	0s
H	39	35				3071.6500000	2865.60293	6.71%	28.8	0s
H	71	45				3038.3200000	2875.16483	5.37%	22.5	0s
H	127	73				3038.3100000	2875.16483	5.37%	18.8	0s
H	147	82				3028.0000000	2875.16483	5.05%	18.2	0s
H	193	89				3027.9900000	2875.16483	5.05%	17.5	0s

Cutting planes:  
 Learned: 1  
 Cover: 6  
 Implied bound: 21  
 Clique: 1  
 MIR: 27  
 StrongCG: 6  
 GUB cover: 1

Explored 1105 nodes (15223 simplex iterations) in 0.24 seconds  
 Thread count was 6 (of 6 available processors)

Solution count 7: 3027.99 3028 3038.31 ... 4731.62

Optimal solution found (tolerance 1.00e-04)  
 Best objective 3.02799000000e+03, best bound 3.02799000000e+03, gap 0.0000%

Figure 22 GUROBI solution –  $\pi$ -hub 1

### II – Hub 2

Figure 23 displays the optimal solution, 8252.52. After performing the calculations, the fuel cost is \$912,778.99, labour cost is \$166,859.17, and truck cost is \$495,026.82. The total transportation cost for  $\pi$ -hub 2 is \$1,574,664.98.

	Nodes		Current Node			Objective Bounds			Work	
	Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
	0	0	6554.14960	0	11	11148.0600	6554.14960	41.2%	-	0s
H	0	0				11031.650000	6554.14960	40.6%	-	0s
H	0	0				9611.870000	6554.14960	31.8%	-	0s
H	0	0				8252.500000	6554.14960	20.6%	-	0s
	0	0	6830.91074	0	25	8252.50000	6830.91074	17.2%	-	0s
	0	0	7050.39300	0	24	8252.50000	7050.39300	14.6%	-	0s
	0	0	7050.39300	0	24	8252.50000	7050.39300	14.6%	-	0s
	0	0	8216.22190	0	11	8252.50000	8216.22190	0.44%	-	0s
	0	0	8216.22190	0	11	8252.50000	8216.22190	0.44%	-	0s
	0	0	8216.22190	0	16	8252.50000	8216.22190	0.44%	-	0s
	0	0	8220.68122	0	9	8252.50000	8220.68122	0.39%	-	0s
	0	0	cutoff	0		8252.50000	8252.50000	0.00%	-	0s

Cutting planes:  
 Learned: 1  
 Gomory: 1  
 Implied bound: 4

Explored 1 nodes (316 simplex iterations) in 0.04 seconds  
 Thread count was 6 (of 6 available processors)

Solution count 5: 8252.5 8252.5 9611.87 ... 11148.1

Optimal solution found (tolerance 1.00e-04)  
 Best objective 8.252500000000e+03, best bound 8.252500000000e+03, gap 0.0000%

Figure 23 GUROBI solution –  $\pi$ -hub 2

	Nodes		Current Node			Objective Bounds			Work	
	Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
	0	0	4676.60470	0	16	7017.14000	4676.60470	33.4%	-	0s
H	0	0				5947.260000	4676.60470	21.4%	-	0s
	0	0	4842.17868	0	17	5947.26000	4842.17868	18.6%	-	0s
	0	0	4876.26005	0	20	5947.26000	4876.26005	18.0%	-	0s
	0	0	4876.26005	0	21	5947.26000	4876.26005	18.0%	-	0s
	0	0	4893.83369	0	16	5947.26000	4893.83369	17.7%	-	0s
	0	0	5134.69000	0	12	5947.26000	5134.69000	13.7%	-	0s
	0	0	5941.24929	0	11	5947.26000	5941.24929	0.10%	-	0s
	0	0	5941.24929	0	8	5947.26000	5941.24929	0.10%	-	0s
	0	0	5941.24929	0	9	5947.26000	5941.24929	0.10%	-	0s
	0	0	cutoff	0		5947.26000	5947.26000	0.00%	-	0s

Cutting planes:  
 Learned: 2  
 Gomory: 1  
 Implied bound: 1  
 MIR: 1  
 Zero half: 1

Explored 1 nodes (434 simplex iterations) in 0.05 seconds  
 Thread count was 6 (of 6 available processors)

Solution count 4: 5947.26 5947.26 5947.26 7017.14

Optimal solution found (tolerance 1.00e-04)  
 Best objective 5.947260000000e+03, best bound 5.947260000000e+03, gap 0.0000%

Figure 24 GUROBI solution –  $\pi$ -hub 3

## **II – Hub 3**

As shown in the Figure 24, the optimal solution obtained is 5947.28. After performing the calculations, the fuel cost is \$657,804.79, labour cost is \$120,249.00, and truck cost is \$356,746.83. The total transportation cost for  $\pi$ -hub 3 is \$1,134,800.62.

A “split delivery” would happen when the size (weight and/or volume) of an order (a load of possibly different products consolidated for a particular customer, i.e. consignee) is bigger than the vehicle’s transport capacity. The delivery of such larger freight orders can be handled by utilizing a mix of full truck load (FTL) and less-than-truck load (LTL); for example, see Ülkü (2012). As it can be observed in Table 9, are some additional trucks required under Model H (first leg). These additional trucks are for split deliveries, i.e.  $\pi$ -containers from suppliers to the  $\pi$ -hubs which utilize full trucks loads are dispatched in full. Only those that require split deliveries because of less than truck load utilization is consolidated for stage 1 in Model H. Adding the direct full truck load direct dispatch costs to the  $\pi$ - hubs and accumulating the results obtained from the Model H.1, the system-wide transportation cost of Model H turns out to be \$4,391,559.64.

## **COMPARISON BETWEEN THREE MODELS**

In this section, the performances of system wide optimization of all three proposed methods i.e., PI system (Model P), peddling (Model S), and hybrid PI system (Model H), are compared. Before starting the comparison, it is essential to specify that our comparison metric be purely based on the primary objective value, i.e., the total transportation cost, which includes fuel cost, labor cost, and truck cost. Table 9 provides a summary of all the freight delivery networks.

Table 9 Computational analysis

Model P - Physical Internet	Distance	Fuel cost	Labour cost	Truck cost	Total cost
Network objective	16173.87	\$ 1,788,927.73	\$ 327,022.19	\$ 970,187.98	\$ 3,086,137.90
Total Costs (2 X Network Objective)	32347.73432	\$ 3,577,855.46	\$ 654,044.39	\$ 1,940,375.95	
				<b>Total Config 1</b>	<b>\$ 6,172,275.80</b>
Model S - Peddling (Per Plant)	Distance	Fuel cost	Labour cost	Truck cost	Total cost
Demand point 1	10395.59231	\$ 1,149,815.51	\$ 210,190.26	\$ 623,578.68	\$ 1,983,584.45
Demand point 2	6910.97944	\$ 764,396.21	\$ 139,734.28	\$ 414,554.48	\$ 1,318,684.97
Demand point 3	5529.559893	\$ 611,602.84	\$ 111,803.12	\$ 331,690.16	\$ 1,055,096.11
Demand point 4	7360.984794	\$ 814,169.53	\$ 148,833.01	\$ 441,548.02	\$ 1,404,550.56
Demand point 5	5633.870333	\$ 623,140.20	\$ 113,912.19	\$ 337,947.21	\$ 1,074,999.60
Total Costs	35830.98677	\$ 3,963,124.29	\$ 724,472.87	\$ 2,149,318.54	
				<b>Total Config 2</b>	<b>\$ 6,836,915.70</b>
Model H - PI & Peddling					
First leg( Peddling)	Distance	Fuel cost	Labour cost	Truck cost	Total cost
PI - Hub 1	3027.98	\$ 334,912.86	\$ 61,223.23	\$ 181,633.06	\$ 577,769.15
PI - Hub 2	8252.52	\$ 912,778.99	\$ 166,859.17	\$ 495,026.82	\$ 1,574,664.98
PI - Hub 3	5947.28	\$ 657,804.79	\$ 120,249.00	\$ 356,746.83	\$ 1,134,800.62
Additional trucks (Hub1)	103.90	\$ 11,492.32	\$ 2,100.84	\$ 6,232.62	\$ 19,825.78
Additional trucks (Hub2)	4571.81	\$ 505,670.05	\$ 92,438.24	\$ 274,239.70	\$ 872,347.98
Additional trucks (Hub3)	1111.84	\$ 122,976.70	\$ 22,480.57	\$ 66,693.87	\$ 212,151.13
Total Costs	23015.34	\$ 2,545,635.70	\$ 465,351.04	\$ 1,380,572.90	\$ 4,391,559.64
Second leg(PI)	Distance	Fuel cost	Labour cost	Truck cost	Total cost
PI - Hub 1	2970.89	\$ 328,598.59	\$ 60,068.96	\$ 178,208.65	\$ 566,876.21
PI - Hub 2	2794.16	\$ 309,051.44	\$ 56,495.68	\$ 167,607.66	\$ 533,154.78
PI - Hub 3	3171.980087	\$ 350,840.22	\$ 64,134.81	\$ 190,270.94	\$ 605,245.97
Total Costs	8937.04	\$ 988,490.26	\$ 180,699.45	\$ 536,087.26	\$ 1,705,276.96
				<b>Total Config 3</b>	<b>\$ 6,096,836.60</b>

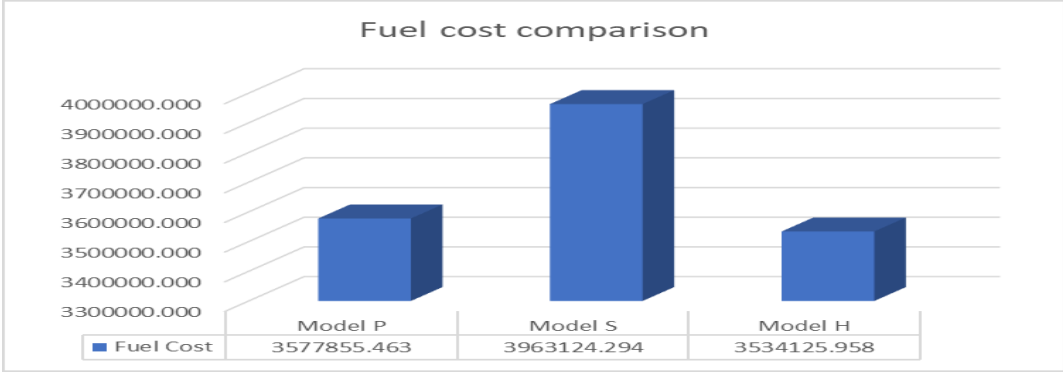


Figure 25 Fuel cost analysis

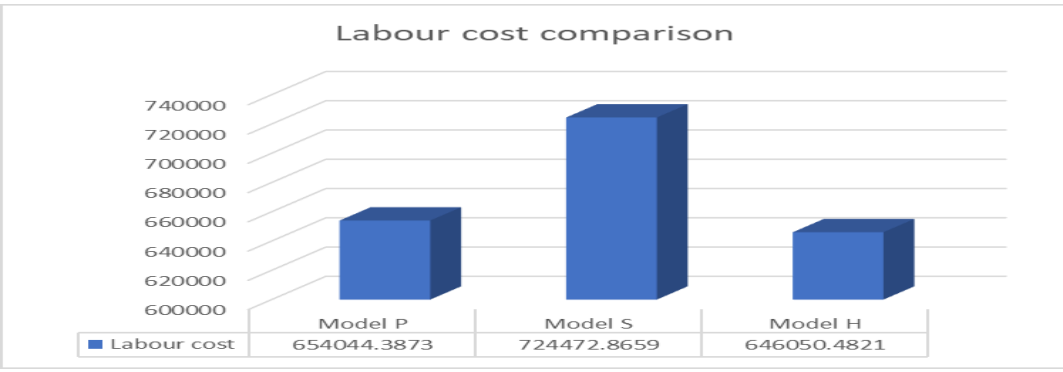


Figure 26 Labour cost analysis

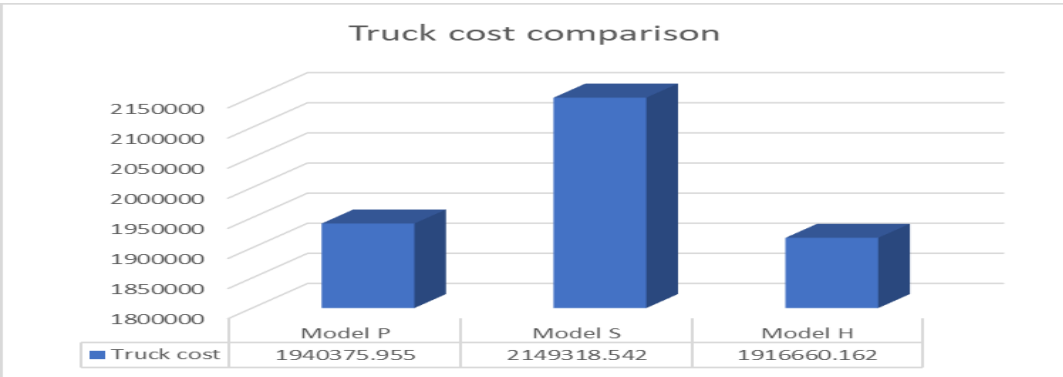


Figure 27 Truck cost analysis

Figure 25, 26 and 27 depicts the comparison between the three models on the performance metric of fuel cost, labour cost and truck cost. In Figure 28, system-wide



transportation cost analysis is depicted, which shows that Model H and Model P show an improvement of 10.82% and 9.72% respectively in contrast to Model S. Higher degree of improvement in Model H is due to integration of peddling (Model S) on the first leg which saves the empty backhauls of the trucks. The results also depict that in Model P, there is a possibility of freight delivery of less than truckloads, which is eliminated by applying the Hybrid approach.

