Development of a REILP Approach for Long-Term Planning of WRM System in Saudi Arabia

by

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Abstract

Water resources in Saudi Arabia are very limited. However, the population is steadily growing at a high rate. Since the yearly rainfall rate is very low in most regions of the country, the non-renewable groundwater has exceedingly consumed which resulted in a huge threat for this precious resource. In Saudi Arabia, the largest consumption of water comes from the agricultural, domestic, and industrial sectors, respectively. Without long-term planning and optimal allocation of scarce water resources among a variety of users, the country will continue to face many problems related to water in the long run.

In this study, a risk explicit interval-parameter linear programming (REILP) approach will be developed and applied to the long-term planning of the water resources management (WRM) system in Saudi Arabia. The approach can effectively reflect the interactions between overall cost-benefit and risk level of WRM system.

List of Abbreviations and Symbols Used

$A^{\pm}$	A set of interval parameters of ILP constraints
AY_{ct}	Average yield of crop c in period t
$B^{\pm}$	A set of right-hand side of parameters of ILP
BWC	Best-worst case algorithm
$C^{\pm}$	A set of interval parameters of ILP objective function
CCP	Chance-constrained programming
CDF	Cumulative distribution Function
C_{ij}	Unit cost of water from water resource *i* and consumed by sector *j*
DP	Dynamic programming
DW	Desalinated water
EP_{1j}	Electricity unit price from the desalination plant to the user *j*
FLP	Fuzzy linear programming
FMP	Fuzzy mathematical programming
FRA	Fuzzy Relation Analysis
I_{ict}	Irrigation water quantity from water resource *i* to crop *c* in period *t*
ILP	Interval-parameter linear programming
G20	The Group of Twenty
GLP	Grey linear programming

IPMP	Interval-parameter mathematical programming
IRWR	Internal Renewable Water Resources
ITSFP	Inexact two- stage fuzzy-stochastic programming
ITSP	Inexact two- stage stochastic programming
IWRM	Integrated water resources management
L_{ict}	Allowable land irrigated by water resource i for crop c in planning period t
LP	Linear programming
MED	Multi-effect distillation
MILP	Mix-integer linear programming
MOA	Ministry of Agriculture
MOAW	Ministry of Agriculture and Water
MOEP	Ministry of Economy and Planning
MOFNE	Ministry of Finance and National Economy
MOMRA	Ministry of Municipal and Rural Affairs
MSF	Multi stage flash
MW	Ministry of Water
MOWE	Ministry of Water and Electricity
NGW	Non-renewable groundwater
NRL	Normalized risk level

P	Feasible decision space of ILP solved by 2-Step algorithm
P_{ct}	Price of crop *c* in period *t*
P_{et}	Market price of petrochemical product e in period t
P_{iet}	Quantity of petrochemical products *e* generated through using water resources *i* in period *t*
PDF	Probability distributions function
P_{ij}	Price of one cubic meter of water delivered from water resource *i* to user *j*
P^l	Smallest decision space of ILP solved by 2-Step algorithm
P^u	Largest decision space of ILP solved by 2-Step algorithm
Q	Decision space of ILP solved by BWC algorithm
Q^l	Smallest decision space of ILP solved by BWC algorithm
Q^u	Largest decision space of ILP solved by BWC algorithm
Q_{1jt}	Quantity of generated power from only desalination plants and consumed by sector *j* during period *t*
RAW	Reclaimed agricultural wastewater
REILP	Risk explicit interval-parameter linear programming
RGWSW	Renewable groundwater and surface water
RO	Reverse osmosis
RW	Reclaimed wastewater
SAMA	Saudi Arabian Monetary Agency

SMP	Stochastic mathematical programming
TAGC	Total agriculture cost
TAGR	Total agriculture revenues
TDOC	Total domestic cost
TDOR	Total domestic revenues
$TDoW_t$	Total water demand for the domestic sector in period *t*
TDW_t	Total water supply from the desalination water in period *t*
TINC	Total industrial cost
$TInW_t$	Total water demand for the industrial sector in period *t*
TINR	Total industrial revenues
TLP	Traditional linear programming
TMDL	Total Maximum Daily Load
TNB	Total net-benefit
$TNGW_t$	Total water supply from the non-renewable groundwater in period *t*
$TRGWSW_t$	Total water supply from the renewable groundwater and surface water in period *t*
TRW_t	Total water supply from the reclaimed water in period *t*
USA	United States of America
WRM	Water resources management
WWTP	Wastewater treatment plant

$X^{\pm}$	A set of decision variables of ILP
X_{ijt}	Water allocation from resource i to the user j in the planning period t
$X^{\pm}_{opt}$	Optimum decision variable values
X^{-}_{opt}	Lower bound of optimum decision variable values
X^{+}_{opt}	Upper bound of optimum decision variable values
a_{ij}	Interval parameters of ILP constraints
b_i	Right-hand side of parameters of ILP
c_j	Interval parameters of ILP objective function
$f^{\pm}$	Value of objective function
$f^{\pm}_{jopt}$	Optimum objective function interval
f^{+}_{jopt}	Upper bound of optimum objective function
f^{-}_{jopt}	Lower bound of optimum objective function
$f_{2\text{-}stepopt}$	Optimum objective function solved by 2-Step algorithm
f_{BWCopt}	Optimum objective function solved by BWC algorithm
k_1	Number of positive c_j parameters
λ_0	Aspiration level
λ_{ij}	Risk level variables
ξ	Risk function
x_j	Decision variables of ILP

$x^{\pm}_{jopt}$	Optimum decision variable intervals of x_j
x^{+}_{jopt}	Upper bound of optimum decision variable x_j
x^{-}_{jopt}	Lower bound of optimum decision variable x_j
$^{-}$	“-” superscript represents the lower bound of an interval-parameter or variable
$^{+}$	“+” superscripts represents the upper bound of an interval-parameter or variable

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CHAPTER 1

INTRODUCTION

1.1. Statement of the Problem

1.1.1. Water Resources Management (WRM) System

Water plays an essential role in the survival of all species on planet earth. However, the sources of water in some parts of the world have not had effective management and planning in order to sustain them sufficiently. This is most common in so-called developing countries.

According to Loucks et al. (1981), water resources planning and management are considered as support tools of decision makers for, (1) suitable allocation of water to different users at required locations and times, (2) to safeguard against water crises such as flood and drought and, (3) conservation of adequate water quality. Hence, efficient planning and management for these components can enhance the continuity of this significantly vital source (i.e., water). Additionally, appropriate application of these components can help humanity to protect itself from water resource management related disasters.

Recently, important changes in the water utility industry have exacerbated the challenges of water resource planning and management. The intense rivalry between water use for different purposes, such as drinking, domestic usage, industry and agriculture has been highlighted as one of these important changes (Weng, 2005). Thus, effective application of strategic planning and management of water resources is vitally important in order to deal with these new challenges.

Moreover, The Global Water Partnership (2000) defined Integrated Water Resources Management (IWRM) as "a process which promotes the co-ordinated development and management of water, land and related resources in order to maximise the resultant economic and social welfare in an equitable manner without compromising the sustainability of vital ecosystems". Based on this definition, any useful WRM system should take into account three components: (a) getting the highest economic benefits of the system, if it is possible; (b) fair allocation of water among different users; (c) maintaining the sustainability of critical ecosystems during implementation of the plan. Thus, combining the definition of IWRM with the purposes of water resources planning and management, which have been mentioned above by Loucks et al. (1981), could help decision makers to take the most appropriate approach for enhancing the life expectancy of water resources.

WRM systems include two major factors that should be taken into account in any related research: water quantity and quality (Biswas, 2008). The aim of this study is to obtain an appropriate plan for sufficient water quantity distribution of water resources in Saudi Arabia, and this will be in parallel with the economic benefit of the water system. Therefore, the suggested plan will be focused on WRM parameters and will evaluate the most suitable method to ensure fair allocation to different users. In addition, because WRM has different sources and users, measurements of water allocation should include several economic aspects, such as: (1) land use and required water quantity for agriculture, (2) industrial products that depend on water, (3) domestic water tariff and the alternative resources. . Hence, this research will be of interest to WRM authorities in Saudi Arabia because of its combination of water conservation and economic feasibility strategies.

Furthermore, WRM has environmental and social impacts, in addition to economic effects, which need to be appraised by system analysis methods to create effective system design and operating policies (Yeh, 1996). Consideration of all these parameters in WRM system might make the process complicated, especially if the stakeholders' satisfaction with the whole system is also important. As a result, any study, which deals with WRM, requires the use of optimization techniques to create a general framework that can deal with these complexities (Li et al., 2008). Some of these complexities are called "uncertainties". According to Reckhow and Chapra (1983), uncertainty is defined as "a state or condition of incomplete or unreliable knowledge". Also, these researchers have added that risk and decision analysis are required methods when the uncertainty is exist in relation with the economic outcomes of events such as cost, benefit and loss. Thus, uncertainty often occurs in some present or future data that should be collected for WRM system, and it is preferable to evaluate the associated risks by using optimization methods.

1.1.2. Optimization Model Development for WRM

In the past, optimization methods such as fuzzy (FMP), stochastic (SMP) and interval-parameter mathematical programming (IPMP) were developed in order to deal with the uncertainties in WRM (Slowinski, 1986; Kindler, 1992; Chang et al., 1996; Huang, 1996; Huang, 1998; Ferrero et al., 1998; Huang & Loucks, 2000; Wang et al., 2003; Li et al., 2006; Li et al., 2007; Li et al., 2008; Liu et al., 2011). These methods have been used to generate useful results for decision-makers regarding assumed and real-world dilemmas. Afterwards, suggested solutions should be provided to stakeholders in order to create specific regulations as a process of decision-making.

IPMP is considered as the most useful method, if it would be compared with FMP and SMP in terms of data quality and requirements. This is because FMP demands the information of membership, and SMP requests the distribution of parameters, both of which are not easy to find. Therefore, these methods might not be helpful in dealing with real-world issues, which makes them impractical in dealing with such kind of applications. While at the same time, IPMP does not involve all these previous requirements (Pei, 2011). Interval-parameter linear programming (ILP) is considered as a sub-type of IPMP. In fact, because of the effectiveness of ILP in tackling uncertainties, it has been used in practical applications that are shown as interval values with known upper and lower limits (Han et al., 2011). Because of this advantage, a number of civil and environmental researchers have chosen the ILP in order to deal with modeling issues (Ben-Israel & Robers, 1970; Huang & Moore, 1993; Chinneck & Ramadan, 2000; Yeh & Tung, 2003; Xu et al., 2010). WRM systems are considered as a part of civil and environmental areas that sometimes need to be converted into sub-models (e.g., the ILP) for planning purposes. Various algorithms have been developed in order to assist the treatment of ILP such as: Monte Carlo simulation (Mooney, 1997; Zio, 2012), Best-Worst Case algorithm (BWC) (Tong, 1994; Chinneck & Ramadan, 2000), and 2-Step algorithm (Huang & Moore, 1993).

Monte Carlo simulation is an analytical mathematics method that provides a better understanding of statistic's sampling distribution of random samples (Mooney, 1997). In this method, a classic linear programming (LP) model is formed by randomly determining values for each parameter through their interval range (Pei, 2011). Meanwhile, Monte Carlo simulation is considered as an effective method to provide solutions of interest to complex large-scale problems; however, it requires intensive time (i.e., millions of times) to run the model (Zio, 2012). Therefore, since any real WRM system consists of many decision variables and constraints, it is not often realistic to run its models millions of times.

In order to obtain more reasonable solution methods that can treat the real environmental and civil planning issues, 2-Step algorithm and BWC analysis algorithm have been created (Huang & Moore, 1993; Tong, 1994; Chinneck & Ramadan, 2000). The main purpose of the BWC and 2-Step algorithm is to characterize the most optimistic and the most pessimistic solutions by reframing the original model using extreme constraints. In fact, these algorithms are similar in the way of formulating their sub-models and different in their results. This is because the BWC algorithm is treating all the parameters without discrimination, while in the 2-Step algorithm the choice of extreme parameter values (i.e., lower or upper bounds of coefficients) for decision variables in the objective function is dependent on their different signs (Pei, 2011). To clarify, the 2-Step method divides the x_j to x_j^+ and x_j^-, while the BWC has only one x_j for both reformulated sub-models.

For ILP applications, both the BWC and 2-Step algorithm have been used in significant areas of research. These algorithms can generate an interval solution space that has upper and lower bounds, which helps decision makers to choose any number through this interval solution space. Nevertheless, feasibility and optimality of these algorithms should be checked to see if there are any disadvantages in their solutions. This is because their solutions might be used in the decision alternative for implementation, and this requires certainty about their reliability. Hence, checking the validity of both BWC and 2-Step algorithm is necessary in order to ensure the conformity of the ILP modeling results.

Furthermore, because of the inability of ILP solutions to reflect the connection and trade-off between decision risks and system return under an interval-type uncertainty environment, a risk explicit interval-parameter linear programming (REILP) model has been developed by Zou et al. (2010) in order to achieve this goal and alert decision makers about potential risks of any selection that they may make.

1.1.3. WRM System in Saudi Arabia

Saudi Arabia is considered to be one of the driest and hottest countries in the world, which the average yearly rainfall in the most of the country ranging from 80 mm to 140 mm, and summer temperatures usually surpassing 45 degrees Celsius (Alkolibi, 2002). In addition, water resources in Saudi Arabia are very limited because its desert environment. Meanwhile, the population is consistently growing at a high rate, which means more water is required to cover this increase. In 1970, the population of Saudi Arabia was just 7.7 million, while in 2004, the population reached 22.5 million. This represents an increase of 192% across the period from 1970 to 2004. Furthermore, the population is expected to reach 41 million by 2025 (MOFNE, 2004). Thus, this can reflect how the issue of efficient WRM is critically important in Saudi Arabia. This is even further compounded by the fact that the average consumption rate for each citizen of the country is 248.7 liters per day, which represents the third highest per capita consumption rate in the world after the United States of America (USA) and Canada (SAMA, 2010). Therefore, all these factors could show why it is necessary to have strategic planning to effectively face this challenge.

Prior to 2001, the responsible governmental sectors for water issues in Saudi Arabia were the Ministry of Agriculture and Water (MOAW) and the Ministry of Municipal and Rural Affairs (MOMRA) and other government agenesis (Abderrahman, 2006a). One of the major concerns regarding water was about the groundwater that has been used extensively by drilling wells for agriculture purposes. However, in July 2001, the government of Saudi Arabia decided to establish a new ministry that called the Ministry of Water (MW). Then, in May 2003, the government merged water and electricity sectors under one ministry. This Ministry is called the Ministry of Water and Electricity (MOWE) (Abderrahman, 2006b). One of the most important reasons to make these changes and establish this ministry was to make a long-term plan for water with applying the IWRM principles. Hence, these administrative changes indicate the high-level importance that the government placed on this issue.

According to the Saudi Arabian Monetary Agency (SAMA) (2010), the National Water Plan of Saudi Arabia was issued during the 9th Five-Year Development Plan, which was in the beginning of 2010. This plan was based on detailed water studies of some water formations, and defining water sources and their quantity as well. Furthermore, studying and improving a strategy of IWRM is a key issue in the plan. Therefore, this can be considered as a positive indicator of serious thinking that has started to appear by water authorities regarding the WRM system in Saudi Arabia, and the role of researchers in this area of national planning and development. This research is intended to provide a meaningful contribution to these national planning and development efforts.

In order to design a successful long-term plan for WRM system in Saudi Arabia, some environmental, economic, and social factors should be taken into account. This research will be focused on the economic and environmental factors in parallel with the degree of risk for any decision that might be chosen regarding WRM in Saudi Arabia.

Since 1975, the cost of domestic water production and distribution plus the sanitation services has increase to become more than US$ 100 billion. In addition, the domestic water tariff in Saudi Arabia is very low compared to the costs of providing water, and the entire yearly revenues is approximately equal to 2.5% of annual cost. Moreover, it has been projected that Saudi Arabia will spend about US$ 130 billion due to meeting the growing water needs and its overall utilities by the year 2022 (Abderrahman, 2006a). These factors emphasize the value of finding a better way to reduce the expenses of WRM system, while getting the maximum benefits and rationalize the water use at the same time.

Usually, a balance must be established between the system investment and the environmental impacts. Unfortunately, the environment often gets neglected in this equation while economic growth is prioritized. However, finding an optimal solution that can combine between the economic benefits and being environmentally sustainable is more reliable (Pei, 2011).

Since 1995, Mohorjy and Grigg (1995) were calling to establish and apply a suitable planning for WRM system in Saudi Arabia. In addition, Abderrahman (2001) has emphasized the need for implementing of WRM solutions in balance with energy resources and the significance of them for the future of the country. Hence, most of the studies regarding WRM system in Saudi Arabia have focused on the regulations and general recommendations for the government of Saudi Arabia that have to be taken into account. Alternatively, this research will examine various ILP methods and check their validity to see whether they can be applied on WRM system in Saudi Arabia or not. Also, the claim that ILP could not reflect the linkage between decision risks and system performance will be addressed by the developed REILP approach.

1.2. Objective

This study aims to provide a meaningful contribution to the existing research on WRM system. This will be carried out by developing and applying a risk explicit interval-parameter linear programming (REILP) model on the case study which is the planning and management of Saudi Arabia's water resource systems. This study encompasses the following objectives:

- Check the validity of two ILP solution algorithms, i.e., Best-Worse Case algorithm and the 2-Step interactive algorithm. To find the validities of the two previous algorithms, a numerical example will be created and solved by Monte-Carlo simulation, BWC and 2-Step algorithm individually. Then, compare the interval solutions by concentrating on their feasibility and optimality.

- Development and implementation of a risk explicit interval-parameter linear programming (REILP) model into the case study (i.e., the WRM system in Saudi Arabia), by trying to find the best long-term planning for it. Moreover, this model has been selected because of its effectiveness in reflecting the trade-off between the system return and decision risk. Thus, effective strategic plans could be produced and presented to the authorities in Saudi Arabia, in order to help maintain and enhance the sustainability of the WRM system.

1.3. Structure of the Thesis

The thesis is consisted of seven chapters. Chapter one includes an overview about the main elements of the thesis, which are the WRM system in Saudi Arabia and the ILP models. It is an attempt in to illustrate the reasons of doing this research, and why it is important. In addition, it provides some details about the past and current situation of WRM in Saudi Arabia. As a result, the need for applying an ILP approach, which can provide a long-term planning of WRM system, becomes vital. Moreover, the proposed approach could give a balance between the decision risks and the system return, which is crucial for the decision makers.

Chapter two presents a review of the relevant literature focused on the previous optimization techniques that have been applied to the planning and management of the WRM systems. The main issue is the uncertainty data and its role in the problems that relates to the WRM system, and how the optimization techniques deal with it. Chapter three provides a brief description of 2-Step and Best-Worst Case (BWC) algorithm. This chapter includes a numerical example that aims to illustrate both algorithms, and more importantly, make a comparison between them through examining their solution optimality and feasibility.

Chapter four, the proposed REILP model is described. This contains the model formulation and how it was developed. After that, an example is provided and solved by the REILP to show the advantages of this kind of ILP models. Chapter five presents more details about the critical water situation followed by an explanation for the main elements of the WRM system in Saudi Arabia. These elements include the four water resources and the three users, and the interaction between them represents the decision variables. Then, the model constraints are illustrated to complete the parameters of the ILP model. Then, the model formulation of REILP on this case study is delivered, which includes all the equations and brief explanation for each one.

Chapter six, the results gained from the ILP and REILP model are offered. The difference between two models will be illustrated. Then, a discussion about the consequences of the modeling results to the planning of WRM system in Saudi Arabia is presented. This discussion involves the evaluation of two REILP scenarios from different perspectives. The thesis concludes in chapter seven with a summary of this research followed by conclusions. Then, various recommendations are provided regarding the case study and future studies.

CHAPTER 2

LITERATURE REVIEW

2.1. Previous WRM System and Planning Methods

Since the middle of the 20th century, the non-structural approach has been one of the favourite choices for both water resources managers and scientists. This approach was a kind of cooperation between different sectors such as hydrologists, economists, and sociologists in order to establish a coherent framework leading to the integrity of WRM system (McKinney et al., 1999). Some economic optimization methods have been presented in order to rationalize water use and sustainability in WRM. For example, reconsideration of the price of water that provided to each sector, which could help to save the water amount by modifying its price (Jordan, 1994; Loehman, 2008). Thus, this type of WRM concentrates on what regulations should be applied or modified in order to modify end-user consumption habits. If end-users can be made to understand that excess water usage will require them to pay more, the threat of increased expenditure might persuade them to consume the available water more carefully.

Along with the non-structural approach, several mathematical optimization methods have been used for designing and planning of water resources systems. The linear programming (LP) model was one of the most famous methods to deal with WRM system (Maas et al., 1962; Hall & Dracup, 1970; Major & Lenton, 1979; Jacovkis et al., 1989). Furthermore, Randall and Cleland (1997) developed a mixed-integer linear programming (MILP) model and applied it to the long-term planning of the water supply system for the Alameda County Water District, California. Recently, the MILP has been also used in Syros Island, Greece in order to optimize WRM by minimizing water production and distribution costs (Liu et al., 2012).

Another programming method that has been applied to the WRM field is the dynamic programming (DP) model. Hall and Buras (1961) have used this approach to suggest an 'optimum policy' that could lead to water resources development. Additionally, Georgopoulou et al. (2001) developed a decision aid tool using DP, and recommended its application to the demand-stressed Mediterranean coastal regions for the purpose of finding an alternative water supply strategy. This strategy aimed to develop a sustainable water management plan. Yang et al. (2001) presented a multi-objective optimization model, by using multi-phase linear programming that aimed to compromise and lessen the objectives for the multi-period conjunctive water use optimization model. Their model has been applied to the planning of the sustainable water resources development, which contains ecological, environmental and economic goals for the Shiyang catchment in northwestern China.

As mentioned above, the methods that deal with WRM system have been improved over time while striving to integrate various factors (e.g., environment, economic). However, because of the failure to consider the uncertainty within the traditional deterministic models, Dantzig and Infanger (2011) stated that these kinds of models do not mirror the real dynamic behaviour of practical problems. Hence, it is better to develop models that are capable of dealing with real world problems.

2.2. Optimization Methods That Handle with Uncertainties

While the need for the integration of WRM system has increased, it has been realized that some of the previous optimization techniques are incapable of handling such complex systems. This is because of the number of factors that have complex relationships among them, such as climate, hydrology, environment, society, and economics (Wang, 2005). Also, the difficulties of any WRM scheme are caused by their multiperiod, multilayer, and multiobjective characteristics (Yin et al., 1999). Hence, most of these elements should be taken into account by decision makers in order to plan an effective allocation of water resources incorporating positive economic feasibility.

However, according to the definition of the uncertainty that stated in (section 1.1.1), part of this complexity could be described as uncertainty. This is because the data or the model of the system could be incomplete or unreliable, which emphasized the significant of finding appropriate optimization techniques that can deal with this situation. For this reason, several approaches have been developed and applied to WRM. How each approach has tried to deal with uncertainties has depended on the features of the system. To illustrate, the fuzzy mathematical programing (FMP) is required to use when the model's parameters are fuzzy sets. While the stochastic mathematical programming (SMP) is preferred when the data is controlled by the probability distributions. In contrast, the interval-parameter mathematical programming (IPMP) does not contain all these requirements, and it is applied when the parameters are recognized as intervals with upper and lower bounds (Zou et al., 2010; Pei, 2011).

2.2.1. Fuzzy Mathematical Programming

In 1965, Zadeh defined the fuzzy theory as “a ‘class’ with a continuum of grades of membership”, and each grade is varying from zero to one (Zadeh, 1965). Since that time, many FMP, which were designed based on this theory, have been developed and applied within several real world management systems. These applications include the WRM system. Bogardi et al. (1983) presented one of the first studies that used fuzzy sets in the area of WRM in order to present a multiobjective planning model by merging two objectives (i.e., fuzzy environmental and economic) for the management of an aquifer and mining area under water hazard. Then, an allocation model was designed by using both partially FMP and satisficing theories, which could lead to an agreement between water managers and users regarding water use rationalization choices (Kindler, 1992).

Jairaj and Vedula (2000) applied FMP to study the multireservoir system in the Upper Cauvery River basin, south India, and compared the model results with the stochastic dynamic programming (SDP) model. While in the same year, Raju and Kumar (2000) developed a fuzzy linear programming (FLP) irrigation planning model of Sri Ram Sagar project, Andhra Pradesh, India. This model was designed as a multiobjective model by merging three conflicting goals, which are (1) maximization of net benefits, (2) labor employment, and (3) crop production, in order to solve the planning and management problems. Yin et al. (1999) used Fuzzy Relation Analysis (FRA) model to analyze different alternatives that were related to WRM of the Great Lakes-St. Lawrence River Basin, which is located along the border between the USA and Canada. Nevertheless, FMP has a weak point, which is the difficulty of defining the membership functions of parameters. Consequently, the modeling process could output in unwanted results if there is any imprecision among the membership functions would happen (Pei, 2011).

2.2.2. Stochastic Mathematical Programming

According to Sengupta (1966), an ordinary linear programming is called stochastic linear programming (SLP) when one or more of the parameters whether they are an objective function, constraint, or resource; are recognized only by their probability distributions. When comparing stochastic programming solutions to deterministic approximations (e.g., LP), for actual operations, stochastic programs that explicitly include randomness could be more valuable (Birge, 1995). Though, it is not easy at all to solve a model represented by random variables in all its parameters, and this usually leads to infeasibility issues (Pei, 2011).

The development of stochastic algorithms concerned about the future inflows is considered as one of the biggest challenges in the area of water resource systems optimization (Hooper et al., 1991). Mishra and Desai (2005) have chosen SMP and related models for predicting droughts due to their random nature, which could help in the planning and management of water resources systems.

In 1958, the idea of a new SMP method was presented by Charnes et al. (1958), and they later named it chance-constrained programming (CCP). In this method, the constraints are desired to hold at least a particular level of probability, but not significantly with the probability itself (Seppälä, 1972). These constraints should be uncertain with a confidence level in the right-hand side. In summary, the main function of CCP is to convert the parameters of stochastic optimization model that exist as coefficients of the decision variables or on the right-hand side into a corresponding deterministic optimization model (Moghaddam & DePuy, 2011).

Some types of SMP have merged with other optimization methods (e.g., FMP, IPMP) in order to deal with uncertainty more effectively. For instance, to reflect uncertainties that expressed as both intervals and probability distributions, incorporation between inexact optimization and two-stage stochastic programming has been done by Huang and Lucks (2000), and applied to WRM filed. This method has been called as an inexact two- stage stochastic programming (ITSP), and it aims to make simpler sub-models with fewer computational requirements. Furthermore, because of the restrictions of ITSP in handling more difficulties in WRM, Lu et al. (2008) advanced an inexact two- stage fuzzy-stochastic programming (ITSFP) method for WRM. This method has developed in order to include flexible punishment policies with different rates under several fuzzy events in case of fluctuating runoff levels, which is economically important for both water managers and water users.

In general, SMP has both positive and negative aspects that can arise when it is applied to optimization processes. The most important benefit of the SMP methods is its ability to let the decision makers get a comprehensive view over the impacts of uncertainties in addition to the interactions between uncertain inputs and output results. Of course, this feature is happened because the SMP itself does not easily lessen the complexity of the programming problems (Huang, 1994). On the other hand, SMP is not very effective in dealing with large-scale model. This is because the difficulty of solving such a model with all uncertain parameters that are shown as probability density functions (PDFs), while the non-PDF data cannot immediately apply to SMP models (Lu et al., 2008).

2.2.3. Interval-Parameter Mathematical Programming

In 1966, interval number ($X^{\pm}$) was introduced by Moore as an extension of the real numbers that have been used in the interval analysis (Moore, 1966). The advantage of interval number is to avoid using only crisp number, which gives more flexibility to decision makers in order to make their range of approximations regarding any application parameters (Hajiagha et al., 2013). Then, Interval-parameter mathematical programming (IPMP) has been considered as a development of interval analysis. Moreover, interval-parameter linear programming (ILP) is a branch of IPMP that has been widely applied on different case studies as an optimization technique to deal with uncertainties (Ben-Israel &

Robers, 1970; Ishibuchi & Tanaka, 1990; Huang et al., 1992; Huang & Moore, 1993; Chinneck & Ramadan, 2000; Pei, 2011).

With regards to real-world applications, some advantages of ILP have been found when compared with both SMP and FMP in order to reflect uncertainties (e.g., lower computational requirements, no distribution information or membership functions are required for model parameters, and simplicity of results interpretation) (Ishibuchi & Tanaka, 1990; Huang, 1996; Oliveira & Antunes, 2007). For this reason, a development of ILP was chosen in this research as the main method to apply in the case study. In addition, with recognized lower and upper bounds of uncertainties that are represented as interval values, ILP was recognized as a useful approach in real-world applications to handle these uncertainties that came with an anonymous distribution function (Han et al., 2011).

Regarding previous studies that used ILP methods for WRM system, Huang and Moore (1993) developed an approach of traditional linear programming (TLP) that can handle the uncertainty of both parameters and variables effectively, which they called grey linear programming (GLP). This approach was applied for the first time to WRM in Xiamen, China, in order to solve the water quantity and quality issues. The model of the previous case study has been illustrated in more details by Huang (1996) with a different mode of sensitivity analysis to manifest the trade-offs between environmental and economic goals. Hence, GLP is similar in its proprieties to ILP and it can be considered as another form of ILP.

In 2008, an optimal land use management for better source water protection under uncertainty by using ILP was conducted by Liu et al. (2008). This study was applied to Songhuaba watershed, which is located in southwestern China, in order to achieve several goals. These goals include maximization of both land-use distribution and local economic profits, while reducing the passive environmental impacts and satisfying the future needs of water at the same time.

Aside from these examples of applying ILP on WRM, some studies have combined FLP, SLP, or both of them with ILP in order to enhance the optimization process of WRM under uncertainty. For instance, Chang et al. (1996) integrated ILP and FLP together for treating the complexity of WRM and to develop sustainable management policies. This scheme was recommended to obtain more realistic and flexible optimal solutions for WRM problems. Furthermore, a hybrid inexact-stochastic approach, which merged between ILP and SLP, was developed by Huang (1998) to mix more uncertain information through the model framework. This method was applied to water management model and it was suggested by the author to deal with real-world problems that contain a lot of uncertainties. Similarly, Huang and Loucks (2000) presented a hybrid of ILP and SLP to reflect uncertainties that exist as intervals and probability distributions.

Moreover, incorporation of these three linear programming (i.e., ILP, FLP, and SLP) was introduced by Maqsood et al. (2005) to allocate water efficiently among competing sectors. The main advantage of this method is to effectively merge all different system uncertainties that were expressed as discrete intervals, possibility distributions, and probability distributions into the solution process. Thus, it can be clearly seen how the incorporation or integration of linear programming methods can lead to better schemes in order to tackle uncertainties in WRM and to get more useful results.

