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**THE SENSITIVITY OF EXCHANGE RATE PREMIA TO
CONSUMPTION, DIVIDENDS AND EARNINGS**

By

Richard Watuwa

**Submitted in partial fulfilment of the requirements for the degree of Doctor
of Philosophy**

at

Dalhousie University

Halifax, Nova Scotia, Canada

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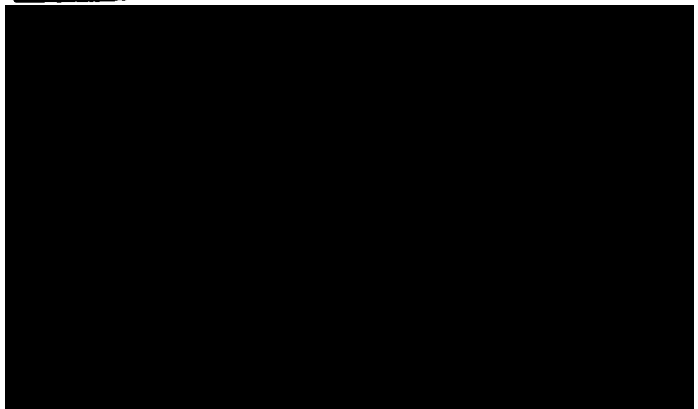
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Dedicated to my daughter, Miriam and to the memory of my parents

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ABSTRACT

We investigate the possibility that dividends and earnings per share contain enough information to explain the time-varying risk premium in foreign exchange markets.

In the context of a model of foreign exchange determination, we show using GMM estimation, that dividend based stochastic discount factors are a better measure of households' intertemporal marginal rate of substitution than consumption based discount factors. An earnings based discount factor performs better than both dividend and consumption based discount factors.

We also use the calibration methodology to investigate the quantitative implications of the model based on US-Canadian data. While the earnings model increases the volatility of the risk premium relative to the consumption model, it does not account for the volatility and persistence in the data.

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Chapter 1

INTRODUCTION

1.1 Pricing Anomalies in Stock Markets and Foreign Exchange Markets.

The central objective of asset pricing theory is to explain values or prices of claims to uncertain payments. On the basis of data on the U.S. economy however, the standard theory throws up some empirical implications which have so far proved either very difficult to rationalize or that can only be explained within its framework by relying on clearly implausible assumptions. These empirical results are what have come to be known as asset pricing anomalies. They connote continuing weaknesses in modeling asset pricing behavior in economics. In a closed economy context, they involve prices and returns in stock markets. In an open economy context on the other hand, they involve exchange rates and speculative returns in foreign exchange markets.

In the case of stock markets an important risk premium is the spread between the return on equity and the risk free return. The observed premium in the U.S. annual data over the past century averages 6 percentage points. According to the standard theory, the right measure of this risk is consumption risk. The measured consumption risk associated with the stock market using the same data is however too small to explain

this magnitude of the observed risk premium. In order to provide a plausible explanation, an unusually high level of risk aversion has to be assumed. This is the equity premium puzzle.

In foreign exchange markets spot exchange rates are the prices at which foreign currencies trade in the spot market where there is immediate delivery of amounts traded. In practice delivery is in two business days. Forward rates, on the other hand, are prices at which currencies for future delivery are traded. Standard maturities of forward contracts are 1 or 2 weeks and 1, 3, 6 and 12 months. The spread between the forward rate and the future spot rate is the forward premium. A parallel puzzle here takes the form of a persistent failure of forward exchange rates to forecast future spot exchange rates. In addition to this, regressions of the spot rate on the forward premium provide overwhelming evidence that predictions of changes in the spot rate by the forward rate bear a negative sign, which is the wrong sign. If the forward rate exceeds the current spot rate for instance, the spot rate should then be expected to fall. This result is also difficult to rationalize within the standard theory of international finance. In estimation, like the case of the equity premium puzzle, an implausible level of risk aversion is again required in order to explain the forward premium. This is the forward rate puzzle, which is the main focus of this study.

In this study, three versions of the neoclassical model of exchange rate determination of Lucas (1982) are used to investigate the performance of the

Consumption based capital asset pricing model (CCAPM) in markets for foreign exchange. The model is set up to determine exchange rates and investigated empirically in turn. This will be done by carrying out euler equation econometric tests to assess the potential of extensions on the basic model for solving the forward rate puzzle. In particular, modifications of the canonical model will be evaluated in terms of the magnitude of the coefficient of risk aversion, the precision with which it is estimated, whether or not the model is rejected overall by the data . To complement these tests the ability to replicate the volatility and autocorrelation in the data is examined further using the calibration methodology.