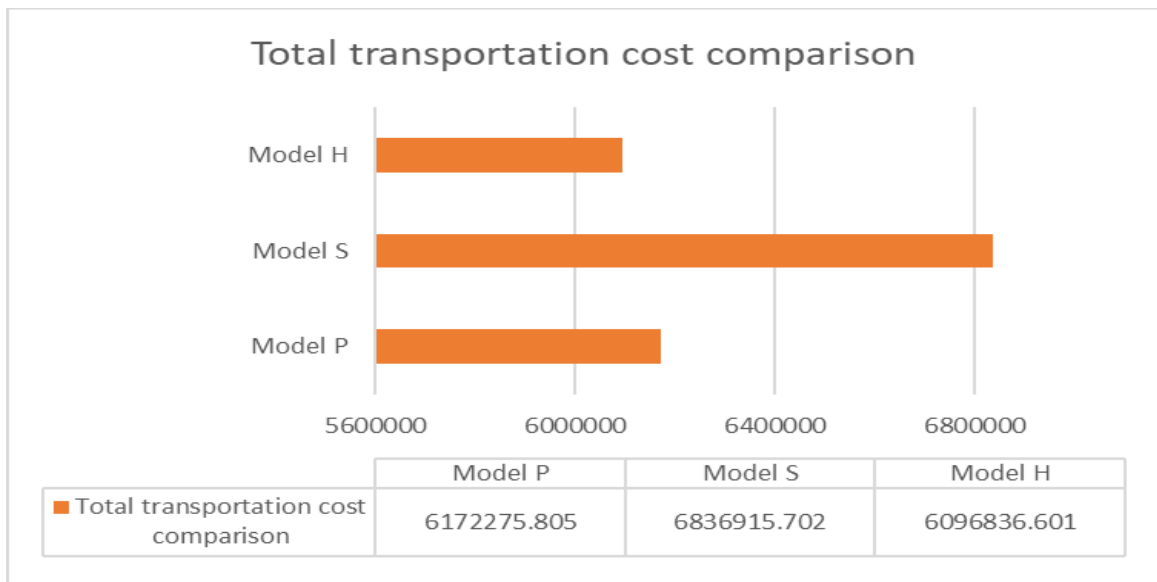


Figure 28 System-wide transportation cost analysis

Figure 29 depicts the comparison analysis of the three models on the basis of average vehicle capacity utilization or average fill rate of a vehicle (“vehicle utilization,” hereafter). It shows that Model S overall utilizes 85.7% of vehicles capacity. Model P vehicle utilization is 63.2% on the first leg and 88.9% on the second leg. Model H utilization is 88.9% for the whole network. Model H shows a better systemwide vehicle utilization rate as compared to Model P and Model S. Integration of both the models in

hybrid configuration offers the benefits of higher vehicle utilization on the first leg due to implementation of peddling and on the second leg due to consolidation at  $\pi$ -hubs.

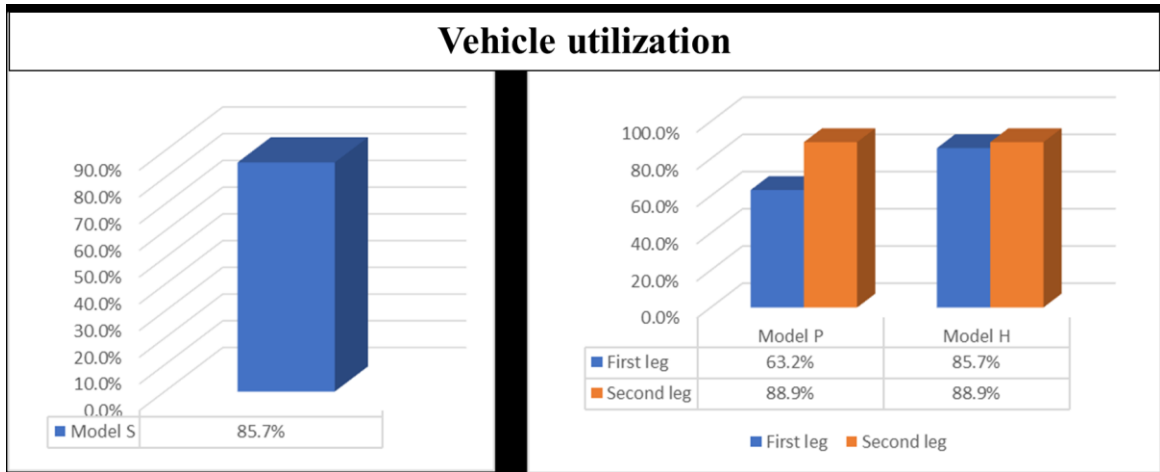


Figure 29 Vehicle utilization analysis (P, S, H)

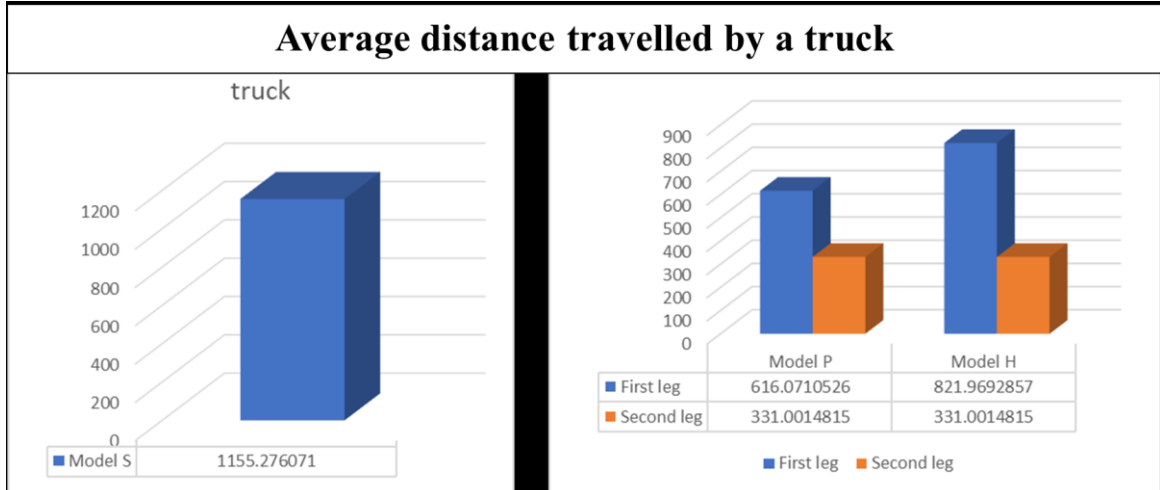


Figure 30 Average distance travelled by a vehicle

In contrast to vehicle utilization, average distance travelled by a truck in Model S is highest as compared to Model P and H (Figure 30), due to long hauls between suppliers and demand points without transit points. However, Model P shows reduction

in average distance travelled for the whole network as compared to Model S, due to consolidation of  $\pi$ -containers at  $\pi$ -hubs. Model H shows an increase in average distance travelled on the first leg due to peddling as compared to Model P, but similar results on second leg after consolidation at  $\pi$ -hubs. Even though an increase of average distance travelled is observed on the first leg of Model H, its higher vehicle utilization has more impact on the results making it the best out of the three.

## CHAPTER 5 CONCLUDING REMARKS

This study investigates numerous research papers regarding PI, which enlightened us with the core concept of PI. Tremblay et al. (2015) stated on PI that, "It is a hyper-connected global logistics system enabling seamless open asset sharing and flow consolidation." The reason being its intensive connectivity among digital, physical, operational, business, legal, and interpersonal layers. Imitating the digital internet, being its principal concept, has inspired the researchers not only to reduce empty mileage but also making sure each vehicle is loaded to its full potential. One substantial segment of PI is the PI container (or  $\pi$  containers), which are like the data packets of the digital Internet. These are world-standard, smart, green modular containers for transport, handling, and packaging purposes, which assist in seamless open asset sharing and consolidation across interconnected networks and modes. Another vital part of PI is the open  $\pi$ -hubs, which would replace the standalone warehouses, leading to less use of energy and resources. A study estimated that if only a fourth of the current distribution infrastructure in the US was rebuilt according to the principles of PI, an annual savings of \$100 billion could be achieved, and carbon dioxide output reduced by one-third (University of Arkansas, 2012).

We adapted our Model P from one of the recent researches conducted in cross-docking and consolidated shipment (Küçükoğlu & Öztürk, 2017), with some rendering to elevate it to become a part of the PI infrastructure. A test case with a gathered data set was conducted. The results gave us a brief insight into the full-scale system. The research findings show that on the basis of systemwide transportation cost Model P (PI system) shows an improvement of 9.7213%, and Model H (Hybrid PI system) shows an improvement of 10.8248% in comparison to Model S, that is our standard model based on

the concept of peddling. It is important to recognize that these results are based on the case study data. For further generalization, either more case studies or sensitivity analysis for varied data parameters is required. An interesting result is the fact that Model H performed better than Model P, which suggests that the PI configuration can potentially be improved by peddling, especially when there are suppliers in close proximity with partial truck loads.

For the distance parameters, we have considered geodesic distances, which gives a close approximation to the actual network distances. The reason being the network span from the North West to the South East of Mexico. The use of actual network distances will affect the values of the results obtained but the comparison analysis method will remain the same.

Run time of solving the models varies from couple of second to a few hours, due to the network selection which consists of 30 suppliers, five demand points, four  $\pi$ -hubs in a two-echelon framework operating on a set of 100  $\pi$ -containers. If we wish to increase the number of nodes,  $\pi$ -containers or multiple series of  $\pi$ -hubs, the runtime will go up substantially, since these formulations are NP-hard.

Results obtained from implementation of the models and their comparison show that benefits of PI in terms of system wide cost reduction is higher than standard/conventional model (peddling), due to higher vehicle utilization and reduced distance travelled because of consolidation at  $\pi$ -hubs. The hybrid model shows even better results a compared to PI and standard model, due to much higher vehicle utilization because of integration of peddling on the first leg and it still enables the network to have reduction in distance travelled by trucks because of consolidation at  $\pi$ -hubs. This reduction in travel distance reduces the GHG emissions from the trucks, enabling the positive impact on

environmental sustainability. The consolidation at  $\pi$ -hubs enables drivers to get back to their homes early, which provides them more family time and time to rest. This enables drivers to work more efficiently and reduces the possibility of getting distracted en route because of restlessness, adding the positive impact on social sustainability.

This thesis did not include the cost of material handling at the  $\pi$ -hubs. At the same time, the consolidation costs in peddling was not included either. Adding both can improve the comparative analysis of the configurations.

For future extensions, the stochastic version of the problem can be developed to add uncertainties in the model such as delay in lead time at supplier or  $\pi$ -hubs end, varying demand orders with multi period simulation. Trade-offs between conventional (CO), PI, and hybrid systems with a complete perspective of PI infrastructure, including all the fixed and variable costs involved at  $\pi$ -hubs and  $\pi$ -movers. Development of metaheuristics solutions using big data, to assess the real-time scenarios within time constraints to achieve an optimal solution with inclusion of inter hub consolidation with bin packing and sorting inside the PI hubs is another promising research avenue.

Introduction of innovative technologies such as Big Data, Internet of Things (IOT), and Industry 4.0, to the infrastructure for more accurate implementation of the PI system. Researchers from different fields with different backgrounds can engage in cooperation to pursue interdisciplinary research applying the PI vision to, for example, the study of humanitarian logistics. These studies will provoke the firms to collaborate and lead together towards the development of PI infrastructure as it introduces dimensions of socio-environmental sustainability in addition to traditional economic considerations.