In this research, a development of ILP scheme has been chosen to apply on WRM system of Saudi Arabia since this type of programming has an established track-record for effectively dealing with such systems. Validity checking of three different ILP algorithms will be conducted, and these algorithms are Monte Carlo simulation algorithm, 2-step algorithm, and best-worst case (BWC) algorithm. The purpose of this checking is to compare their feasible and optimal space solutions and determine if there are any negative flaws that could affect their use. Regarding the first algorithm, Rubinstein (1981) has described Monte Carlo simulation as a method that aims to generate the outcomes by depending on the iterative random sampling. However, when dealing with a considerable number of uncertainties that exist in complicated systems, the method becomes undesirable because of its huge computational requirement (Pei, 2011).

On the other hand, Huang and Moore (1993) and Tong (1994) have presented 2-Step algorithm and BWC analysis algorithm, respectively; in order to deal with complicated planning issues and get more appropriate solution. The main purpose of both algorithms is to reframe the original model into two sub-models using extreme constraints and get the most conservative and the most aggressive solutions of the model. However, the results of these algorithms are not equal because they differ in how their sub-models are formulated. Moreover, Zhou et al. (2009) conducted a study to compare various ILP models, and found that BWC and 2-Step algorithms have limitations. Thus, ensuring the validity of these two algorithms is required to prove their applicability in the complicated systems and whether they include an indicator for degree of risk or not. This could be realized by comparing their optimal results with Monte Carlo simulation result, and by checking the feasibility of their results.

CHAPTER 3

CHECKING THE VALIDITY OF ILP ALGORITHMS

3.1. Current ILP Solution Algorithms

Definition 3.1.1: An ***interval-parameter linear programming (ILP)*** model (for maximized problems) is defined as (Huang et al., 1992):

$$Max\ f^{\pm} = C^{\pm} X^{\pm} \tag{3.1.1}$$

$$s.t.\ \ A^{\pm} X^{\pm} \le B^{\pm} \tag{3.1.2}$$

$$X^{\pm} \ge 0 \tag{3.1.3}$$

where,

$C^{\pm} = [c_1^{\pm}, c_2^{\pm}, ..., c_n^{\pm}]$

$X^{\pm} = [x_1^{\pm}, x_2^{\pm}, ..., x_n^{\pm}]^T$

$B^{\pm} = [b_1^{\pm}, b_2^{\pm}, ..., b_m^{\pm}]^T$

$A^{\pm} = \{a_{ij}^{\pm}\}$, i=1, 2, …, m; j=1, 2,…, n.

For minimized problems, the ILP model is as the following:

$$Min\ f^{\pm} = C^{\pm} X^{\pm} \tag{3.1.4}$$

$$s.t. \quad A^{\pm}X^{\pm} \geq B^{\pm} \tag{3.1.5}$$

$$X^{\pm} \geq 0 \tag{3.1.6}$$

Where the superscripts “-” and “+” represent lower and upper bounds of an interval-parameter or variable, respectively.

For the ILP problems, their optimal solutions will be intervals because the objective function and constraints have interval parameters that reflect uncertainties in the model. The optimal solutions are written such that:

$$f_{opt}^{\pm} = [f_{opt}^{-}, f_{opt}^{+}] \tag{3.1.7}$$

$$X_{opt}^{\pm} = [x_{1opt}^{\pm}, x_{2opt}^{\pm}, \ldots, x_{nopt}^{\pm}] \tag{3.1.8}$$

$$x_{jopt}^{\pm} = [x_{jopt}^{-}, x_{jopt}^{+}], \quad j = 1, 2, \ldots, n \tag{3.1.9}$$

Due to the existence of intervals in the ILP model, software could not directly solve it. As a result, switching the ILP models into their deterministic formats will help the software to identify and solve the model. In this study, three solution algorithms were used for solving ILP models. These algorithms are: (1) Monte Carlo simulation algorithm, (2) 2-Step algorithm, and (3) Best-Worst Case (BWC) algorithm. They are explained in the following context.

3.1.1 Monte Carlo Simulation Algorithm

Monte Carlo simulation is an analytical computational algorithm that presents an understanding of the statistic's sampling distribution of random samples (Mooney, 1997). In this method, a classic linear programming (LP) model is formed by randomly determining values for each parameter through their interval range (Pei, 2011). Monte Carlo simulation is considered as an effective method to solve a variety of different scale problems; however, it requires intensive time (i.e., millions of times usually) to run the model (Zio, 2012). Therefore, it is quite often not realistic to apply this method to large-scale real world environmental systems planning and management problems (such as WRM). However, for small ILP problems with few decision variables and constraints, the Monte Carlo simulation can produce relatively accurate solutions within acceptable computation time, and this merit makes it a superior algorithm be used in in the validity checking of the other ILP algorithms.

3.1.2 2-Step Algorithm

The 2-Step algorithm was developed by Huang and Moore in 1993 (Huang & Moore, 1993).This algorithm consists of two interactive steps. The original ILP model needs to be divided into two sub-models, which correspond to the upper and lower bound of the objective function. For a maximization problem, the sub-model in correspondence to the upper bound of the objective function should be solved firstly followed by solving the sub-model that is equivalent to the lower bound of the objective function (Huang & Moore, 1993).

In the initial objective function $f^{\pm} = C^{\pm}X^{\pm}$ of the ILP model, if k_1 of the *n* interval coefficients $c_j^{\pm}$ (where $j = 1, 2, \ldots, n$) are not negative, but the remainder are negative, the *n* coefficients can be reorganized such that: $c_j^{\pm} \geq 0$ (where $j = 1, 2, \ldots, k_1$) and $c_j^{\pm} < 0$ (where $j = k_1+1, k_1+2, \ldots, n$). When the objective function is maximizing, the sub-model that is related to the upper bound is expressed as:

$$Max \quad f^{+} = \sum_{j=1}^{k_1} c_j^{+} x_j^{+} + \sum_{j=k_1+1}^{n} c_j^{+} x_j^{-} \tag{3.1.10}$$

$$s.t. \quad \sum_{j=1}^{k_1} \left|a_{ij}\right|^{-} Sign(a_{ij}^{-}) x_j^{+} + \sum_{j=k_1+1}^{n} \left|a_{ij}\right|^{+} Sign(a_{ij}^{+}) x_j^{-} \leq b_i^{+}, \quad \forall i \tag{3.1.11}$$

$$x_j^{\pm} \geq 0, \quad \forall j \tag{3.1.12}$$

The sub-model (3.1.10)-(3.1.12) can be considered as a classic linear programing model. Usually, this kind of model can be easily solved by any present algorithm (e.g. the Simplex method). Then, the second step is to solve the sub-model that is related to the lower bound of the objective function. This sub-model is expressed as:

$$Max \quad f^{-} = \sum_{j=1}^{k_1} c_j^{-} x_j^{-} + \sum_{j=k_1+1}^{n} c_j^{-} x_j^{+} \tag{3.1.13}$$

$$s.t. \quad \sum_{j=1}^{k_1} \left|a_{ij}\right|^{+} Sign(a_{ij}^{+}) x_j^{-} + \sum_{j=k_1+1}^{n} \left|a_{ij}\right|^{-} Sign(a_{ij}^{-}) x_j^{+} \leq b_i^{-}, \quad \forall i \tag{3.1.14}$$

$$x_j^{\pm} \geq 0, \quad \forall j \tag{3.1.15}$$

$$x_j^{-} \leq x_{jopt}^{+}, \quad j = 1, 2, \ldots, k_1 \tag{3.1.16}$$

$$x_j^{+} \geq x_{jopt}^{-}, \quad j = k_1+1, k_1+2, \ldots, n \tag{3.1.17}$$

where x_{jopt}^{+} and x_{jopt}^{-} , in which j is $(\mathrm{j} = 1,2,\ldots,\mathrm{k}_1)$ and $(\mathrm{j} = \mathrm{k}_1 + 1, \mathrm{k}_1 + 2, \ldots, \mathrm{n})$, respectively, are the optimal solutions produced by the sub-model (3.1.10)-(3.1.12).

The sub-model (3.1.13)-(3.1.17) is as a classic linear programing, which can be easily solved by simplex method. Hence, the optimal interval solutions of the ILP model that is found by the 2-Step method could be written as $x_{jopt}^{\pm} = [x_{jopt}^{-}, x_{jopt}^{+}]$ and $f_{opt}^{\pm} = [f_{opt}^{-}, f_{opt}^{+}]$. For minimization applications, the procedure for arranging two sub-models is opposite. The first step of the process is to formulate the sub-model corresponding to the lower bound of the objective function:

$$Min \quad f^{-} = \sum_{j=1}^{k_1} c_j^{-} x_j^{-} + \sum_{j=k_1+1}^{n} c_j^{-} x_j^{+} \tag{3.1.18}$$

$$s.t. \quad \sum_{j=1}^{k_1} \left| a_{ij} \right|^{+} Sign(a_{ij}^{+}) x_j^{-} + \sum_{j=k_1+1}^{n} \left| a_{ij} \right|^{-} Sign(a_{ij}^{-}) x_j^{+} \le b_i^{+}, \quad \forall i \tag{3.1.19}$$

$$x_j^{\pm} \ge 0, \quad \forall j \tag{3.1.20}$$

Next, the sub-model corresponding to the lower bound of the objective function can then be formulated as:

$$Min \quad f^{+} = \sum_{j=1}^{k_1} c_j^{+} x_j^{+} + \sum_{j=k_1+1}^{n} c_j^{+} x_j^{-} \tag{3.1.21}$$

$$s.t. \quad \sum_{j=1}^{k_1} \left| a_{ij} \right|^{-} Sign(a_{ij}^{-}) x_j^{+} + \sum_{j=k_1+1}^{n} \left| a_{ij} \right|^{+} Sign(a_{ij}^{+}) x_j^{-} \le b_i^{-}, \quad \forall i \tag{3.1.22}$$

$$x_j^{\pm} \ge 0, \quad \forall j \tag{3.1.23}$$

$$x_j^+ \geq x_{jopt}^-, \quad j = 1,2,...,k_1 \tag{3.1.24}$$

$$x_j^- \leq x_{jopt}^+, \quad j = k_1 + 1, k_1 + 2,...,n \tag{3.1.25}$$

By solving two sub-models (3.1.18)-(3.1.25), the optimal interval solutions of the original ILP model could be obtained.

3.1.3. Best-Worst Case Algorithm

Similar to the 2-Step algorithm, the Best-Worst Case algorithm (BWC) (Tong, 1994; Chinneck & Ramadan, 2000) can solve the ILP model by converting it into two sub-models. However, there is a major difference between the two algorithms. In the 2-Step algorithm, the choice of extreme parameter values (i.e., lower or upper bounds of coefficients) for decision variables in the objective function is based on their different signs (i.e., positive or negative coefficients inc_j). In contrast, no discrimination exists in the treatment of BWC for all the parameters (Pei, 2011). In other words, the BWC has only one x_j for both sub-models, while 2-Step method divides the x_j to x_j^+ and x_j^-, which will be applied again as a constraint in the second sub-model.

For a maximization problem, the solution of the model begins with formulating the Best-Case sub-model that is equivalent to the upper bound of the objective function as:

$$\text{Max} \quad f^+ = c_j^+ x_j \tag{3.1.26}$$

$$\text{s.t.} \quad a_{ij}^- x_j \leq b_i^+, \quad \forall i \tag{3.1.27}$$

$$x_j \geq 0, \quad \forall j \tag{3.1.28}$$

Then, the Worst-Case sub-model that is equivalent to the lower bound of the objective function is formulated as follows:

$$\text{Max} \quad f^- = c_j^- x_j \tag{3.1.29}$$

$$\text{s.t.} \quad a_{ij}^+ x_j \le b_i^-, \quad \forall i \tag{3.1.30}$$

$$x_j \ge 0, \quad \forall j \tag{3.1.31}$$

For a minimization problem, the best-case sub-model corresponds to the minimization of the lower bound of the objective function of the original ILP model:

$$\text{Min} \quad f^- = c_j^- x_j \tag{3.1.32}$$

$$\text{s.t.} \quad a_{ij}^+ x_j \ge b_i^-, \quad \forall i \tag{3.1.33}$$

$$x_j \ge 0, \quad \forall j \tag{3.1.34}$$

Then, the worst-case sub-model is formulated to minimize the upper bound of the objective function of the original ILP model:

$$\text{Min} \quad f^+ = c_j^+ x_j \tag{3.1.35}$$

$$\text{s.t.} \quad a_{ij}^- x_j \ge b_i^+, \quad \forall i \tag{3.1.36}$$

$$x_j \ge 0, \quad \forall j \tag{3.1.37}$$

The above model formulations show that the BWC algorithm is designed to obtain the "best" or "worst" extreme solutions of the original ILP model. To be specific, the best-case sub-model of a maximization problem, as described in the model (3.1.26-3.1.31), aims to get the values of the upper bound of the original ILP model (3.1.26), while the constraints endeavor to assign the largest decision space from which to search for the optimal solution (3.1.27); the worst-case sub-model aims to find the values of the lower bound of the original ILP model (3.1.29), and its constraints limit the minimum decision space. On the contrary, the best-case sub-model of a minimization problem, as described in the model (3.1.32-3.1.37), aims to find the values of the lower bound of the original ILP model (3.1.32), while the constraints (3.1.33) endeavor to assign a smallest decision space. The worst-case sub-model seeks to find the values of the upper bound of the original ILP model (3.1.35), and its constraints gives the largest decision space (Pei, 2011).

3.1.4. Comparison between 2-Step and BWC Algorithms

(1) Model Equivalence

Theorem 3.1.1: For an ILP problem, the BWC algorithm corresponds to the 2-Step algorithm if and only if $\mathrm{Sign}\left(a_{ij}^{\pm}\right) = \mathrm{Sign}\left(c_j^{\pm}\right), \qquad \forall i, j$ (Pei, 2011):

$$\mathrm{Max} \quad f^{\pm} = \sum_{j=1}^{k_1} c_j^{\pm} x_j^{\pm} + \sum_{j=k_1+1}^{n} c_j^{\pm} x_j^{\pm} \tag{3.1.38}$$

$$\mathrm{s.t.} \quad \sum_{j=1}^{k_1} a_{ij}^{\pm} x_j^{\pm} + \sum_{j=k_1+1}^{n} a_{ij}^{\pm} x_j^{\pm} \leq b_i^{\pm}, \qquad \forall i \tag{3.1.39}$$

$$x_j^{\pm} \geq 0, \qquad \forall j \tag{3.1.40}$$

Proof: A general ILP model (3.1.38-3.1.40) is provided as a clarifying example to prove the theorem 3.1.1. Let assume that $c_j^{\pm} > 0$ for *j=1,2,...,k₁* and $c_j^{\pm} < 0$ for *j= k₁+1, k₁+2,...,n.* In addition, let $\mathrm{Sign}\left(c_j^{\pm}\right) = 1$ for *j=1, 2,..., k1* and $\mathrm{Sign}\left(c_j^{\pm}\right) = -1$ for *j= k₁+1, k₁+2,..., n*. Therefore, by referring to the condition of Theorem 3.1.1, it is found that

$\mathrm{Sign}\left(a_{ij}^{\pm}\right) = \mathrm{Sign}\left(c_j^{\pm}\right) = 1$ for $j=1,2,\ldots,k_1$ and $\mathrm{Sign}\left(a_{ij}^{\pm}\right) = \mathrm{Sign}\left(c_j^{\pm}\right) = -1$ for $j= k_1+1, k_1+2,\ldots,n$. The detailed proof of Theorem 3.1.1 is given below:

Sub-model #1(best-case sub-model objective function):

$$\begin{aligned} \mathrm{Max} \quad f^{+} &= \sum_{j=1}^{k_1} c_j^{+} x_j^{+} + \sum_{j=k_1+1}^{n} c_j^{+} x_j^{-} \qquad (3.1.41) \\ &= \sum_{j=1}^{k_1} c_j^{+} x_j + \sum_{j=k_1+1}^{n} c_j^{+} x_j \\ &= \sum_{j=1}^{n} c_j^{+} x_j \end{aligned}$$

Sub-model #2(worst-case sub-model objective function):

$$\begin{aligned} \mathrm{Max} \quad f^{-} &= \sum_{j=1}^{k_1} c_j^{-} x_j^{-} + \sum_{j=k_1+1}^{n} c_j^{-} x_j^{+} \qquad (3.1.42) \\ &= \sum_{j=1}^{k_1} c_j^{-} x_j + \sum_{j=k_1+1}^{n} c_j^{-} x_j \\ &= \sum_{j=1}^{n} c_j^{-} x_j \end{aligned}$$

In the first step of the proof, the two algorithms should provide equivalent objective functions to each other by reformulating the two sub-models. The 2-Step algorithm rearranges the decision variables according to the signs of the equivalent coefficients ($c_j^{\pm}$); while at the same time it reframes the objective function of the original ILP model, as shown in model (3.1.41 and 3.1.42). Hence, it can be clearly noticed that the upper-bound objective function (i.e., f^{+}) reframed by the 2-Step algorithm corresponds to the objective function of the best-case sub-model from the BWC algorithm, as mentioned in model (3.1.41). In addition, the lower-bound objective function (i.e., f^-) reformulated by the 2-Step algorithm corresponds to the objective function of the worst-case sub-model from the BWC algorithm, as shown in model (3.1.42). However, the upper- or lower-bound sign related to the decision variables could be ignored mathematically, as the only treatment, without changing the model formulation.

In addition to the proof of the equivalent objective functions for both algorithms, the corresponding decision spaces bounded by their own constraints should prove that they are identical to each other under the same condition ofSign $(a_{ij}^{\pm}) = \text{Sign } (c_j^{\pm})$.

The feasible decision space of 2-Step algorithm for the sub-model #1 (f^+) is defined as:

$$\mathrm{P}^u = \left\{ X^{\pm} \middle| \sum_{j=1}^{k_1} \left|a_{ij}\right|^- Sign(a_{ij}^{\pm}) x_j^+ + \sum_{j=k_1+1}^{n} \left|a_{ij}\right|^+ Sign(a_{ij}^{\pm}) x_j^- \leq b_i^+, x_j^-, x_j^+ \in X^{\pm}, X^{\pm} \geq 0, \forall i \right\} \quad (3.1.43)$$

The feasible decision space of BWC algorithm for the best-case sub-model is defined as:

$$Q^u = \left\{ X^{\pm} \middle| \sum_{j=1}^{k_1} a_{ij}^- x_j + \sum_{j=k_1+1}^{n} a_{ij}^- x_j \leq b_i^+, x_j^-, x_j^+ \leq b_i^+, x_j^-, x_j^+, \in X^{\pm}, X^{\pm} \geq 0, \forall i \right\} \quad (3.1.44)$$

Since the condition is $Sign(a_{ij}^{\pm}) = Sign(c_j^{\pm})$, it is known that Sign $(a_{ij}^{\pm})$ =1 for $j=1,2,\ldots,k_1$ and Sign $(a_{ij}^{\pm})$ = -1 for $j = k_1+1, k_1+2,\ldots,n$. Also, $\left|a_{ij}\right|^- = a_{ij}^-$ for $j=1,2,\ldots,k_1$ because all the coefficients are positive, and $\left|a_{ij}\right|^+ = -a_{ij}^-$ for $j = k_1+1, k_1+2,\ldots,n$ because all the coefficients are negative. As a result, the first two parts of model (3.1.43) become:

$$\sum_{j=1}^{k_1} \left|a_{ij}\right|^{-} Sign(a_{ij}^{\pm})x_j^{+} = \sum_{j=1}^{k_1}(a_{ij}^{-})(1)x_j^{+} = \sum_{j=1}^{k_1} a_{ij}^{-}x_j^{+}, \qquad \forall \mathrm{i} \tag{3.1.45}$$

$$\sum_{j=k_1+1}^{n} \left|a_{ij}\right|^{+} Sign(a_{ij}^{\pm})x_j^{-} = \sum_{j=1}^{k_1}(-a_{ij}^{-})(-1)x_j^{-} = \sum_{j=k_1+1}^{n} a_{ij}^{-}x_j^{-}, \quad \forall \mathrm{i} \tag{3.1.46}$$

Now, by combining (3.1.45) and (3.1.46), the outcome will be:

$$P^{u} = \left\{ X^{\pm} \left| \sum_{j=1}^{k_1} a_{ij}^{-}x_j^{+} + \sum_{j=k_1+1}^{n} a_{ij}^{-}x_j^{-} \le b_i^{+}, x_j^{-}, x_j^{+} \in X^{\pm}, X^{\pm} \ge 0, \forall i \right. \right\} \tag{3.1.47}$$

Thus, the final P^u (3.1.47) can be clearly equal to Q^u (3.1.44) if the signs of the upper- or lower-bound that are connected with decision variables are neglected. That also means the feasible decision spaces bounded by two algorithms, respectively, for sub-model #1 and best-case sub-model are identical to each other. Therefore, the 2-Step algorithm could be equal to the BWC algorithm if the condition $Sign(a_{ij}^{\pm}) = Sign(c_j^{\pm})$ is met.

Then, the previous theorem needs to prove that it can be still held only if $Sign(a_{ij}^{\pm}) = Sign(c_j^{\pm})$, which means if there is no equality between the signs, the theorem cannot be applicable. This can be done by assuming that there is another $a_{ij}^{\pm}$ (where *i=p, j=q*), and $Sign(a_{ij}^{\pm}) \ne Sign(c_j^{\pm})$.

If $q > k_1$, the sign of $(a_{pq}^{\pm}) = -1$ and $\left|a_{pq}\right|^{-} = -a_{pq}^{+}$. After that, by following the similar process of equations (3.1.45) to (3.1.47), it is obvious that $P^u \ne Q^u$, which shows that the

2-Step algorithm is not equivalent to the BWC algorithm. Likewise, the condition also holds for $q \le k_1$. As a result, theorem 3.1.1 has been proved because the equivalent situation between the two algorithms is only applicable if $Sign(a_{ij}^{\pm}) = Sign(c_j^{\pm})$. Hence, if any ILP problem fulfills the condition of $Sign(a_{ij}^{\pm}) = Sign(c_j^{\pm}), \forall i, j,$ and is solved by these two algorithms, the solutions would be the same. However, the solutions will be different if the condition is not satisfied.

(2) Feasible Decision Space

Theorem 3.1.2: Assume an ILP problem has interval inequalities as:

$$Q = \left\{ X^{\pm} \left| \sum_{j=1}^{n} a_{ij}^{\pm} x_j \le b_i^{\pm}, x_j \in X^{\pm}, X^{\pm} \ge 0, \forall i \right. \right\} \tag{3.1.48}$$

The largest and smallest feasible decision spaces equivalent to the respective upper bound and lower bound of the objective function can be provided as:

$$Q^u = \left\{ X^{\pm} \left| \sum_{j=1}^{n} a_{ij}^{-} x_j \le b_i^{+}, x_j \in X^{\pm}, X^{\pm} \ge 0, \forall i \right. \right\} \tag{3.1.49}$$

$$Q^l = \left\{ X^{\pm} \left| \sum_{j=1}^{n} a_{ij}^{+} x_j \le b_i^{-}, x_j \in X^{\pm}, X^{\pm} \ge 0, \forall i \right. \right\} \tag{3.1.50}$$

Tong first declared this theorem in 1994, but he did not prove it (Tong, 1994). In 2000, Chinneck and Ramadan have proved it by using the form of minimization problems (Chinneck & Ramadan, 2000).

An ILP model can obtain the largest and smallest feasible decision space by using theorem 3.1.2. Any value between the particular lower and upper bound of the interval parameters can be possibly taken, and this is the practical clarification for an ILP model. When this occurs, the ILP problems convert to a classic LP problem. Also, the classic LP has a feasible decision space placed between the smallest and largest feasible decision space of the original ILP problem. Thus, the classic LP can be defined as an event model of the ILP problem.

Definition 3.1.2: An ***event model*** of an ILP is defined as "a classic LP model where the interval parameters in $A^{\pm}, B^{\pm}$ and $C^{\pm}$ take a specific set of crisp values within their respective lower and upper bounds".Clearly, from this definition, the 2-Step algorithm and BWC algorithm produce two sub-models, which can both be considered as particular event models symbolizing two opposite extreme conditions of the original model, respectively. Meanwhile, Q^u and Q^l mentioned in models (3.1.49)-(3.1.50) represent the constraints of the best-case sub-model and worst-case sub model by BWC, respectively. To illustrate, the largest and smallest feasible spaces of the original ILP model can be extracted from the feasible decision spaces bounded by the BWC algorithm, while the feasible decision spaces produced by the 2-Step algorithm are surrounded by the BWC feasible spaces, and only represent two general event model situations. To conclude, it can be clearly seen that the feasible decision space of an ILP delimited by BWC is larger or equal to (when $Sign(a_{ij}^{\pm}) = Sign(c_j^{\pm})$, $\forall i, j$, according to Theorem 3.1.1) the feasible decision space restricted by the 2-Step algorithm (Pei, 2011).

(3) Optimal Solution

Theorem 3.1.3: The optimal solutions gained by the BWC algorithm can be different from the optimal solutions found by the 2-Step algorithm, such that:

$$f^{+}_{BWCopt} \geq f^{+}_{2-stepopt} \tag{3.1.51}$$

$$f^{-}_{BWCopt} \leq f^{-}_{2-stepopt} \tag{3.1.52}$$

Proof: It has been proved in Theorem 3.1.1, that the two sub-models reframed by the 2-Step algorithm and BWC algorithm are equivalent when $Sign(a^{\pm}_{ij}) = Sign(c^{\pm}_{j})$, $\forall i, j$. This means the solutions gained from both algorithms must be the same (Pei, 2011) as follows: $f^{+}_{BWCopt} = f^{+}_{2-stepopt}$ and $f^{-}_{BWCopt} = f^{-}_{2-stepopt}$

In a general case, when $\exists i, j,\ Sign(a^{\pm}_{ij}) \neq Sign(c^{\pm}_{j})$, this theorem still holds. Moreover, Theorem 3.1.2 shows that Q^{u} in model 3.1.49 represents the feasible decision space for solving the upper bound objective function of the BWC algorithm (i.e. best-case sub-model). Based on Definition 3.1.2, the feasible decision space for solving sub-model #1 from the 2-Step algorithm is surrounded by Q^{u} (i.e. smaller than Q^{u}) when $\exists i, j$, $Sign(a^{\pm}_{ij}) \neq Sign(c^{\pm}_{j})$. As a result, the maximum objective function value gained by the 2-Step algorithm (i.e., $f^{+}_{2-stepopt}$) is equal to or less than that gained by the BWC algorithm (i.e., f^{+}_{BWCopt}), and it can be written as $f^{+}_{2-stepopt} \leq f^{+}_{BWCopt}$. Likewise, the minimum objective function value obtained by the 2-Step algorithm is equal or greater than the one from the BWC algorithm (i.e., $f^{-}_{2-stepopt} \geq f^{-}_{BWCopt}$).

Remark 3.1.1: These two algorithms mentioned above were designed to account for system uncertainties in an ILP problem. According to Theorems 3.1.2 and 3.1.3, the optimal solutions investigated by the 2-Step algorithm are found in a smaller feasible decision space, while the formulation method of the left-hand sides of the sub-model constraints is the reason for the reduction of feasible decision space. This space reduction means that the 2-Step algorithm randomly neglects some system uncertainties to a particular level, which has not been mathematically or theoretically justified. Therefore, the dealing of BWC algorithm with uncertainties presented as intervals appears to be better than the 2-Step algorithm (Pei, 2011).

Remark 3.1.2: Both an interval optimal solution for each decision variable and an interval objective function value, are founded by using the optimal solutions produced from each sub-model of the two algorithms. For instance, in order to solve a maximization ILP problem using the 2-Step algorithm, the upper bound sub-model is firstly solved to get the upper bound solutions for decision variables x^{+}_{jopt} (where $j = 1,2,...,k_1$), and the lower bound solutions for decision variables x^{-}_{jopt} (where $j = k_1 + 1, k_1 + 2,...,n$). After that, the lower bound sub-model is solved to find the lower and upper bound solutions for decision variables (i.e., x^{-}_{jopt} where $j = 1,2,...,k_1$ and x^{+}_{jopt} where $j = k_1 + 1, k_1 + 2,...,n$), respectively. By following these steps, all the solutions will be combined in order to get the final interval optimal solution for the original ILP problem (i.e., $x_{jopt}^{\pm} = [x_{jopt}^{-}, x_{jopt}^{+}], \forall j, j = 1,2,...,n$). Similarly, the optimal value of the objective function of the original ILP model is an interval (i.e., $f_{opt}^{\pm} = [f_{opt}^{-}, f_{opt}^{+}]$), where f_{opt}^{+} and f_{opt}^{-} represent the upper- and lower- bound values of the original objective function, respectively, as follows:

$$f_{opt}^{+} = f(x_{1opt}^{+}, x_{2opt}^{+}, ..., x_{k_1opt}^{+}, x_{(k_1+1)opt}^{-}, x_{(k_1+2)opt}^{-}, ..., x_{(n)opt}^{-}) \tag{3.1.53}$$

$$f_{opt}^{-} = f(x_{1opt}^{-}, x_{2opt}^{-}, ..., x_{k_1opt}^{-}, x_{(k_1+1)opt}^{+}, x_{(k_1+2)opt}^{+}, ..., x_{(n)opt}^{+}) \tag{3.1.54}$$

From the model solutions, and by compromising between the execution of the objective function and constraints, more realistic schemes can be found by decision-makers (Zou et al., 2000)**.** While at the same time, decision makers can select any values through the possible interval ranges to achieve their goal for various policies, which helps them to control the system risk and economic return. (Huang & Moore, 1993; Yeh & Tung, 2003)**.**

3.2. Checking the Validity of 2-Step Algorithm and BWC Algorithms

Because of the contribution of the ILP models in dealing with many practical applications, it is considered as an effective method (Ben-Israel & Robers, 1970; Xu et al., 2010; Han et al., 2011), while the 2-Step and the BWC algorithms are also commonly used to treat these models (Huang & Moore, 1993; Huang, 1998; Chinneck & Ramadan, 2000). It is clearly known from remark 3.1.1 that during reformulation of the sub-model constraints in the 2-Step algorithm, the algorithm itself neglects some of the system uncertainties. This oversight might be considered as a deficiency and may cause feasibility and optimality concerns for the obtained interval optimal solutions. This leads to an attempt to check the validity of these two algorithms (i.e., BWC and 2-Step). In the following context, a numerical example provided to exercise this attempt, which could help testify if there are any infeasible or missing optimal solutions in the obtained interval optimal solution.