The first model is a direct test of the CCAPM in foreign exchange markets using aggregate consumption growth to evaluate the intertemporal marginal rate of substitution. This proceeds along the lines of Mark (1985) where the generalized method of moments (GMM) approach was used. In this approach, euler equations derived from the neoclassical model of exchange rate determination (Lucas 1982) are estimated, assuming time separable preferences of the constant relative risk aversion type. The current and past consumption ratio and past values of realized profits from foreign exchange speculation were used as instruments. The sample period in Mark's study is March 1973 to July 1983, at monthly intervals. This sample period is changed in the current study, so that the estimation period is March 1973 to December 1995 at a

quarterly interval for GMM estimation. The corresponding period for the calibrated model is March 1973 to March 1997 with quarterly data.

The second model to be evaluated represents a departure from the use of aggregate consumption growth to evaluate the inter-temporal marginal rate of substitution in the pricing kernel. Instead dividend growth is used. Dividend-based discount factors have been previously used in studies on stock returns (Abel 1988, Cecchetti et. al, 1997). The econometric method used here is the generalized method of moments estimation.

The third model that is tested is based on growth of earnings per share as a measure of the intertemporal marginal rate of substitution. Earnings are computed as net cash income of a company after payment of taxes, dividends on preferred shares and bond interest, apportioned to each share of the company. Analysts use them to calculate current growth as well as future potential. Since earnings provide a good reflection of net income per share, they are usually monitored by investors as a gauge for corporate operating performance and expected future dividends. The contribution in this study is to introduce earnings and dividend discount factors. There are no previous applications of earnings discount factors applied to the estimation of the Capital Asset Pricing Model in foreign exchange markets.

A complementary strategy is to calibrate the model. Given reasonable values of deep parameters, does a simulation of the model generate exchange rate data that matches in some fashion the statistical properties of observed data in the sample? The

statistical measures considered in this case are the first-order autocorrelation and volatility of the gross depreciation rate, S_{t+1}/S_t , the realized speculative or forward profits, $F_t - S_{t+1}/S_t$ and the forward premium, F_t/S_t . Also of interest is the slope coefficient of a regression of gross depreciation on the forward premium. The calibration is carried out using quarterly data for the US and Canada from the first quarter of 1973 to the first quarter of 1997.

In light of results from these models, three questions can be answered. That is from the first model it can be clarified as to whether the rejection of the model estimated in Mark (1985) was specific to the period 1973 to 1986, which constituted the sample.

Secondly, from the outcome of the dividend growth based model, it will be possible to assess what advantage if any might be gained by moving away from the consumption-based discount factor towards a dividend based discount factor. In other words, is the Dividend CAPM empirically any better than the Consumption CAPM respecting speculative returns in the foreign exchange market?

Thirdly, in the same vein as the case of the dividend discount factor model, it will be determined to what extent the adoption of an earnings discount factor alters the model's empirical performance.

Furthermore, the calibration approach will be used to highlight the effect of using earnings and dividend growth on the statistical properties of the implied forward

premium. In the end then, the study elicits the specific role of re-defining the discount factor in determining foreign exchange prices. But first we lay out the basics of asset pricing theory to show there is nothing inherently amiss with the underlying theory of risk and return.

1.2 Asset Pricing Theory.

The basic theoretical models used to explain asset returns in the financial economics literature were challenged by Mehra and Prescott (1985), who showed that these economic models were unable to account for historically observed rates of return on stocks and short term bonds in the U.S.A. In particular, for the period 1889 to 1978, the real rate of return on stocks averaged 6.98 percent per annum. The real rate of return on short term bonds, on the other hand, averaged 0.80 percent per annum. Accordingly the equity premium, which is equal to the rate of return on stocks minus the rate of return on bills (short term government bonds), averaged 6.18 percent per annum. The relatively large equity premium called for an explanation.

A basic insight that was alluded to is that of a trade-off between risk and return. In order to bear additional risk, investors who are risk averse require additional return. Fundamentals of asset market theories that explained risk premia and financial asset pricing were provided by Hicks(1946), Markowitz (1959) and Tobin (1958). They built

a 'mean-variance' micro-model whereby investors evaluated portfolios in terms of means and variances of portfolio returns. By assuming either normally distributed portfolio returns or von Neumann - Morgenstern utility functions they were able to concentrate on only the mean and variance of the portfolios such that investors would choose those portfolios that generated the highest level of returns for a given variance, i.e. mean variance efficient portfolios. Portfolio choice could as a result be analyzed by simply examining properties of the mean variance efficient set.

Against this background, the micro-model was studied further by Sharpe (1964), Lintner (1965) and Mossin (1969) and extended by way of aggregation into a model of equilibrium in the capital markets. This is the Capital Asset Pricing Model (CAPM). In order to outline the CAPM, the security market line equation describing the necessary and sufficient conditions for the mean variance efficiency of a portfolio p , is derived following Ross (1995). Suppose that to expand a portfolio by adding one unit of an asset requires financing at an interest rate, r . The net benefit resulting from this addition is the extra expected return it generates, E_i , minus the financing cost. The change, Δx , adds to the expected return on the portfolio, E_p , by the amount of the risk premium on the asset, that is the expected return on the asset, E_i , less the cost of financing, r .