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# APPENDIX I

## i. MODEL P

#Sets

#Module1

set S; #Set of suppliers

set C; #Set of PI hubs

set PR; #Set of products

set T; #Set of trucks, TR in paper

set dem{i in PR}; #Supplier of product i

set TL{k1 in T}; #Supplier of truck k1

set SL{s in S}; #Trucks at supplier s, redundant but needed because GUSEK does not accept cst[TL[k],c]

#Module2

set D; #Set of destinations

set TP; #Set of trucks, TR' in paper

set demP{i in PR}; #Destination of product i

set TLP{k2 in TP}; #Pi Hub of truck k2

set SLP{c in C}; #Trucks at PI hub c, redundant but needed because GUSEK does not accept cst'[TL'[k],c]

set prD{d in D}; #Products going to a destination

#Parameters

#Module1

param q{i in PR}; #Size of PI container carrying product i

param cst{s in S, c in C}; #Cost of shipping one full container from s to c

#Module2

param cstP{c in C, d in D}; #Cost of shipping one full container from c to d

#Variables

#Module1

var z{i in PR, k1 in T, c in C}, binary;

```

var v{k1 in T, c in C}, binary;
#Module2
var zP{i in PR, k2 in TP, d in D}, binary;
var vP{k2 in TP, d in D}, binary;
#Objective function
minimize cost: sum{s in S, c in C, k1 in SL[s]} cst[s,c] * v[k1,c] +
                sum{c in C,d in D, k2 in SLP[c]} cstP[c,d] * vP[k2,d]; #C1
#Constraints
#Module1
s.t. C2{i in PR}      : sum{k1 in T, c in C: TL[k1] within dem[i]} z[i,k1,c] = 1;
s.t. C3{k1 in T}      : sum{c in C} v[k1,c] <= 1;
s.t. C4{k1 in T,c in C}: sum{i in PR: dem[i] within TL[k1]}z[i,k1,c] <= 1000*v[k1,c];
s.t. D1{k1 in T}      : sum{i in PR, c in C: TL[k1] within dem[i]} z[i,k1,c]* q[i] <= 1;
#Module2
#s.t. C5{i in PR}      : sum{k2 in TP, d in demP[i]} zP[i,k2,d] = 1;
s.t. C5{i in PR}      : sum{k2 in TP, d in demP[i]} zP[i,k2,d] = 1;
s.t. C6{k2 in TP}      : sum{d in D} vP[k2,d] <= 1;
#s.t. C7{k2 in TP,d in D}: sum{i in PR:demP[i] == d} zP[i,k2,d] <= 1000*vP[k2,d];
s.t. C7{k2 in TP,d in D}: sum{i in prD[d]} zP[i,k2,d] <= 1000*vP[k2,d];
s.t. D2{k2 in TP}      : sum{i in PR, d in demP[i]} zP[i,k2,d]* q[i] <= 1;
#Flow Balance
s.t. C8{i in PR,c in C} : sum{k1 in T:TL[k1] within dem[i]}z[i,k1,c] = sum{k2 in
SLP[c]} sum{d in demP[i]}zP[i,k2,d];
solve;
display z, v, zP, vP;
data;
#Sets
#Module1
set S:= S1    S2    S3    S4    S5    S6    S7    S8    S9    S10   S11
        S12   S13   S14   S15   S16   S17   S18   S19   S20   S21   S22
        S23   S24   S25   S26   S27   S28   S29   S30;

```

```

set C:= C1    C2    C3    C4;

set PR:= PR1 PR2  PR3  PR4  PR5  PR6  PR7  PR8  PR9  PR10 PR11
        PR12 PR13 PR14 PR15 PR16 PR17 PR18 PR19 PR20 PR21 PR22
        PR23 PR24 PR25 PR26 PR27 PR28 PR29 PR30 PR31 PR32 PR33
        PR34 PR35 PR36 PR37 PR38 PR39 PR40 PR41 PR42 PR43 PR44
        PR45 PR46 PR47 PR48 PR49 PR50 PR51 PR52 PR53 PR54 PR55
        PR56 PR57 PR58 PR59 PR60 PR61 PR62 PR63 PR64 PR65 PR66
        PR67 PR68 PR69 PR70 PR71 PR72 PR73 PR74 PR75 PR76 PR77
        PR78 PR79 PR80 PR81 PR82 PR83 PR84 PR85 PR86 PR87 PR88
        PR89 PR90 PR91 PR92 PR93 PR94 PR95 PR96 PR97 PR98 PR99
        PR100;

set T:= TR1  TR2  TR3  TR4  TR5  TR6  TR7  TR8  TR9  TR10 TR11
        TR12 TR13 TR14 TR15 TR16 TR17 TR18 TR19 TR20 TR21 TR22
        TR23 TR24 TR25 TR26 TR27 TR28 TR29 TR30 TR31 TR32 TR33
        TR34 TR35 TR36 TR37 TR38 TR39 TR40 TR41 TR42 TR43 TR44
        TR45;

#Module2

set D := D1    D2    D3    D4    D5;

set TP:= TRP1TRP2 TRP3 TRP4 TRP5 TRP6 TRP7 TRP8 TRP9 TRP10TRP11
        TRP12 TRP13 TRP14 TRP15 TRP16 TRP17 TRP18 TRP19 TRP20 TRP21 TRP22
        TRP23 TRP24 TRP25 TRP26 TRP27 TRP28 TRP29 TRP30 TRP31 TRP32 TRP33
        TRP34 TRP35;

#Parameters
set dem["PR12"] := S6 ;
#Module1
set dem["PR13"] := S20 ;
set dem["PR1"] := S6 ; set dem["PR14"] := S26 ;
set dem["PR2"] := S9 ; set dem["PR15"] := S15 ;
set dem["PR3"] := S13 ; set dem["PR16"] := S12 ;
set dem["PR4"] := S14 ; set dem["PR17"] := S24 ;
set dem["PR5"] := S14 ; set dem["PR18"] := S15 ;
set dem["PR6"] := S26 ; set dem["PR19"] := S7 ;
set dem["PR7"] := S14 ; set dem["PR20"] := S10 ;
set dem["PR8"] := S22 ; set dem["PR21"] := S16 ;
set dem["PR9"] := S22 ; set dem["PR22"] := S2 ;
set dem["PR10"] := S10 ; set dem["PR23"] := S24 ;
set dem["PR11"] := S4 ; set dem["PR24"] := S6 ;

```

set	dem["PR25"] :=	S11 ;	set	dem["PR55"] :=	S21 ;
set	dem["PR26"] :=	S7 ;	set	dem["PR56"] :=	S11 ;
set	dem["PR27"] :=	S20 ;	set	dem["PR57"] :=	S16 ;
set	dem["PR28"] :=	S3 ;	set	dem["PR58"] :=	S30 ;
set	dem["PR29"] :=	S30 ;	set	dem["PR59"] :=	S3 ;
set	dem["PR30"] :=	S1 ;	set	dem["PR60"] :=	S20 ;
set	dem["PR31"] :=	S11 ;	set	dem["PR61"] :=	S16 ;
set	dem["PR32"] :=	S3 ;	set	dem["PR62"] :=	S9 ;
set	dem["PR33"] :=	S9 ;	set	dem["PR63"] :=	S21 ;
set	dem["PR34"] :=	S23 ;	set	dem["PR64"] :=	S28 ;
set	dem["PR35"] :=	S14 ;	set	dem["PR65"] :=	S10 ;
set	dem["PR36"] :=	S8 ;	set	dem["PR66"] :=	S9 ;
set	dem["PR37"] :=	S23 ;	set	dem["PR67"] :=	S25 ;
set	dem["PR38"] :=	S23 ;	set	dem["PR68"] :=	S19 ;
set	dem["PR39"] :=	S28 ;	set	dem["PR69"] :=	S14 ;
set	dem["PR40"] :=	S1 ;	set	dem["PR70"] :=	S29 ;
set	dem["PR41"] :=	S25 ;	set	dem["PR71"] :=	S19 ;
set	dem["PR42"] :=	S1 ;	set	dem["PR72"] :=	S17 ;
set	dem["PR43"] :=	S6 ;	set	dem["PR73"] :=	S3 ;
set	dem["PR44"] :=	S30 ;	set	dem["PR74"] :=	S22 ;
set	dem["PR45"] :=	S9 ;	set	dem["PR75"] :=	S9 ;
set	dem["PR46"] :=	S27 ;	set	dem["PR76"] :=	S28 ;
set	dem["PR47"] :=	S26 ;	set	dem["PR77"] :=	S7 ;
set	dem["PR48"] :=	S8 ;	set	dem["PR78"] :=	S3 ;
set	dem["PR49"] :=	S20 ;	set	dem["PR79"] :=	S25 ;
set	dem["PR50"] :=	S4 ;	set	dem["PR80"] :=	S11 ;
set	dem["PR51"] :=	S15 ;	set	dem["PR81"] :=	S5 ;
set	dem["PR52"] :=	S28 ;	set	dem["PR82"] :=	S22 ;
set	dem["PR53"] :=	S17 ;	set	dem["PR83"] :=	S11 ;
set	dem["PR54"] :=	S7 ;	set	dem["PR84"] :=	S24 ;



set	dem["PR85"] :=	S3	;	set	TL["TR15"] :=	S8	;
set	dem["PR86"] :=	S29	;	set	TL["TR16"] :=	S9	;
set	dem["PR87"] :=	S18	;	set	TL["TR17"] :=	S9	;
set	dem["PR88"] :=	S25	;	set	TL["TR18"] :=	S10	;
set	dem["PR89"] :=	S4	;	set	TL["TR19"] :=	S11	;
set	dem["PR90"] :=	S16	;	set	TL["TR20"] :=	S11	;
set	dem["PR91"] :=	S24	;	set	TL["TR21"] :=	S12	;
set	dem["PR92"] :=	S18	;	set	TL["TR22"] :=	S13	;
set	dem["PR93"] :=	S20	;	set	TL["TR23"] :=	S14	;
set	dem["PR94"] :=	S13	;	set	TL["TR24"] :=	S14	;
set	dem["PR95"] :=	S21	;	set	TL["TR25"] :=	S15	;
set	dem["PR96"] :=	S29	;	set	TL["TR26"] :=	S15	;
set	dem["PR97"] :=	S7	;	set	TL["TR27"] :=	S16	;
set	dem["PR98"] :=	S11	;	set	TL["TR28"] :=	S17	;
set	dem["PR99"] :=	S30	;	set	TL["TR29"] :=	S18	;
set	dem["PR100"] :=	S27	;	set	TL["TR30"] :=	S19	;
set	TL["TR1"] :=	S1	;	set	TL["TR31"] :=	S20	;
set	TL["TR2"] :=	S1	;	set	TL["TR32"] :=	S21	;
set	TL["TR3"] :=	S2	;	set	TL["TR33"] :=	S22	;
set	TL["TR4"] :=	S2	;	set	TL["TR34"] :=	S23	;
set	TL["TR5"] :=	S3	;	set	TL["TR35"] :=	S24	;
set	TL["TR6"] :=	S3	;	set	TL["TR36"] :=	S24	;
set	TL["TR7"] :=	S4	;	set	TL["TR37"] :=	S25	;
set	TL["TR8"] :=	S4	;	set	TL["TR38"] :=	S26	;
set	TL["TR9"] :=	S5	;	set	TL["TR39"] :=	S27	;
set	TL["TR10"] :=	S5	;	set	TL["TR40"] :=	S28	;
set	TL["TR11"] :=	S6	;	set	TL["TR41"] :=	S29	;
set	TL["TR12"] :=	S6	;	set	TL["TR42"] :=	S29	;
set	TL["TR13"] :=	S7	;	set	TL["TR43"] :=	S29	;
set	TL["TR14"] :=	S7	;	set	TL["TR44"] :=	S30	;

```

set    TL["TR45"] :=S30    ;
set    SL["S1"] :=  TR1  TR2  ;
set    SL["S2"] :=  TR3  TR4  ;
set    SL["S3"] :=  TR5  TR6  ;
set    SL["S4"] :=  TR7  TR8  ;
set    SL["S5"] :=  TR9  TR10 ;
set    SL["S6"] :=  TR11 TR12 ;
set    SL["S7"] :=  TR13 TR14 ;
set    SL["S8"] :=  TR15      ;
set    SL["S9"] :=  TR16 TR17 ;
set    SL["S10"] := TR18      ;
set    SL["S11"] := TR19 TR20 ;
set    SL["S12"] := TR21      ;
set    SL["S13"] := TR22      ;
set    SL["S14"] := TR23 TR24 ;
set    SL["S15"] := TR25 TR26 ;
set    SL["S16"] := TR27      ;
set    SL["S17"] := TR28      ;
set    SL["S18"] := TR29      ;
set    SL["S19"] := TR30      ;
set    SL["S20"] := TR31      ;
set    SL["S21"] := TR32      ;
set    SL["S22"] := TR33      ;
set    SL["S23"] := TR34      ;
set    SL["S24"] := TR35 TR36 ;
set    SL["S25"] := TR37      ;
set    SL["S26"] := TR38      ;
set    SL["S27"] := TR39      ;
set    SL["S28"] := TR40      ;
set    SL["S29"] :=TR41 TR42 TR43;
set    SL["S30"] :=  TR44  TR45 ;
param q:=
PR1  0.40
PR2  0.09
PR3  0.07
PR4  0.01
PR5  0.05
PR6  0.25
PR7  0.36
PR8  0.17
PR9  0.06
PR10 0.17
PR11 0.23
PR12 0.41
PR13 0.28
PR14 0.10
PR15 0.15
PR16 0.42
PR17 0.36
PR18 0.48
PR19 0.49
PR20 0.13
PR21 0.15
PR22 0.48
PR23 0.21
PR24 0.35
PR25 0.11
PR26 0.39