3.2.1. A Numerical Example for Validity Checking

To examine the validity of both algorithms, a minimized ILP model with two decision variables and two constraints was created as the numerical example to illustrate the validity checking exercise:

$$Min \quad f = [2.5,4]x_1 + x_2 \tag{3.2.1}$$

$$s.t. \quad x_1 - [1.6,1.8]x_2 \geq [3,4] \tag{3.2.2}$$

$$x_1 + [1.2,1.9]x_2 \geq [6,7] \tag{3.2.3}$$

$$x_1, x_2 \geq 0 \tag{3.2.4}$$

As mentioned before, the validity checking will include a comparison between the optimal solutions provided by Monte Carlo Simulation, 2-step algorithm and BWC algorithm, respectively. Monte Carlo Simulation method requires generating and solving a large number of event models to obtain the optimal solutions. Each of these event models represents a classic deterministic LP model which could be easily solved to obtain a solution set for both decision variables and objective function. The numerical example was solved by Monte Carlo Simulation method first, where 10 million event models were generated and solved, and the obtained interval solutions are: f = [11.85,24.05], x_1 = [4.39, 5.78], x_2 = [0.54, 1.31].

(1) 2-Step Algorithm Solution

According to the 2-step algorithm provided in Section 3.1.2, a minimization problem should start with solving the sub-model that corresponds to the lower bound of the objective function, then solving the sub-model that corresponds to the upper bound of the objective function. This can be described as:

Sub-model #1:

$$Min \quad f^- = 2.5x_1^- + x_2^- \tag{3.2.5}$$

$$s.t. \quad x_1^- - 1.8x_2^- \geq 3 \tag{3.2.6}$$

$$x_1^- + 1.9x_2^+ \geq 6 \tag{3.2.7}$$

$$x_1^-, x_2^- \geq 0 \tag{3.2.8}$$

Sub-model #2:

$$Min \quad f^+ = 4x_1^+ + x_2^+ \tag{3.2.9}$$

$$s.t. \quad x_1^+ - 1.6x_2^+ \geq 4 \tag{3.2.10}$$

$$x_1^+ + 1.2x_2^+ \geq 7 \tag{3.2.11}$$

$$x_1^+, x_2^+ \geq 0 \tag{3.2.12}$$

$$x_1^+ \geq x_{1opt}^- \tag{3.2.13}$$

$$x_2^+ \geq x_{2opt}^- \tag{3.2.14}$$

It can be noted that in sub-model #2, x_{1opt}^- in constraint (3.2.13) and x_{2opt}^- in constraint (3.2.14) have been taken from the optimal solutions of decision variables of Sub-model #1. Both sub-models are classic LP models. As a result, the optimal interval solutions obtained by the 2-step algorithm are $f = [11.96,\ 23.93], x_1 = [4.46,\ 5.71],\ x_2 = [0.81,\ 1.07]$.

(2) BWC Algorithm Solution

According to the BWC algorithm provided in Section 3.1.3, the numerical example should be reformulated to two sub-models that correspond to the best-case and worst-case, respectively. This can be written as:

Best-Case Sub-model:

$$Min \quad f^- = 2.5x_1^- + x_2^- \tag{3.2.15}$$

$$s.t. \quad x_1^- - 1.6x_2^- \geq 3 \tag{3.2.16}$$

$$x_1^- + 1.9x_2^+ \geq 6 \tag{3.2.17}$$

$$x_1^-, x_2^- \geq 0 \tag{3.2.18}$$

Worst-Case Sub-Model:

$$Min \quad f^+ = 4x_1^+ + x_2^+ \tag{3.2.19}$$

$$s.t. \quad x_1^+ - 1.8x_2^+ \geq 4 \tag{3.2.20}$$

$$x_1^+ + 1.2x_2^+ \geq 7 \tag{3.2.21}$$

$$x_1^+, x_2^+ \geq 0 \tag{3.2.22}$$

Both sub-models are classic LP models and could be solved easily. Thus, the optimal interval solutions gained by this algorithm are $f = [11.79,\ 24.20], x_1 = [4.37,\ 5.8]$, $x_2 = [0.86,\ 1.00]$.

3.2.2. Results of Validity Checking and the Interpretation

(1) Optimality Checking

Even though the Monte Carlo Simulation algorithm has been run for 10 million times, the optimal solution space presented by the Monte Carlo Simulation algorithm must be narrower than the real solution space, or might be very close to it (Pei, 2011). Meanwhile, two facts should be taken into account: (1) the optimal solutions of event models of Monte Carlo Simulation represent a series of true optimal solution sets of the original model, where the infeasibility of solution does not exist, (2) the optimal solution space of Monte Carlo Simulation method should be covered by the optimal solution spaces provided by both 2-Step and BWC algorithm. This could be illustrated mathematically as follows:

$$x_{1opt}^{-} \le 4.39 \le 5.78 \le x_{1opt}^{+}, x_{2opt}^{-} \le 0.54 \le 1.31 \le x_{2opt}^{+}, f_{opt}^{-} \le 11.85 \le 24.05 \le f_{opt}^{+} \quad (3.2.23)$$

Hence, if the previous relationship (3.2.23) cannot be fulfilled, two possible interpretations would be suggested: (1) the 2-Step algorithm or the BWC algorithm is missing some optimal solution pairs, (2) some pair points which are included in the optimal solutions of both algorithms are infeasible (Pei, 2011). These two interpretations are important to check the validity of both algorithms in order to decide their applicability for real-world issues. As a result, a comparison is conducted between the results of all algorithms as provided in

Table 3-1. This comparison is based on the interval range of each optimal solution to examine whether it is larger than the Monte Carlo simulation algorithm or smaller.

Table 3-1 Optimal results of all three ILP algorithms

Algorithms	x_1		x_2		Objective Function	
	x_1^-	x_1^+	x_2^-	x_2^+	f^-	f^+
Monte Carlo Simulation	4.39	5.78	0.54	1.31	11.85	24.05
2-Step	**4.46**	**5.71**	**0.81**	**1.07**	**11.96**	**23.93**
BWC	4.37	5.8	**0.86**	**1.00**	11.79	24.20

By referring to Table 3-1, it can be seen clearly that the interval ranges of optimal solutions obtained by 2-Step algorithm are smaller than the ranges of Monte Carlo simulation algorithm. This includes both decision variable x_1, x_2 and objective function f. This indicates that the 2-step algorithm has missed some optimal solution pairs (i.e., x_1, x_2). As a result, the 2-step algorithm fails the validity checking in terms of the solution optimality. The way how the two sub-models are formulated might be the cause of missing some optimal solutions.

On the other hand, it is observed that the interval ranges of optimal solutions gained by the BWC algorithm is larger than the ranges of Monte Carlo simulation algorithm in terms of decision variable x_1 and objective function f. The BWC algorithm was designed to provide the best and worst cases of the ILP model, and this observation is in line with this design. However, the interval optimal solution range of the decision variable x_2 is smaller than that from the Monte Carlo simulation algorithm and 2-step algorithm. This might indicate that some optimal solutions are missing, and this is opposite to the design purpose of the BWC algorithm. It is apparently that the BWC algorithm has its own flaw and cannot get the corresponding extreme solutions. The BWC algorithm fails the validity checking in terms of the solution optimality as well.

(2) Feasibility Checking

Figure 3-1 shows the solution space of the 2-Step algorithm for the numerical example (3.2.1). It includes the feasible decision space and optimal solution space given by this algorithm. To illustrate, each line in this figure represents one constraint given by the sub-models. Line EFGH represents one boundary of the feasible decision space given by the constraint (3.2.6), while line ABCD represents the constraint (3.2.10). On the other hand, two dotted lines located beside each of the previous lines symbolize two BWC sub-models' constraints delimited by the same original constraints. The lines IFBJ and KGCL represent the boundaries of feasible decision space bounded by two sub-models' constraints (3.2.7) and (3.2.11), respectively. In the meanwhile, these lines represent the feasible decision space of BWC sub-models' constraints (3.2.17) and (3.2.21) since the coefficients of both algorithms for these constraints are positive. In Figure 3-1, the quadrangle GFBC represents the feasible optimal solution space generated by 2-Step algorithm, and this quadrangle consists of the intersections of four lines (i.e., KGCL, IFBJ, EFGH, and

ABCD). Hence, the whole feasible decision space, in addition to the quadrangle GFBC, can be divided into different regions:

- The triangle DCL represents the absolute feasible region that satisfies all the constraints.
- The areas of the two triangles which are located above line EFGH and below line IFBJ represent the infeasible regions that violate at least one constraint.
- The area limited by these points in sequence H, G, F, B, J, L, C, D represents the softly feasible region, which indicates that the solution sets (x_1, x_2) are not guaranteed to meet all the constraints.

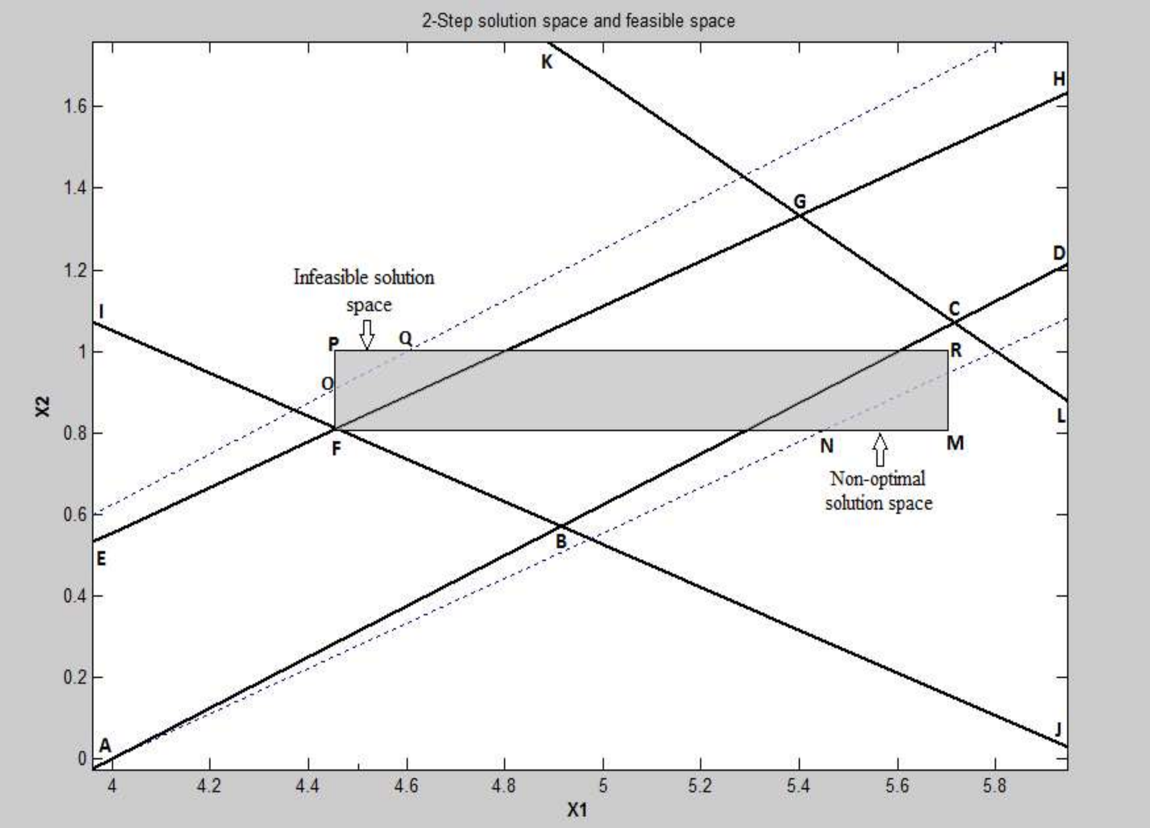

Figure 3-1 2-Step solution space and feasible space

Moreover, the optimal solution region for decision variable sets (x_1, x_2) generated by 2-Step algorithm is the rectangular grey area in Figure 3-1. It can be seen clearly that the majority of the 2-Step optimal solutions are found in the softly feasible region. Also, this rectangle contains an infeasible solution space (i.e., the triangle OPQ), located in the left corner above the dotted line. Therefore, any optimal solution pairs obtained from the 2-Step algorithm and located in this triangle means that at least one constraint has been violated and these solution pairs are infeasible for the numerical example model. On the other hand, this grey rectangle contains a non-optimal solution space in the right corner below the dotted line (i.e., the triangle NRM), which means any solution pairs existed in this area are softly feasible but not optimal.

Figure 3-2 shows the solution space of the BWC algorithm for the numerical example. It includes the feasible decision space and optimal solution space given by the BWC algorithm. To illustrate, each line in this figure represents one constraint given by the sub-models. Line E'F'G'H' represents one boundary of the feasible decision space given by the constraint (3.2.16), while line A'B'C'D' represents constraint (3.2.20). On the other hand, the two dotted lines located beside each of the previous lines symbolize two equivalent constraints from the 2-Step algorithm. The lines I'F'B'J' and K'G'C'L' represent the boundaries of feasible decision space limited by two BWC sub-models' constraints (3.2.17) and (3.2.21), respectively. In the meanwhile, these lines represent the feasible decision space of 2-Step sub-models' constraints (3.2.7) and (3.2.11). In addition, the quadrangle G'F'B'C' represents the feasible optimal solution space generated by the BWC algorithm.

Furthermore, this quadrangle consists of the intersections of four lines (i.e. K'G'C'L', I'F'B'J', E'F'G'H', and A'B'C'D').

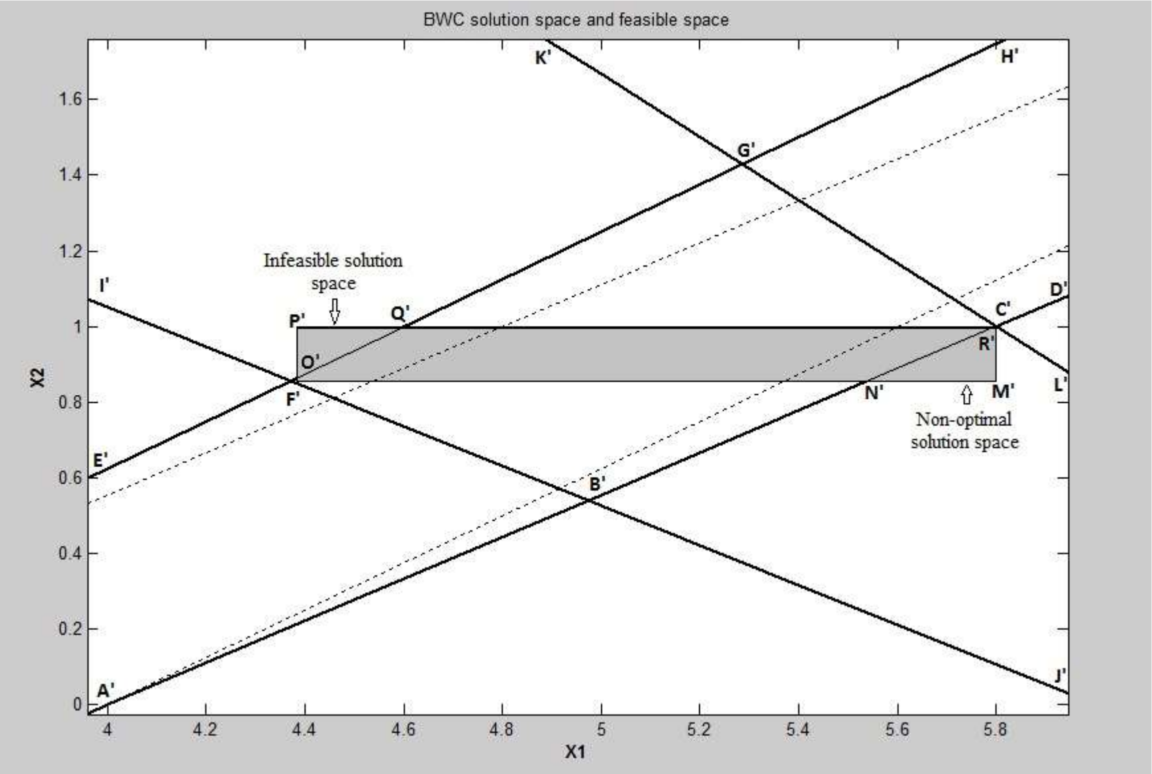

Figure 3-2 BWC solution space and feasible space

Thus, the whole feasible decision space, in addition to the quadrangle G'F'B'C', can be divided into various categories:

- The triangle D'C'L' represents the absolute feasible region that satisfies all the constraints.
- The areas of the two triangles which are located above line E'F'G'H' and below line I'F'B'J' represent the infeasible regions that violate at least one original constraint.

- The area limited by these points in sequence H', G', F', B', J', L', C', D' represents the softly feasible region, where the solution sets (x_1, x_2) are not guaranteed to meet all the constraints.

Furthermore, the rectangular grey area in

Figure 3-2 represents the optimal interval solution provided by the BWC algorithm. Obviously, the majority of the BWC optimal solutions are found in the softly feasible region, as well as the 2-Step optimal solutions. Also, this rectangle contains an infeasible solution space (i.e., triangle O'P'Q') which is located in the left corner and above line E'F'G'H'. On the other side, this grey rectangle generated by the BWC algorithm contains also a non-optimal solution space, which is located at the right corner and below the A'B'C'D' line (i.e., the triangle N'R'M').

The above validity checking results indicate that both 2-step algorithm and BWC algorithm have flaws, i.e., both algorithms could generate infeasible solutions and/or non-optimal solutions. When the modeling results are applied to practical problems, the decision makers should take extra cautions regarding these infeasible solutions and non-optimal solutions to reduce the level of risks caused by these decisions.

Based on these results from the numerical example that has been designed to examine the validity of 2-Step algorithm and BWC algorithm, two conclusions are evident: (1) the optimal solutions generated by the two algorithms (i.e., 2-Step algorithm and BWC algorithm) are not valid all the time, and might include some infeasible solutions space which can be appeared in portion of the results, (2) the gained interval solutions miss or do not include some optimal solutions. Hence, these algorithms are not valid for application into real-world problems. In order to tackle this situation, two options are suggested: (1) redeveloping both algorithms in order to avoid their limitations that appear in the optimal and feasible solution, (2) finding a way to merge some useful components such as taking the decision risks into account, especially if these algorithms are going to deal with uncertainty in real life applications.

CHAPTER 4

REILP MODEL DEVELOPMENT

4.1. REILP Modeling Approach

Since the validity checking of ILP algorithms, which was provided in the previous chapter, has proved that some flaws exist in the solutions of 2-Step and BWC. These flaws include non-feasible and non-optimal solutions which threatened the violation of the model’s constraints. This result made the dependence on these algorithms to be not the best option for decision makers particularly if they are going to implement these ILP schemes in real-world problems. In addition, these algorithms lack the ability to address the correlations between the decision risks and system performance, which is quite significant from the decision makers’ standpoints. Therefore, developing a new algorithm or improving existing ones that can overcome these flaws is required in order to deal with realistic issues.

For this reason, this research has chosen a Risk Explicit Interval Linear Programming (REILP) to be the methodology that will be applied to the WRM system in Saudi Arabia. This method improves the existing ILP method through combining the advantages of ILP algorithms and the risk degree of each suggested solution into one modeling framework. The REILP model was developed by Zou et al. (2010) and is provided below:

By referring to Definition 3.1.1, which describes the original ILP, an event model can be expressed as:

$$Max \qquad f = \sum_{j=1}^{n}[c_j^- + \lambda_0(c_j^+ - c_j^-)]x_j \tag{4.1.1}$$

$$s.t. \qquad \sum_{j=1}^{n}[a_{ij}^+ - \lambda_{ij}(a_{ij}^+ - a_{ij}^-)]x_j - [b_i^- + \eta_i(b_i^+ - b_i^-)] \le 0, \forall i \tag{4.1.2}$$

$$x_j \ge 0, \forall j \tag{4.1.3}$$

$$0 \le \lambda_0 \le 1 \tag{4.1.4}$$

$$0 \le \lambda_{ij} \le 1, \forall i, j \tag{4.1.5}$$

$$0 \le \eta_i \le 1, \forall i \tag{4.1.6}$$

Apparently, equations (4.1.1) to (4.1.6) represents a classic LP model, which corresponds to a specific set of crisp value of each coefficient given λ_0, λ_{ij} and η_i. The terms of equations (4.1.1) to (4.1.6) can be reorganized as:

$$Max \qquad f = \sum_{j=1}^{n}[c_j^- x_j + \lambda_0(c_j^+ - c_j^-)x_j] \tag{4.1.7}$$

$$s.t. \qquad \sum_{j=1}^{n} a_{ij}^+ x_j - b_i^- \le \sum_{j=1}^{n} \lambda_{ij}(a_{ij}^+ - a_{ij}^-)x_j + \eta_i(b_i^+ - b_i^-), \forall i \tag{4.1.8}$$

$$x_j \ge 0, \forall j \tag{4.1.9}$$

$$0 \le \lambda_0 \le 1 \tag{4.1.10}$$

$$0 \le \lambda_{ij} \le 1, \forall i, j \tag{4.1.11}$$

$$0 \le \eta_i \le 1, \forall i \tag{4.1.12}$$

Let $\mu = \lambda_0(c_j^+ - c_j^-)x_j$, and $\xi_i = \sum_{j=1}^{n} \lambda_{ij}(a_{ij}^+ - a_{ij}^-)x_j + \eta_i(b_i^+ - b_i^-)$, where $i = 1,2,...,m$. The model can then be reformulated as:

$$Max \quad f = \sum_{j=1}^{n} c_j^- x_j + \mu \tag{4.1.13}$$

$$s.t. \quad \sum_{j=1}^{n} a_{ij}^+ x_j - b_i^- \le \xi_i, \forall i \tag{4.1.14}$$

$$x_j \ge 0, \forall j \tag{4.1.15}$$

When μ and ξ_i are equal to 0, the model (4.1.13) to (4.1.15) becomes the worst-case sub-model of the BWC algorithm (i.e., most pessimistic case). The solution resulted from this case in an interval decision environment would have no risk of violating the constraints. This is because the formulation of this model has guaranteed satisfying the narrowest constraints. On the other hand, when ξ_i gets values greater than 0, the constraints become relaxed to allow obtain optimal solutions for fulfilling higher system return. Meanwhile, a specific level of risk to violate the constraints would be appeared in the solution itself. Clearly, the larger the ξ_i, the higher risk would be correlated with the solutions until ξ_i reaches these values: $\lambda_{ij} = 1 \ (\forall i, j)$ and $\eta_i = 1 \ (\forall i)$, which represent the best-case sub-model of the BWC algorithm (i.e., most optimistic case). Hence, the ξ_i can be an indicator for the risk level of a decision in terms of violating the constraints.

Definition 4.1.1: Function $\xi_i = \sum_{j=1}^{n} \lambda_{ij}(a_{ij}^+ - a_{ij}^-)x_j + \eta_i(b_i^+ - b_i^-)$ is defined as the *risk function* for constraint *i* in an ILP problem.

From Equations (4.1.13) to (4.1.15), it can be understood that (1) when $\xi_i = 0$, there is no risk of violating the corresponding constraint by the decision which is based on the optimal solution, and (2) when $\xi_i > 0$, there is a level of risk of violating the corresponding constraint by the decision which is based on the optimal solution. This level of risk is proportion to the value of ξ_i.

While the original ILP is to maximize the objective function or, in other words, system returns (i.e., equation (4.1.1)), the decision risk represents an opposite factor in practical decision making. Thus, a favorable decision would be obtained through maximizing the system return while minimizing the risk function. As a result, this could lead to a multi-objective optimization problem as follows:

$$Max \quad f = \sum_{j=1}^{n} c_j^- x_j + \mu \tag{4.1.16}$$

$$Min. \quad \xi_i = \oplus_i \left[\sum_{j=1}^{n} \lambda_{ij} (a_{ij}^+ - a_{ij}^-) x_j + \eta_i (b_i^+ - b_i^-) \right] \tag{4.1.17}$$

$$s.t. \quad \sum_{j=1}^{n} a_{ij}^+ x_j - b_i^- \le \xi_i, \forall i \tag{4.1.18}$$

$$x_j \ge 0, \forall j1 \tag{4.1.19}$$

Where $\oplus$ is a general arithmetic operator which can be a simple addition, a weighted addition, simple arithmetic mean, weighted arithmetic mean, or a max operator. The subscript for $\oplus_i$, suggests that the operator would be implemented across constraints for the whole optimization problem in order to acquire a unified risk function. Hence, the multi-objective programming problem can be solved by reformulating the model as:

$$\text{Min.} \quad \xi = \oplus_i \left[\sum_{j=1}^{n} \lambda_{ij} (a_{ij}^{+} - a_{ij}^{-}) x_j + \eta_i (b_i^{+} - b_i^{-}) \right] \tag{4.1.20}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} c_j^{-} x_j + \mu \geq f_{opt}^{-} + \lambda_0 (f_{opt}^{+} - f_{opt}^{-}) \tag{4.1.21}$$

$$\sum_{j=1}^{n} a_{ij}^{+} x_j - b_i^{-} \leq \xi_i, \forall i \tag{4.1.22}$$

$$\lambda_0 = \lambda_{pre} \tag{4.1.23}$$

$$0 \leq \lambda_{ij} \leq 1 \tag{4.1.24}$$

$$x_j \geq 0, \forall j \tag{4.1.25}$$

$$0 \leq \eta_i \leq 1, \forall i \tag{4.1.26}$$

Definition 4.1.2: The optimization model (4.1.20) to (4.1.26), which is derived from the original ILP model and includes an objective function of risk-minimization, is defined as a Risk Explicit ILP (REILP) model.

After defining the REILP model, some elements of this method need to be illustrated. First of all, the constraint-wise risk function $\sum_{j=1}^{n} \lambda_{ij} (a_{ij}^{+} - a_{ij}^{-}) x_j + \eta_i (b_i^{+} - b_i^{-})$ for each individual constraint can vary from that of another constraint by order of magnitude due to different categories of b_i as well as the incorporation of interactions among $\lambda_{ij}, a_{ij}^{+}, a_{ij}^{-}, x_j, \eta_i, b_i^{+}$ and b_i^{-} in the function. Consequently, converting the constraint-wise risk function into comparable quantity becomes necessary. A feasible option is to scale each constraint-wise risk function by $\frac{1}{b_i^{-}}$, which basically represents a fractional risk factor from the most pessimistic situation. However, more revised scheme can be improved in implementation in order to best mirror the decision environment for the specific case.

Another REILP element is the aspiration level of decision makers given the uncertainty in the optimization model, which is denoted as λ_0 and also represents the degree of aggressiveness. Thereby, when $\lambda_0 = 0$, the model is corresponding to the least aggressive situation where the most conservative and safe solution is anticipated. Vice versa, when $\lambda_0 = 1$, the model is corresponding to the most aggressive situation where the most optimistic solutions but meanwhile risky solutions are to be expected. Nevertheless, since the λ_0 is pre-defined by decision makers, the balance situation when $0 < \lambda_0 < 1$ is more desirable for them to handle with real-world problems rather than the previous extreme cases. Hence, the mission is to obtain the optimal solutions with minimum risk level for a desired degree of aggressiveness.

Lastly, the REILP model is a non-linear programming problem. This is because the introduction of risk variables (i.e., λ_0 and λ_{ij}) to represent the complex non-linear interactions of uncertainties between different variables and terms in a constraint. It is obvious that for a particular constraint, if a large λ_{ij} is correlated with a small x_j, the result would have small contribution to the risk in the decision. In contrast, if the λ_{ij} is associated with a large x_j, it would make a significant contribution to the overall risk of decision making.

4.2. Solution Process for the REILP Model

For solving the models formulated in Section 4.1, the solution procedure contains the next steps (Zou et al., 2010):

[Step 1] Use the BWC algorithm to convert the original ILP model into two sub-models and solve both of them in order to obtain the solutions of the lower bound and the upper bound of the objective function of the original ILP model.

[Step 2] Use the solutions of objective function obtained in Step 1 to formulate a REILP model as given in equations (4.1.20) to (4.1.26).

[Step 3] Solve the model for a series of predefined aspiration level values to get the corresponding optimal solutions for optimal risk levels and decision variable. The solving process in this research is done by running the REILP model into the LINGO software.

4.3. An Illustrative Example of REILP

A numerical example has been presented by Zou et al. (2010) in order to explain the process of REILP and its benefit in dealing with real-world problems. The problem was about a land-use management by controlling the distribution size of nutrient and getting the maximum benefits of the system. In this hypothetical example, there are two types of crop that will be produced in total 1,200 acre of lands in a watershed. Each crop has its interval of unit productivity and net profit. The details of this example are shown below in Table 4-1.

Table 4-1 the intervals of each allocation factor of the land-use management problem

Allocation Factors	Crop 1	Crop2
Unit Productivity (kg/acre)	[4,326, 4,920]	[3,480, 4,120]
Net Profit ($/kg)	[0.26, 0.3]	[0.22, 0.29]
Nitrogen Loading Rate (kg/acre/year)	[4.3, 5.2]	[3.2, 3.6]
Phosphorus Loading Rate (kg/acre/year)	[0.42, 0.48]	[0.27, 0.32]

In addition to these factors, some conditions have been taken in this regard from a Total Maximum Daily Load (TMDL) study. These conditions include that the total loading of nitrogen and phosphorus discharged into the lake cannot be greater than 4,144 and 379 kg/year, respectively, without considering an explicit margin of safety. Nevertheless, by considering 10% as a margin of safety, the maximum allowable loading for nitrogen and phosphorus are 3,730 and 341 kg/year, respectively. Meanwhile, The watershed authorities need a land use planning scheme to optimally allocate lands to different crops in order to maximize the crop production profit while satisfying the environmental requirements in terms of nitrogen and phosphorus discharges.

In order to apply the solution process of REILP model, this land-use problem should be solved firstly by BWC algorithm to get the upper and lower bound of the objective function. Hence, The ILP model can be formulated as:

$$Max \quad f = [0.26,0.3]*[4326,4920]*X_1 + [0.22,0.29]*[3480,4120]*X_2 \tag{4.1.27}$$

$$\Rightarrow Max \quad f = [1125,1476]*X_1 + [765,1194.8]*X_2 i$$

$$s.t. \quad X_1 + X_2 \le 1200$$

$$[4.3,5.2]*X_1 + [3.2,3.6]*X_2 \le [3730,4144]$$

$$[0.42,0.48]*X_1 + [0.27,0.32]*X_2 \le [341,379]$$

$$X_1, X_2 \ge 0$$

By applying the BWC algorithm, the optimal solutions of the ILP is: *f* = [803250, 1511470]. These values are taken and applied to formulate a REILP model for the original land-use ILP model as follows:

$$\begin{aligned} Min \quad \xi = {} & r_3(5.2-4.3)X_1/3730 + r_4(3.6-3.2)X_2/3730 + r_5(4144-3730)/3730 \\ & + r_6(0.48-0.42)X_1/341 + r_7(0.32-0.27)X_2/341 + r_8(379-341)/341 \end{aligned}$$

$$\begin{aligned} s.t. \quad & (1125 + r_0(1476-1125))X_1 + (765 + r_0(1194.8-765))X_2 \ge 803250 \\ & + r_0(1511470-803250) \end{aligned}$$

$$X_1 + X_2 \le 1200$$

$$5.2X_1 + 3.6X_2 - 3730 \le r_3(5.2-4.3)X_1 + r_4(3.6-3.2)X_2 + r_5(4144-3730)$$

$$0.48X_1 + 0.32X_2 - 341 \le r_6(0.48-0.42)X_1 + r_7(0.32-0.27)X_2 + r_8(379-341)$$

$$X_1, X_2 \ge 0$$

$$0 \le r_0, r_3, r_4, r_5, r_6, r_7, r_8 \le 1 \tag{4.1.28}$$

Where, r_0 is the pre-defined aspiration level that its value is chosen by decision makers. Model (4.1.27) has been solved and presented the results in Table 4-2 (Zou et al., 2010). r_0 has taken values from 0 to 1 with a step of 0.1 in order to show the relation between the risk level and the system return, which is obvious in this example. Meanwhile, when r_0 equal 0 or 1, the solutions represent the lower and upper bounds of the ILP model, which is the same value that was generated from model (4.1.26). The last row in Table 4-2 is the Normalized Risk Level (NRL), which can be calculated by multiplying the value of risk function by a number that let the lowest NRL value close to 0 and the highest close to 1 (Pei, 2011). It can be seen also in Table 4-2 that each aspiration level can provide crisp (i.e., single valued) solution which is not applicable to the traditional ILP approach.

Table 4-2 Optimal solutions under different values of aspiration level

r_0	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Profit (10^5\$)	8.03	8.74	9.45	10.20	10.90	11.60	12.30	13.00	13.70	14.40	15.10
X_1 (acre)	531	0	0	0	0	0	2	66	132	202	276
X_2 (acre)	277	1082	1110	1136	1160	1181	1198	1134	1068	998	924
NRL	0.00	0.10	0.20	0.28	0.36	0.43	0.50	0.60	0.70	0.81	0.93

Source: (Zou et al., 2010)

In conclusion, the REILP scheme is effective in reflecting the risk level of decisions provided by the ILP model. The idea of this approach is very helpful for decision makers in choosing best decisions under considerations of balancing system return and decision risks. The decision makers could either choose their decisions or at least give the stakeholders their advice in making decisions with the provisions of the consequence and risk of each option. Moreover, this is the feature that both BWC algorithm and 2-Step algorithm don’t have. This is definitely in parallel with the unsatisfied results that generated from BWC and 2-Step algorithms, which has illustrated in Chapter 3, in terms of the lack of feasibility and optimality in their solutions.

CHAPTER 5

WRM SYSTEM IN SAUDI ARABIA AND REILP MODEL FORMULATION

The aim of this study is to provide a real and specific allocation plan for the WRM sector in Saudi Arabia. Although many goals with general tools have been assigned in the past (Mohorjy & Grigg, 1995; Alkolibi, 2002; Abderrahman, 2006a; Al-Salamah et al., 2011), little concern has been given to the particular steps that should be taken in order to fulfill the target. Therefore, applying an effective approach such as REILP is desired for the purpose of suggesting appropriate allocation plans for WRM, which could help the water authorities in Saudi Arabia to support and enhance their efforts for this vital issue. In this study, a REILP model is formulated to make a long-term planning for the WRM system in Saudi Arabia. The plan consists of 6 periods beginning from the year of 2004. Each period represents 5 years. The data was taken from the report of the 9th Development plan of the Ministry of Economy and Planning (MOEP) (2010).

In this report, the amount of supply and demand of water that aims to reach in 2014 is provided, as well as the supply and demand of 2004 and 2009. Furthermore, the monetary parameters and coefficients are all converted into the U.S. dollar. The whole system includes the returns from the three water use sectors (i.e., domestic, agriculture, industry and their components), and the total cost of each water source. Hence, a calculation of the net benefit of the system with consideration of effective water allocation in a sustainable way is provided with different scenario. This research aims to give the decision makers of the country a chance to see the two sides of each suggested scenarios in order to get a long term WRM plan.

5.1. Overview of WRM System in Saudi Arabia

Saudi Arabia is one of the largest countries in the Middle East, which was considered as a water-stressed region and expects to suffer from a dwindling in the water supply (Alkolibi, 2002). The total area of Saudi Arabia is 2.25 million km^2, while 40% of these areas are desert lands (Abderrahman, 2006a). Meanwhile, the country is surrounded by two seas, which are the Red Sea in the west, and the Arabian Gulf in the east. Because of the critical water situation of Saudi Arabia, the desalination plants have been established on the coasts of these seas. Moreover, Saudi Arabia is an arid region with low rate of annual rainfall, where most of the country regions are lower than 150 mm/year as shown in Figure 5-1, and limited water resources. In addition, the population are increasing with a high rate, and this can be seen obviously by comparing the population in the 1970s, which were around 7 million and became approximately 25 million in 2009 (MOEP, 2010). Hence, all these aforementioned factors can indicate how crucial it is the need to have an effective WRM plan to avoid a shortage of water for the future generations.

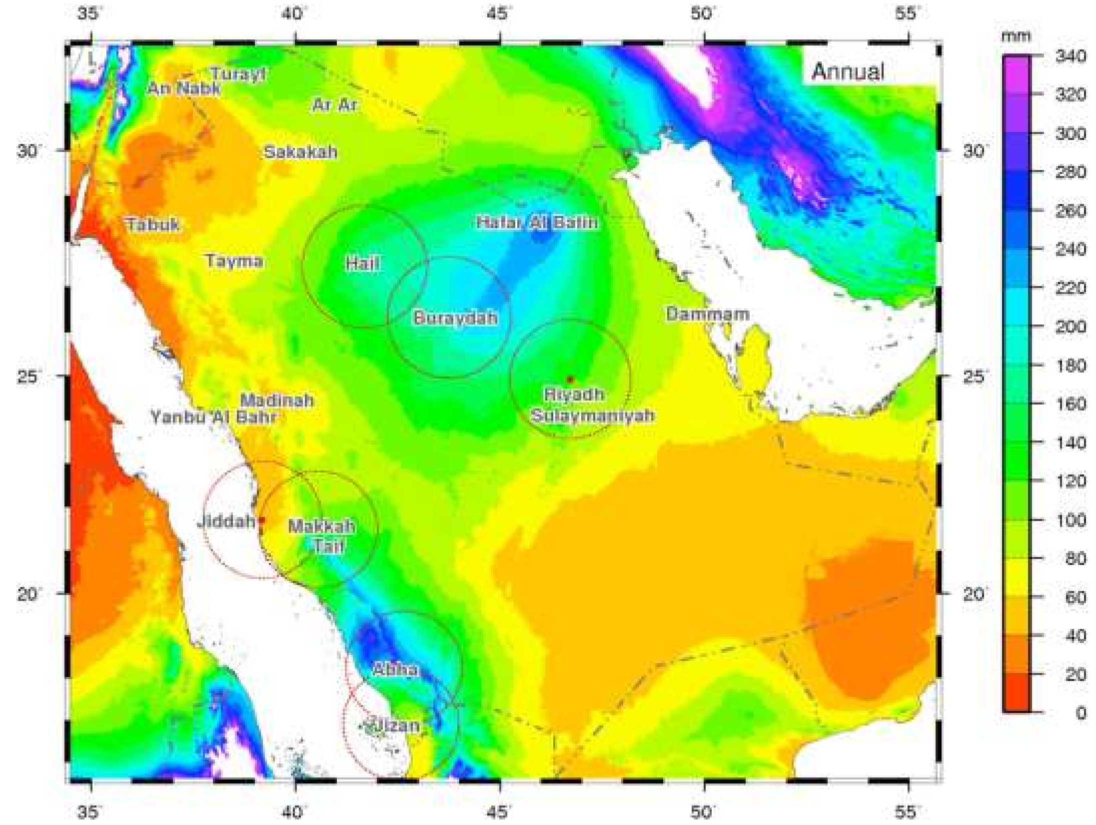

Figure 5-1 Annual rainfall rate of Saudi Arabia during 50 years (1950-2000) based on rain gauge observations

(Source: Research Applications Laboratory, 2008)

The water resources in Saudi Arabia have categorized into five types: (1) Non-Renewable Ground Water (NGW), (2) Renewable Ground Water and Surface Water (RGWSW), (3) Desalinated Water (DW), (4) Reclaimed Wastewater (RW), (5) Reclaimed Agricultural Water. However, the last two types represent a total of less than 2% of water supply in Saudi Arabia and have some similar features for their origins, thus were combined into one category of RW.

On the other hand, there are three main water user sectors in Saudi Arabia, and they are agriculture, domestic, and industry. During the last 30 years, the agricultural sector consumed the largest quantity of water followed by the domestic and industrial sectors, respectively. However, since two decades, the government of Saudi Arabia has taken various measures to reduce the extensive use of water for agricultural purposes. For example, the subsidies that were provided to the farmers have been decreased gradually since the late of 1990s due to the depletion of groundwater (FAO, 2009). Also, the government has banned the extraction and use of water from the farmlands as long as the associated water has been detected with issues such as water quality deteriorations or water table declines (MOEP, 2010).

Beginning in 1994, measures have been taken to rationalize the water use in Saudi Arabia such as the introduction of water tariffs (Zaharani et al., 2011). Before 1994, there were no tariffs or charges for water consumption because the country is governed by the Islamic regulations, which dictate water should be free for everyone. However, the overuse of water, even for the purposes of worshiping, is prohibited in Islam. As a result, water tariffs were introduced to encourage the public consume water carefully. Another measure is to reuse the treated wastewater for landscape irrigation, and industrial cooling (Zaharani et al., 2011).

While these measures have been taken to minimize water use, many problems remain and represent a threat to the WRM system in Saudi Arabia. For instance, water supply networks are facing an increase in leakage in several regions of the country, which has been estimated to be 30% of the total delivered amount (Abderrahman, 2006a; MOEP, 2010). Furthermore, the index of water scarcity, which is the ratio of water demand to existing renewable water supply, in Saudi Arabia is regarded as one of the highest in the world (Alkolibi, 2002). In addition, more attention has been given to the water supply management rather than water demand management and conservation (Abderrahman, 2006b).

Hence, the MOWE authorities realized the significance of having a strategy for handling the water resources supply and allocation, and they decided to improve the general national water plan in 2009. This plan was designed for a 5-year period and consisted of 4 elements: (a) updating the national water plan, (b) improving the strategy of integrated water resources management, (c) researching how to improve water resources, (d) applying rainwater harvest systems in the country (SAMA, 2010). In addition, the country has invested over US$43 billion for the water use sectors only between 2004 and 2009 (MOEP, 2010). Thus, these elements can obviously reflect the concern for WRM system and the need for integrated efforts from all stakeholders (e.g., authorities, researchers, farmers, society, etc.).

Furthermore, the estimation of the future population is an important factor for the WRM system in order to understand the future needs of the country. Therefore, a constant population increase rate of 2.4%, which was the rate between 1992 and 2004 (MOEP, 2010), will be the main estimation of the population growth during the entire planning horizon of this study. The results of this estimation are presented in Table 5-1.

Table 5-1 Population growth during the planning horizon

Year	Population (million)
2004	22.67
2009	25.39
2014	28.44
2019	31.85
2024	35.67
2029	39.95
2034	44.75

In the next sections, a brief description of water suppliers and demands that represent the WRM system in Saudi Arabia is provided. This description includes the future estimation of each element of the WRM system in order to recognize its limitations and then formulate the REILP model. This process is very important in terms of designing long-term allocation plans.

5.1.1. Water Supply

As mentioned above, the water supply in Saudi Arabia is provided by five water resources. Table 5-2 presents the past and forecast amount of water supply for 2004, 2009 and 2014. This table shows that the dependence on the NGW is high, which is one of the major problems for the WRM system in Saudi Arabia. This is because of the extensive use of this resource and the difficulties in replenishing it. Therefore, the Ministry of Water and

Electricity (MOWE) succeeded in decreasing the quantity between 2004 and 2009, and is trying to reduce its percentage from 62.4% in 2009, to 55% in 2014. This decrease can be ameliorated by enlarging the DW facilities, which is scheduled to happen in 2014. This will increase water production from 1048 million m^3/year to 2070 million m^3/year.

Table 5-2 Average annual growth rate of water supply (2004-2014)

Source: (MOEP, 2010)

Water Resource	2004		2009		Average Annual Growth Rate (%)	2014		Average Annual Growth Rate (%)
	MCM / year	Share (%)	MCM / year	Share (%)		MCM / year	Share (%)	
NGW	13490	66.5	11551	62.4	**-3.1**	8976	55.0	**-4.9**
RGWSW	5410	26.7	5541	29.9	**0.5**	4644	28.5	**-3.5**
DW	1070	5.3	1048	5.7	**-0.4**	2070	12.7	**14.6**
RW	260	1.3	325	1.8	**4.6**	570	3.5	**11.9**
RAW	40	0.2	42	0.2	**1.0**	47	0.3	**2.3**
Total Available Water Resources	**20270**	**100.0**	**18507**	**100.0**	**-1.8**	**16307**	**100.0**	**-2.5**

Additionally, the RGWSW quantity has been increased slightly (i.e., 0.5%) between 2004 and 2009, and it is projected to decrease from 5541 million m^3/year to 4644 million m^3/year by 2014. This is because of the desire of water authorities to replenish the water loss in nearby aquifers by using RGWSW. The last water resources in this study will be RW combined with RAW. The use of RW has increased slightly between 2004 and 2009. The quantity of RW was 325 million m^3/year in 2009, which represents a small quantity that is equal to 1.8% of the total water supply. However, it is projected to increase gradually in the future to be 3.5%. Meanwhile, the last type of water resources in Table 5-2, which is the reclaimed agricultural water (RAW), will be combined with RW due to the small percentage of RAW and the similarity in the treatment method.

Most importantly, the average annual growth rate that appears in Table 5-2 will be the base for the future estimation of water supply. Since the ILP model contains data in intervals, the average annual growth rate between 2004 and 2009, and between 2009 and 2014 will be applied toward the entire planning horizon. As a result, the water supply will be divided into lower and upper bounds based on real data which was provided by the MOEP. Hence, this study can be more realistic by providing reasonable future estimations and contribute effectively in designing the long-term plan. The next sections briefly illustrate the past and current situation of each water resource in the WRM system of Saudi Arabia followed by the future estimations of water quantity.

5.1.1.1. Non-renewable Groundwater

In Saudi Arabia, the main source for producing water is Non-Renewable Groundwater (NGW), also known as deep groundwater, which is found in sedimentary rock areas such as geological water stored in aquifers (Alkolibi, 2002). There are six major aquifers that contain groundwater in Saudi Arabia, and they are positioned in the eastern and central regions. The recharge of these aquifers is a very slow process, while their water is being consumed rapidly (FAO, 2009). To illustrate, the estimated amount of NGW is about 2185 billion m^3, while the total recharge is 2762 million m^3 (Abderrahman, 2005). Furthermore, 11551 million m^3/year of deep groundwater were consumed in 2009, while it is expected to reduce in 2014 by supplying 8976 million m^3/year. As a result, the sustainability of this vital resource is very low. This is because the withdrawal quantity of the non-renewable groundwater is around four times of the total recharge, which reflects a negatively on the deep groundwater management.

In addition to these indicators, it can be seen in Table 5-2 that NGW has the highest rate of water supplies in Saudi Arabia with a consumption rate of 62.4% in 2009. This rate is expected to decrease to 55% in 2014. However, NGW will remain the largest water source for the WRM system in Saudi Arabia. This is because the major user of this water is the agriculture sector (Kajenthira et al., 2012), which is the largest user in the country.

In this study, an interval non-renewable groundwater ratio with a range of [-4.9,-3.1] has been used to estimate the future quantity of NGW for the planning horizon in order to provide the interval range of the NGW. These percentages have been taken from Table 5-2, which represents the average annual growth rate of two 5-year periods (i.e., from 2004 to 2014) (MOEP, 2010). The results are presented in Table 5-3, and represent the expected amount across the planning horizon.

Table 5-3 Estimate of non-renewable groundwater quantity during the planning horizon

Year	Lower Bound Quantity (MCM / year)	Upper Bound Quantity (MCM / year)	Average Quantity (MCM / year)
2009	11000.00	12102.00	11551.00
2014	8721.01	9760.60	9240.80
2019	6976.80	7808.48	7392.64
2024	5581.44	6246.78	5914.11
2029	4465.15	4997.42	4731.29
2034	3572.12	3997.94	3785.03

5.1.1.2. Surface Water and Renewable Groundwater

Shallow groundwater, which is a renewable source, is found in sedimentary, weathered and fractured rock (Alkolibi, 2002). Deep groundwater is consumed more than the shallow groundwater. In Table 5-2, RGWSW are in the second position among water suppliers in Saudi Arabia. The total estimation of renewable surface water resources is 2.2 km^3/year, while the majority of this water is recharging the aquifers due to infiltration. On the other hand, the total estimation of renewable groundwater resources is also 2.2 km^3/year; however, the overlap between these resources is approximately around 2 km^3/year. Thus, the total Internal Renewable Water Resources (IRWR) is estimated to be 2.4 km^3/year (FAO, 2009).

There are no permanent rivers are located in Saudi Arabia, although runoff happens few times after sudden rare storms. Generally, the distribution of annual surface runoff has been divided as follows: 30% for agriculture use, 45% for recharging the groundwater aquifers and 25% is evaporated (Rizaiza & Allam, 1989). However, these rates have changed recently due to the construction of new dams and the awareness of getting a better WRM system. In fact, the main goal of reconsidering these rates is to reduce the dependence on NGW for agriculture purposes. Furthermore, it can be seen in Figure 5-1 that most of the country received less than 150 mm of rainfall yearly, while the north east and small parts of the central region and the south west had around 300 mm/year. The south western region, which represents 2% of the total area of the country, has the highest runoff rate among the country regions at 40% (FAO, 2009).

Therefore, most of the dams have been constructed in this area in order to benefit from this water either by recharging the aquifers or using it for agricultural purposes. For these reasons, dams have become more important in Saudi Arabia in the last years. The total number of dams in 2004 was 210 with a storage capacity of 832 MCM and number has increased to 302 dams in 2009 with a storage capacity of 1354 MCM. According to Chowdhery and Al-Zahrani [(Characterizing water resources and trends of sector wise water consumptions in Saudi Arabia, 2013) Journal of King Saud University – Engineering Sciences (under review)], 73.66% of the water quantity stored in these dams is used for recharging the aquifers, while 22.5% and 3.8% are used for drinking and irrigating purposes, respectively. It is estimated that the storage capacity of the newly constructed dams will be 2500 MCM by the end of 2014 (MOEP, 2010). Meanwhile, the total cost of this resource has the lowest cost of US \$0.20/m^3 (Abderrahman, 2001). Hence, this huge quantity and low treatment cost could enhance the amount of groundwater recharging and improve the economic feasibility and sustainability of the WRM system, which lead to sustain the water consumption of the country by better allocation.

In this study, an interval of RGWSW ratio with a range of [-3.5, 0.5] will be multiplied by the average quantity of this resource in order to calculate the amount of the horizon plan. These percentages were taken from the average annual growth rate in Table 5-2. Moreover, because this study is focused on sustainability, the balance between water demand and supply should be taken into account. In addition, since the surface water and shallow ground water are renewable, the rate of changing their quantity (i.e., -3.5% and 0.5%) in the next table is assumed to be fixed from 2014 for each of the two periods of the horizon plan. The predicted water supplies from this resource are presented in Table 5-4. The reason for the slightly declining is to recharge the deep groundwater by drilling some wells through the aquifers and inject the RGWSW for this purpose. This option is better than keep this water behind the dams, which leads to evaporate it especially with the high temperatures of the country in the summer.

Table 5-4 Estimate of renewable groundwater & surface water quantity for the planning horizon

Year	Lower bound quantity (MCM / year)	Upper Bound Quantity (MCM / year)	Average Quantity (MCM / year)
2009	5000.00	6082.00	5541.00
2014	4571.33	5679.53	5125.43
2019	4571.33	5679.53	5125.43
2024	3771.34	5821.51	4796.43
2029	3771.34	5821.51	4796.43
2034	3111.36	5967.05	4539.20

5.1.1.3. Desalinated Water

Because of the limited renewable water resources in Saudi Arabia, and due to the location of the Arabian Peninsula, Desalinated Water (DW) has become a main water resource. This resource is managed by the Saline Water Conversion Corporation (SWCC), which was established by the government of Saudi Arabia in 1972. Desalinated water accounts for 35% of domestic and industrial water consumptions in Saudi Arabia (Ouda, 2013a). However, it is predicted that the desalination production will be approximately 54% of the total domestic and industrial demand in 2025 (Abderrahman, 2001). While the total cost of producing this resource is considered as the highest cost among other water resources, this did not affect the plans to develop this sector. This is because of the wealth of the country and the scarcity of the other options.

As a result, Saudi Arabia became the largest producer of DW in the world (FAO, 2009). In 2009, the total number of desalination plants was 30 with water production capacity of 2878 thousand m^3/day. The number of plants is expected to rise to 44 by 2014 with water production capacity of 5671 thousand m^3/day (MOEP, 2010). These plants are located on the east and the west coasts of the country as shown in Figure 5-2. The coverage of these plants includes cities in the west, the east and the middle of the country, while the other cities depend on either NGW or RGWSW. Figure 5-2 also shows that DW is running through pipelines for a distance of approximately 450 Km. This helps to explain the high total cost of DW in parallel with the energy that consumed to get this kind of water.

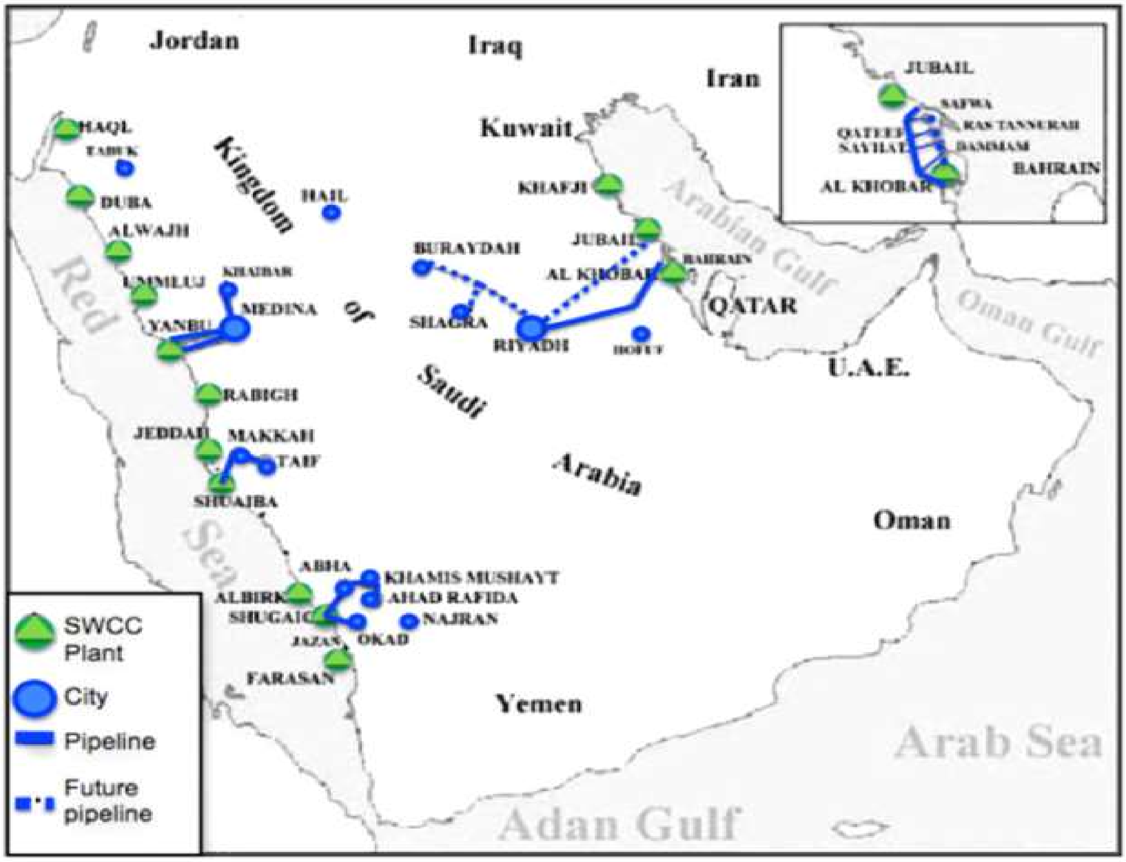

Figure 5-2 Desalination plants and pipelines controlled by SWCC
(Source: SWCC, 2010)

In 2008, three main technologies were used for desalination in these plants: 46% were treated by using multi-stage flash (MSF) systems, while 41% treated the sea water and brackish water by reverse osmosis (RO) and 10% used multi-effect distillation (MED) (ESCWA, 2009). These plants can be divided into two types: (a) single purpose, which just treats water; (b) dual purpose, which usually includes treating the sea water plus generating electricity.

In addition to the details of desalination plants, the average cost of desalinated water production in Saudi Arabia is US \$0.80/m^3, and the transmission cost is US \$0.29/m^3 (Ouda, 2013b). According to Abderrahman (2006a), the estimated cost of DW treatment, transportation and distribution is US \$1.41/ m^3. This cost is applied in this study as the upper bound value because it includes the whole process expenses. For a mid-sized plant that also provides electricity, the sea water desalination cost is about US \$0.90/m^3 (Abderrahman, 2001).

On the other hand, the cost that water users to pay for the government (i.e., tariff) is very cheap, starting at US \$0.027/m^3 for the first 50 m^3/month and ending with US \$1.6/m^3 for any quantity that exceeds 301 m^3/month. This system is called block rate and has been adopted by Saudi Arabia’s water authorities (Ouda, 2013b). In this system, the water tariff increases in specific intervals of water consumption. For example, the interval of water quantity of [50,100] m^3/month has a price of US \$0.04/m^3. The purpose of this system is to keep the users concerned about their water use amount to avoid paying more money for it. However, some previous studies suggested that the current water tariff system is not very effective because of the lower prices that have been assigned (Alkolibi, 2002; Abderrahman, 2006a; Al-Zahrani & Baig, 2011; Ouda, 2013a & 2013b).

In this study, an interval of DW ratio with a range of [-0.4, 14.6] will be multiplied by the average quantity of this resource for the purpose of calculating the required amount of water for the horizon plan. These percentages were taken from the average annual growth rate shown in Table 5-2. At the same time, it is important to know that desalination plants costs a lot of money and time to construct, and large energy consumption is required for their operation. All these previous factors are not in favour of achieving the goal of sustainability. As a result, the rate of increase or decrease (i.e., -0.4% and 14.6%) in the next table will be applied for the desalinated water quantity of 2024. Then, these rates will get decrease to half (i.e., -0.2% and 7.3%) and applied once to the rest of the long-term plan. This method of calculation is more realistic. Also, the desalination plants have a life expectancy that varies from one to another, and this is the reason for the declining rate in the lower bound column after 2019. These details are shown in Table 5-5.

Table 5-5 Estimate of desalinated water quantity for the planning horizon

Year	Lower Bound Quantity (MCM / year)	Upper Bound Quantity (MCM / year)	Average Quantity (MCM / year)
2009	1048.00	1048.00	1048.00
2014	2000.00	2140.00	2070.00
2019	2000.00	2140.00	2070.00
2024	1980.00	2921.10	2450.55
2029	1980.00	2921.10	2450.55
2034	1980.00	2921.10	2450.55

5.1.1.4. Reclaimed Wastewater

Wastewater was considered an obstacle for the environment in the past and caused the spread of epidemics due to the difficulty of properly disposing of it (Lofrano & Brown, 2010). This was manifested more in the developing countries because of either the lack of research or budgetary constraints. Saudi Arabia is one of the countries, which was not interested in using reclaimed wastewater, because of a lack of knowledge about its appropriate use and the resistance of the society, who considered it as toxic garbage. In this research, the RW and reclaimed agriculture wastewater represent together just 2% of water supply in Saudi Arabia in 2009, and because they have some similar features, they are combined under one category called ‘reclaimed wastewater’ (RW).

Recently, the use of this type of water has increased from 300 million m^3/year in 2004 to 367 million m^3/year in 2009 (Table 5-2), and this quantity is projected to be 617 million m^3/year in 2014. This represents a doubling of the share percentage among water resources from 2% in 2009 to 4% by 2014. This way of dealing with treated wastewater reflects the degree of awareness among governmental authorities and researchers regarding this alternative resource. For example, Abdurrahman (2006a) has suggested that the wastewater in Riyadh, which is the capital city of Saudi Arabia, be treated and reused in large volumes as one of the possible solutions for water and sanitation problems of the city.

In 2003, the number of sewage treatment plants in operation was 70, while the use of the treated wastewater, is mainly for irrigation of non-edible and landscape plants, plus the industrial cooling (FAO, 2009). However, this degree of treating, types of use, and quantity are not enough for a country that faces a real water shortage situation. In fact, this type of water could be an effective solution through any WRM system that aims to be sustainable. This is because the quantity of this water will rise as long with the population. This water can be reused and combined with water conservation policies in order to save both groundwater and DW production (Kajenthira et al., 2012).

According to Abo-Rizaiza (1999), the cost of secondary treatment is between US \$0.29-0.59/m^3, while the cost of tertiary treatment is around US \$0.27-1.23/m^3. These prices should be combined with the transportation cost in order to get the total cost. These calculations have been done by Kajenthira et al. (2012), who indicated that the price for domestic wastewater treatment will be in the range of US \$0.13-2.50/m^3. The average of these values is assumed in this research where RW treatment cost is US \$0.7-0.9/m^3, which represents the lower and upper bound values, respectively. Hence, relying on reclaimed wastewater for the water fields that are not directly connected to food production (i.e., non-potable use) would cause benefits for both the national economy and rationalize the NGW.

In this study, an interval of both reclaimed wastewater and reclaimed agriculture wastewater ratio with a range of [5.6, 14.2] will be applied to the quantity of this resource for the future years. These percentages have been taken from Table 5-2. The results are provided in Table 5-6, and they present the expected amount of the horizon plan.