```

PR27	0.00	PR52	0.50	PR77	0.23
PR28	0.13	PR53	0.40	PR78	0.21
PR29	0.10	PR54	0.14	PR79	0.24
PR30	0.28	PR55	0.11	PR80	0.09
PR31	0.06	PR56	0.24	PR81	0.32
PR32	0.31	PR57	0.20	PR82	0.46
PR33	0.37	PR58	0.45	PR83	0.04
PR34	0.12	PR59	0.47	PR84	0.35
PR35	0.46	PR60	0.40	PR85	0.39
PR36	0.32	PR61	0.01	PR86	0.46
PR37	0.22	PR62	0.12	PR87	0.05
PR38	0.49	PR63	0.28	PR88	0.26
PR39	0.00	PR64	0.03	PR89	0.18
PR40	0.40	PR65	0.45	PR90	0.01
PR41	0.18	PR66	0.06	PR91	0.31
PR42	0.17	PR67	0.06	PR92	0.25
PR43	0.07	PR68	0.16	PR93	0.01
PR44	0.35	PR69	0.39	PR94	0.08
PR45	0.31	PR70	0.36	PR95	0.38
PR46	0.10	PR71	0.40	PR96	0.30
PR47	0.49	PR72	0.35	PR97	0.16
PR48	0.34	PR73	0.34	PR98	0.09
PR49	0.01	PR74	0.07	PR99	0.41
PR50	0.00	PR75	0.13	PR100	0.24;
PR51	0.31	PR76	0.29		

param cst: C1 C2 C3 C4:=

S1 597.1620962 252.9461093 2008.164414 75.49158562

S2 588.7135171 272.7088828 1989.338723 55.4657387

S3	789.9594495	11.75213764	2269.923157	339.0306011
S4	790.1712238	11.93721398	2270.075006	339.1567507
S5	307.6915785	1078.06985	1484.807443	867.5190457
S6	726.3614207	87.62839466	2177.056064	243.138152
S7	769.5848169	12.24022177	2246.513847	315.9002088
S8	619.6675189	215.7180162	2044.753688	112.6990671
S9	75.59317425	738.9662935	1614.482094	500.3375289
S10	792.8828874	16.34289727	2274.81377	344.2863317
S11	1623.914368	2258.471538	0	1935.250612
S12	23.30128017	774.8455439	1642.546348	565.7725154
S13	41.73855026	742.7226944	1664.639662	535.6301569
S14	74.36354741	747.8023006	1605.597606	507.5410197
S15	426.810337	420.4580457	1840.807071	134.7207336
S16	587.0244201	352.6909771	1921.726141	47.04263447
S17	35.77607549	754.8115447	1659.576183	548.7748736
S18	31.51733934	752.8248008	1655.105963	543.6557821
S19	33.77401336	754.4684722	1657.685186	547.2723974
S20	150.9261333	917.7768303	1480.787347	673.9184573
S21	16.64964803	773.2452117	1638.65565	561.5880907
S22	578.5012773	359.2868077	1913.376285	46.15882131
S23	41.47329808	741.3555829	1663.543421	533.179004
S24	555.9220566	385.8378578	1883.880683	61.64203185
S25	18.39051116	766.8714387	1642.169393	555.0922292
S26	16.43962751	763.5162414	1639.368131	549.0957306
S27	27.19563553	752.0181899	1640.145781	534.0755631
S28	20.26011281	763.2933618	1626.62842	541.6605925
S29	51.9515964	746.3627944	1675.436332	546.5193865
S30	35.20742193	754.5078683	1659.069432	548.0938858

;

#Module2

```
set demP[ "PR1" ] := D2 ; set demP[ "PR30" ] := D5 ;
set demP[ "PR2" ] := D1 ; set demP[ "PR31" ] := D1 ;
set demP[ "PR3" ] := D1 ; set demP[ "PR32" ] := D1 ;
set demP[ "PR4" ] := D1 ; set demP[ "PR33" ] := D3 ;
set demP[ "PR5" ] := D5 ; set demP[ "PR34" ] := D4 ;
set demP[ "PR6" ] := D5 ; set demP[ "PR35" ] := D3 ;
set demP[ "PR7" ] := D1 ; set demP[ "PR36" ] := D1 ;
set demP[ "PR8" ] := D2 ; set demP[ "PR37" ] := D2 ;
set demP[ "PR9" ] := D2 ; set demP[ "PR38" ] := D3 ;
set demP[ "PR10" ] := D4 ; set demP[ "PR39" ] := D5 ;
set demP[ "PR11" ] := D4 ; set demP[ "PR40" ] := D3 ;
set demP[ "PR12" ] := D5 ; set demP[ "PR41" ] := D3 ;
set demP[ "PR13" ] := D5 ; set demP[ "PR42" ] := D4 ;
set demP[ "PR14" ] := D1 ; set demP[ "PR43" ] := D3 ;
set demP[ "PR15" ] := D4 ; set demP[ "PR44" ] := D1 ;
set demP[ "PR16" ] := D1 ; set demP[ "PR45" ] := D1 ;
set demP[ "PR17" ] := D5 ; set demP[ "PR46" ] := D5 ;
set demP[ "PR18" ] := D1 ; set demP[ "PR47" ] := D3 ;
set demP[ "PR19" ] := D1 ; set demP[ "PR48" ] := D5 ;
set demP[ "PR20" ] := D2 ; set demP[ "PR49" ] := D1 ;
set demP[ "PR21" ] := D4 ; set demP[ "PR50" ] := D3 ;
set demP[ "PR22" ] := D4 ; set demP[ "PR51" ] := D5 ;
set demP[ "PR23" ] := D5 ; set demP[ "PR52" ] := D1 ;
set demP[ "PR24" ] := D5 ; set demP[ "PR53" ] := D4 ;
set demP[ "PR25" ] := D2 ; set demP[ "PR54" ] := D1 ;
set demP[ "PR26" ] := D3 ; set demP[ "PR55" ] := D4 ;
set demP[ "PR27" ] := D3 ; set demP[ "PR56" ] := D4 ;
set demP[ "PR28" ] := D5 ; set demP[ "PR57" ] := D5 ;
set demP[ "PR29" ] := D5 ; set demP[ "PR58" ] := D4 ;
```

```

set demP[ "PR59" ] := D2 ; set demP[ "PR80" ] := D2 ;
set demP[ "PR60" ] := D4 ; set demP[ "PR81" ] := D5 ;
set demP[ "PR61" ] := D2 ; set demP[ "PR82" ] := D5 ;
set demP[ "PR62" ] := D4 ; set demP[ "PR83" ] := D3 ;
set demP[ "PR63" ] := D4 ; set demP[ "PR84" ] := D2 ;
set demP[ "PR64" ] := D5 ; set demP[ "PR85" ] := D4 ;
set demP[ "PR65" ] := D1 ; set demP[ "PR86" ] := D5 ;
set demP[ "PR66" ] := D5 ; set demP[ "PR87" ] := D2 ;
set demP[ "PR67" ] := D2 ; set demP[ "PR88" ] := D5 ;
set demP[ "PR68" ] := D2 ; set demP[ "PR89" ] := D3 ;
set demP[ "PR69" ] := D1 ; set demP[ "PR90" ] := D2 ;
set demP[ "PR70" ] := D2 ; set demP[ "PR91" ] := D2 ;
set demP[ "PR71" ] := D1 ; set demP[ "PR92" ] := D4 ;
set demP[ "PR72" ] := D3 ; set demP[ "PR93" ] := D4 ;
set demP[ "PR73" ] := D4 ; set demP[ "PR94" ] := D3 ;
set demP[ "PR74" ] := D1 ; set demP[ "PR95" ] := D3 ;
set demP[ "PR75" ] := D2 ; set demP[ "PR96" ] := D4 ;
set demP[ "PR76" ] := D4 ; set demP[ "PR97" ] := D3 ;
set demP[ "PR77" ] := D2 ; set demP[ "PR98" ] := D3 ;
set demP[ "PR78" ] := D4 ; set demP[ "PR99" ] := D4 ;
set demP[ "PR79" ] := D5 ; set demP[ "PR100" ] := D4 ;

```

```

set prD["D1"] := PR2 PR3 PR4 PR7 PR14 PR16 PR18 PR19 PR31
PR32 PR36 PR44 PR45 PR49 PR52 PR54 PR65 PR69 PR71 PR74;
set prD["D2"] := PR1 PR8 PR9 PR20 PR25 PR37 PR59 PR61 PR67
PR68 PR70 PR75 PR77 PR80 PR84 PR87 PR90 PR91;
set prD["D3"] := PR26 PR27 PR33 PR35 PR38 PR40 PR41 PR43 PR47
PR50 PR72 PR83 PR89 PR94 PR95 PR97 PR98;

```

```

set prD["D4"] := PR10 PR11 PR15 PR21 PR22 PR34 PR42 PR53 PR55
PR56 PR58 PR60 PR62 PR63 PR73 PR76 PR78 PR85 PR92 PR93
PR96 PR99 PR100;

set prD["D5"] := PR5 PR6 PR12 PR13 PR17 PR23 PR24 PR28 PR29
PR30 PR39 PR46 PR48 PR51 PR57 PR64 PR66 PR79 PR81 PR82
PR86 PR88 ;

```

```

set TLP[" TRP1 "] := C1 ;      set TLP[" TRP24 "] := C3 ;
set TLP[" TRP2 "] := C1 ;      set TLP[" TRP25 "] := C3 ;
set TLP[" TRP3 "] := C1 ;      set TLP[" TRP26 "] := C3 ;
set TLP[" TRP4 "] := C1 ;      set TLP[" TRP27 "] := C4 ;
set TLP[" TRP5 "] := C1 ;      set TLP[" TRP28 "] := C4 ;
set TLP[" TRP6 "] := C1 ;      set TLP[" TRP29 "] := C4 ;
set TLP[" TRP7 "] := C1 ;      set TLP[" TRP30 "] := C4 ;
set TLP[" TRP8 "] := C1 ;      set TLP[" TRP31 "] := C4 ;
set TLP[" TRP9 "] := C1 ;      set TLP[" TRP32 "] := C4 ;
set TLP[" TRP10 "] := C2 ;     set TLP[" TRP33 "] := C4 ;
set TLP[" TRP11 "] := C2 ;     set TLP[" TRP34 "] := C4 ;
set TLP[" TRP12 "] := C2 ;     set TLP[" TRP35 "] := C4 ;
set TLP[" TRP13 "] := C2 ;
set TLP[" TRP14 "] := C2 ;
set TLP[" TRP15 "] := C2 ;
set TLP[" TRP16 "] := C2 ;
set TLP[" TRP17 "] := C2 ;
set TLP[" TRP18 "] := C2 ;
set TLP[" TRP19 "] := C3 ;
set TLP[" TRP20 "] := C3 ;
set TLP[" TRP21 "] := C3 ;
set TLP[" TRP22 "] := C3 ;
set TLP[" TRP23 "] := C3 ;

```

```

set   SLP["C1"] := TRP1 TRP2 TRP3 TRP4 TRP5 TRP6 TRP7 TRP8 TRP9
      ;
set   SLP["C2"] := TRP10 TRP11 TRP12 TRP13 TRP14 TRP15 TRP16 TRP17 TRP18
      ;
set   SLP["C3"] := TRP19 TRP20 TRP21 TRP22 TRP23 TRP24 TRP25 TRP26
      ;
set   SLP["C4"] := TRP27 TRP28 TRP29 TRP30 TRP31 TRP32 TRP33 TRP34 TRP35
      ;

param cstP:D1 D2 D3 D4 D5:=
C1    789.974143   796.9210297  555.8827644  21.95095002  103.1889164
C2    11.97488531  59.72829641   385.7672308   760.4325319  732.1501676
C3    2270.269685  2301.170251   1883.931066   1629.409394  1605.007803
C4    339.4613726  378.2486128   61.55319953   539.3548557  484.1158248
      ;

end;

```

## ii. MODEL S

Model S requires implementation on all the demand points. Here, we present the basic GUSEK model used for all the cases, with the example of demand point 1 and provide the data sets for rest of the demand points in subsections.

### Basic GUSEK code (Demand point 1):

```
set S;
```



```

set Sprime;
param cst{i in S, j in S};
param demand{i in Sprime};
var x{i in S, j in S}, binary;
var a{i in Sprime}, >=0;
minimize cost: sum{i in S, j in S} cst[i,j]*x[i,j];
s.t. C1 {j in S}: sum{i in S} x[i,j] = 1;
s.t. C2 {i in S}: sum{j in S} x[i,j] = 1;
s.t. C3 {i in Sprime}: a[i] >=0;
s.t. C4 {i in Sprime}: a[i] <=1-demand[i];
s.t. C5 {i in Sprime,j in Sprime}: a[i] + demand[i] - a[j] - 1 + x[i,j] <=0;
solve;
display x,a;
data;
set S:= S2    S7    S8    S9    S10    S11    S12    S13    S14    S15    S19
        S20    S22    S26    S28    S30    T31    T32    T33    T34    T35    T36;
set Sprime:=S2    S7    S8    S9    S10    S11    S12    S13    S14    S15
        S19    S20    S22    S26    S28    S30    ;
param cst:S2 S7  S8  S9  S10  S11  S12  S13  S14  S15  S19
        S20  S22  S26  S28  S30  T31  T32  T33  T34  T35  T36:=
S2    10000 260.493493  57.25282319  532.3452058  288.893476  1989.338723
      591.7372724  560.7324314  540.0311001  171.3219546  572.5640408
      708.7686443  92.85968259  576.0577218  570.1848943  573.2196348
      284.0829431  284.0829431  284.0829431  284.0829431  284.0829431
      284.0829431
S7    260.493493  10000 203.5244452  729.2078897  28.43524969  2246.513847
      766.0006292  733.8651428  738.0147071  408.854159  745.634496
      908.1274319  347.0594633  754.4919127  754.0607017  745.696772
      23.7628355  23.7628355  23.7628355  23.7628355  23.7628355
      23.7628355
S8    57.25282319  203.5244452  10000 567.5691179  231.9501071  2044.753688
      620.8186185  589.209801  575.6539856  216.5055406  601.1343173
      745.7912304  148.0452608  606.2024708  601.8599863  601.6259382
      227.1776921  227.1776921  227.1776921  227.1776921  227.1776921
      227.1776921