Table 5-6 Estimate of reclaimed wastewater quantity for the planning horizon

Year	Lower Bound Quantity (MCM / year)	Upper Bound Quantity (MCM / year)	Average Quantity (MCM / year)
2009	367.00	367.00	367.00
2014	600.00	634.00	617.00
2019	789.76	1055.07	922.42
2024	1010.89	1804.17	1407.53
2029	1293.94	3085.13	2189.54
2034	1656.25	5275.57	3465.91

5.1.2. Water Demand and Users

Having provided highlights on Saudi Arabia's water supply and resources, this thesis now presents some details about the history of water demand in order to provide a comprehensive understanding of the WRM system. There are three main sectors that consume water: agricultural, domestic, and industrial. As a result of increasing population, water demand has increased yearly. However, a dramatic expansion in water demand occurred from the 1980s to the late 1990s. This expansion was due to an agricultural strategy that was assigned without consideration for the water supply future.

Additionally, the rising living standards from 1970 to the present have placed a heavy load on domestic water demand. Lastly, the industrial sector is expected to increase its use. This sector has been consuming non-renewable or costly desalination water for the last decades. This trajectory is not economically feasible and should be changed soon due to the water situation of the country. By balancing all these main sectors' needs and the available water resources via strategic planning, the life expectancy of the limited groundwater could be increased in parallel with the economic revenues.

Water consumption for the last 15 years, plus forecasting for the next 5 years is provided below in

Figure 5-3. It can be noticed how agriculture consumes most of the water during that period, while the municipal and industrial uses are secondary. This situation has created a detailed focus on agricultural water allocation and its benefit. Meanwhile, the increase of municipal water use is normal due to population increase, but some regulations are needed to reduce the average water use per citizen per day (l/c/d) since it is high compared with many other countries. Additionally, water consumption by the industrial sector has risen slightly which does not represent a big concern. However, the main issue with this sector is the source of water and its toxic residues.

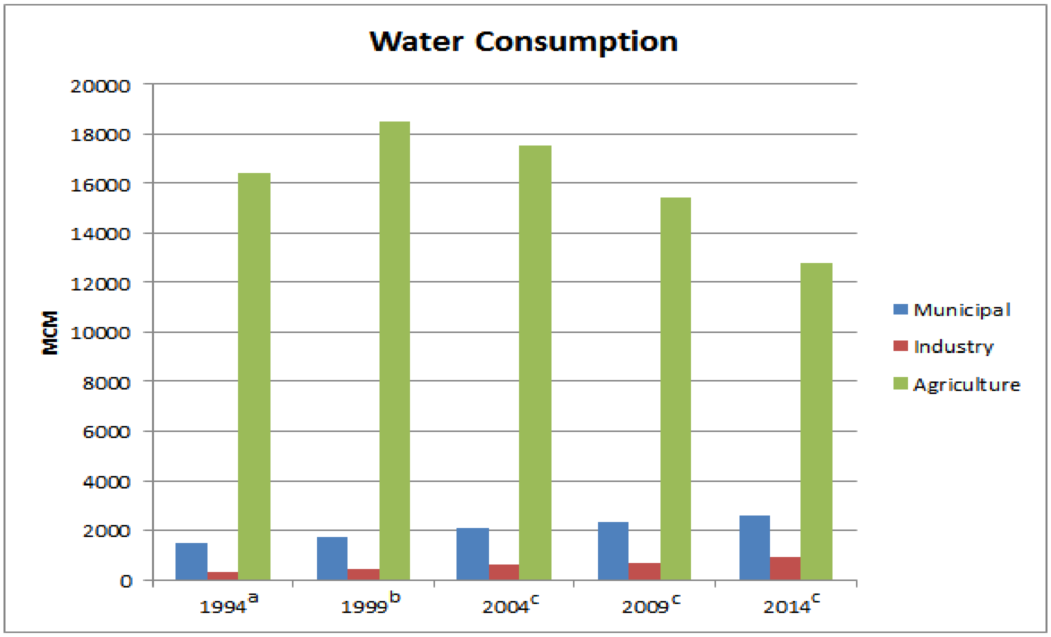

Figure 5-3 Water consumption among main sectors of Saudi Arabia (1994-2014)
[a] (MOEP, 1995), [b] (MOEP, 2000), [c] (MOEP, 2010)

Table 5-7 presents a comparison between the past demand of 2004 and 2009 and the future demand of 2014 among the three sectors. It is indicated that the agricultural water uses have been decreased annually, while both domestic and industrial water uses have been increased gradually every year. Similar increases and decreases for all three sectors are projected to happen in 2014. The domestic water use increase is due to both the high rate of population growth, which requires more domestic water as well as the high living standards to satisfy their needs. This could be achieved by either developing new distribution stations or expanding the old ones. Also, the country is trying to expand its economic varieties by enhancing the industrial sector without relying only on oil sales, and the industrial water demand needs to be increased accordingly. All these water users, the aspects of their water demand, and the net costs and benefits will be illustrated in the next sections.

Table 5-7 Average annual growth rate of water demand (2004-2014)

Water Demand	2004		2009		Average Annual Growth Rate (%)	2014		Average Annual Growth Rate (%)
	MCM / year	Share (%)	MCM / year	Share (%)		MCM / year	Share (%)	
Municipal Purposes	2100	10.4	2330	12.6	**2.1**	2583	15.8	**2.1**
Industrial Purposes	640	3.1	713	3.9	**2.2**	930	5.7	**5.5**
Agricultural Purposes	17530	86.5	15464	83.5	**-2.5**	12794	78.5	**-3.7**
Total Water Demand	**20270**	**100.0**	**18507**	**100.0**	**-1.8**	**16307**	**100.0**	**-2.5**

Source: (MOEP, 2010)

5.1.2.1. Agricultural Sector

Since the largest water consumer of Saudi Arabia is agriculture, it is more appropriate to start with it in this section. Although this country is considered a desert region with a low rainfall rate and without permanent rivers, the wealth that came from oil revenues has let the governmental authorities think about food self-sufficiency. This is because they can provide the required technology for drilling wells and irrigation to the farmers plus subsidies in order to achieve this goal. However, the other consequences of this action were not considered realistically. Also, there was a belief that the groundwater quantity is massive, which was true at that time, but it is non-renewable, which means it should at least be used carefully. As a result, a notable lowering of the water table has been observed. For example, Layla Lake which is located in the middle of the country (i.e., 350 km South West of Riyadh: the capital city) was completely depleted by 2002 because of the overuse of both its water and nearby groundwater for agricultural purposes. Therefore, the main concern of this research is how to allocate the irrigation water efficiently, since agriculture is the major user of water.

Because of the dramatic increase in the price of oil during the 1980s, the wealth of Saudi Arabia, which is considered as one of the biggest oil providers in the world, has flourished largely. Many infrastructure projects have been implemented and the whole country has developed in several areas such as educational, industrial, economical and health sectors due to this sudden wealth. Then, in order to integrate the strength of the country, some considerations about food self-sufficiency have appeared. Therefore, the government started to support farmers by giving them loans without interest and based on long terms (e.g., 30 years). In addition, permissions for drilling water wells have been distributed among farmers without any consideration for the sustainability of this water. At the same time, modern technology for irrigation and farming has been imported to enhance the process and get better crops. All these factors have led to extensive use of groundwater, which represents 95% of the total water use for agriculture (FAO, 2009).

In Figure 5-4, the trend of water consumption shows how the dramatic expansion in the farming process has happened. Water quantity for agriculture use was below 2 billion m^3/year between 1975 and 1980. In just five years after the governmental plan for enriching farming products began the water amount tripled and reached 6 billion m^3/year. In 1990, the biggest jump of agricultural water use compared to previous years took place by consuming 17.100 billion m^3/year. This quantity was required since the cultivated area has grown from fewer than 0.4 million ha in 1971 to 1.62 million ha in 1992 (Abderrahman, 2001). The main crop at the end of this period was cereals, particularly, wheat which was sold by farmers to the government with a support price that was higher than the import price for the same crop.

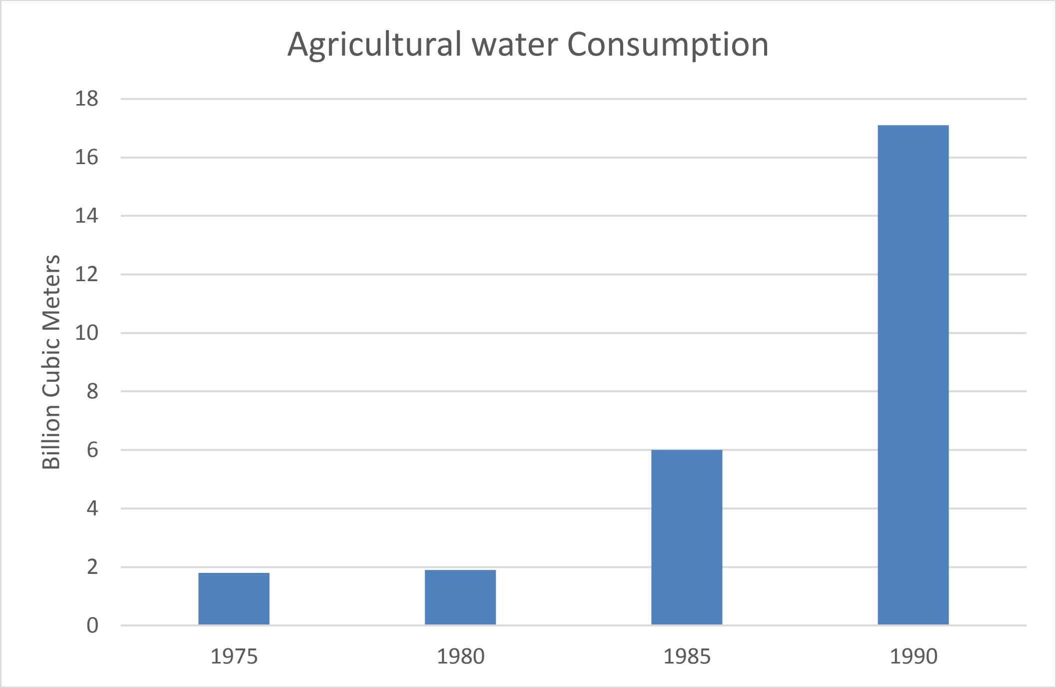

Figure 5-4 Agricultural water consumption in Saudi Arabia (1975-1990)
(Source: MOEP, 2005)

However, the drop in the water table because of water overuse allowed the water and agriculture authorities to realize the negative impact of the previous policies. As a result, a governmental decree in 1993 banned wheat exports and stopped subsidies provided to local farmers for this type of crops (Ouda, 2014). In general, this step was very important and reflected the awareness of the government regarding the protection of water resources. But the impact of this decision was temporary and in 1994 the agricultural water quantity decreased to 16.300 billion m^3/year as shown in

Figure 5-3. Agricultural water consumption did not lessen as was expected in 1999 because for two reasons: (1) some farmers who had already established their farms continued to plant wheat rather than replacing their crops since it still had economic benefits, (2) most of the remaining farmers replaced wheat crops by forage crops which consumed more water for the whole year, while wheat was only planted during the winter. Regarding this last reason, the forage rate has grown year by year and this can be seen in Table 5-8, where the forage cultivated area has increased from 160,356 ha to 184,462 ha in only one year (i.e., 2009-2010).

Table 5-8 Cultivated area of main crops in Saudi Arabia

Crop	Area (hectares)		
	2009	2010	% Change
Cereals	328,725	286,932	-12.7
Vegetables	106,761	108,845	2.0
Dates	161,975	155,118	-4.2
Total Fruits	239,147	226,443	-5.3
Green Fodder	160,356	184,462	15.0

Source: (SAMA, 2010)

Consequently, water consumption of agriculture has fluctuated around 18 billion m^3 from 1994 to 2004 as shown in

Figure 5-3, which means more policies were required to control the water use situation. In 2008, the government announced that the support of wheat would stop completely by 2016 with a reduction of 12.5% each year for its production from the year of this decision. This reduction rate can be clearly observed between 2009 and 2010 as is illustrated in Table 5-8. Even though this rate refers to the cultivated area, this can be an indicator for decreasing production. In addition, this rule includes a strict ban of forage exporting and prevents the ministry of agriculture (MOA) from providing any new licenses for planting either forage or wheat. Meanwhile, the government will give subsidies to investors who import either crop in order to meet the needs of the country. As a result, water consumption for agriculture purposes was lower than 16 billion m^3/year in 2009 and the plan is for it to be 13 billion m^3/year in 2014 as demonstrated in
Figure 5-3. This is a significant step towards designing a long term plan for water allocation, and it has been considered carefully in this study.

While the water usage issues of forage and wheat crops, which represent the major part of cereal crops, have been discussed above, vegetable and fruit crops are planted in Saudi Arabia with a less negative impact on water resources. This is because the balance between domestic needs and the quantity of these crops has been taken into account. Table 5-8 indicated that the increase in the cultivated area of vegetable between 2009 and 2010 is only 2%, which is close to the rate of increase of the population of the country. At the same time, the cultivated area of total fruits, of which 68.5% are dates, has been reduced. The major decrease came from the dates, which is a good sign since dates are the largest water consumer among the fruit crops in Saudi Arabia. One of the biggest concerns regarding agricultural water is that dates have been exported widely in spite of the bad situation of water resources of the country. Therefore, revising the regulation of exporting dates is obligatory and reducing the yearly date crop to only cover the country’s needs is suggested in this research.

Aside from all these crop production issues, the limitations of each crop cultivated area have been estimated in order to be proportional with the allowable agricultural water. This estimation is helpful also in calculating the net cost and benefit of this sector, and it is based on the areas and the change rates that happened between 2009 and 2010, (Table 5-8). In the previous table, two crops, which are cereal and fruit, have reduced their cultivated area, while the forage and vegetable have increased. Therefore, the same rate will be applied for the future years with exception of forage crop since its increase rate, which is 15%, is already high. The reason for the future decrease is that forage crop consumes a lot of water and this study aims to balance the use of water in order to design an effective long-term water allocation plan. Thus, the reduction in the cultivated area of this crop becomes necessary. Meanwhile, the increase of vegetable crop will remain the same to cover the public needs and because vegetable crop does not consume massive amounts of water during the entire year. All these crop limitations can be found in Table 5-9.

Table 5-9 Limitations of cultivated area of main crops during the planning horizon

Year	**Forage** (ha/year)			**Cereal** (ha/year)		
	Low.	Upp.	Ave.	Low.	Upp.	Ave.
2009	160000	160700	160356	328000	329500	328725
2014	280000	281246	280623	119000	120968.6	119984.6
2019	350000	351556	350778.8	63000.8	64782.8	63891.8
2024	368000	368634	368317.7	33000.4	35022.4	34022.4
2029	276000	276476	276238.3	22000.3	24220.3	23220.3
2034	138000	138240	138119.1	15000.8	16721.8	15847.8

Year	**Vegetable** (ha/year)			**Fruit** (ha/year)		
	Low.	Upp.	Ave.	Low.	Upp.	Ave.
2009	106000	107500	106761	238447	239847	239147
2014	117000	117880	117437.1	175000	176546	175773.0
2019	129000	129360	129180.8	152000	153844	152922.5
2024	142000	142196	142098.9	133000	133400	133042.6
2029	156000	156616	156308.8	115000	116494	115747.1
2034	171000	172878	171939.7	100000	101400	100700.0

Lastly, reorganization of the source of water for agriculture is important in order to protect water resources of Saudi Arabia. This is because the water situation is unsustainable at its current rate. The main water resource for irrigation in 2000 was fossil groundwater and this represents more than 95% of the total irrigation water (FAO, 2009). This is a problem in a country with limited water resources. However, there have been efforts to increase the use of RW which was only 47 million m^3/year in 2009. Therefore, this study has taken this option into account and designed a WRM system by increasing the quantity of RW gradually for each 5 year period during the long-term plan. The RW should have tertiary treatment and its use will be specific for unrestricted agricultural irrigation, which includes salad crops and vegetable eaten row, while any RW with secondary treatment will be used in landscaping (KAUST- KICP, 2012). The use of NGW will be planned to decrease while the use of RGWSW will be close to their maximum available amount in order to replenish the required agricultural water quantity.

5.1.2.2. Domestic Sector

In a country whose temperature surpasses 45 Celsius degree for around 8 months yearly, the use of water for cooling purposes is predictable. This is the reality of Saudi Arabia where air conditioners exist everywhere in order to enhance the living environment. Some of these air conditioners are water-based. In addition, dust storms that happen suddenly and regularly in recent years during these months force people into consuming a lot of water to clean their houses and properties. Furthermore, the high population growth rate with rising living standards of the citizens from 1970 to the present places a heavy load on water demand. For these reasons, the average consumption rate for each citizen of the country is 248.7 liters per day, which represents the third highest per capita consumption rate in the world after the United States of America (USA) and Canada (SAMA, 2010). The high water consumption of both USA and Canada is reasonable because they are rich in water resources and have high rainfall rates throughout the year; however, the case is different in Saudi Arabia with limited water resources plus a low rain fall rate, and this consumption should be changed.

In order to reduce the average water use per citizen per day (l/c/d), a few measures have been taken such as publishing a public campaign about rationalizing water consumption through media. This has been supplemented with distribution of tools for water conservation to each house and company by MOWE. These steps are useful to raise public awareness, but alone they are not enough. A review of the water tariff is required because the entire yearly revenues are approximately 2.5% of the annual treatment cost. The idea of a block rate system which is applied right now is useful, but the price of water needs to be increased in order to make the citizens deal with water carefully.

Another measure would be to specify a water amount for each household and commercial building which is proportional to the number of people who are living in or working there. Also, it is suggested to use the RW for non-potable purposes such as flushing toilets, which is one of the biggest water consumers, air conditioners and increasing the use of this water for landscaping. These uses do not have direct contact with humans' skin and might be acceptable to the public. However, these suggestions need cooperation between health organizations and municipalities in order to monitor the implementation and avoid health risks.

Moreover, the water leakage in the capital city through the distribution network is estimated to be 20-30% of the total water flow (Abderrahman, 2006a), which reflects the poor state of the country's infrastructure. Similarly, the rate of water leakage in the water collection systems for the whole country is 20% (KAUST- KICP, 2012). Indeed, the lack of adequate water infrastructure was obvious during several floods which happened in recent years in different cities resulting in loss of life. As a consequence, many projects for enhancing the infrastructure are under construction in the major cities of the country. These projects will reduce many risks that threaten human lives, water resources sustainability and the environment. In addition, the application of a comprehensive plan will be easier with these steps.

Regarding water resources for domestic use in Saudi Arabia, the largest water quantity came from deep groundwater and DW. Deep groundwater is a non-renewable source and the continuously using it at the same rate may lead to depletion of it within a few decades. Therefore, the country is planning to increase the use of DW and will depend on it more than deep groundwater for domestic use. Although DW is the most expensive resource and consumes high energy, the need to save the precious groundwater which requires lower treatment and cost is a better option in order to sustain the WRM system. This can happen by recharging the deep aquifers by RW, while the research for treating the sea water by lower energy methods is in progress.

Aside from the previously mentioned water resources, both shallow groundwater and surface water are considered as renewable water and they are consumed for municipal use but only as a small percentage of the total water supply. These resources are supposed to grow gradually in Saudi Arabia as a result of constructing new dams, which are designed to store rainfall water in order to use it for agricultural and municipal purposes and also to recharge the shallow groundwater. Therefore, the WRM system can be sustainable and feasible economically at least in the regions that have a high rainfall rate (i.e., the south west of the country), which will lead to reduce the dependence on other water resources. Finally, the use for RW for non-potable domestic use is suggested to increase slightly in this study especially in the middle and the north of Saudi Arabia where the NGW is the main resource for domestic use. Hence, the long-term plan that is provided in this research can contribute effectively in developing the water environment and economy.

By referring to

Figure 5-3, it can be noticed that the increase in municipal use is steady between 1994 and 2009. Also, Table 5-7 shows that the growth of domestic water demands in 2014 will be only 2.1%. As a result, this study recommends an increase in the municipal water demand from 2% to 2.2% in order to design the long-term plan. The domestic water demand of this research can be viewed below in Table 5-10, where only the water use in 2009 has the same values for all three columns since it is a real value, while the growth rate in the rest is fluctuating between 2% and 2.2%. In the model formulation, the water supply should be equal or greater than water demand in order to avoid any water shortage in any period.

Table 5-10 Domestic water demand during the planning horizon

Year	Lower Water Quantity (MCM /year)	Upper Water Quantity (MCM/year)	Average Water Quantity (MCM /year)
2009	2330.00	2330.00	2330.00
2014	2563.00	2603.00	2583.00
2019	2819.30	2889.33	2854.32
2024	3139.75	3168.29	3154.02
2029	3469.42	3500.96	3485.19
2034	3833.71	3868.56	3851.13

5.1.2.3. Industrial Sector

The last user in the WRM system of Saudi Arabia is the industrial sector. Since the country does not have many industrial factories, water consumption for this sector is the smallest among other users. This can be seen clearly in Figure 5-4 where the water need for industrial activities has not exceeded 720 m^3/year since 1975 and it is expected to reach 940 m^3/year in 2014. This is because the economy of the country is based on oil exports and this source provides enough money to import industrial products from abroad. In fact, 80% of budget revenues and 90% of export earnings of Saudi Arabia are from the petroleum sector (CIA, 2014). Meanwhile, Saudi Arabia has two huge industrial cities which require water for different purposes on the east and the west costs (i.e., Jubail and Yanbua, respectively). One of the main industrial products of the country is petrochemical products, which are exported to different countries and bring high economic benefits. Ethylene and propylene products represent the majority of petrochemical products and their quantities are expected to increase gradually during the study frame time.

In this study, the focus will be on the treatment cost of the expected industrial water during the long-term plan plus the economic benefits gained from ethylene and propylene products. The selling price of the surplus electricity that is generated by some desalination seawater plants is considered, as well. In fact, the 30% of the electricity that is generated from the purification process will not be counted in this research since it is used for the facilities of the desalination plants directly, while 70% is sold by the Saudi Electricity Company to different customers.

Currently, industrial water sources include groundwater, DW, and small percentage of RW. While the quantity of deep groundwater is limited and the treatment cost of DW is high, reused water quantity could be guaranteed and its cost is lower than the purification of seawater. Thus, relying more on reclaimed water for industrial use is projected in this research in order to make the WRM of Saudi Arabia an efficient system.

In order to design a long-term water allocation plan, the quantity of industrial water demand needs to be estimated. This can be done by combining the history of the water demand of this sector with the latest forecasting. For this research, this data is presented in Table 5-7, where the average annual growth rate that is forecasted between 2009 and 2014 is an increase of 5.5% in the industrial water demand. However, this rate was different according to a real data, and was only 2.2% between 2004 and 2009 (MOEP, 2010). Therefore, any estimation for industrial water demand should take into account these rates, and this is the base of this study for the planning horizon. This forecasting can be seen in Table 5-11, where the lower bound of industrial water demand has an increase of 2.2% every year during the long-term plan, while the upper bound will grow yearly with a rate of 5.5%. This planning strategy is important in order to guarantee that the water supply will cover the water demand during each period of the suggested plan.

Table 5-11 Industrial water demand during the planning horizon

Year	Lower Water Quantity (MCM /year)	Upper Water Quantity (MCM /year)	Average Water Quantity (MCM /year)
2009	713.00	713.00	713.00
2014	930.00	950.00	940.00
2019	1032.30	1211.25	1121.78
2024	1145.85	1544.34	1345.10
2029	1271.90	1969.04	1620.47
2034	1411.81	2510.52	1961.16

5.2. Model Input Data

As discussed in the previous sections, there are four main water resources and three main users in the WRM system of Saudi Arabia.

Figure 5-5 shows the suggested framework for the WRM system of the country. The first row represents water resources which are organized from the left to the right depending on their future rates, while the rest columns present in a vertical direction from top to bottom the largest water consumer to the smallest one. All these elements have different factors that will be included in the model. These factors are illustrated in the next sections and are used as the model input data.

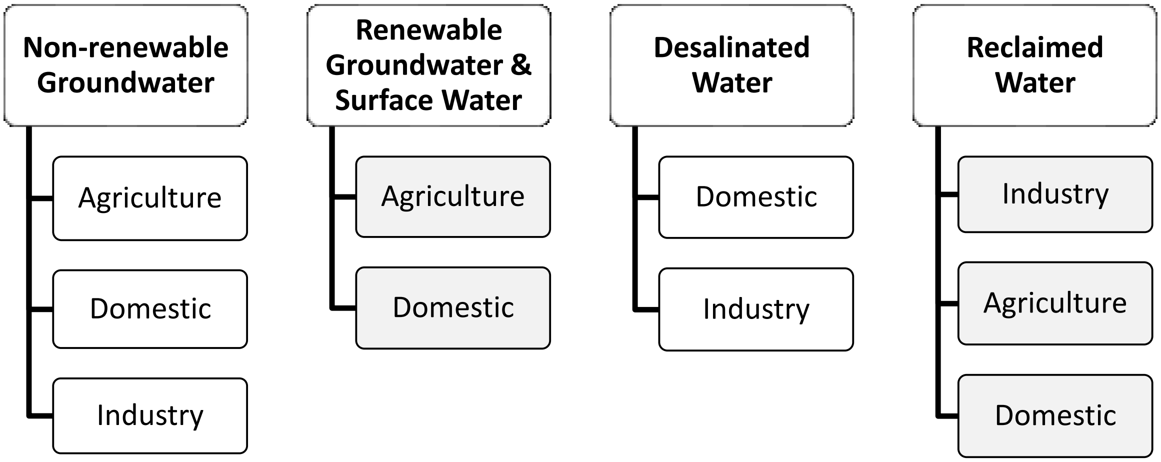

Figure 5-5 Main water resources and users in Saudi Arabia

5.2.1. Water Total Cost and Selling Price

In general, each water resource has its own total cost and selling price. There are several water treatment methods, which have different cost depending on the source of water. The capital and operating cost of any water treatment system is reliant on the load of suspended solids, organics, the degree of salinity and the desired water quality (Gienger & Kranzmann, 1995). Also, the production and the distribution cost should be accounted for in order to get the total cost of each source of water. In this study, total water costs and selling prices were collected or estimated in order to compare between different cases for the economic feasibility of WRM system of Saudi Arabia. Table 5-12 shows the upper and lower value of both total cost and selling price for each water resource of the country. The total costs for all water resources are accounted for in this study, while the selling price is specific to the domestic use.

Table 5-12 Water resources treatment costs and market prices

Water Recourse	Total Cost		Market Price	
	Lower Value (US $/m^3)	Upper Value (US $/m^3)	Lower Value (US $/m^3)	Upper Value (US $/m^3)
NGW	0.42[a]	0.5	0.1	0.15
RGWSW	0.2[b]	0.3	0.07	0.12
DW	1.3	1.41[a]	0.04	0.08
RW	0.7	0.9	0.06	0.1

[a] (Abderrahman, 2006a), [b] (Abderrahman, 2001)

While these values are difficult to be the same for the whole country, an average value was either taken from scientific paper (Table 5-12) or estimated based on the direction of the long term plan. The total costs of NGW, DW amd RGWSW are US 0.42$/m^3, US 1.41$/m^3 and US 0.2$/m^3, respectively (Abderrahman W. A., 2001; Abderrahman W. A., 2006a). It can be observed that the DW total cost is the highest among water resources. In contrast, the lower value of the total cost of DW (i.e., US 1.3$/m^3) is estimated to be lower than current cost due to the research that is taking place in the country to find new and cost effective purification technologies.

Moreover, the total cost of NGW and RGWSW are much lower. The total estimation costs of the upper values of both NGW and RGWSW, which are shown in Table 5-12, are estimated to be higher than current cost due to the decrease in the water table in many regions of Saudi Arabia which means more energy is required for the pumping process. These prices should be considered concurrently with new regulations to control the consumption of both water resources. This is because NGW quantity is projected to be lower than the current amount, but its affordability may encourage its usage. On the other hand, the low-cost pricing of SW and RGW are maintained because they are renewable. It is expected that this low-cost pricing would lead to increased usage by the agriculture sector in order to achieve the sustainability of the WRM.

Furthermore, the average total cost of wastewater is estimated to be US 0.7-0.9$/m^3. This estimation is based on an evaluation of minimal transportation and costs of domestic wastewater treatment which is between US 0.13$ and US 2.5$/ m^3, and the cost is varying depend on the treatment degree and the size of treatment plan (Kajenthira et al., 2012). The resulting price could be attractive for both agricultural and industrial sectors since most of their water use does not require high water quality. Hence, the sustainability of WRM system can be achieved by applying such strategies.

With regard to the selling price of the treated water for the domestic sector, the only water that currently has a fixed price is DW and NGW, where the block rate system is applied. For instance, the interval of water quantity of [50,100] m^3/month is sold for US \$0.04/m^3. This rate has been chosen in this study as the lower value for DW, while the upper value is US \$0.08/m^3. This upper value is still low compared to the total cost, but since the most of this water is going to the domestic sector, it is better to raise the price slightly in order to minimally impact the poor people of the society. At the same time, anyone who will consume more water will be required to pay more money by referring to the block rate system. This system ensures that citizens are aware of their consumption.

On the other hand, the selling price of NGW should be increased to be between US 0.1\$/m^3 and US 0.15\$/m^3. The purpose of this increase is to convince water users to minimize their use for this type of water. Furthermore, since the shallow groundwater and the surface water are renewable and their treatment and transportation costs are low compared to previous water resources, their selling price is planned to be the lowest. This policy will ensure maximized efficiency by domestic consumers who comprise one of the main water consumption sectors. The suggested price is between US 0.07\$/m^3 and US 0.12\$/m^3. However, in comparison with the DW price, the price of RGWSW is slightly higher and the reason for that is to keep the priority of this water to agriculture use since it is easier to allocate among nearby farms. Meanwhile, the DW does not reach all the regions of Saudi

Arabia, and in this case the use of either NGW or RGWSW by the domestic sector becomes necessary.

Finally, the price of RW which provides to the domestic sector is suggested to be between US 0.06$/ m^3 and US 0.1$/ m^3. This price is the lowest after the DW price in order to encourage the public to use this kind of water. Nevertheless, the use of RW is limited and should be only for non-potable use such as the irrigation of gardens or trees inside homes and around the buildings, landscaping and in toilets.

5.2.2. Average Crop Yield and Price

As mentioned in section 5.1.2.1, Saudi Arabia has four main crops. Each one of these crops has its own average yield and price. The average yield refers to how many tons of each crop can be planted in each hectare, and its unit is ton/ha. In this research, the data of two different years (i.e., 2007 and 2009) was collected from valid sources[1] by taking the average of different plants that can be sorted under one crop.

For example, both tomatoes and potatoes had their own average yield but can be categorized as vegetables. This data is shown in Table 5-13 where the average yield of each crop has been estimated to be the average of the real data that were taken in 2007 and 2009. The results of these calculations have been estimated to be constant during the planning horizon of this study with the exception of cereal crop. This is because wheat,

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[1] (SAMA, 2010; Alabdulkader, Al-amoud, & Awad, 2012)

which represents the majority of cereal production, will be phased out completely by 2016. Therefore, the average yield of cereal crops will be decreased as a result of this decision.