```

S9	532.3452058	729.2078897	567.5691179	10000	753.802043	1614.482094
	93.67365386	83.53740445	9.291731156	365.6184061	87.61658119	
	179.2642444	514.5649714	74.06655944	56.37122736	89.5889954	
	750.5572656	750.5572656	750.5572656	750.5572656	750.5572656	
	750.5572656					
S10	288.893476	28.43524969	231.9501071	753.802043	10000	2274.81377
	788.8065136	756.7001407	762.6638798	436.699626	768.4211185	
	932.4989387	375.1405069	777.6549144	777.6338479	768.4378534	
	5.050708557	5.050708557	5.050708557	5.050708557	5.050708557	
	5.050708557					
S11	1989.338723	2246.513847	2044.753688	1614.482094	2274.81377	10000
	1642.546348	1664.639662	1605.597606	1840.807071	1657.685186	
	1480.787347	1913.376285	1639.368131	1626.62842	1659.069432	
	2270.269685	2270.269685	2270.269685	2270.269685	2270.269685	
	2270.269685					
S12	591.7372724	766.0006292	620.8186185	93.67365386	788.8065136	
	1642.546348	10000	32.14935195	93.95722683	432.549403	20.39546494
	165.4565662	585.1270996	19.95120594	37.71718322	20.5449826	
	786.0193532	786.0193532	786.0193532	786.0193532	786.0193532	
	786.0193532					
S13	560.7324314	733.8651428	589.209801	83.53740445	756.7001407	
	1664.639662	32.14935195	10000	86.97337465	402.9439193	11.93839371
	192.5477808	555.9008	25.51380445	39.61954892	12.49069595	
	753.9043685	753.9043685	753.9043685	753.9043685	753.9043685	
	753.9043685					
S14	540.0311001	738.0147071	575.6539856	9.291731156	762.6638798	
	1605.597606	93.95722683	86.97337465	10000	372.8254104	89.89256557
	170.7280672	521.2361589	74.87967999	56.24796353	91.89810458	
	759.4046838	759.4046838	759.4046838	759.4046838	759.4046838	
	759.4046838					
S15	171.3219546	408.854159	216.5055406	365.6184061	436.699626	
	1840.807071	432.549403	402.9439193	372.8254104	10000	414.4435255
	539.6464273	153.4445608	415.4660494	407.4834317	415.3521454	
	432.406046	432.406046	432.406046	432.406046	432.406046	
	432.406046					
S19	572.5640408	745.634496	601.1343173	87.61658119	768.4211185	
	1657.685186	20.39546494	11.93839371	89.89256557	414.4435255	10000
	183.2761914	567.3108212	19.31584066	37.34837984	2.018657841	
	765.637742	765.637742	765.637742	765.637742	765.637742	
	765.637742					
S20	708.7686443	908.1274319	745.7912304	179.2642444	932.4989387	
	1480.787347	165.4565662	192.5477808	170.7280672	539.6464273	

	183.2761914	10000	684.2492053	167.3528992	160.5433013	184.2615234	
	929.3187763	929.3187763	929.3187763	929.3187763	929.3187763	929.3187763	
	929.3187763						
S22	92.85968259	347.0594633	148.0452608	514.5649714	375.1405069		
	1913.376285	585.1270996	555.9008	521.2361589	153.4445608		
	567.3108212	684.2492053	10000	567.6828034	558.8980838	568.2674943	
	370.1942087	370.1942087	370.1942087	370.1942087	370.1942087		
	370.1942087						
S26	576.0577218	754.4919127	606.2024708	74.06655944	777.6549144		
	1639.368131	19.95120594	25.51380445	74.87967999	415.4660494		
	19.31584066	167.3528992	567.6828034	10000	19.34580329	21.1144608	
	774.7791898	774.7791898	774.7791898	774.7791898	774.7791898		
	774.7791898						
S28	570.1848943	754.0607017	601.8599863	56.37122736	777.6338479		
	1626.62842	37.71718322	39.61954892	56.24796353	407.4834317		
	37.34837984	160.5433013	558.8980838	19.34580329	10000	39.31531566	
	774.655285	774.655285	774.655285	774.655285	774.655285		
	774.655285						
S30	573.2196348	745.696772	601.6259382	89.5889954	768.4378534		
	1659.069432	20.5449826	12.49069595	91.89810458	415.3521454		
	2.018657841	184.2615234	568.2674943	21.1144608	39.31531566	10000	
	765.6656044	765.6656044	765.6656044	765.6656044	765.6656044		
	765.6656044						
T31	284.0829431	23.7628355	227.1776921	750.5572656	5.050708557		
	2270.269685	786.0193532	753.9043685	759.4046838	432.406046		
	765.637742	929.3187763	370.1942087	774.7791898	774.655285		
	765.6656044	0	0	0	0		
T32	284.0829431	23.7628355	227.1776921	750.5572656	5.050708557		
	2270.269685	786.0193532	753.9043685	759.4046838	432.406046		
	765.637742	929.3187763	370.1942087	774.7791898	774.655285		
	765.6656044	0	0	0	0		
T33	284.0829431	23.7628355	227.1776921	750.5572656	5.050708557		
	2270.269685	786.0193532	753.9043685	759.4046838	432.406046		
	765.637742	929.3187763	370.1942087	774.7791898	774.655285		
	765.6656044	0	0	0	0		
T34	284.0829431	23.7628355	227.1776921	750.5572656	5.050708557		
	2270.269685	786.0193532	753.9043685	759.4046838	432.406046		
	765.637742	929.3187763	370.1942087	774.7791898	774.655285		
	765.6656044	0	0	0	0		
T35	284.0829431	23.7628355	227.1776921	750.5572656	5.050708557		
	2270.269685	786.0193532	753.9043685	759.4046838	432.406046		

	765.637742	929.3187763	370.1942087	774.7791898	774.655285
	765.6656044	0	0	0	0
T36	284.0829431	23.7628355	227.1776921	750.5572656	5.050708557
	2270.269685	786.0193532	753.9043685	759.4046838	432.406046
	765.637742	929.3187763	370.1942087	774.7791898	774.655285
	765.6656044	0	0	0	0;

param demand:=

S2 0.31

S7 0.63

S8 0.32

S9 0.4

S10 0.45

S11 0.06

S12 0.42

S13 0.08

S14 0.75

S15 0.48

S19 0.4

S20 0.01

S22 0.07

S26 0.1

S28 0.5

S30 0.35

;

end ;

Data for rest of the demand points are as follows:

## Demand point 2

```

set S:= S3    S6    S7    S10   S11   S16   S18   S19   S22   S23   S24
         S25   S29   T31   T32   T33   T34   T35   T36;

set Sprime:=S3    S6    S7    S10   S11   S16   S18   S19   S22   S23
         S24   S25   S29;

param cst:S3 S6    S7    S10   S11   S16   S18   S19   S22   S23   S24
         S25   S29   T31   T32   T33   T34   T35   T36:=

S3    10000 97.55929591 23.40931354 5.634278943 2269.923157 363.0162536
      764.0258267 765.6437782 369.7135261 752.5558264 396.4598063
      778.090907 757.4534532 54.39963984 54.39963984 54.39963984
      54.39963984 54.39963984 54.39963984

S6    97.55929591 10000 75.5633219 103.0706601 2177.056064 265.5689056
      702.3593053 704.5293461 272.3843852 690.966039 299.4872006
      715.9467632 698.1661301 144.3644981 144.3644981 144.3644981
      144.3644981 144.3644981 144.3644981

S7    23.40931354 75.5633219 10000 28.43524969 2246.513847 340.4766177
      743.9298125 745.634496 347.0594633 732.463516 373.5979205
      757.9308039 737.7250324 70.35293231 70.35293231 70.35293231
      70.35293231 70.35293231 70.35293231

S10   5.634278943 103.0706601 28.43524969 10000 2274.81377 368.4694529
      766.8378909 768.4211185 375.1405069 755.3672399 401.8283414
      780.9277424 760.1211182 49.38192727 49.38192727 49.38192727
      49.38192727 49.38192727 49.38192727

S11   2269.923157 2177.056064 2246.513847 2274.81377 10000 1921.726141
      1655.105963 1657.685186 1913.376285 1663.543421 1883.880683
      1642.169393 1675.436332 2301.170251 2301.170251 2301.170251
      2301.170251 2301.170251 2301.170251

S16   363.0162536 265.5689056 340.4766177 368.4694529 1921.726141 10000
      571.7016909 575.6031281 9.392282905 561.5678294 39.27968727
      582.497711 576.0952543 406.0036241 406.0036241 406.0036241
      406.0036241 406.0036241 406.0036241

S18   764.0258267 702.3593053 743.9298125 766.8378909 1655.105963
      571.7016909 10000 5.656531424 563.3779514 11.47082983 541.8244603
  
```

	14.61183632	23.26890527	769.8071147	769.8071147	769.8071147				
	769.8071147	769.8071147	769.8071147						
S19	765.6437782	704.5293461	745.634496	768.4211185	1657.685186				
	575.6031281	5.656531424	10000	567.3108212	14.09849003	545.909747			
	15.61797409	18.82366337	771.0382561	771.0382561	771.0382561				
	771.0382561	771.0382561	771.0382561						
S22	369.7135261	272.3843852	347.0594633	375.1405069	1913.376285				
	9.392282905	563.3779514	567.3108212	10000	553.2864116	30.14758835			
	574.1000284	567.9527721	412.102085	412.102085	412.102085				
	412.102085	412.102085	412.102085						
S23	752.5558264	690.966039	732.463516	755.3672399	1663.543421				
	561.5678294	11.47082983	14.09849003	553.2864116	10000	531.9517934			
	25.85727948	22.55484897	758.356394	758.356394	758.356394				
	758.356394	758.356394	758.356394						
S24	396.4598063	299.4872006	373.5979205	401.8283414	1883.880683				
	39.27968727	541.8244603	545.909747	30.14758835	531.9517934	10000			
	552.1669211	547.2812674	437.5450111	437.5450111	437.5450111				
	437.5450111	437.5450111	437.5450111						
S25	778.090907	715.9467632	757.9308039	780.9277424	1642.169393				
	582.497711	14.61183632	15.61797409	574.1000284	25.85727948				
	552.1669211	10000	33.56212767	784.1148737	784.1148737	784.1148737			
	784.1148737	784.1148737	784.1148737						
S29	757.4534532	698.1661301	737.7250324	760.1211182	1675.436332				
	576.0952543	23.26890527	18.82366337	567.9527721	22.55484897				
	547.2812674	33.56212767	10000	761.6643319	761.6643319	761.6643319			
	761.6643319	761.6643319	761.6643319						
T31	54.39963984	144.3644981	70.35293231	49.38192727	2301.170251				
	406.0036241	769.8071147	771.0382561	412.102085	758.356394				
	437.5450111	784.1148737	761.6643319	0	0	0	0	0	
	0								
T32	54.39963984	144.3644981	70.35293231	49.38192727	2301.170251				
	406.0036241	769.8071147	771.0382561	412.102085	758.356394				
	437.5450111	784.1148737	761.6643319	0	0	0	0	0	
	0								
T33	54.39963984	144.3644981	70.35293231	49.38192727	2301.170251				
	406.0036241	769.8071147	771.0382561	412.102085	758.356394				
	437.5450111	784.1148737	761.6643319	0	0	0	0	0	
	0								
T34	54.39963984	144.3644981	70.35293231	49.38192727	2301.170251				
	406.0036241	769.8071147	771.0382561	412.102085	758.356394				

	437.5450111	784.1148737	761.6643319	0	0	0	0	0
	0							
T35	54.39963984	144.3644981	70.35293231	49.38192727	2301.170251			
	406.0036241	769.8071147	771.0382561	412.102085	758.356394			
	437.5450111	784.1148737	761.6643319	0	0	0	0	0
	0							
T36	54.39963984	144.3644981	70.35293231	49.38192727	2301.170251			
	406.0036241	769.8071147	771.0382561	412.102085	758.356394			
	437.5450111	784.1148737	761.6643319	0	0	0	0	0
	0							

;

param demand:=

S3 0.47

S6 0.4

S7 0.23

S10 0.26

S11 0.2

S16 0.02

S18 0.05

S19 0.16

S22 0.23

S23 0.22

S24 0.66

S25 0.06

S29 0.36

;

### Demand point 3

set S:= S1 S4 S6 S7 S9 S12 S13 S14 S17 S20 S21  
 S23 S25 S26 T31 T32 T33 T34 T35 T36;

set Sprime:=S1 S4 S6 S7 S9 S12 S13 S14 S17 S20  
 S21 S23 S25 S26;

param cst:S1 S4 S6 S7 S9 S12 S13 S14 S17 S20 S21  
 S23 S25 S26 T31 T32 T33 T34 T35 T36:=

S1	10000	264.093266	168.9709538	240.7451182	542.4252109	599.4639162	568.2005307	550.2756626	581.2470309	719.6668055	596.0131797	566.0439645	589.4517056	584.1826184	135.4246631	135.4246631	135.4246631	135.4246631
S4	264.093266	10000	97.65550177	23.56180609	750.6903744	786.2383906	754.1219724	759.5348911	766.1861178	929.4641695	784.6705972	752.7691313	778.3038718	774.9823376	396.5048053	396.5048053	396.5048053	396.5048053
S6	168.9709538	97.65550177	10000	75.5633219	680.8276698	724.7250869	692.6292501	689.3625829	705.1724902	860.0791955	722.441642	690.966039	715.9467632	711.8702395	299.4196278	299.4196278	299.4196278	299.4196278
S7	240.7451182	23.56180609	75.5633219	10000	729.2078897	766.0006292	733.8651428	738.0147071	746.0121333	908.1274319	764.3213788	732.463516	757.9308039	754.4919127	373.5272788	373.5272788	373.5272788	373.5272788
S9	542.4252109	750.6903744	680.8276698	729.2078897	10000	93.67365386	83.53740445	9.291731156	90.66175291	179.2642444	86.49691394	80.40293634	82.42431827	74.06655944	490.5472217	490.5472217	490.5472217	490.5472217
S12	599.4639162	786.2383906	724.7250869	766.0006292	93.67365386	10000	32.14935195	93.95722683	20.42084347	165.4565662	7.38046507	33.77639209	11.8348401	19.95120594	563.3009231	563.3009231	563.3009231	563.3009231
S13	568.2005307	754.1219724	692.6292501	733.8651428	83.53740445	10000	32.14935195	10000	86.97337465	13.14471672	192.5477808	3.316088619	25.2177131	25.51380445	534.5991633	534.5991633	534.5991633	534.5991633
S14	550.2756626	759.5348911	689.3625829	738.0147071	9.291731156	10000	93.95722683	86.97337465	10000	92.96006923	170.7280672	86.64133481						



	83.96666624	83.15673797	74.87967999	496.9835821	496.9835821			
	496.9835821	496.9835821	496.9835821	496.9835821				
S17	581.2470309	766.1861178	705.1724902	746.0121333	90.66175291			
	20.42084347	13.14471672	92.96006923	10000	184.5298631	21.49881833		
	15.75892894	17.41785182	21.93751304	547.6588755	547.6588755			
	547.6588755	547.6588755	547.6588755	547.6588755				
S20	719.6668055	929.4641695	860.0791955	908.1274319	179.2642444			
	165.4565662	192.5477808	170.7280672	184.5298631	10000	163.0313786		
	192.3990076	167.8160535	167.3528992	658.4750482	658.4750482			
	658.4750482	658.4750482	658.4750482	658.4750482				
S21	596.0131797	784.6705972	722.441642	764.3213788	86.49691394			
	7.38046507	31.05956803	86.64133481	21.49881833	163.0313786	10000		
	32.00064935	6.563872698	13.4268764	558.4768636	558.4768636			
	558.4768636	558.4768636	558.4768636	558.4768636				
S23	566.0439645	752.7691313	690.966039	732.463516	80.40293634			
	33.77639209	3.316088619	83.96666624	15.75892894	192.3990076			
	32.00064935	10000	25.85727948	25.06342342	531.9069824	531.9069824		
	531.9069824	531.9069824	531.9069824	531.9069824				
S25	589.4517056	778.3038718	715.9467632	757.9308039	82.42431827			
	11.8348401	25.2177131	83.15673797	17.41785182	167.8160535			
	6.563872698	25.85727948	10000	8.359960417	552.1247272	552.1247272		
	552.1247272	552.1247272	552.1247272	552.1247272				
S26	584.1826184	774.9823376	711.8702395	754.4919127	74.06655944			
	19.95120594	25.51380445	74.87967999	21.93751304	167.3528992			
	13.4268764	25.06342342	8.359960417	10000	545.5154609	545.5154609		
	545.5154609	545.5154609	545.5154609	545.5154609				
T31	135.4246631	396.5048053	299.4196278	373.5272788	490.5472217			
	563.3009231	534.5991633	496.9835821	547.6588755	658.4750482			
	558.4768636	531.9069824	552.1247272	545.5154609	0	0	0	
	0	0	0					
T32	135.4246631	396.5048053	299.4196278	373.5272788	490.5472217			
	563.3009231	534.5991633	496.9835821	547.6588755	658.4750482			
	558.4768636	531.9069824	552.1247272	545.5154609	0	0	0	
	0	0	0					
T33	135.4246631	396.5048053	299.4196278	373.5272788	490.5472217			
	563.3009231	534.5991633	496.9835821	547.6588755	658.4750482			
	558.4768636	531.9069824	552.1247272	545.5154609	0	0	0	
	0	0	0					
T34	135.4246631	396.5048053	299.4196278	373.5272788	490.5472217			
	563.3009231	534.5991633	496.9835821	547.6588755	658.4750482			

	558.4768636	531.9069824	552.1247272	545.5154609	0	0	0
	0	0	0				
T35	135.4246631	396.5048053	299.4196278	373.5272788	490.5472217		
	563.3009231	534.5991633	496.9835821	547.6588755	658.4750482		
	558.4768636	531.9069824	552.1247272	545.5154609	0	0	0
	0	0	0				
T36	135.4246631	396.5048053	299.4196278	373.5272788	490.5472217		
	563.3009231	534.5991633	496.9835821	547.6588755	658.4750482		
	558.4768636	531.9069824	552.1247272	545.5154609	0	0	0
	0	0	0				

;

param demand:=

S1 0.4  
S4 0.33  
S6 0.07  
S7 0.55  
S9 0.37  
S12 0.13  
S13 0.08  
S14 0.46  
S17 0.35  
S20 0.15  
S21 0.38  
S23 0.49  
S25 0.18  
S26 0.49

;

### Demand point 4

set S:= S1 S2 S3 S4 S9 S10 S11 S15 S16 S17 S18  
 S20 S21 S23 S27 S28 S29 S30 T31 T32 T33 T34  
 T35 T36;

set Sprime:=S1 S2 S3 S4 S9 S10 S11 S15 S16 S17  
 S18 S20 S21 S23 S27 S28 S29 S30;

param cst:S1 S2 S3 S4 S9 S10 S11 S15 S16 S17 S18  
 S20 S21 S23 S27 S28 S29 S30 T31 T32 T33 T34  
 T35 T36:=

S1	10000	20.19683283	263.9622572	264.093266	542.4252109	269.1642984	2008.164414	184.9494123	108.3686556	581.2470309	577.0131043	719.6668055	596.0131797	566.0439645	570.0252538	578.8996335	577.0406792	580.6712365	576.3819004	576.3819004	576.3819004
S2	20.19683283	10000	283.6576358	283.7856835	532.3452058	288.893476	1989.338723	171.3219546	89.01924603	573.8246625	569.3455881	708.7686443	588.0834128	558.4931287	561.6470748	570.1848943	570.1385166	573.2196348	567.7211237	567.7211237	567.7211237
S3	263.9622572	283.6576358	10000	0.222479495	750.4850624	5.634278943	2269.923157	432.1242127	363.0162536	765.9721356	764.0258267	929.2575191	784.457501	752.5558264	763.3221838	774.628939	757.4534532	765.6734545	771.7642986	771.7642986	771.7642986
S4	264.093266	283.7856835	0.222479495	10000	750.6903744	5.587545983	2270.075006	432.2925327	363.1204481	766.1861178	764.2391201	929.4641695	784.6705972	752.7691313	763.5334704	774.8395257	757.6685147	765.8873542	771.9749719	771.9749719	771.9749719
S9	542.4252109	532.3452058	750.4850624	750.6903744	10000	753.802043	1614.482094	365.6184061	523.3559622	90.66175291	81.97216812	179.2642444	86.49691394	80.40293634	59.439304	56.37122736	102.8194348	89.5889954	55.96620948	55.96620948	55.96620948
S10	269.1642984	288.893476	5.634278943	5.587545983	753.802043	10000	2274.81377	436.699626	368.4694529	768.7299066	766.8378909	932.4989387	787.2842477	755.3672399	766.2800846	777.6338479	760.1211182	768.4378534	774.7635064	774.7635064	774.7635064

S11	2008.164414	1989.338723	2269.923157	2270.075006	1614.482094	
	2274.81377	10000	1840.807071	1921.726141	1659.576183	1655.105963
	1480.787347	1638.65565	1663.543421	1640.145781	1626.62842	
	1675.436332	1659.069432	1629.409394	1629.409394	1629.409394	
	1629.409394	1629.409394	1629.409394			
S15	184.9494123	171.3219546	432.1242127	432.2925327	365.6184061	
	436.699626	1840.807071	10000	161.4466632	416.0725878	410.6298421
	539.6464273	428.13073	400.3847917	400.2353163	407.4834317	
	414.6844702	415.3521454	405.2297312	405.2297312	405.2297312	
	405.2297312	405.2297312	405.2297312			
S16	108.3686556	89.01924603	363.0162536	363.1204481	523.3559622	
	368.4694529	1921.726141	161.4466632	10000	577.280826	571.7016909
	693.3171572	588.9158826	561.5678294	560.6934964	567.4566342	
	576.0952543	576.5460057	565.2945161	565.2945161	565.2945161	
	565.2945161	565.2945161	565.2945161			
S17	581.2470309	573.8246625	765.9721356	766.1861178	90.66175291	
	768.7299066	1659.576183	416.0725878	577.280826	10000	8.689593852
	184.5298631	21.49881833	15.75892894	31.60339877	40.26826505	
	16.3178498	1.074249151	38.76212482	38.76212482	38.76212482	
	38.76212482	38.76212482	38.76212482			
S18	577.0131043	569.3455881	764.0258267	764.2391201	81.97216812	
	766.8378909	1655.105963	410.6298421	571.7016909	8.689593852	10000
	182.0981688	20.57134177	11.47082983	23.01009017	32.53886906	
	23.26890527	7.616939414	30.76886627	30.76886627	30.76886627	
	30.76886627	30.76886627	30.76886627			
S20	719.6668055	708.7686443	929.2575191	929.4641695	179.2642444	
	932.4989387	1480.787347	539.6464273	693.3171572	184.5298631	
	182.0981688	10000	163.0313786	192.3990076	173.6458591	160.5433013
	199.1825794	184.2615234	163.5556597	163.5556597	163.5556597	
	163.5556597	163.5556597	163.5556597			
S21	596.0131797	588.0834128	784.457501	784.6705972	86.49691394	
	787.2842477	1638.65565	428.13073	588.9158826	21.49881833	
	20.57134177	163.0313786	10000	32.00064935	29.73208022	30.39337347
	36.79903044	21.24747584	30.53230462	30.53230462	30.53230462	
	30.53230462	30.53230462	30.53230462			
S23	566.0439645	558.4931287	752.5558264	752.7691313	80.40293634	
	755.3672399	1663.543421	400.3847917	561.5678294	15.75892894	
	11.47082983	192.3990076	32.00064935	10000	25.36322407	37.78801993
	22.55484897	14.98430879	35.36619278	35.36619278	35.36619278	
	35.36619278	35.36619278	35.36619278			
S27	570.0252538	561.6470748	763.3221838	763.5334704	59.439304	
	766.2800846	1640.145781	400.2353163	560.6934964	31.60339877	

	23.01009017	173.6458591	29.73208022	25.36322407	10000	13.61929571			
	45.58625191	30.54671408	10.74807758	10.74807758	10.74807758				
	10.74807758	10.74807758	10.74807758						
S28	578.8996335	570.1848943	774.628939	774.8395257	56.37122736				
	777.6338479	1626.62842	407.4834317	567.4566342	40.26826505				
	32.53886906	160.5433013	30.39337347	37.78801993	13.61929571	10000			
	55.80694845	39.31531566	3.012712864	3.012712864	3.012712864				
	3.012712864	3.012712864	3.012712864						
S29	577.0406792	570.1385166	757.4534532	757.6685147	102.8194348				
	760.1211182	1675.436332	414.6844702	576.0952543	16.3178498				
	23.26890527	199.1825794	36.79903044	22.55484897	45.58625191				
	55.80694845	10000	17.05307871	54.0088875	54.0088875	54.0088875			
	54.0088875	54.0088875	54.0088875						
S30	580.6712365	573.2196348	765.6734545	765.8873542	89.5889954				
	768.4378534	1659.069432	415.3521454	576.5460057	1.074249151				
	7.616939414	184.2615234	21.24747584	14.98430879	30.54671408				
	39.31531566	17.05307871	10000	37.77826533	37.77826533	37.77826533			
	37.77826533	37.77826533	37.77826533						
T31	576.3819004	567.7211237	771.7642986	771.9749719	55.96620948				
	774.7635064	1629.409394	405.2297312	565.2945161	38.76212482				
	30.76886627	163.5556597	30.53230462	35.36619278	10.74807758				
	3.012712864	54.0088875	37.77826533	0	0	0	0	0	0
	0								
T32	576.3819004	567.7211237	771.7642986	771.9749719	55.96620948				
	774.7635064	1629.409394	405.2297312	565.2945161	38.76212482				
	30.76886627	163.5556597	30.53230462	35.36619278	10.74807758				
	3.012712864	54.0088875	37.77826533	0	0	0	0	0	0
	0								
T33	576.3819004	567.7211237	771.7642986	771.9749719	55.96620948				
	774.7635064	1629.409394	405.2297312	565.2945161	38.76212482				
	30.76886627	163.5556597	30.53230462	35.36619278	10.74807758				
	3.012712864	54.0088875	37.77826533	0	0	0	0	0	0
	0								
T34	576.3819004	567.7211237	771.7642986	771.9749719	55.96620948				
	774.7635064	1629.409394	405.2297312	565.2945161	38.76212482				
	30.76886627	163.5556597	30.53230462	35.36619278	10.74807758				
	3.012712864	54.0088875	37.77826533	0	0	0	0	0	0
	0								
T35	576.3819004	567.7211237	771.7642986	771.9749719	55.96620948				
	774.7635064	1629.409394	405.2297312	565.2945161	38.76212482				
	30.76886627	163.5556597	30.53230462	35.36619278	10.74807758				

	3.012712864	54.0088875	37.77826533	0	0	0	0	0
	0							
T36	576.3819004	567.7211237	771.7642986	771.9749719	55.96620948			
	774.7635064	1629.409394	405.2297312	565.2945161	38.76212482			
	30.76886627	163.5556597	30.53230462	35.36619278	10.74807758			
	3.012712864	54.0088875	37.77826533	0	0	0	0	0
	0							

;

param demand:=

S1 0.17

S2 0.48

S3 0.94

S4 0.23

S9 0.12

S10 0.17

S11 0.24

S15 0.15

S16 0.15

S17 0.4

S18 0.25

S20 0.41

S21 0.39

S23 0.12

S27 0.24

S28 0.29

S29 0.3

S30 0.86

;