Table 5-13 Crops average yield for the planning horizon

Year	Average Yield (ton/ha)			
	Forage	Cereal	Vegetable	Fruit
2007[a]	9.915	2.57	20.1	13.14
2009[b]	18.55	4.84	25.07	6.77
2014	14.23	3.71	22.58	9.66
2019	14.23	2.6	22.58	9.66
2024	14.23	2.6	22.58	9.66
2029	14.23	2.6	22.58	9.66
2034	14.23	2.6	22.58	9.66

[a] (Alabdulkader et al., 2012), [b] (SAMA, 2010)

On the other hand, each plant has its own price while the same process of calculating the average yield has been applied in order to get the average price of each crop. The main difference in the average price is that the selling price will not be constant during the planning horizon because of the variation in the economic conditions such as inflation. Therefore, an upper and lower value for each crop has been estimated and this can be viewed in Table 5-14 where each crop has three different prices. These prices include the average price which was used as an indicator to get the upper and lower values by increase and decrease the average price based on assumed variations.

Regarding the values of each period, all agricultural products have been assumed to increase in their average value due to the increase in population and the reduction in the local production of some crops such as cereals and forages. The reason for this reduction is the new regulations that have been declared in order to minimize the use of NGW, which is currently the main water supply for the agricultural sector. Hence, the demand for local products will be greater than the supply, and this will lead to an increase in their prices.

Table 5-14 Lower, upper and average values of crops during the planning horizon

Year	**Forage** (US $/ton)			**Cereal** (US $/ton)		
	Low.	Upp.	Ave.	Low.	Upp.	Ave.
2007[a]	390	410	401.5	370	382	376.9
2009	390	410	401.5	370	382	376.9
2014	431.5	471.5	451.5	406.9	446.9	426.9
2019	481.5	521.5	501.5	930	970	950
2024	531.5	571.5	551.5	940	980	960
2029	581.5	621.5	601.5	950	990	970
2034	631.5	671.5	651.5	960	1000	980

Year	**Vegetable** (US $/ton)			**Fruit** (US $/ton)		
	Low.	Upp.	Ave.	Low.	Upp.	Ave.
2007[a]	405.7	425.7	415.7	1100	1150	1126.53
2009	405.7	425.7	415.7	1100	1150	1126.53
2011[b]	501.5	541.5	521.5	1686.5	1726.5	1706.5
2014	501.5	541.5	521.5	1686.5	1726.5	1706.5
2019	551.5	591.5	571.5	1736.5	1776.5	1756.5
2024	601.5	641.5	621.5	1786.5	1826.5	1806.5
2029	651.5	691.5	671.5	1836.5	1876.5	1856.5
2034	701.5	741.5	721.5	1886.5	1926.5	1906.5

[a] (Alabdulkader et al., 2012), [b] (MOA, 2012)

The base for the crop prices has been calculated by accounting for the average price of different crops that belong to the same category. This data was collected in 2007 (Alabdulkader et al., 2012), while the data presented in Table 5-14 was collected in 2011 by MOA (2012) and it was specified for only vegetable and fruit crops. Meanwhile, the average price for cereal crops has been estimated to dramatically increase in 2019 because of the banning of wheat beginning in 2016. The price of wheat is low compared to other cereal crops while it accounts for the highest percentage of cereal crop production. This will lead to this high increase in price. Thus, the average yield and the price of each crop should be considered as a model input data in parallel with total water cost for the whole agricultural sector. Then, the decision variables of this sector, which are the cultivated land area and total irrigation water quantity, would be easier to calculate if the average yield and the price for each crop are known. These decision variables that will be accounted for in the model and they can help the WRM decision makers to choose better options for agriculture water use, while comparing the net cost and benefit of each option. These variables are explained in the next sections.

5.2.3. Irrigation Water Quantity

In order to calculate the total quantity of agricultural water for each period of the study, the irrigation quantity has been estimated from each water resources to each crop (m^3/ha). These assumptions can be viewed in Table 5-15. This study is focused on reducing the use of NGW and increasing the use of both RGWSW and RW for agricultural purposes and the estimation of irrigation quantity follows the same focus. There are two components to achieving this goal: (1) minimizing the allowable irrigation quantity for each crop cultivated area as it is shown in Table 5-15, where the allowable quantity of NGW will decrease beginning in 2024 for most of the crops while the allowable amount of RGWSW and RW will be constant, (2) since the use of NGW for agriculture use was 97% in 2000 (FAO, 2009), this study suggests decreasing the total amount of NGW while simultaneously effecting a gradual increase in the total amount of RGWSW and RW for agricultural purposes. Hence, the sustainability of the WRM system of Saudi Arabia can be accomplished by implementing this strategy.

Table 5-15 Assumption of irrigation quantity from different water resources for each crop during the planning horizon

Forage

Year	NGW (m^3/ha)			RGWSW & RW (m^3/ha)		
	Low.	Upp.	Ave.	Low.	Upp.	Ave.
2009	13980	14000	13990	13980	14000	13990
2014	13980	14000	13990	13980	14000	13990
2019	13980	14000	13990	13980	14000	13990
2024	9990	10000	9995	13980	14000	13990
2029	9990	10000	9995	13980	14000	13990
2034	9990	10000	9995	13980	14000	13990

Cereal

Year	NGW (m^3/ha)			RGWSW & RW (m^3/ha)		
	Low.	Upp.	Ave.	Low.	Upp.	Ave.
2009	8980	9000	8990	8980	9000	8990
2014	8900	9000	8990	8900	9000	8990
2019	8900	9000	8990	8900	9000	8990
2024	7400	7500	7450	8980	9000	8990
2029	7400	7500	7450	8900	9000	8990
2034	7400	7500	7450	8900	9000	8990

Vegetable

Year	NGW (m^3/ha)			RGWSW & RW (m^3/ha)		
	Low.	Upp.	Ave.	Low.	Upp.	Ave.
2009	10980	11000	10990	10980	11000	10990
2014	10980	11000	10990	10980	11000	10990
2019	10980	11000	10990	10980	11000	10990
2024	8990	9000	8950	10980	11000	10990
2029	8990	9000	8950	10980	11000	10990
2034	8990	9000	8950	10980	11000	10990

Fruit

Year	NGW (m^3/ha)			RGWSW & RW (m^3/ha)		
	Low.	Upp.	Ave.	Low.	Upp.	Ave.
2009	18900	19000	18950	18900	19000	18950
2014	16900	17000	16950	16900	17000	16950
2019	15950	16000	15975	15950	16000	15975
2024	12980	13000	12990	15950	16000	15975
2029	12980	13000	12990	15950	16000	15975
2034	12980	13000	12990	15950	16000	15975

Since the required water for each crop is not the same, the estimation for allowable irrigation quantity for any hectare has been assumed to be different. This is because there are different factors that can affect the irrigation water requirement such as weather conditions, soil types, type of crop and irrigation method (Ali, 2010). Hence, the difference in the water irrigation quantities for each type of crop can be understood.

On the other hand, Saudi Arabia is one of the world's largest producers of date fruit (Al-Farsi & Lee, 2008) which require massive quantities of water compared to other crops. This has led to fruit crops in general being categorized as the largest consumer of irrigated water. Additionally the watermelon, which is considered as a high water consumption crop, is cultivated in the country. However, the rate of allowable water consumption for fruits and vegetable will be decreased gradually because future plans to use green houses, which consumes much less water.

The forage crops occupy second position of water consumption. This is because they are planted in several regions where some of them need large amounts of water leading to an increase in the average allowable water quantity. There are plans to reduce the production of local forage crops in favor of imported forage crops. Local forage crops will also be restricted to regions with lower water needs. This strategy will allow for the reduction in the average irrigation water requirement by 2024. As mentioned earlier, the price and quantity of industrial products whose production consumes water is another factor in the planning of an effective WRM system for Saudi Arabia.

5.2.4. Price and Quantity of Petrochemical Products

Ethylene and propylene are the main petrochemicals produced in Saudi Arabia. The factories that produce these products utilize significant quantities of water for cooling purposes. The prices of both products are expected to increase gradually during the study timeframe. The data for selling prices in 2007, 2008 and 2009 was collected from Aljazira Capital report (2011), while a forecast of the 2014 prices has been provided by the research department of the National Commercial Bank Capital (NCBC, 2010). Subsequently, a fluctuation in the selling price has been assumed for the whole plan and these values can be seen in Table 5-16.

Table 5-16 Lower, upper and average values of petrochemical prices during the planning horizon

Year	Ethylene (US $/ton)			Propylene (US $/ton)		
	Low.	Upp.	Ave.	Low.	Upp.	Ave.
2007[a]	-----	-----	1125	-----	-----	1040
2008[a]	-----	-----	1380	-----	-----	1500
2009[a]	830	870	850	880	920	900
2014[b]	1151.6	1251.6	1201.6	1136.13	1236.13	1186.13
2019	970	1070	1020	1000	1100	1050
2024	800	900	850	850	950	900
2029	1151.6	1251.6	1201.6	1136.13	1236.13	1186.13
2034	970	1070	1020	1000	1100	1050

[a] (Aljazira Capital , 2011), [b] (NCBC, 2010)

The reason for the variations in the price is because of the relationship between petrochemical products and the price of oil. Whenever the price of oil decreases, there is a corresponding decrease in the price of the ethylene and propylene. Presently, the value of oil fluctuates for various reasons such as political conflicts in the exporting countries or the economic situations in the importing countries. These factors are unpredictable most of the time which means these changes need to be considered in order to develop realistic plans. Therefore, the same average price of 2009, 2014 and 2019 will be assumed again in 2024, 2029, and 2034 as a random assumption to cover the long-term plan of this study.

Moreover, the estimation of the quantity of the petrochemical products across the timeframe of this study is required in order to have minimum and maximum limitations for the economic benefit. This is demonstrated in Table 5-17 where the data of the past quantity of both products has been presented as only an average quantity for the periods that have been excluded from this research (i.e., 2007 and 2011), while the production quantity of 2014 was estimated by Kuwait Finance House Research Limited (KFHR, 2013). Regarding the remaining years, the average quantity has been assumed to increase by 1% yearly as the region is having a massive increase in these kinds of products, while the lower and upper bounds of each period are estimated randomly.

Table 5-17 Lower, upper and average quantities of ethylene and propylene products during the planning horizon

Year	Ethylene (million ton/year)			Propylene (million ton/year)		
	Low.	Upp.	Ave.	Low.	Upp.	Ave.
2007[a]	-----	-----	9.796	-----	-----	2.49
2009[b]	13.1	13.1	13.1	5.94	5.94	5.94
2011[a]	-----	-----	15.387	-----	-----	6.391
2014	19.00	20.00	19.50[c]	6.05	7.05	6.55[c]
2019	19.79	20.79	20.29	6.57	7.57	7.07
2024	20.83	21.83	21.33	6.93	7.93	7.43
2029	21.91	22.91	22.41	7.31	8.31	7.81
2034	23.06	24.06	23.56	7.71	8.71	8.21

[a] (GPCA, 2012), [b] (Aljazira Capital , 2011), [c] (KFHR, 2013)

5.2.5. Price and Quantity of Power Generation by DW

While the desalination cost is expensive and high energy consumption, some plants can provide economic benefits by generating electric power. These plants called dual-purpose plants and operated by MSF system, where there are six plants in Saudi Arabia of this kind (SWCC, 2012). This number is supposed to grow as a result of a real plan of constructing new plants in order to increase the water quantity of DW. In addition, a percentage of 30% of the generated power is used to operate some plants and their facilities, while 70% is sold by Saudi Electricity Company to different users. These users include domestic, industrial and agricultural sectors with the rate of 75.8%, 17.9% and 2.6%, respectively (SEC, 2010). In this study, only 70% of the power generation will be considered in accounting the net benefit of electricity. The quantity of this power is estimated based on the extension of DW which means more power will be generated in the future.

The electricity price varied based on the consuming quantity and the type of the user, and this is illustrated in Table 5-18 . For the domestic sector, if the customer, which includes the commercial, residential and governmental user, is consuming between 4.001 and 6 MW·h/month, the price will be US 32$/MW·h. Any more consumption will cost more which is similar in the idea of the block rate system. In this research, It has assumed that the average power consumption for the domestic sector is between 4.001 and 7 MW·h /month which cost US [32, 40] $/MW·h. Regarding the agricultural and industrial sector, their price will be estimated to be fixed (i.e., US [30, 34] $/MW·h), which is the reality for any power consumption required more than 5.001 MW·h/month. Furthermore, the reason for keeping the same cost for the power consumption during the planning horizon of this study is that the government, which owns the highest share rate of the Saudi Electricity Company, has no intention to increase this amount of money.

Table 5-18 Price of power generation during the planning horizon

Year	Domestic Use (US $/ MW·h)		Agriculture Use (US $/ MW·h)		Industrial Use (US $/ MW·h)	
	Low.	Upp.	Low.	Upp.	Low.	Upp.
2009	32	40	30	34	30	34
2014	32	40	30	34	30	34
2019	32	40	30	34	30	34
2024	32	40	30	34	30	34
2029	32	40	30	34	30	34
2034	32	40	30	34	30	34

Beside the price of power generation, getting estimation for the future production quantity is important, too. This is because it can be accounted for the net benefit and the cost of this area which help in calculating the economic profits and losses for the whole WRM system of Saudi Arabia. The estimation of future power production can be assumed based on the future of DW quantity. These estimations can be seen below in Table 5-19.

Table 5-19 Lower, upper and average bounds of power quantity during the planning horizon

Year	Lower Bound Quantity (million MW·h /year)	Upper Bound Quantity (million MW·h /year)	Average Bound Quantity (million MW·h /year)
2009	14.7	14.7	14.7
2014	22.40	23.97	23.18
2019	22.40	23.97	23.18
2024	22.18	32.72	27.45
2029	22.18	32.72	27.45
2034	22.18	32.72	27.45

5.3. REILP Model Formulation for WRM System in Saudi Arabia

Based on the previous sections of this chapter, the main elements of WRM system of Saudi Arabia include four water supply resources (i.e., NGW, RGWSW, DW, and RW) and three water consumers (i.e., agriculture, industry, and domestic).

Figure 5-6 presents a water allocation flow chart to describe the interrelations and linkages among these elements. Both NGW and RGWSW are treated in RO plants before they distributed to the water users. The main difference between these two resources is that the

NGW supplies all three water users, while RGWSW are consumed only by the domestic and agricultural sectors, which is shown as dotted line to illustrate that the water allocation is only from NGW.

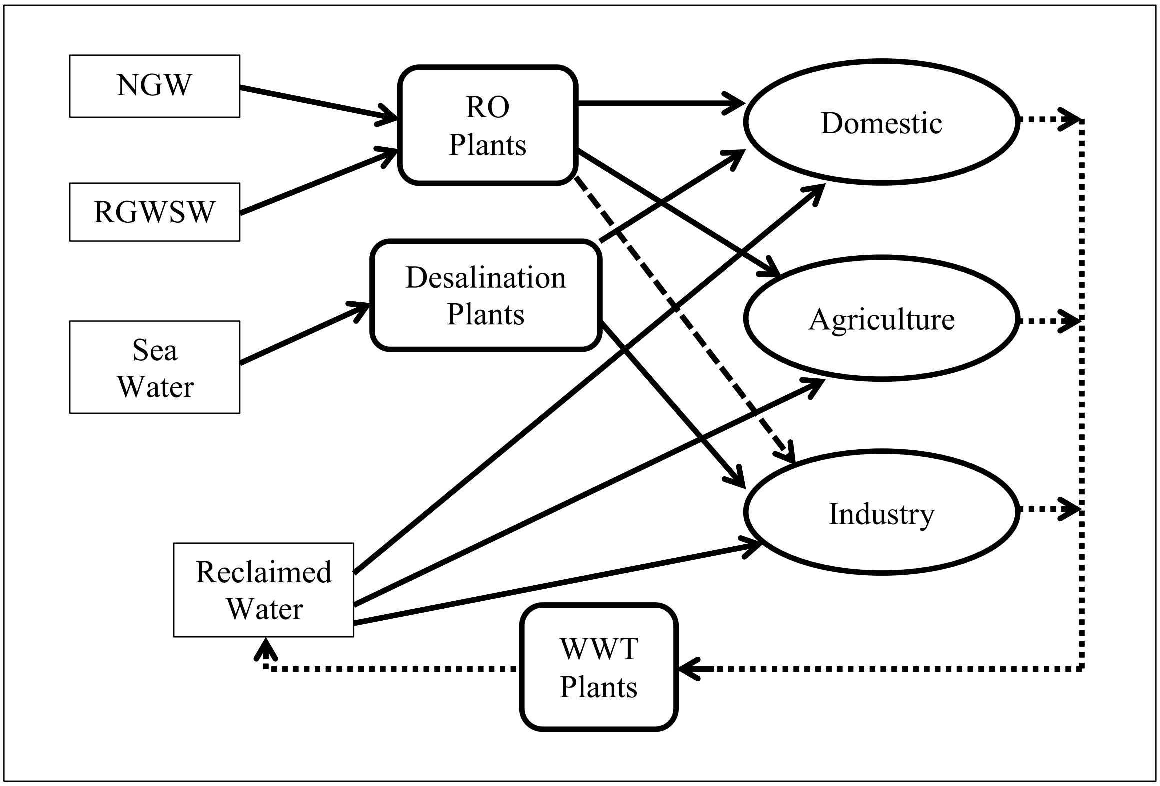

Figure 5-6 Water allocation flow chart

Meanwhile, the sea water is treated in the desalination plants by three different technologies (i.e., MSF, RO, and MED), and the output of the desalinated water is used by both domestic and industrial sectors. All water residues generated from the three sectors are collected and treated by wastewater treatment plants (WWTP). Some of the treated wastewater is used as the RW by all three sectors while the rest is discharged in different ways. Next, the WRM model is designed by refereeing to the components of these main elements as decision variables, with an objective of maximizing total net benefits.

5.3.1. Objective Function

The main objective of the proposed model is to maximize the total system net benefit, which is calculated by the total revenues produced by all three sectors minus the total cost of water consumptions. In this study, there are four different types of decision variables. One type of the decision variables, denoted as X_{ijt}, represents the water allocation from resource i to the user j in the planning period t, where i= 1, 2, 3, 4 representing DW, NGW, RGWSW, RW, respectively; while j=1, 2, 3, where j=1 representing domestic, agricultural, and industrial water users. Another type is the decision variable that denoted as L_{ict}, and represents the allowable land which can be used for crop c and irrigated by water resource i in different planning period t. Most of the elements in the objective function contain a unit cost of water. These units are taken from Table 5-12. In this study, the objective function is given as follows:

$$\text{Maximize Total Net Benefit} = (\text{I}) + (\text{II}) + (\text{III}) + (\text{IV}) - (\text{V}) - (\text{VI}) - (\text{VII}) \qquad (5.3.1)$$

Where, (I) is the revenues from the agricultural activities;

(II) is the benefit generated from the domestic use of waters;

(III) is the revenues from petrochemical industries;

(IV) is the revenues of power generated from DW plants;

(V) is the irrigation water cost;

(VI) is the domestic water cost; and

(VII) is the industrial water cost.

(I) Agricultural Revenues

$$\sum_{i=2}^{4}\sum_{c=1}^{4}\sum_{t=1}^{6} L_{ict} AY_{ct} P_{ct} \times 5 \qquad (5.3.2)$$

Where, (5.3.2) is the total revenues generated from crop production through consuming water resources *i* by crop *c* in each planning period *t*. L_{ict} is the area of land cultivated per year for crop *c* (ha/year), where *c*=1, 2, 3, 4, representing different types of crops: 1 for forage, 2 for cereal, 3 for vegetable, and 4 for fruit crop, and *i* represents the water resources available for crop irrigation. As mentioned earlier, the DW is not used for irrigation purposes in Saudi Arabia; therefore, *i* = 2, 3, 4, representing, NGW, RGWSW, RW, respectively. AY_{ct} is the average yield of crop c in period t (ton/ha) and P_{ct} is the price of crop *c* in period *t* (US \$/ton), where *t*=1, 2,…, 6. Each period is 5-year long and the total planning horizon is 30 years.

(II) Domestic Revenues

$$\sum_{i=1}^{4}\sum_{t=1}^{6} X_{i1t} P_{i1} \times 5 \qquad (5.3.3)$$

This revenue is generated through selling water to the domestic users. X_{i1t} is the quantity of water consumed by domestic users per year (m^3/year) in period *t*. P_{i1} is the price of one cubic meter of water delivered to domestic users (i.e., j=1) from different water resources i (US \$/ m^3), and because this price is assumed to be fixed during the entire planning horizon, the period t is not included.

(III) Petrochemical Revenues

$$\sum_{i=1,2,4} \sum_{e=1}^{2} \sum_{t=1}^{6} Pc_{iet} P_{et} \times 5 \tag{5.3.4}$$

Where, (5.3.4) is the total revenue generated by the petrochemical industries through consuming water from three water resources (i.e., NRW, DW, and RW) during the planning horizon t. In this expression, Pc_{iet} is the quantity of petrochemical products e generated through using water resources i for cooling purposes in period t (ton/year), where, e=1 for ethylene, and e=2 for propylene, while P_{et} is the market price of petrochemical product e in period t (US \$/ton).

(IV) Power Revenues

$$\sum_{j=1}^{3} \sum_{t=1}^{6} Q_{1jt} EP_{1j} \times 5 \tag{5.3.5}$$

Where, (5.3.5) is the total power revenues. Q_{1jt} is the quantity of power that generated from only MSF desalination plants (i.e., i=1) and consumed by sector j during each planning period t (MW·h/year). EP_{1j} is the electricity unit price which its value is changeable based on the user j (US \$/ MW·h). All the prices are shown in Table 5-18.

(V) Irrigation Water Cost

$$\sum_{i=2}^{4}\sum_{c=1}^{4}\sum_{t=1}^{6} L_{ict} I_{ict} C_{i2} \times 5 \tag{5.3.6}$$

$$\sum_{i=2}^{4}\sum_{c=1}^{4}\sum_{t=1}^{6} L_{ict} I_{ict} \cong \sum_{i=2}^{4}\sum_{t=1}^{6} X_{i2t} \tag{5.3.7}$$

Where, (5.3.6) is the total water irrigation cost per year (US \$/year) from water resource *i* to crop *c* during each period *t*. I_{ict} is the irrigation water quantity from water resource *i* to crop *c* (m^3/ha) in period *t*, as provided in Table 5-15. C_{i2} is the unit cost of irrigation water consumed from water resource *i* (US \$/ m^3). Equation (5.3.7) is important to know the total agricultural water quantity, which is taken from each source *i* during period *t* in order to calculate the total agricultural water which is denoted as X_{i2t} where *j*=2.

(VI) Domestic Water Cost

$$\sum_{i=1}^{4}\sum_{t=1}^{6} X_{i1t} C_{i1} \times 5 \tag{5.3.8}$$

Where, (5.3.8) is the total cost of domestic water, from each water resource *i* in period *t* (US \$/ year). X_{i1t} is the domestic water quantity from resource *i* to the domestic sector, where *j*=1, in period *t* (m^3/year). C_{i1} is the unit cost of domestic water consumed from water resource *i* (US \$/ m^3). This cost includes treatment, collection, and distribution cost.

(VII) Industrial Water Cost

$$\sum_{i=1,2,4}\sum_{t=1}^{6} X_{i3t} C_{i3} \times 5 \tag{5.3.9}$$

Where, (5.3.9) is the total cost of water used by the industries from three water resources (i=1, 2, 4) in period t (US \$/year). X_{i3t} is the quantity of water delivered from resource i to the industries (i.e., j=3) in period t (m^3/year). C_{i3} is the unit cost of industrial water consumed from water resource i (US \$/ m^3). This cost includes treatment, collection, and distribution cost.

5.3.2. Constraints

(1) Water Supply Limitation Constraints

$$X_{11t} + X_{13t} \le TDW_t, \qquad \forall t \tag{5.3.10}$$

$$\sum_{j=1}^{3} X_{2jt} \le TNGW_t, \qquad \forall t \tag{5.3.11}$$

$$X_{31t} + X_{32t} \le TRGWSW_t, \qquad \forall t \tag{5.3.12}$$

$$\sum_{j=1}^{3} X_{4jt} \le TRW_t, \qquad \forall t \tag{5.3.13}$$

To understand equations (5.3.10) to (5.3.13), it is better to have a look at

Figure 5-5, where all water resources have been matched to the suitable users. Equation (5.3.10) is one of the water limitation constraints where the total water supply from DW

(TDW_t) in any period t should be equal or greater than the domestic and industrial water supplies quantity (i.e., X_{11t} and X_{13t}, respectively) in period t. Equation (5.3.11) is another water limitation constraint where the NGW supply to each sector j during period t should be equal or less than the total water supply from NGW ($TNGW_t$) in the same period t. Similarly, Equations (5.3.12) and (5.3.13) are the total water supply of RGWSW and RW, respectively, which should be equal or bigger than water use of each related sector j in any period t. However, the main difference here is that RGWSW is only used by domestic and agricultural sector, while RW is planned to be consumed by all sectors. The upper and lower bound values of TDW_t, $TNGW_t$, $RGWSW_t$, and RW_t are shown in Table 5-5, Table 5-3, Table 5-4 and Table 5-6, respectively.

(2) Industrial Production Limitation Constraints

$$\sum_{i=1,2,4} Pc_{iet} \leq TPc_{iet}, \qquad \forall t; e. \tag{5.3.14}$$

$$\sum_{j=1}^{3} Q_{1jt} \leq TEl_{1jt}, \qquad \forall t, e. \tag{5.3.15}$$

Where, (5.3.14) is the petrochemical production limitation constraint. In fact, this equation is important to ensure that the total petrochemical production of each product e (TPc_{iet}), which their limitations have been stated in Table 5-17, should be equal or greater than the ethylene and propylene products during each period t (ton/year). Meanwhile, Pc_{iet} is the quantity of each petrochemical product e, which are using industrial water from only three water resources *i* (i.e., NRW, DW, and RW) for cooling purposes during the planning horizon *t* (ton/year). These products are only two: *e*=1 for ethylene, and *e*=2 for propylene.

On the other hand, constraint (5.3.15) represents the total power generation through the treatment of sea water in the desalination plants (*i*=1) in each period *t* (TEl_{1jt}), which is shown in Table 5-19. This quantity should be equal or greater than the provided power to each sector *j* in each period *t* (MW·h /year), and it is symbolized as Q_{1jt}.

(3) Crop Cultivated Area Limitation Constraints

$$\sum_{i=2}^{4} L_{ict} \le TCA_{ict}, \qquad \forall t; c. \tag{5.3.16}$$

Where, (5.3.16) is representing the limitation of crop cultivated area from only three water resource (i.e., i=2, 3, 4) in period *t*. This area, which is denoted by L_{ict}, should be equal or less than the total suggested area TCA_{ict} that have been illustrated in Table 5-9.

(4) Water Demand Limitation Constraints

$$\sum_{i=1}^{4} X_{i1t} \ge TDoW_t, \qquad \forall t \tag{5.3.17}$$

$$\sum_{i=1,2,4} X_{i3t} \ge TInW_t, \qquad \forall t \tag{5.3.18}$$

Equation (5.3.17) represents the limitation constraint for domestic water demand. X_{i1t} is the water quantity from each water resources *i* to the domestic sector, where *j*=1, during any period *t*. The sum of this water supply should be equal or greater than the total water demand for domestic sector ($TDoW_t$) in period *t* in order to avoid any shortage of water. The same process is applied to constraints (5.3.18), but with replacing the domestic use by the industrial use. The total water demands for the domestic and industrial sectors are shown in Table 5-10 and Table 5-11.

(5) Technical Constraints

$$X_{ijt}, L_{ict}, P_{iet}, Q_{1jt} \geq 0, \qquad \forall i; j; t; c; e. \tag{5.3.19}$$

Equation (5.3.19) is the technical constraint, where all the decision variables must be equal to or greater than 0.

CHAPTER 6

RESULTS AND DISCUSSIONS

After illustrating the methodology and the case study in the last two chapters, the results of this thesis will be presented. The first step is to solve the model of the WRM system by using the BWC algorithm, which can be called also as ILP. Then, the obtained results of this algorithm are going to be the base of the REILP model formulation. Both methods will be solved by LINGO software. From the REILP results, two scenarios will be selected in order to evaluate them. The main evaluations will be for the risk level, economic return, and water allocation, following by an examination for the environmental impact and social perspective. The last two factors are significant to give the stakeholders of the WRM system a full perception of the issue, which would lead to their cooperation with the selected plan.

6.1. ILP Results

As discussed in Chapter 3, the traditional ILP model should be reformulated into two sub-models. Then, their solution will represent the upper and lower bound of both decision variables and the objective function. Table 6-1 provides the results of the BWC algorithm for the case study, which include the total revenues and the costs of all three sectors (i.e., domestic, agriculture and industrial sectors, respectively). The first row represents the best-case of BWC algorithm to obtain the highest net-benefit of the WRM system of Saudi Arabia, while the second row represents the worst-case in terms of the system benefit.

Table 6-1 BWC results for the WRM system of Saudi Arabia during the planning horizon

BWC	Total Net Benefit (10^9\$)	Total Domestic Revenues (10^9\$)	Total Agriculture Revenues (10^9\$)	Total Industrial Revenues (10^9\$)	Total Domestic Cost (10^9\$)	Total Agriculture Cost (10^9\$)	Total Industrial Cost (10^9\$)
Best-Case	932.80	9.31	198.00	943.12	91.67	93.66	32.30
Worst-Case	764.74	5.76	186.82	803.09	88.93	114.04	27.95

According to Table 6-1, both cases provided huge net benefits; however, the difference between the two cases is large — around US \$ 168 billion. Moreover, the net benefits of the two solutions are led by the industrial sector which provides the largest benefits and includes the profits of both petrochemical products and power generation. Meanwhile, since the industrial sector is only accounting for the industrial water cost, its total cost is the lowest among all sectors. On the other hand, the total profits of the agricultural sector are almost double the total cost in the best-case, while the gap is still huge in the worst-case. This explains the reason for this sector remaining the largest water consumer. In fact, most of farms are owned by the public, and are not easy to control, while the industrial sector is owned by the government. Furthermore, the lowest total benefits are coming from the domestic sector, while the total cost for this user is much higher with a huge gap between benefits and costs. This leads to money losses by this sector and makes it the weakest part in the WRM system.

6.1.1. Water Allocation Quantity Based on the BWC Results

The previous details of the ILP results were illustrated in terms of the net benefit of the WRM system, while also knowing the water allocation quantity that is delivered to each user of each case during the planning horizon is important.

Figure 6-1 shows the results of the BWC algorithm regarding water allocation to each sector with the upper and lower bound of each case. While agricultural water use will remain above 9000 MCM during the first 3 periods, a gradual decrease in the water quantity will occur beginning in 2024 due to various factors such as the dependence on sufficient water irrigation methods and also importing some crops to cover the country's needs. Hence, it is important to note that there is no big difference between the lower and upper bound of the agricultural water quantity in
Figure 6-1. This is because the design of this long-term plan has considered the country's minimum need for agriculture products and going below these amounts might cause several economic and social impacts.

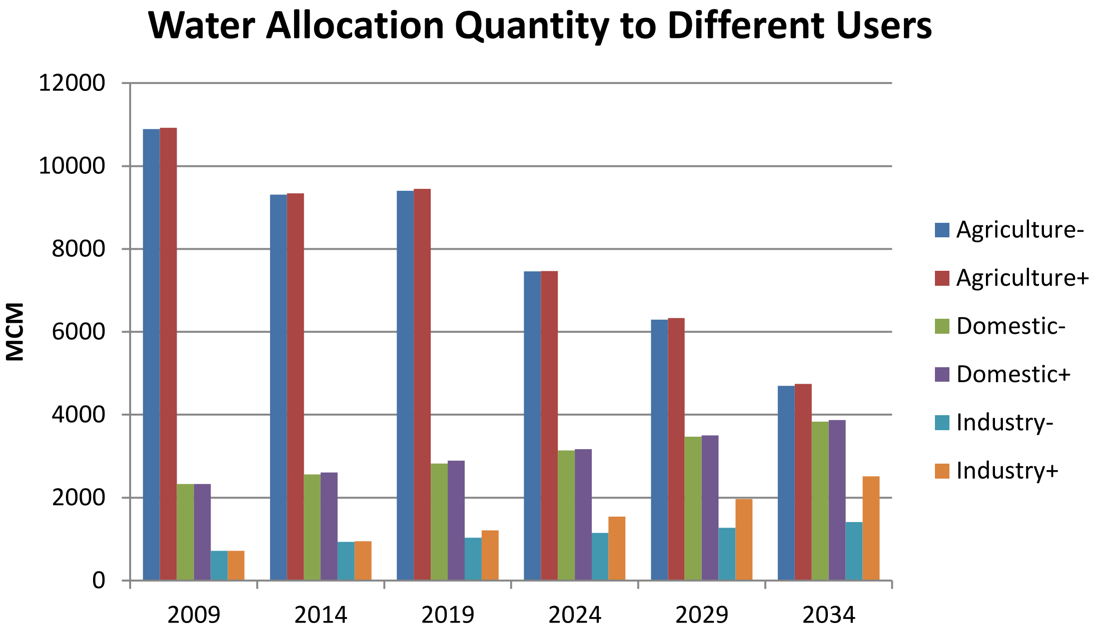

Figure 6-1 Water allocation quantity to each sector during the planning horizon (BWC)

On the other hand, domestic water use will remain the second largest consumer of water in terms of quantity during the whole planning horizon. Furthermore, the difference between the lower and upper bound water quantities in both cases are almost stable. This is because water is considered as the lifeblood of humans, and satisfying their basic needs with regard to it is mandatory. Meanwhile, water use by the industrial sector is the lowest among all sectors during the long-term plan. Nevertheless, the gap between the lower and upper bound of water quantity will increase from 2019 and reach its maximum limit in 2034. The future of industrial sector is not very clear since the economy of Saudi Arabia is dependent on oil exports, and the current industrial activities do not require a massive amount of water. Hence, the possibilities of water use by this sector are hard to predict, leading to the difference between lower and upper bound of water quantity.

In addition to the description of water consumption by each sector, the total water quantity for each period that is generated from the ILP solutions should be taken into account. Figure 6-2 shows the total water quantity that should be reached in each period during the 30-year planning horizon based on the best-case and worst-case solutions. These quantities include all four water resources (i.e., NGW, RGWSW, DW and RW) which are delivered to all three sectors. Even though the best-case solution is achieving a huge net-benefit for the WRM system, the water consumption might not be the in the ideal situation. A decrease in water amount will happen in four periods through the designed plan, with the exception of a sudden increase that will take place in 2019. This increase is due to the increase of RW consumption, while the use of RGWSW is almost the same of the previous period. The positive side of the best-case result is that the water quantity will remain below 14 MCM during the planning horizon, which can enhance the chances to satisfy to country's water demand.

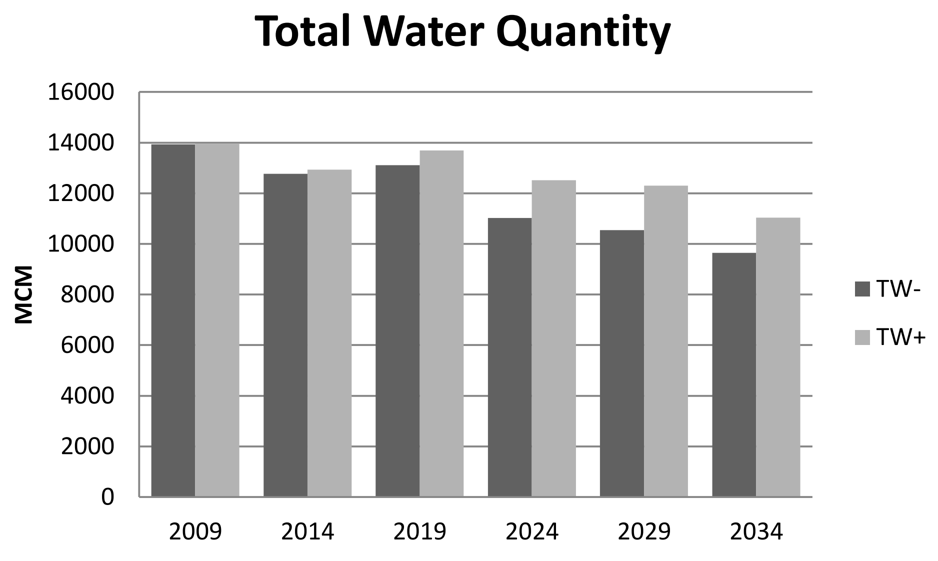

Figure 6-2 Total water quantity based on the lower and upper bounds of BWC

In contrast, the water quantity that is based on the worst-case solutions is going to be below 11 MCM from 2024 to the end of the long-term plan. This quantity might negatively affect the development of the country, given water's critical role for various products. In addition, the same situation that is expected between 2009 and 2019 (i.e., a decrease then increase) is repeated reflecting unbalance in the WRM system. This is because the same reason for the best-case solution. To indicate which case is better, the risk factor must be known in order to address the uncertainty in the system. Unfortunately, most ILP models do not include this factor, which makes them ineffective for decision makers.

6.1.2. Distribution of Water Ratio Based on BWC solutions

Determining the rate of each water resource in any WRM system is necessary in order to improve its sustainability. This can be calculated by referring to the total water quantity (Figure 6-2) and multiplying it by the water ratio in Figure 6-3 (i.e., best-case solution), and Figure 6-4 (worst-case solution). As Figure 6-3 and Figure 6-4 show that the NGW rate is decreasing gradually during the long-term plan in each case, and this is considered as one of the main objectives for this research, and is also significant for water authorities of the country. At the same time, the quantity of RW is increasing in both cases as well, while the growing rate in the best-case is higher than in the worst-case. Hence, the RW is a key element in the sustainability of the WRM system since most of it can be reused at a constant rate.

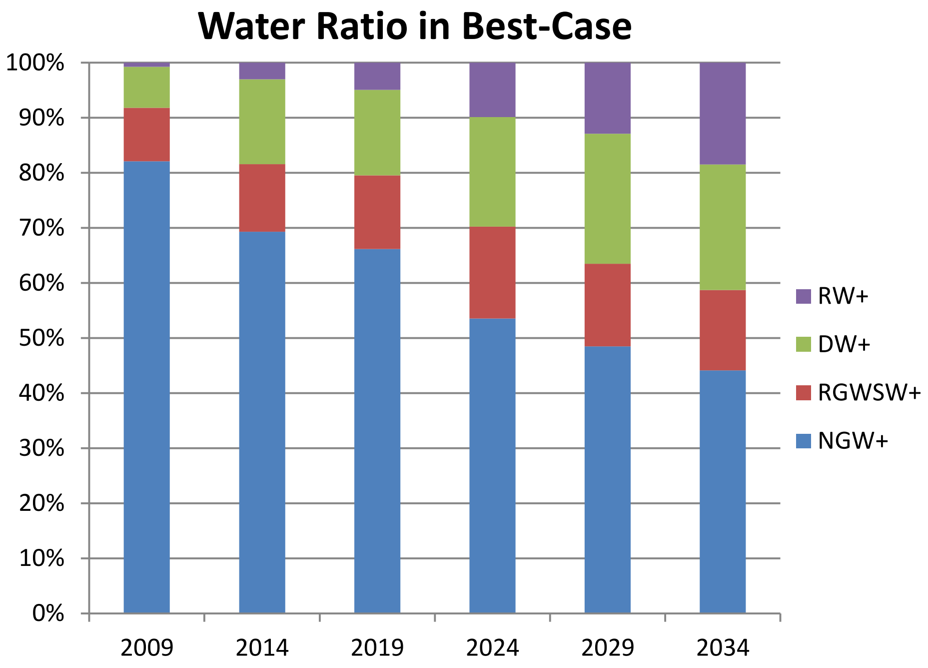

Figure 6-3 Distribution of water resources ratio based on the best-case solution

On the other hand, the rate of RGWSW is increasing slightly in the first 15 years and remains at almost the same rate in the final 15 years projected primarily because part of it depends on the rainfall rate and this is out of human control. The increase of the first 10 years is due to the decease of relying on NGW and using the RGWSW as a one of the alternatives to offset the water demand. Regarding the rate of DW, it is increasing more in the best-case situation because of the large number of water purification plants that will be built if the economy of the country stays strong. Otherwise, the cost of these plants and their operation is almost the highest among all water resources and any instability in the economic situation might decrease its rate as is predicted in Figure 6-4.

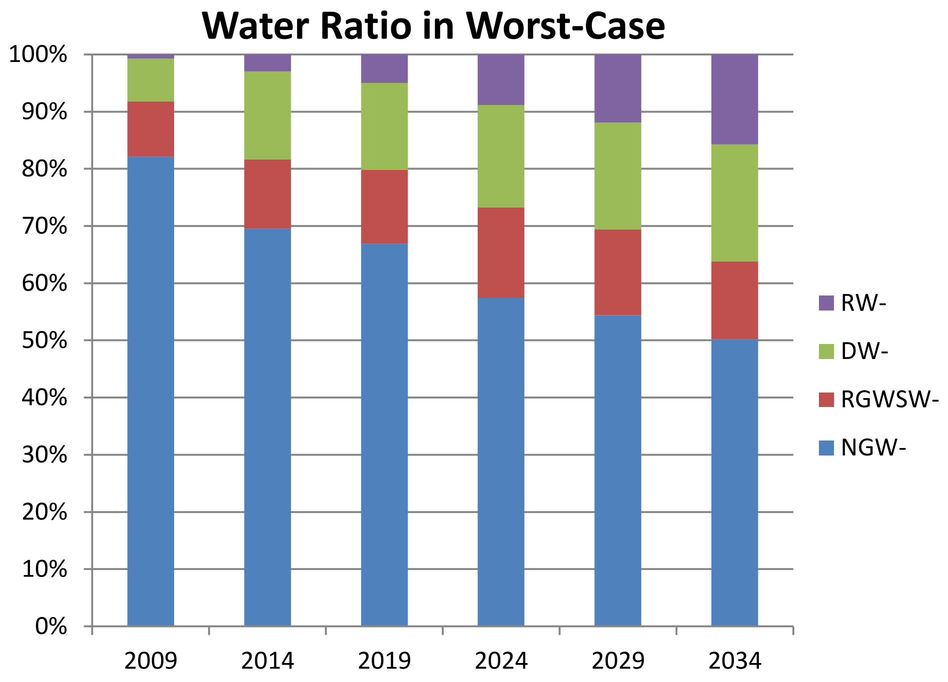

Figure 6-4 Distribution of water resources ratio based on the worst-case solution

Finally, the results of the BWC algorithm are useful in finding the extreme limits of any environmental system. Nonetheless, it includes some flaws such as the non-feasible and non-optimal solutions that have been shown in Chapter 3. In addition, the risk level of violating the constraints is ambiguous, which makes the solutions given by these methods undesirable to decision makers. However, the BWC solution is the base of the formulation of the REILP scheme, which does not contain all these disadvantages, and can provide the risk level for each aspiration level. As a result, REILP is used in this research in order to improve the WRM system of Saudi Arabia by presenting different scenarios with an indication of the risk level of each one.

6.2. REILP Results under Different r_0

In order to apply the REILP model, the solutions of the BWC algorithm should be included in the model formulation as mentioned in 4.2. Then, the results of the REILP model are based on the pre-defined value of the aspiration level, which has been calculated from 0 to 1 with a step of 0.1. The values of the objective function that are generated from the REILP approach are shown in Table 6-2. These results include the total of: (1) net-benefit (TNB), (2) domestic revenues (TDOR), (3) agriculture revenues (TAGR), (4) industrial revenues (TINR), (5) domestic cost (TDOC), (6) agriculture cost (TAGC), (7) industrial cost (TINC); for each aspiration level in US billion dollars (10^9\$). The industrial benefit combines between the benefits of petrochemical products and power generated by the DW plants. Most importantly, the risk function value, which is shown in the second column of Table 6-2, for each aspiration level can incorporate the risk level with each solution or option. These solutions below can be very useful for both decision makers and stakeholders in order to understand the features of each alternative.

Table 6-2 REILP results for the objective function under different aspiration levels

Aspiration Level	Risk Function	TNB (10^9\$)	TDOR (10^9\$)	TAGR (10^9\$)	TINR (10^9\$)	TDOC (10^9\$)	TAGC (10^9\$)	TINC (10^9\$)
0	0.00	764.74	5.75	172.38	803.09	73.88	84.95	57.65
0.1	0.04	781.55	6.15	173.26	810.86	74.81	86.84	47.07
0.2	0.04	798.35	7.05	172.63	819.63	78.75	85.72	36.49
0.3	0.08	815.16	6.95	175.01	826.40	76.69	90.61	25.90
0.4	0.12	831.96	7.36	175.88	844.02	77.63	92.50	25.17
0.5	0.18	848.77	6.76	171.75	868.83	78.56	91.39	28.63
0.6	0.22	865.57	8.16	177.63	881.65	79.50	96.27	26.09
0.7	0.25	882.38	7.06	183.00	897.46	78.44	97.16	29.54
0.8	0.33	899.19	8.96	179.38	919.27	81.38	100.05	27.00
0.9	0.45	915.99	9.36	180.25	938.08	82.31	101.93	27.46
1	0.61	932.80	9.76	181.13	956.89	83.25	103.82	27.92

As mentioned in the ILP results, the highest net-benefit of the WRM system is US \$ 932.8 billion when the aspiration level is equal to 1, the risk function is 0.61, which is considered as a high risk for violating the constraints of the model. In contrast, the lowest net-benefit of WRM system is US \$ 764.74 billion when the aspiration level is equal to 0; while the risk function in this case is 0, reflecting no risk of violating the constraints of the model. Meanwhile, the big difference in the total net-benefit between the two solutions, which is equal to US \$ 168.06 billion, makes the lowest net-benefit unfavorable even if the risk level is very low.

The main results of Table 6-2 are illustrated in Figure 6-5 in order to indicate the relationship between the pre-defined aspiration level with risk function and total net benefit of the WRM system. This demonstrated that any increase in aspiration level causes a rise in both risk function and total net-benefit. Also, the relationship between the aspiration levels with the total net-benefit is linear, while it is non-linear with the risk function. In addition, the solutions with risk level lower than 0.2 and aspiration level from 0 to 0.5 can be sorted as safe options for decision makers in terms of violating the constraints of the model. Vice versa, the higher risk function value that increase gradually can be categorized as medium- and aggressive-risk, respectively.

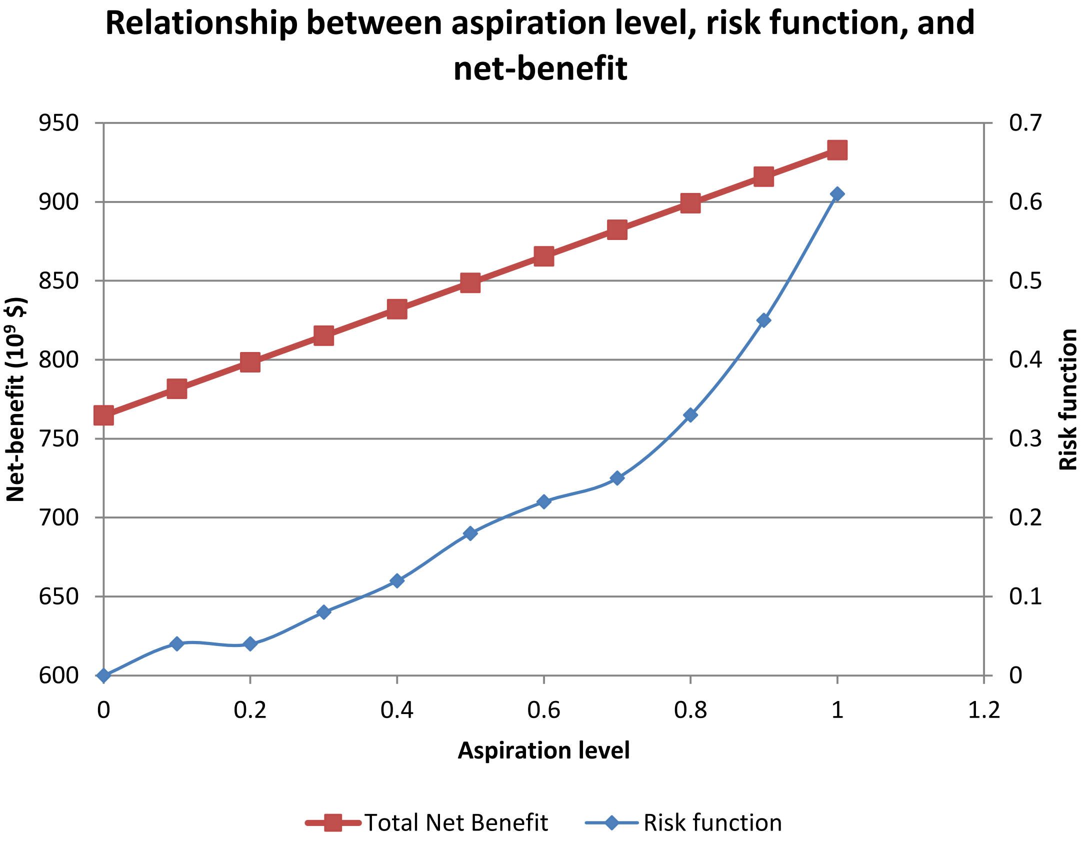

Figure 6-5 Relationship between aspiration level with risk function and net-benefit

6.2.1. Analysis of the total Revenues and Costs

For analysis of the benefits and costs of the WRM system based on REILP model, the whole system has been divided into two groups. The first group, shown in Table 6-3, presents the revenues of each water resource to different sectors. The second group, shown in Table 6-4, provides the distribution of the costs and ends by collecting the total net-benefit. The values in these tables are based on the biggest and smallest values of each parameter determined by all aspiration levels of the REILP model

Table 6-3 Distribution of total system revenues from the REILP model

Sector and Water Resources	Benefit (10^9\$)	Percentage (%)
Revenues of Domestic Sector		
DW	[1.94, 3.87]	[0.20, 0.34]
NGW	[2.96, 4.44]	[0.30, 0.39]
RGWSW	[0.61, 1.05]	[0.06, 0.09]
RW	[0.24, 0.41]	[0.02, 0.04]
Revenues of Agricultural Sector		
DW	[0, 0]	[0, 0]
NGW	[140.69, 147.89]	[14.32, 12.87]
RGWSW	[28.36, 29.81]	[2.89, 2.59]
RW	[4.49, 4.72]	[0.46, 0.41]
Revenues of Petrochemical Products		
DW	[150.32, 176.07]	[15.30, 15.32]
NGW	[282.45, 321.98]	[28.75, 28.02]
RGWSW	[0, 0]	[0, 0]
RW	[351.16, 435.35]	[35.75, 37.89]
Revenues of Power Generated by DW		
Domestic	[15.28, 19.11]	[1.56, 1.66]
Agriculture	[0.49, 0.56]	[0.05, 0.051]
Industry	[3.38, 3.84]	[0.34, 0.33]
Gross System Revenues	[982.38, 1149.07]	[100, 100]

Among all revenues gained from the WRM system in Table 6-3, the petrochemical products contribute the highest percentage of the total system benefit (i.e., [79.8, 81.23] % of the total benefit), with a priority on using the RW in order to cool their factories. This is a good sign from the economic point of view and the sustainable condition. Since the economy of Saudi Arabia depends on oil exports, the petrochemical sector will remain second in importance. In addition, this result shows that the use of RW will increase and possibly become the main source for the industrial sector, reflecting one of the chief goals for water authorities of the country since the quantity of RW is sustainable and this can enhance the WRM system overall.

The profits gained by using the NGW are the next largest, indicating that the use of this precious resource will continue at a large quantity unless its consumption amount is decreased gradually during the planning horizon. The use of DW in the petrochemical products provides the lowest profit to the system – not surprising given that this resource is the most expensive compared to other water resources and should be used for limited purposes that required high water quality.

Regarding the profits of power generation from DW, the arrangement of the power benefit in Table 6-3 is different due to that the power is only generated from one water resource. However, it is sold to all three main sectors of this study Domestic use for power is the main user and contributes in 1.56% to 1.66% of the total benefit of the WRM system, which is equal to US $ 15.28 billion and US $ 19.11 billion, respectively. Industrial and agricultural power consumption provides lower benefits. This is because the industrial activities in Saudi Arabia are limited in compared to the industrial countries, while the agriculture sector, with most farmers using diesel-powered electric generators, does not need a lot of electrical energy.

After illustrating the total revenues from the industrial sector, which includes the petrochemical products and power sales, consuming water for agricultural purposes provides large revenues to the WRM system in Saudi Arabia. These revenues have an interval that ranges between 15.87% and 17.67% of the total benefit. The profit gained from the consumption of NGW is the highest while that of RW is the lowest, with non-use of DW for this sector. Regarding the NGW use for this sector, since it has been the largest water supply for many years, it will be better to decrease its use gradually in order to treat the socioeconomic impacts of the farmers. This can happen by providing alternative water resources and convincing the farmers about the negative effects of continuing to use the NGW at a high rate.

Meanwhile, the crop production profits that come from using RGWSW positive because of the sustainability of this resource, and the focus on increasing its use is one of the main objectives of this research. At the same time, the RW profits are low since it requires tertiary treatment, which is expensive and does not exist currently in every WWTP, and its use is restricted to some crops. Next, Table 6-4 will illustrate the REILP results of the total system cost that should be subtracted from the total benefit of the system in order to find the lower and upper bound of the system net-benefit.

Table 6-4 Distribution of total system cost and net-benefit from the REILP model

Sector and Water Resources	Cost (10^9\$)	Percentage (%)
Cost of Domestic Sector		
DW	[56.86, 62.19]	[26.13, 28.76]
NGW	[12.44, 14.81]	[5.71, 6.85]
RGWSW	[1.74, 2.61]	[0.80, 1.21]
RW	[2.84, 3.65]	[1.30, 1.69]
Cost of Agricultural Sector		
DW	[0, 0]	[0, 0]
NGW	[74.91, 88.98]	[34.42, 41.16]
RGWSW	[7.57, 11.35]	[3.48, 5.25]
RW	[3.65, 4.69]	[1.68, 2.17]
Cost of Industrial Sector		
DW	[17.22, 8.79]	[7.91, 4.06]
NGW	[8.68, 5.66]	[3.99, 2.62]
RGWSW	[0, 0]	[0, 0]
RW	[31.75, 13.47]	[14.59, 6.23]
Gross System Costs	[217.65, 216.19]	[100, 100]
Total Net-Benefit	[764.74, 932.88]	

Table 6-4 shows the costs of domestic, agricultural and industrial water use of the WRM system. The agricultural sector represents the highest cost with an interval ranging from 39.57% to 48.58% of the total system cost. This cost comes from only three water resources, which are NGW, RGWSW and RW. The main water resource for agriculture is NGW and it required US \$ 74.91 billion to US \$ 88.98 billion, which represents [34.42, 41.16%] of the total water agricultural cost. This might be considered as a negative sign since this research is aimed to reduce dependence on this resource. However, both solutions provide a gradual decrease for this resource, which could enhance the sustainability of this type of water.

On the other hand, the use of sustainable water resources (i.e., RGWSW and RW) have been increased; their costs range from US $ 7.57 to 11.35 billion for RGWSW, and from US $ 3.65 to US $ 4.69 billion for RW. Moreover, there is variation between the costs of each cubic meter of water depends on the water resource. In other words, the cost of RW is almost three times the cost of RGWSW, which give an indicator for the water quantity that should be consumed for each resource.

The total percentage of domestic water cost in Table 6-4 is [33.94, 38.51%]. Even though the domestic water quantity is the lowest among all sectors, the high expenses of DW, which is the main water resource for municipal use, produces this result. The use of NGW is in second place due to the difficulties of transporting DW to some regions that are far away from the sea. In contrast, the use of RGWSW, which is cheap compared to other water resources, is widespread in high elevation regions because most of these regions have abundant rainfall. RW is the last water resources for the municipal use, which is provided by the treatment of the residual water of all other resources. However, because of the high treatment cost of this water, it appears to be in third place in terms of total expenses.

In comparison to agricultural and domestic water costs, industrial water has the lowest total cost. Meanwhile, RW comes higher in terms of the cost of industrial water. This result is showing in the both cases of REILP solutions. This is because the increase use of RW for industrial activates is one of the main objectives of this research. While this type of water is sustainable, the quantity of industrial water is projected to increase in order to vary the economic benefits of the country. This is due to that the country cannot depend on oil production for ever.

Then, the cost of using DW comes second. This is because some industrial products need only high quality water and many of desalination plants are located near the biggest industrial areas. Therefore, the use of this type of water becomes a priority. However, in the few and small industrial cities that are located in the middle of the country, the use of NGW is required because it needs sufficient water quality and its total cost is lower than the cost of transporting DW. Furthermore, the use of NGW remains minimal for this sector, which indicates the effectiveness of this plan in converting the WRM system to be closer to the sustainable situation by relying more on other water alternatives.

To conclude, the total water cost for WRM system is divided between three sectors: agriculture, domestic, and industry. The domestic sector uses all four water resources, while the industrial and agricultural sectors use only three water resources. The gross system costs from the whole previous sectors ranges from US $ 216.19 billion to US $ 217.65 billion. These costs should be subtracted from the total system benefits that shown in Table 6-3 in order to obtain the total net-benefits that are provided at the end of Table 6-4 which are equal to US $ [764.74, 932.88] billion. Thus, the difference between the two values of the total net-benefit is massive, and it needs to be reconsidered in order to have a situation in the middle.

6.2.2. Examination of Two REILP Scenarios

After illustrating the WRM system in context of the best and worst views of REILP, it is now important to select two scenarios between these two cases and examine them. This step is more realistic in terms of ILP solutions, and these scenarios can reflect the effects of each option in terms of diverse factors such as environmental and social aspects. This could happen by comparing the influence of each water quantity in the provided REILP scenarios.

One of these scenarios represents the conservative option, which involves a low net-benefit from the system by consuming more water in order to meet the demand of the three sectors, while the second scenario is the opposite and represents the aggressive choice. In the latter case, the priority would be for the high net-benefit without regard for the future of water demand, which might lead ultimately to water scarcity. These two scenarios are shown below in Figure 6-6 and are characterized by two aspiration levels. The green curve above the solution columns is related to the forecasting of water demand during the planning horizon. It can be seen clearly in Figure 6-6 that the conservative solution includes lower water quantity than the aggressive solution, which means that the aggressive option is closer to water demand for each period in the planning horizon, especially from 2019 to the end of the long-term plan.

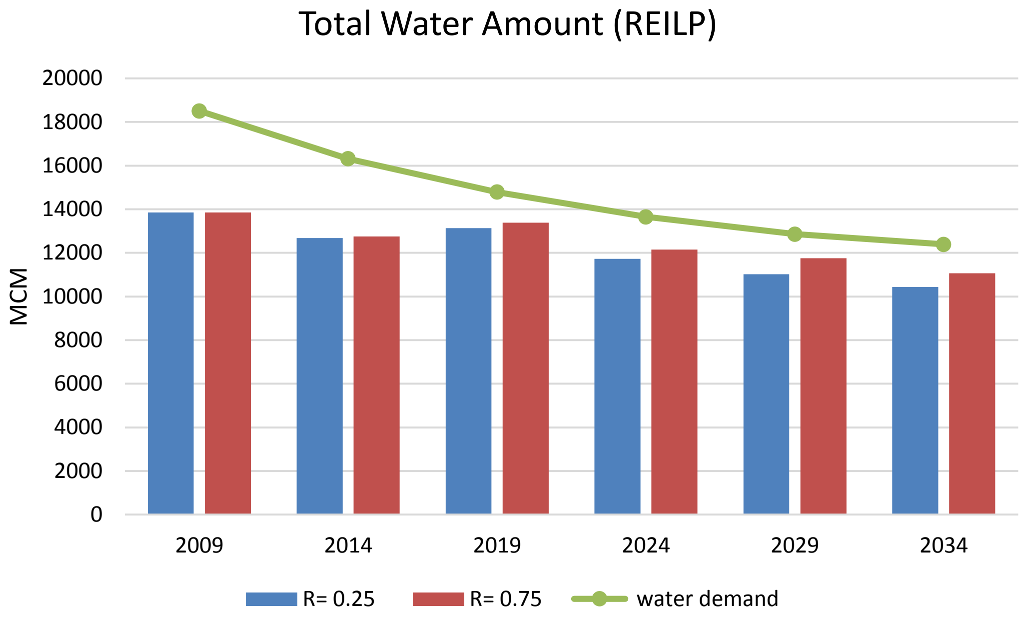

Figure 6-6 Water quantity based on two REILP solutions

The comparison between two scenarios starts with presenting the total system cost and benefit, which is related to the economic view. This includes determining the highest and lowest values for both options. Then, a discussion for these scenarios will be provided in terms of the preference of each selection from an ILP, environmental, and social point of views with focus on the water quantity of each option that shown in Figure 6-6. This discussion aims to integrate the factors of WRM system of Saudi Arabia in order to give the public and decision makers a chance to evaluate each option and its effects on the long-term.