## Demand point 5

set S:= S1 S2 S5 S6 S8 S9 S14 S15 S16 S19 S22  
 S24 S25 S26 S27 S28 S29 S30 T31 T32 T33 T34  
 T35 T36;

set Sprime:=S1 S2 S5 S6 S8 S9 S14 S15 S16 S19  
 S22 S24 S25 S26 S27 S28 S29 S30;

param cst:S1 S2 S5 S6 S8 S9 S14 S15 S16 S19 S22  
 S24 S25 S26 S27 S28 S29 S30 T31 T32 T33 T34  
 T35 T36:=

S1	10000	20.19683283	904.831187	168.9709538	37.26095138	542.4252109					
	550.2756626	184.9494123	108.3686556	580.078181	112.6289545						
	135.5051511	589.4517056	584.1826184	570.0252538	578.8996335						
	577.0406792	580.6712365	529.0928867	529.0928867	529.0928867						
	529.0928867	529.0928867	529.0928867								
S2	20.19683283	10000	896.3788195	188.1504083	57.25282319	532.3452058					
	540.0311001	171.3219546	89.01924603	572.5640408	92.85968259						
	115.3174714	581.5308203	576.0577218	561.6470748	570.1848943						
	570.1385166	573.2196348	518.2094916	518.2094916	518.2094916						
	518.2094916	518.2094916	518.2094916								
S5	904.831187	896.3788195	10000	1030.521422	926.9404607	368.6706045					
	362.3119306	733.0350251	891.8693494	326.3426041	883.1273639						
	859.2481694	315.8288086	320.7674108	334.8090142	326.2518513						
	332.4159931	326.0007372	388.5348196	388.5348196	388.5348196						
	388.5348196	388.5348196	388.5348196								
S6	168.9709538	188.1504083	1030.521422	10000	132.3190595	680.8276698					
	689.3625829	344.9308651	265.5689056	704.5293461	272.3843852						
	299.4872006	715.9467632	711.8702395	699.3113664	709.8053659						
	698.1661301	704.7682421	671.535024	671.535024	671.535024						
	671.535024	671.535024	671.535024								
S8	37.26095138	57.25282319	926.9404607	132.3190595	10000	567.5691179					
	575.6539856	216.5055406	142.9681602	601.1343173	148.0452608						
	172.154495	611.1366846	606.2024708	592.4752383	601.8599863						
	597.1196951	601.6259382	555.4748533	555.4748533	555.4748533						
	555.4748533	555.4748533	555.4748533								
S9	542.4252109	532.3452058	368.6706045	680.8276698	567.5691179	10000					
	9.291731156	365.6184061	523.3559622	87.61658119	514.5649714						
	490.5798667	82.42431827	74.06655944	59.439304	56.37122736						
	102.8194348	89.5889954	28.22627028	28.22627028	28.22627028						
	28.22627028	28.22627028	28.22627028								

S14	550.2756626	540.0311001	362.3119306	689.3625829	575.6539856	
	9.291731156	10000	372.8254104	530.0673393	89.89256557	521.2361589
	497.014968	83.15673797	74.87967999	61.37476153	56.24796353	
	106.0528415	91.89810458	29.23573673	29.23573673	29.23573673	
	29.23573673	29.23573673	29.23573673			
S15	184.9494123	171.3219546	733.0350251	344.9308651	216.5055406	
	365.6184061	372.8254104	10000	161.4466632	414.4435255	153.4445608
	135.1167625	421.6755181	415.4660494	400.2353163	407.4834317	
	414.6844702	415.3521454	349.5659369	349.5659369	349.5659369	
	349.5659369	349.5659369	349.5659369			
S16	108.3686556	89.01924603	891.8693494	265.5689056	142.9681602	
	523.3559622	530.0673393	161.4466632	10000	575.6031281	9.392282905
	39.27968727	582.497711	576.1195318	560.6934964	567.4566342	
	576.0952543	576.5460057	505.2824368	505.2824368	505.2824368	
	505.2824368	505.2824368	505.2824368			
S19	580.078181	572.5640408	326.3426041	704.5293461	601.1343173	
	87.61658119	89.89256557	414.4435255	575.6031281	10000	567.3108212
	545.909747	15.61797409	19.31584066	28.53511525	37.34837984	
	18.82366337	2.018657841	115.6267664	115.6267664	115.6267664	
	115.6267664	115.6267664	115.6267664			
S22	112.6289545	92.85968259	883.1273639	272.3843852	148.0452608	
	514.5649714	521.2361589	153.4445608	9.392282905	567.3108212	10000
	30.14758835	574.1000284	567.6828034	552.2188284	558.8980838	
	567.9527721	568.2674943	496.3589874	496.3589874	496.3589874	
	496.3589874	496.3589874	496.3589874			
S24	135.5051511	115.3174714	859.2481694	299.4872006	172.154495	
	490.5798667	497.014968	135.1167625	39.27968727	545.909747	
	30.14758835	10000	552.1669211	545.5568264	529.9112454	536.1485846
	547.2812674	546.9336664	471.6088027	471.6088027	471.6088027	
	471.6088027	471.6088027	471.6088027			
S25	589.4517056	581.5308203	315.8288086	715.9467632	611.1366846	
	82.42431827	83.15673797	421.6755181	582.497711	15.61797409	
	574.1000284	552.1669211	10000	8.359960417	24.26388344	27.277629
	33.56212767	16.90310272	110.640342	110.640342	110.640342	
	110.640342	110.640342	110.640342			
S26	584.1826184	576.0577218	320.7674108	711.8702395	606.2024708	
	74.06655944	74.87967999	415.4660494	576.1195318	19.31584066	
	567.6828034	545.5568264	8.359960417	10000	16.30594992	19.34580329
	38.13762354	21.1144608	102.2843749	102.2843749	102.2843749	
	102.2843749	102.2843749	102.2843749			
S27	570.0252538	561.6470748	334.8090142	699.3113664	592.4752383	
	59.439304	61.37476153	400.2353163	560.6934964	28.53511525	



	552.2188284	529.9112454	24.26388344	16.30594992	10000	13.61929571			
	45.58625191	30.54671408	87.60450172	87.60450172	87.60450172				
	87.60450172	87.60450172	87.60450172						
S28	578.8996335	570.1848943	326.2518513	709.8053659	601.8599863				
	56.37122736	56.24796353	407.4834317	567.4566342	37.34837984				
	558.8980838	536.1485846	27.277629	19.34580329	13.61929571	10000			
	55.80694845	39.31531566	84.38646482	84.38646482	84.38646482				
	84.38646482	84.38646482	84.38646482						
S29	577.0406792	570.1385166	332.4159931	698.1661301	597.1196951				
	102.8194348	106.0528415	414.6844702	576.0952543	18.82366337				
	567.9527721	547.2812674	33.56212767	38.13762354	45.58625191				
	55.80694845	10000	17.05307871	130.3029471	130.3029471	130.3029471			
	130.3029471	130.3029471	130.3029471						
S30	580.6712365	573.2196348	326.0007372	704.7682421	601.6259382				
	89.5889954	91.89810458	415.3521454	576.5460057	2.018657841				
	568.2674943	546.9336664	16.90310272	21.1144608	30.54671408				
	39.31531566	17.05307871	10000	117.5829746	117.5829746	117.5829746			
	117.5829746	117.5829746	117.5829746						
T31	529.0928867	518.2094916	388.5348196	671.535024	555.4748533				
	28.22627028	29.23573673	349.5659369	505.2824368	115.6267664				
	496.3589874	471.6088027	110.640342	102.2843749	87.60450172				
	84.38646482	130.3029471	117.5829746	0	0	0	0	0	
	0								
T32	529.0928867	518.2094916	388.5348196	671.535024	555.4748533				
	28.22627028	29.23573673	349.5659369	505.2824368	115.6267664				
	496.3589874	471.6088027	110.640342	102.2843749	87.60450172				
	84.38646482	130.3029471	117.5829746	0	0	0	0	0	
	0								
T33	529.0928867	518.2094916	388.5348196	671.535024	555.4748533				
	28.22627028	29.23573673	349.5659369	505.2824368	115.6267664				
	496.3589874	471.6088027	110.640342	102.2843749	87.60450172				
	84.38646482	130.3029471	117.5829746	0	0	0	0	0	
	0								
T34	529.0928867	518.2094916	388.5348196	671.535024	555.4748533				
	28.22627028	29.23573673	349.5659369	505.2824368	115.6267664				
	496.3589874	471.6088027	110.640342	102.2843749	87.60450172				
	84.38646482	130.3029471	117.5829746	0	0	0	0	0	
	0								
T35	529.0928867	518.2094916	388.5348196	671.535024	555.4748533				
	28.22627028	29.23573673	349.5659369	505.2824368	115.6267664				
	496.3589874	471.6088027	110.640342	102.2843749	87.60450172				

	84.38646482	130.3029471	117.5829746	0	0	0	0	0
	0							
T36	529.0928867	518.2094916	388.5348196	671.535024	555.4748533			
	28.22627028	29.23573673	349.5659369	505.2824368	115.6267664			
	496.3589874	471.6088027	110.640342	102.2843749	87.60450172			
	84.38646482	130.3029471	117.5829746	0	0	0	0	0
	0							

;

param demand:=

S1	0.28
S2	0.13
S5	0.32
S6	0.76
S8	0.34
S9	0.06
S14	0.05
S15	0.31
S16	0.2
S19	0.28
S22	0.46
S24	0.57
S25	0.5
S26	0.25
S27	0.1
S28	0.18
S29	0.46
S30	0.1

;

## i. MODEL H

Model H requires implementation on all the  $\pi$ -hubs. Here, we present the basic GUSEK model used for all the cases, with the example of  $\pi$ -hub 1 and provide the data sets for rest of the demand points in subsections.

### Basic GUSEK code ( $\Pi$ -hub 1):

```
set S;
set Sprime;

param cst{i in S, j in S};
param demand{i in Sprime};

var x{i in S, j in S}, binary;
var a{i in Sprime}, >=0;

minimize cost: sum{i in S, j in S} cst[i,j]*x[i,j];

s.t. C1 {j in S}: sum{i in S} x[i,j] = 1;
s.t. C2 {i in S}: sum{j in S} x[i,j] = 1;
s.t. C3 {i in Sprime}: a[i] >=0;
s.t. C4 {i in Sprime}: a[i] <=1-demand[i];
s.t. C5 {i in Sprime, j in Sprime}: a[i] + demand[i] - a[j] - 1 + x[i,j] <=0;

solve;

display x,a;

data;
```

```

set S:= S5 S9 S13 S14 S17 S18 S19 S20 S21 S23 S25
      S27 S28 S29 S30 T31 T32 T33 T34 T35 T36 T37
      T38 T39 T40 T41;

```

```

set Sprime:=S5 S9 S13 S14 S17 S18 S19 S20 S21 S23
      S25 S27 S28 S29 S30;

```

```

param cst:S5 S9 S13 S14 S17 S18 S19 S20 S21 S23
      S25 S27 S28 S29 S30 T31 T32 T33 T34 T35 T36
      T37 T38 T39 T40 T41:=

```

```

S5 10000 368.6706045 1089.0601 1089.27655 325.5522349 328.8597796
    326.3426041 216.6549036 309.27 340.122036 315.8288086 334.8090142
    326.2518513 332.4159931 326.0007372 307.6915785 307.6915785
    307.6915785 307.6915785 307.6915785 307.6915785 307.6915785
    307.6915785 307.6915785 307.6915785 307.6915785

```

```

S9 368.6706045 10000 750.4850624 750.6903744 90.66175291 81.97216812
    87.61658119 179.2642444 86.49691394 80.40293634 82.42431827
    59.439304 56.37122736 102.8194348 89.5889954 75.59317425
    75.59317425 75.59317425 75.59317425 75.59317425 75.59317425
    75.59317425 75.59317425 75.59317425 75.59317425

```

```

S13 1089.0601 750.4850624 10000 0.222479495 765.9721356
    764.0258267 765.6437782 929.2575191 784.457501 752.5558264
    778.090907 763.3221838 774.628939 757.4534532 765.6734545
    789.9594495 789.9594495 789.9594495 789.9594495 789.9594495
    789.9594495 789.9594495 789.9594495 789.9594495

```

```

S14 1089.27655 750.6903744 0.222479495 10000 766.1861178
    764.2391201 765.8575156 929.4641695 784.6705972 752.7691313
    778.3038718 763.5334704 774.8395257 757.6685147 765.8873542
    790.1712238 790.1712238 790.1712238 790.1712238 790.1712238
    790.1712238 790.1712238 790.1712238 790.1712238

```

```

S17 325.5522349 90.66175291 765.9721356 766.1861178 10000
    8.689593852 3.06832229 184.5298631 21.49881833 15.75892894
    17.41785182 31.60339877 40.26826505 16.3178498 1.074249151
    35.77607549 35.77607549 35.77607549 35.77607549 35.77607549
    35.77607549 35.77607549 35.77607549 35.77607549

```

```

S18 328.8597796 81.97216812 764.0258267 764.2391201 8.689593852
    10000 5.656531424 182.0981688 20.57134177 11.47082983 14.61183632
    23.01009017 32.53886906 23.26890527 7.616939414 31.51733934