6.2.3. Scenario 1: $r_0 = 0.25$

As mentioned above, this scenario is considered as a conservative option, where the priority is not for gaining a high net-benefit of the WRM system of Saudi Arabia, but rather to minimize the risk of violating the constraints of the model. This choice involves a low risk to violate the constraint of the model, and the risk function is only equal to 0.06, while the total net-benefit is equal to US $ 806.75 billion through the twenty five years. The details of the net-benefit and cost of this option are illustrated in Table 6-5, where the percentage of any revenue is positive, and the percentage of any cost is negative. Also, it can be noticed clearly that the industrial sector provides the largest benefit, while the agricultural sector represents the highest cost of the WRM system in Saudi Arabia.

Table 6-5 REILP solutions when aspiration level = 0.25

$R_0 = 0.25$	Value	Percentage %
Risk Function	0.06	-
Total Net-Benefit (10^9\$)	806.75	100
Domestic Revenue (10^9\$)	6.75	+0.84
Industrial Revenue (10^9\$)	822.52	+101.95
Agricultural Revenue (10^9\$)	188.74	+23.39
Domestic Cost (10^9\$)	76.22	-9.45
Industrial Cost (10^9\$)	37.34	-4.63
Agricultural Cost (10^9\$)	97.70	-12.11

6.2.4. Scenario 2: $r_0 = 0.75$

Conversely, the second scenario in this study aims to get the highest net-benefit of the system; along with these objectives comes a higher likelihood of violating the constraints of the model. Table 6-6 shows that the aspiration level in this case is equal to 0.75, while the total net-benefit is equal to US \$ 890.78 billion through the complete period of the planning horizon. Meanwhile, the risk function of this scenario is high and equal to 0.31. Similar to the previous scenario, the largest revenue will be from the industrial sector, while the highest cost will be from the agricultural sector followed by the domestic sector.

Table 6-6 REILP solutions when aspiration level = 0.75

$R_0 = 0.75$	Value	Percentage %
Risk Function	0.31	-
Total Net-Benefit (10^9\$)	890.78	100
Domestic Revenue (10^9\$)	8.67	+0.98
Industrial Revenue (10^9\$)	904.08	+101.49
Agricultural Revenue (10^9\$)	193.64	+21.74
Domestic Cost (10^9\$)	80.91	-9.08
Industrial Cost (10^9\$)	26.77	-3.01
Agricultural Cost (10^9\$)	108.02	-12.13

6.2.5. Scenarios Discussion

While the previous results have been concentrated on the total net-benefit and the value of each sector during the planning horizon, the allocation of water resources and how they relate to water demands should be discussed. This is significant in terms of interpretation of the results and effects upon the decision makers from the LP, environmental, and social views. The economic part is important but it is not the only purpose of this research. Moreover, the integration between economic, environmental, and social aspects will be more convincing for both water authorities and the public, thus making it easier for decision makers to select a scenario. This effort could enhance the sustainability of the WRM system of Saudi Arabia.

Before starting the evaluation of both scenarios, and regardless of their differences, one common feature of the two scenarios is that they both fall short of water demand over the course of the plan. One reason for this is that this research has not taken the livestock water needs plus some industrial activities into account due to the lack of such data. Therefore, the water supplies of missing data have not been considered which cause this decline in both scenarios from reaching water demand. This decrease can be resolved by decision makers by collecting this information and including it in future studies. Furthermore, this gap is reduced after 2014 even that the missing of such data which can reflect the effectiveness of these results.

6.2.5.1. Water Allocation

To examine both scenarios in more detail, it is better to present the allocation percentage of each water resource to the three sectors of each scenario. The conservative option can be seen in Figure 6-7, Figure 6-8, and Figure 6-9. As illustrated in Figure 6-7, the domestic sector will stay dependent on the DW and NGW for the entire period of this study. However, based on the solutions of ILP that were shown in

Figure 6-1 – which is the base of the REILP model formulation – the water quantity overall for domestic use slightly increases every 5 years. This increase is in parallel with the population increase, which is between 2% to 2.2% per year. Also, the big difference between the NGW and DW rates in 2009 and 2014 is because of the current projects intended to double the capacity of desalination plants in 2014. Then, some of these plants will be stopped due to the end of their life expectancy, which is the reason for the gradual decline. Meanwhile, the consumption rate of RGWSW will increase due to the construction of new dams, which can raise the dependence on this source for domestic and agricultural use. Likewise, the use of RW will increase due to the construction of new WWTPs in most of the country's regions (KAUST- KICP, 2012), while the awareness of the public will be raised to accept these alternative resources.

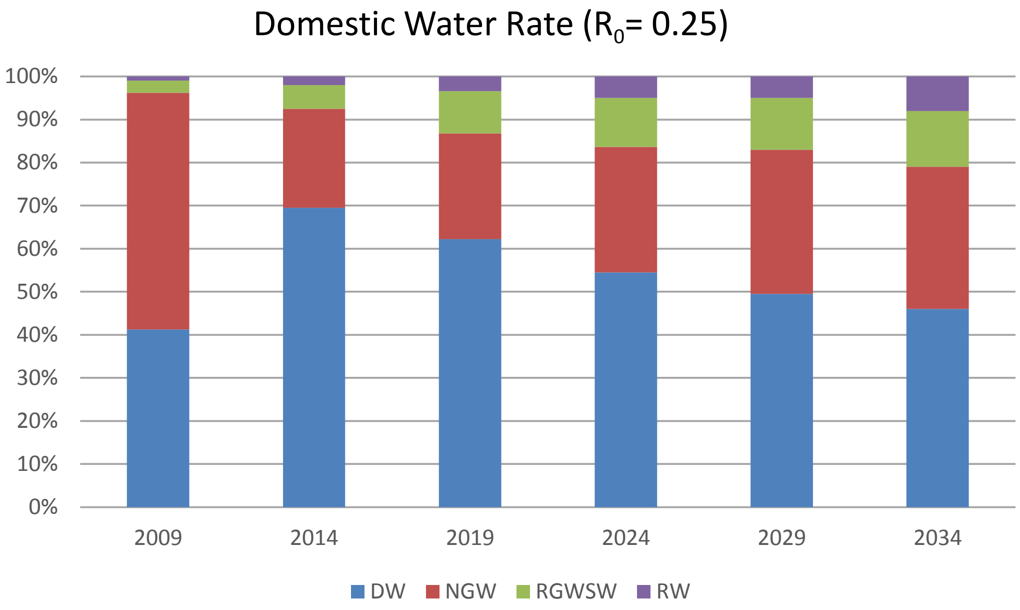

Figure 6-7 Allocation of water resources to the domestic sector when r_0 = 0.25

Regarding the agricultural water use in the first scenario, the total quantity is projected to decrease (

Figure 6-1). This sector will only use three water resources during the planning horizon, and they are illustrated in Figure 6-8. The majority of these resources will be for the use of NGW with slight decline due to providing alternative water resources, such as RW, in some regions. On the other hand, the RGWSW rate will be increased by increasing the dams and reservoirs, and enhancing the RGWSW distribution stations to supply near farms.

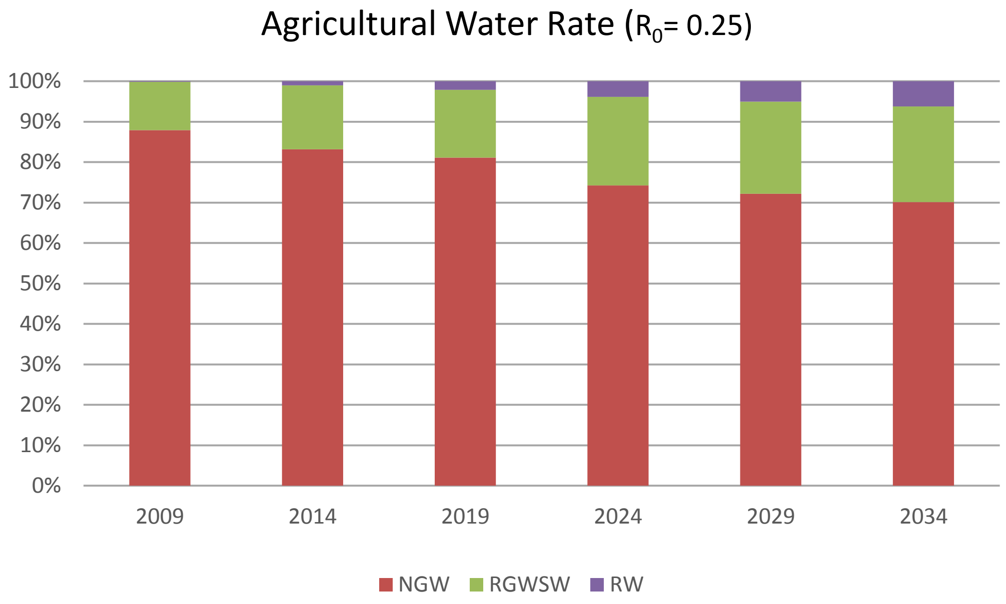

Figure 6-8 Allocation of water resources to the agricultural sector when $r_0 = 0.25$

Lastly, industrial water use is shown below in Figure 6-9. This sector consumes three water resources as well. However, the DW is used instead of the RGWSW. Moreover, there is an opposite relation between the use of RW and NGW. While the use of RW is expected to increase and to be the major water resource for this sector, the NGW rate will dramatically decrease from around 70% to only 15%. This change is very important in order to save NGW as a strategic store for any critical conditions that the country might face in the future, such as wars or shortage in the source of energy. Furthermore, this step is one of the highly recommended goals of this research to solve the situation of the WRM system of Saudi Arabia.

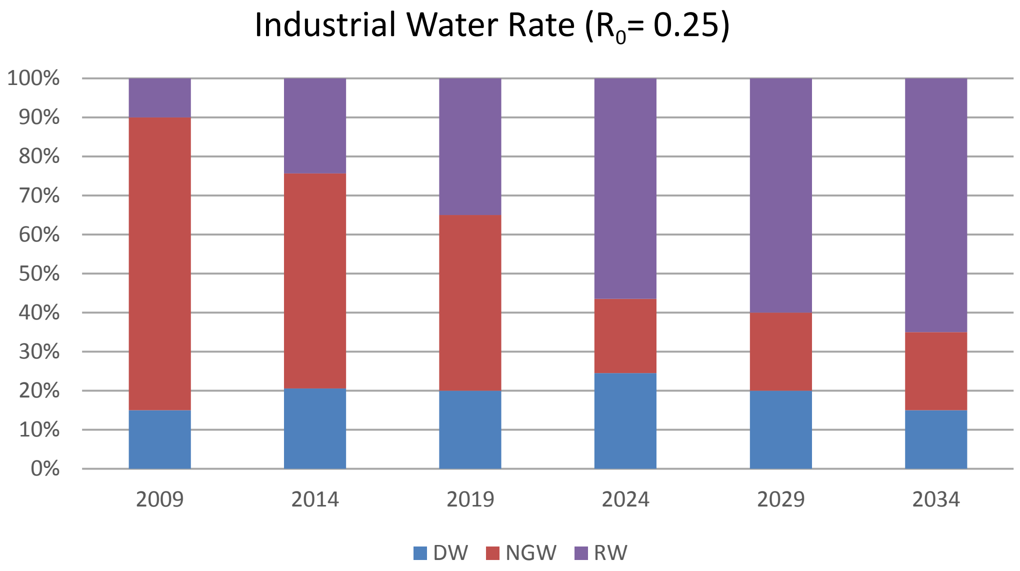

Figure 6-9 Allocation of water resources to the industrial sector when $r_0 = 0.25$

For the second scenario, the allocation rates for all water resources to the three sectors are illustrated in Figure 6-10, Figure 6-11, and Figure 6-12. The allocation rates of the second scenario are almost similar to those of the first scenario. In contrast, the total water quantity in the second scenario is higher than the first, which represents, in addition to the risk of violating the constraints, another risk to be further from the sustainable condition of the WRM of the country. However, this situation might be preferred by the stakeholders because the chances to meet their needs are higher, as can be reflected in the smaller gap between the forecast of water demand and the total water quantity in Figure 6-6. At the same time, the opponents of this scenario might argue about the water situation of the country and the importance of minimizing the water consumption in order to protect this vital resource for the future generations.

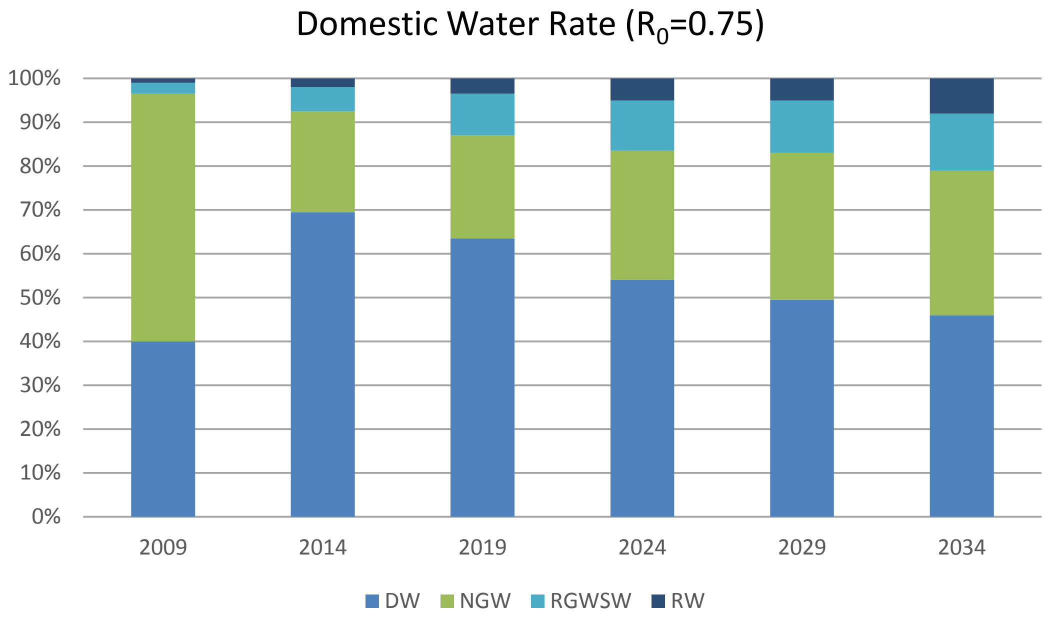

Figure 6-10 Allocation of water resources to the domestic sector when $r_0 = 0.75$

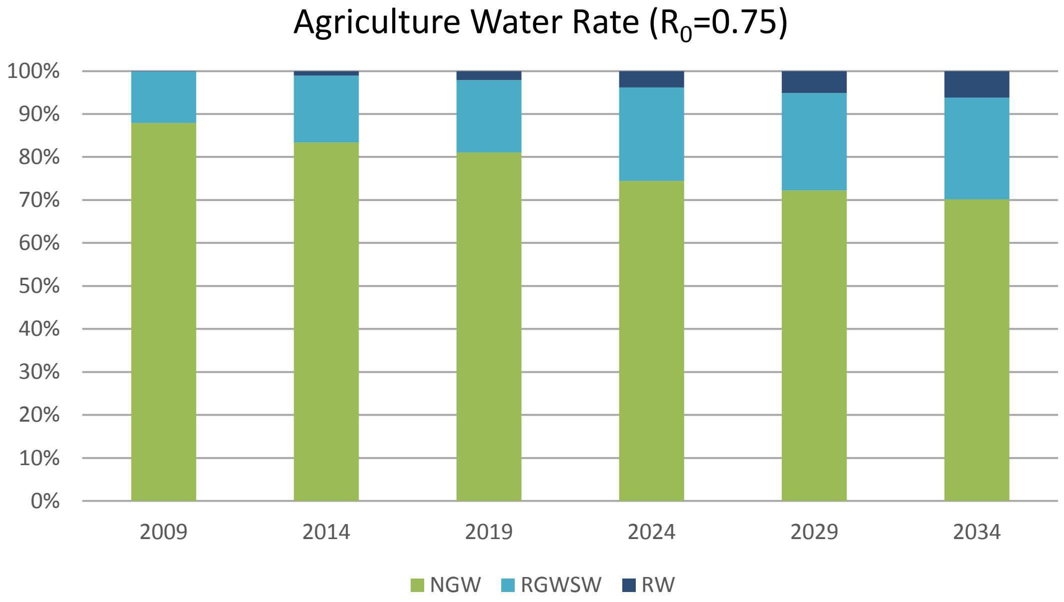

Figure 6-11 Allocation of water resources to the agricultural sector when $r_0 = 0.75$

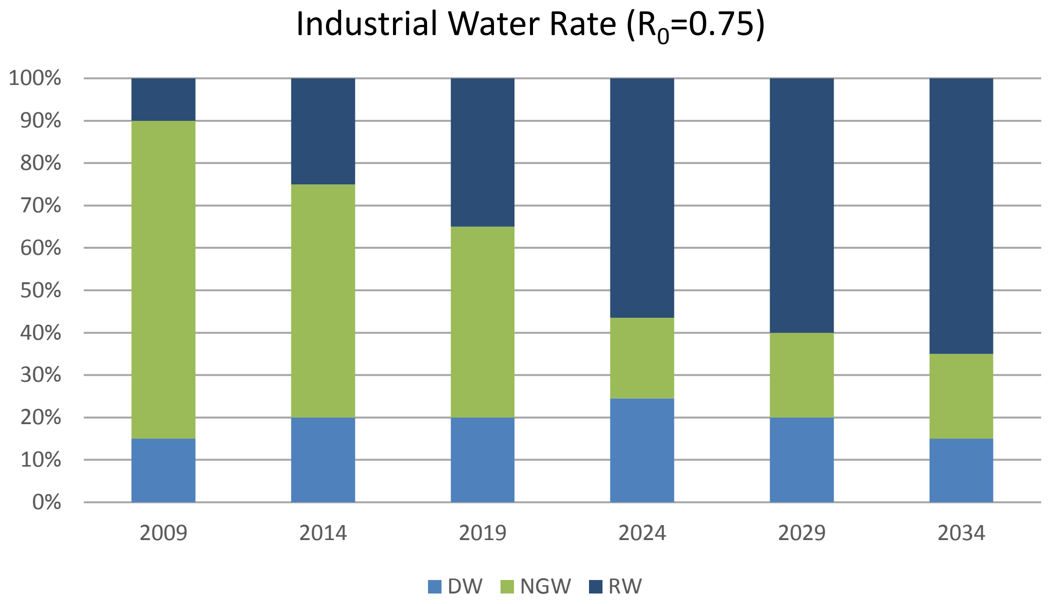

Figure 6-12 Allocation of water resources to the industrial sector when r_0 = 0.75

6.2.5.2. LP Evaluation and Economic Return

A comparison between two scenarios will be provided in order to give the decision makers a chance to know the pros and cons of each option. From an ILP view, the best selection is the lower risk scenario. This is because the results of this scenario are more reliable, and the likelihood of problems regarding the solutions is relatively low. Conversely, the other scenario is inherently more risky in terms of issues that related to inaccurate projections. As a result, the first scenario is called ‘conservative’ since its risk function is low and equal to 0.06 and the second scenario is called ‘aggressive’ since its risk function is high and equal to 0.31. These names are related to the probability of violating the constraints in each scenario.

On the other hand, the conservative scenario has lower economic benefits than the aggressive scenario. The difference between them reaches to US $ 84.03 billion in the whole planning horizon. Indeed, this amount of loss would not be acceptable by any economist and needs a strong argument to be chosen. Moreover, this would be right if the total economy of the country was weak, but Saudi Arabia is one of the Group of Twenty (G20) countries, which are defined as "the premier forum for international economic cooperation, bringing together the world's major advanced and emerging economies" (Australian Presidency, 2014). Thus, the economy of Saudi Arabia is already strong and among the highest economies of the world, which means the sacrifice of this amount of money will not be considered as a huge issue.

6.2.5.3. Environmental Impact

Aside from the economic view, other aspects such as environmental factors should also be included in the evaluation criteria of the WRM system in order to integrate it successfully. It is important to know first which water resources and sectors are making a negative impact on the environment. Regarding the water resources in Saudi Arabia, RGWSW needs lower cost and energy for both treatment and transportation, which suggests it to be the most environmental friendly. The NGW comes in second place in terms of the treatment and energy. However, extensive withdrawal of this resource could cause several negative effects such as decline in the water table, land subsidence, and deterioration of water quality especially in the coastal regions (U.S. Geological Survey, 2014). Hence, using this resource must be limited and strictly controlled by the government authorities to avoid environmental problems.

In addition to the previous conventional resources that came from the hydrological cycle and their relation to the environment, DW and RW are considered as unconventional resources. However, DW is less environmentally friendly than RW. The process of desalination has different environmental impacts based on the treatment process. For instance, the use of fossil energy, which uses thermal technology in some desalination plants in Saudi Arabia, produces large emissions of greenhouse gases (Tsiourtis, 2001). Another example is that marine organisms are threatened by the disposal of high salinity brines, which are generated after the RO desalination process (Elimelech & Phillip, 2011). Thus, since both technologies are used in Saudi Arabia, DW has potential negative effects on the environment of the country. However, the use of this resource is significant due to the water situation of the country, and the option here is to focus on less harmful technology. Further studies are in progress in Saudi Arabia in order to develop new desalination technologies and reduce these impacts.

RW is the last water resource whose effect towards the environment needs to be evaluated. In fact, the central idea of RW is to protect the environment from the waste residual of water by treating and reusing it again instead of getting rid of it with all the pollutants that it has collected from the previous use. The issue with the use of RW is related to the quality degree, since the treatment types produce different water qualities. Therefore, each type of treated water is specific for limited use such as landscape irrigation, industrial use, and aquifer recharging, and these uses should be highly monitored and controlled by water agencies and health organizations. This is because the potential risk that might affect the environment from any seepage, wrong use, or wrong delivery of less treated wastewater. As a result, RW is a smart solution for several environmental problems that might occur in the absence of particular treatments but it needs high attention and management.

Returning to the two scenarios and the environmental impact of each one, the conservative scenario aims to consume less water for the purpose of achieving or at least being closer to the sustainability of the system. In contrast, the aggressive scenario consumes more water, which minimizes the gap between water supply and demand. Therefore, protecting the natural resources by decreasing their use is much better environmentally, which suggests the conservative scenario to be more attractive. However, since the rate of each water resource is almost the same in both scenarios, it can be said that the environmental impact will not be significantly different. This means that both options might be acceptable in environmental terms because they are both satisfying the main objectives of this research. To illustrate, reducing the water quantity of NGW and increasing the use of RGWSW and RW in order to be around the sustainable situation of the WRM system. Meanwhile, the use of DW might be negative to the environment but it cannot be ignored since the country has few water supplies and high water demand.

6.2.5.4. Social Perspective

After discussing the two scenarios from an environmental view, the decision maker should evaluate the two options from a social perspective. First of all, this factor is more significant for the domestic and agricultural sector than the industrial sector since the first two sectors have more freedom to use the water, while the water consumption of the industrial sector based on its needs is easier to control by the government. The difference in the use of DW and RGWSW based on the long-term plan will not pose an issue for the public. This is because the public have gained confidence in these resources through the previous years. Nevertheless, the increase in using RW and the decrease in using NGW might be more difficult to accept by the society.

Regarding RW, many societies around the world have some reluctance against using this resource because of the bad mental image that they have toward its origin. Meanwhile, this issue has been resolved by calling this water by better names for presentation to the public. Additionally, it is important to conduct awareness campaigns about RW in the media, schools, and universities in order to make this option acceptable in society and lessen resistance toward it. The same procedures should be taken with the decrease of NGW consumption especially for the farmers who will be the largest group affected by this measure. On the other hand, the domestic use will be offset by the increase of DW and RGWSW, which is an acceptable alternative.

Beside the explanation of the social view towards each water resource, it is important to evaluate societal preferences for the total water quantity of each scenario. It might be difficult for the society to accept the conservative scenario without being informed regarding the critical situation of water resources of the country, since this option means that they would have to reduce their water consumption, which will inevitably meet resistance. Therefore, a wide campaign would be required to educate the public and raise their awareness about the potential long-term plan. For the aggressive scenario, if the stakeholders of WRM are not convinced by the importance of the first scenario, they would become more satisfied with the aggressive scenario since it is more convenient for them to consume more water. Additionally, their decision would be based on which option could gain a higher economic return. However, the illustration of the other factors of the WRM system should be provided to the public in order to let them recognize the circumstances of each scenario. As a result, the acceptance of any option by the society will help the decision makers to gain support for their selection and help the water authorities to implement the long-term plan.

6.2.5.5. Summary and Conclusion

To sum up the discussion, Table 6-7 is presented below for both scenarios with an evaluation out of five points for each factor of the WRM system that has been included in this study. The conservative scenario is a better option in terms of LP and the environmental impact. This is because the risk level of violating the constraints is very low compared to the aggressive scenario, while the environmental impact is slightly higher because both options aim to reduce the total water quantity with focus on decreasing the consumption of NGW. On the other hand, the aggressive scenario is a better choice from economic and social points of view because the economic return is much higher than the first scenario; while the social factor is a little higher in the aggressive scenario because it has more chances to meet the water demand. In conclusion, both scenarios have advantages and disadvantages, but by collecting the suggested points of each choice, the result is that the conservative scenario has 12 points, while the Aggressive scenario has 11 points.

Table 6-7 Evaluation summary for each factor of the WRM system

Factor	Conservative Scenario	Aggressive Scenario
Linear Programming	4	1
Economic Return	2	4
Environmental Impact	4	3
Social Side	2	3

The distribution of these points is an estimation that might be different from one person to another. However, this research has focused on the optimization techniques, which are the ILP and REILP, and their use in designing a suitable long-term plan. Consequently, the first row in Table 6-7 (i.e., LP factor) should be the keystone for making the decision regarding the WRM system of Saudi Arabia, while the other rows are for illustrating the effects of each option. This evaluation is appropriate for those who lack knowledge of the LP, and provides the opportunity to simplify the whole process for the stakeholders.

CHAPTER 7

SUMMARY AND CONCLUSION

7.1. Summary

In this study, a development of the REILP approach for long-term planning of the WRM system in Saudi Arabia has been conducted. The main advantage of this approach is to produce a solution that can combine the best outlook in terms of both system returns and the risk level. Hence, this would enhance the chances of choosing the right option by decision makers. The selection of this method was based on the validity checking that has been done for old ILP algorithms. The result of this procedure demonstrated that these algorithms have some limitations that exclude them from implementation for real-world problems.

Regarding the case study, the WRM system of Saudi Arabia consists of four water resources and three main users, while the projected time is 30 years. The long-term plan has been designed to get the highest net-benefit of the system, with suitable water allocation from resource i to sector j in time t. This allocation should take into account various measurements, which have been indicated in detail in the previous chapter, towards both water resources and users to treat the water situation of the country. Thus, water resources can be saved for future generations while continuing the effort to reach sustainable conditions.

In order to achieve the goal of this study, two scenarios from the REILP solutions were selected and evaluated. The selection was based on the degree of the risk that is related to the aspiration level, while the evaluation attempts to recognize the impacts of each scenario. This evaluation contains four factors that have direct relation to the WRM system: the LP, economic return, environmental impact, and social perspective. Indeed, the integration of these factors could help both decision makers and stakeholders to assess the WRM system from a wider view.

7.2. Conclusion

A number of conclusions result from the research described above. First, this study proves that the traditional ILP models (i.e., BWC and 2-Step) lack some feasible and optimal solutions. This proof is based on a comparison between the results of both algorithms and the Monte Carlo simulation method to evaluate the validity of the ILP algorithms. A numerical example was presented for this purpose. While the solution of Monte Carlo is more efficient for non-complicated real-world problems, the extensive time to run the model makes it undesirable for real applications. Hence, the generated solutions from the Monte Carlo simulation method in the numerical example are the base to check the optimal solutions of both BWC and 2-Step algorithm against. An improvement for these algorithms is suggested to resolve their flaws.

The second conclusion after the examination of existing ILP algorithms is that the REILP method can avoid the limitations of ILP and integrate between the system return and the decision risk. This step has been achieved by applying the risk explicit ILP model, which includes the process of minimizing the decision risk. At the same time, the original ILP objective function was converted to be a constraint in the same model. Thus, the decision-support process will be easier since the solution is crisp and it can provide the tradeoff between the system return and the risk level only by choosing the pre-defined aspiration level.

The third conclusion reflects the possible effectiveness of application of the developed REILP model to the long-term planning of the WRM system in Saudi Arabia, following determination of the best and worst solutions of the BWC algorithm. The aim of the previous algorithm was to maximize the total net-benefit of the three main water users within a 30-year period. Also, this process was based on the water allocation from four water resources toward the three main sectors as decision variables, with the constraints of the future water supply and demand, industrial production, and crop cultivated area. Next, the traditional ILP was converted to a REILP model, which changed the objective function from maximizing the system return to minimizing the risk function by applying the aspiration level as a critical factor for this process. To give an overview of the REILP procedure for decision makers, the WRM system model was solved eleven times by inserting 11 pre-defined aspiration levels from 0 to 1, with the step of 0.1. This process can make the mission of decision makers easier, especially those who have a modeling background, by providing specific decision support.

The fourth conclusion is that the conservative scenario in terms of the REILP perspective is more preferred than the aggressive scenario. This result was obtained from the comparison that has been conducted for two REILP solutions, where the conservative solution has a lower risk level of violating the constraints than the aggressive scenario. The comparison examined, in addition to the LP, three factors: economic, environmental, and social aspects that related to the WRM system in Saudi Arabia. This comparison aims to get the cooperation of the stakeholders with the long-term plan by giving them an opportunity to recognize the effects of each scenario.

7.3. Recommendation

The main purpose of this study was to design a long-term plan of the WRM system in Saudi Arabia by using a developed optimization technique, which is the REILP method. However, several recommendations should be taken into account for further studies related to this methodology and the case study, as well.

Regarding the REILP model and the uncertainty environment, the solution of REILP is assumed to be absolutely feasible and optimal in terms of system return and decision risk. However, the assumption needs to be further examined for its validity in future studies. Furthermore, the discount factor could be applied in the model to reflect the real value of the US dollar in each period. Additionally, the binary decision variable is recommended to be included in the modeling framework in the future studies especially when the new facilities such as desalination plants and WWTPs need to be constructed and/or the existing facilities needs to be expanded.

On the other hand, more specific water users such as the livestock and the other industrial activities might need to be considered in the case study in order for the modeling results to be more feasible and applicable for the management of water resources system in Saudi Arabia. In addition, because the information of water resources and users that was provided by MOEP (2010) was general for the entire country, the separation of these data to each specific region by the related ministries is suggested for future studies. Moreover, GIS tools might also be helpful for data management, model linkage, and graphical presentation of the modeling results, therefore could provide a user-friendly interface for supporting the relevant authorities in their decision-making process.

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