```

31.51733934 31.51733934 31.51733934 31.51733934 31.51733934  
 31.51733934 31.51733934 31.51733934 31.51733934 31.51733934  
 S19 326.3426041 87.61658119 765.6437782 765.8575156 3.06832229  
 5.656531424 10000 183.2761914 20.40018271 14.09849003 15.61797409  
 28.53511525 37.34837984 18.82366337 2.018657841 33.77401336  
 33.77401336 33.77401336 33.77401336 33.77401336 33.77401336  
 33.77401336 33.77401336 33.77401336 33.77401336 33.77401336  
 S20 216.6549036 179.2642444 929.2575191 929.4641695 184.5298631  
 182.0981688 183.2761914 10000 163.0313786 192.3990076 167.8160535  
 173.6458591 160.5433013 199.1825794 184.2615234 150.9261333  
 150.9261333 150.9261333 150.9261333 150.9261333 150.9261333  
 150.9261333 150.9261333 150.9261333 150.9261333 150.9261333  
 S21 309.27 86.49691394 784.457501 784.6705972 21.49881833  
 20.57134177 20.40018271 163.0313786 10000 32.00064935 6.563872698  
 29.73208022 30.39337347 36.79903044 21.24747584 16.64964803  
 16.64964803 16.64964803 16.64964803 16.64964803 16.64964803  
 16.64964803 16.64964803 16.64964803 16.64964803 16.64964803  
 S23 340.122036 80.40293634 752.5558264 752.7691313 15.75892894  
 11.47082983 14.09849003 192.3990076 32.00064935 10000 25.85727948  
 25.36322407 37.78801993 22.55484897 14.98430879 41.47329808  
 41.47329808 41.47329808 41.47329808 41.47329808 41.47329808  
 41.47329808 41.47329808 41.47329808 41.47329808 41.47329808  
 S25 315.8288086 82.42431827 778.090907 778.3038718 17.41785182  
 14.61183632 15.61797409 167.8160535 6.563872698 25.85727948 10000  
 24.26388344 27.277629 33.56212767 16.90310272 18.39051116  
 18.39051116 18.39051116 18.39051116 18.39051116 18.39051116  
 18.39051116 18.39051116 18.39051116 18.39051116 18.39051116  
 S27 334.8090142 59.439304 763.3221838 763.5334704 31.60339877  
 23.01009017 28.53511525 173.6458591 29.73208022 25.36322407  
 24.26388344 10000 13.61929571 45.58625191 30.54671408 27.19563553  
 27.19563553 27.19563553 27.19563553 27.19563553 27.19563553  
 27.19563553 27.19563553 27.19563553 27.19563553 27.19563553  
 S28 326.2518513 56.37122736 774.628939 774.8395257 40.26826505  
 32.53886906 37.34837984 160.5433013 30.39337347 37.78801993  
 27.277629 13.61929571 10000 55.80694845 39.31531566 20.26011281  
 20.26011281 20.26011281 20.26011281 20.26011281 20.26011281  
 20.26011281 20.26011281 20.26011281 20.26011281 20.26011281  
 S29 332.4159931 102.8194348 757.4534532 757.6685147 16.3178498  
 23.26890527 18.82366337 199.1825794 36.79903044 22.55484897  
 33.56212767 45.58625191 55.80694845 10000 17.05307871 51.9515964  
 51.9515964 51.9515964 51.9515964 51.9515964 51.9515964  
 51.9515964 51.9515964 51.9515964 51.9515964 51.9515964

S30        326.0007372 89.5889954 765.6734545 765.8873542 1.074249151  
7.616939414 2.018657841 184.2615234 21.24747584 14.98430879  
16.90310272 30.54671408 39.31531566 17.05307871 10000 35.20742193  
35.20742193 35.20742193 35.20742193 35.20742193 35.20742193  
35.20742193 35.20742193 35.20742193 35.20742193 35.20742193

T31        307.6915785 75.59317425 789.9594495 790.1712238 35.77607549  
31.51733934 33.77401336 150.9261333 16.64964803 41.47329808  
18.39051116 27.19563553 20.26011281 51.9515964 35.20742193 0  
0        0        0        0        0        0        0        0        0

T32        307.6915785 75.59317425 789.9594495 790.1712238 35.77607549  
31.51733934 33.77401336 150.9261333 16.64964803 41.47329808  
18.39051116 27.19563553 20.26011281 51.9515964 35.20742193 0  
0        0        0        0        0        0        0        0        0

T33        307.6915785 75.59317425 789.9594495 790.1712238 35.77607549  
31.51733934 33.77401336 150.9261333 16.64964803 41.47329808  
18.39051116 27.19563553 20.26011281 51.9515964 35.20742193 0  
0        0        0        0        0        0        0        0        0

T34        307.6915785 75.59317425 789.9594495 790.1712238 35.77607549  
31.51733934 33.77401336 150.9261333 16.64964803 41.47329808  
18.39051116 27.19563553 20.26011281 51.9515964 35.20742193 0  
0        0        0        0        0        0        0        0        0

T35        307.6915785 75.59317425 789.9594495 790.1712238 35.77607549  
31.51733934 33.77401336 150.9261333 16.64964803 41.47329808  
18.39051116 27.19563553 20.26011281 51.9515964 35.20742193 0  
0        0        0        0        0        0        0        0        0

T36        307.6915785 75.59317425 789.9594495 790.1712238 35.77607549  
31.51733934 33.77401336 150.9261333 16.64964803 41.47329808  
18.39051116 27.19563553 20.26011281 51.9515964 35.20742193 0  
0        0        0        0        0        0        0        0        0

T37        307.6915785 75.59317425 789.9594495 790.1712238 35.77607549  
31.51733934 33.77401336 150.9261333 16.64964803 41.47329808  
18.39051116 27.19563553 20.26011281 51.9515964 35.20742193 0  
0        0        0        0        0        0        0        0        0

T38        307.6915785 75.59317425 789.9594495 790.1712238 35.77607549  
31.51733934 33.77401336 150.9261333 16.64964803 41.47329808  
18.39051116 27.19563553 20.26011281 51.9515964 35.20742193 0  
0        0        0        0        0        0        0        0        0

T39        307.6915785 75.59317425 789.9594495 790.1712238 35.77607549  
31.51733934 33.77401336 150.9261333 16.64964803 41.47329808  
18.39051116 27.19563553 20.26011281 51.9515964 35.20742193 0  
0        0        0        0        0        0        0        0        0

T40        307.6915785 75.59317425 789.9594495 790.1712238 35.77607549  
31.51733934 33.77401336 150.9261333 16.64964803 41.47329808  
18.39051116 27.19563553 20.26011281 51.9515964 35.20742193 0  
0        0        0        0        0        0        0        0        0

T41        307.6915785 75.59317425 789.9594495 790.1712238 35.77607549  
31.51733934 33.77401336 150.9261333 16.64964803 41.47329808  
18.39051116 27.19563553 20.26011281 51.9515964 35.20742193 0  
0        0        0        0        0        0        0        0        0;

param demand:=

S5 0.32

S9 0.31

S13        0.15

S14        0.46

S17        0.75

S18        0.3

S19        0.56

S20        0.7

S21        0.77

S23        0.83

S25        0.74

S27        0.34

S28        0.82

S29        0.12

S30        0.55;

end ;

## II-hub 2

```

set S:= S3 S4 S6 S7 S10 S11 S14 S30 T31 T32 T33
      T34 T35 T36 T37
;
set Sprime:=S3 S4 S6 S7 S10 S11;

param cst:S3 S4 S6 S7 S10 S11 S14 S30 T31 T32
      T33 T34 T35 T36 T37:=
S3 10000.00 0.22 97.56 23.41 5.63 2269.92 759.33 765.67 789.96
    789.96 789.96 789.96 789.96 789.96 789.96
S4 0.22 10000.00 97.66 23.56 5.59 2270.08 759.53 765.89 790.17
    790.17 790.17 790.17 790.17 790.17 790.17
S6 97.56 97.66 10000.00 75.56 103.07 2177.06 689.36 704.77 726.36
    726.36 726.36 726.36 726.36 726.36 726.36
S7 23.41 23.56 75.56 10000.00 28.44 2246.51 738.01 745.70 769.58
    769.58 769.58 769.58 769.58 769.58 769.58
S10 5.63 5.59 103.07 28.44 10000.00 2274.81 762.66 768.44
    792.88 792.88 792.88 792.88 792.88 792.88 792.88
S11 2269.92 2270.08 2177.06 2246.51 2274.81
    10000.00 1605.60 1659.07 1623.91 1623.91
    1623.91 1623.91 1623.91 1623.91 1623.91
S14 759.33 759.53 689.36 738.01 762.66 1605.60 10000.00 91.90
    74.36 74.36 74.36 74.36 74.36 74.36 74.36
S30 765.67 765.89 704.77 745.70 768.44 1659.07 91.90 10000.00
    35.21 35.21 35.21 35.21 35.21 35.21 35.21
T31 789.96 790.17 726.36 769.58 792.88 1623.91 74.36 35.21 0.00
    0.00 0.00 0.00 0.00 0.00 0.00
T32 789.96 790.17 726.36 769.58 792.88 1623.91 74.36 35.21 0.00
    0.00 0.00 0.00 0.00 0.00 0.00
T33 789.96 790.17 726.36 769.58 792.88 1623.91 74.36 35.21 0.00
    0.00 0.00 0.00 0.00 0.00 0.00
T34 789.96 790.17 726.36 769.58 792.88 1623.91 74.36 35.21 0.00
    0.00 0.00 0.00 0.00 0.00 0.00
T35 789.96 790.17 726.36 769.58 792.88 1623.91 74.36 35.21 0.00
    0.00 0.00 0.00 0.00 0.00 0.00

```



T36	789.96	790.17	726.36	769.58	792.88	1623.91	74.36	35.21	0.00
	0.00	0.00	0.00	0.00	0.00	0.00			

T37	789.96	790.17	726.36	769.58	792.88	1623.91	74.36	35.21	0.00
	0.00	0.00	0.00	0.00	0.00	0.00			

;

param demand:=

S5 0.32

S9 0.31

S13 0.15

S14 0.46

S17 0.75

S18 0.3

S19 0.56

S20 0.7

S21 0.77

S23 0.83

S25 0.74

S27 0.34

S28 0.82

S29 0.12

S30 0.55;

## II-hub 3

```

set S:= S1 S2 S8 S9 S12 S15 S16 S22 S24 S26 T31
      T32 T33 T34 T35 T36 T37 T38

;

set Sprime:=S1 S2 S8 S9 S12 S15 S16 S22 S24 S26;

param cst:S1 S2 S8 S9 S12 S15 S16 S22 S24 S26
      T31 T32 T33 T34 T35 T36 T37 T38

:=

S1 10000.00 20.20 37.26 542.43 599.46 184.95 108.37 112.63 135.51 584.18
    597.16 597.16 597.16 597.16 597.16 597.16 597.16 597.16

S2 20.20 10000.00 57.25 532.35 591.74 171.32 89.02 92.86 115.32 576.06
    588.71 588.71 588.71 588.71 588.71 588.71 588.71 588.71

S8 37.26 57.25 10000.00 567.57 620.82 216.51 142.97 148.05 172.15 606.20
    619.67 619.67 619.67 619.67 619.67 619.67 619.67 619.67

S9 542.43 532.35 567.57 10000.00 93.67 365.62 523.36 514.56 490.58 74.07
    75.59 75.59 75.59 75.59 75.59 75.59 75.59 75.59

S12 599.46 591.74 620.82 93.67 10000.00 432.55 593.49 585.13 563.34
    19.95 23.30 23.30 23.30 23.30 23.30 23.30 23.30 23.30

S15 184.95 171.32 216.51 365.62 432.55 10000.00 161.45 153.44 135.12
    415.47 426.81 426.81 426.81 426.81 426.81 426.81 426.81 426.81

S16 108.37 89.02 142.97 523.36 593.49 161.45 10000.00 9.39 39.28
    576.12 587.02 587.02 587.02 587.02 587.02 587.02 587.02 587.02

S22 112.63 92.86 148.05 514.56 585.13 153.44 9.39 10000.00 30.15
    567.68 578.50 578.50 578.50 578.50 578.50 578.50 578.50 578.50

S24 135.51 115.32 172.15 490.58 563.34 135.12 39.28 30.15 10000.00
    545.56 555.92 555.92 555.92 555.92 555.92 555.92 555.92 555.92

S26 584.18 576.06 606.20 74.07 19.95 415.47 576.12 567.68 545.56
    10000.00 16.44 16.44 16.44 16.44 16.44 16.44 16.44 16.44

T31 597.16 588.71 619.67 75.59 23.30 426.81 587.02 578.50 555.92 16.44
    0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

T32 597.16 588.71 619.67 75.59 23.30 426.81 587.02 578.50 555.92 16.44
    0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

```

T33	597.16	588.71	619.67	75.59	23.30	426.81	587.02	578.50	555.92	16.44
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T34	597.16	588.71	619.67	75.59	23.30	426.81	587.02	578.50	555.92	16.44
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T35	597.16	588.71	619.67	75.59	23.30	426.81	587.02	578.50	555.92	16.44
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T36	597.16	588.71	619.67	75.59	23.30	426.81	587.02	578.50	555.92	16.44
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T37	597.16	588.71	619.67	75.59	23.30	426.81	587.02	578.50	555.92	16.44
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T38	597.16	588.71	619.67	75.59	23.30	426.81	587.02	578.50	555.92	16.44
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

;

param demand:=

S1 0.85

S2 0.48

S8 0.66

S9 0.77

S12 0.42

S15 0.94

S16 0.37

S22 0.76

S24 0.23

S26 0.84

;