Therefore

$$\Delta E_p = (E_i - r)\Delta x \quad (1.1)$$

The portfolio variance following the addition of Δx of asset i is

$$v + \Delta v = v + 2\Delta x \text{cov}(i, p) + (\Delta x)^2 \text{var}(i) \quad (1.2)$$

where

$\text{var}(i)$: variance of the asset i return

v : variance of current portfolio returns

$\text{cov}(i, p)$: covariance between the return of portfolio p and that on the asset

i .

Δx : addition in holdings of asset i .

Therefore the change in variance is

$$\Delta v = 2(\Delta x)\text{cov}(i, p) + (\Delta x)^2 \text{var}(i) \quad (1.3)$$

For Δx that is very small, the change in variance is approximately given by

$$\Delta v \approx 2(\Delta x)\text{cov}(i, p). \quad (1.4)$$

The trade off between risk and return, the marginal rate of transformation (MRT), can be expressed thus

$$\text{MRT} = \frac{\Delta E_p}{\Delta v} = \frac{(E_i - r)\Delta x}{2(\Delta x)\text{cov}(i, p)} = \frac{(E_i - r)}{2\text{cov}(i, p)} \quad (1.5)$$

Equilibrium for the individual investor is attained by equating this trade-off with his marginal rate of substitution (MRS) between return and risk. Thus if a given

portfolio p is optimal to the investor, then it can serve as a bench mark since its risk-return trade-off must be equal to his MRS.

Suppose the amount of the entire portfolio is modified this time and financed by changing the amount held of the riskless asset. The modification results in a trade-off between risk and return like the one considered before. Thus

$$\text{MRS} = \frac{E_p - r}{2\text{var}(p)} \quad (1.6)$$

Given that for equilibrium every marginal rate of transformation must equal the common marginal rate of substitution, equations (1.5) and (1.6) can be combined as follows

$$\frac{(E_i - r)}{2\text{cov}(i, p)} = \frac{E_p - r}{2\text{var}(p)} \quad (1.7)$$

$$E_i - r = \frac{E_p - r}{2\text{var}(p)} \times 2\text{cov}(i, p) \quad (1.8)$$

$$\Rightarrow E_i - r = (E_p - r)\beta_{ip} \quad (1.9)$$

where $\beta_{ip} = \frac{\text{cov}(i, p)}{\text{var}(p)}$.

β_{ip} is the coefficient of a regression of returns to asset i on returns of portfolio p . Equation (1.9) is called the security market line (SML). The SML equation represents both the necessary and sufficient conditions for mean variance efficiency of portfolio p .

In addition it shows that the risk premium on asset i is proportional to the asset's beta coefficient β_{ip} .

Sharpe (1964), Lintner (1965) and Mossin (1969) observed that it is possible to aggregate the SML and mean variance analysis more or less without altering full capital market equilibrium. Considering that individuals share identical information regarding the mean and variance, every individual's efficient portfolio satisfies equation (1.9). Furthermore, inasmuch as the SML equation is linear in E_p , the expected value of portfolio holding, one can sum weighted SMLs using the proportion of wealth held by individuals in equilibrium as weights. The resulting SML equation is thus for an aggregate portfolio, m , which is a weighted average of investor's portfolios. This gives rise to a portfolio of total assets held relative to their overall market valuation in equilibrium, that is market portfolio m . Consequently, all assets i lie on the market SML,

$$E_i - r = (E_m - r)\beta_{im} \quad (1.10)$$

In other words m is a mean variance efficient portfolio.

The main attributes of the CAPM outlined above are the portfolio's mean variance efficiency and the market portfolio's β coefficient as a determinant of the asset's risk premium. Any asset's characteristics which have an impact only on the variance of returns without influencing its covariance with the market do not affect the asset's

pricing. Accordingly, residuals from a regression of asset returns on market returns which in this case represent idiosyncratic (unsystematic) risk, are orthogonal to the market and hence do not contribute to the formation of the price. It is only the beta that is important for asset pricing.

These results of the CAPM can be understood in terms of the role of diversification and systematic risk. Provided that a portfolio is relatively large and well diversified so that there is no apparent concentration of asset proportions in a limited subset, unsystematic risk may be mitigated in view of the law of large numbers. This would rule out the payment of a premium for unsystematic risk. The remaining component of the optimal portfolio's risk, which is systematic, cannot be removed by diversification alone. It is this systematic risk that necessitates the payment of a risk premium in order to lure risk averse individuals into investing in risky assets. This then implies that assets with no correlation to the market bear no risk premium.

The basic CAPM has been developed into two main variants. The first category is that of multiperiod discrete-time models of Samuelson (1969), Fama (1970), Hakanson (1974), Long (1974), Rubinstein (1974, 1976), Kraus and Litzenberger (1975), Breeden and Litzenberger (1978), Lucas (1978) and Brennan (1979). In these models agents are taken to make consumption and investment choices at fixed points over arbitrarily selected intervals.

The second category comprises the continuous time models developed by Merton (1969, 1971, 1973) and extended by Breeden (1979, 1984, 1986) and Cox, Ingersoll and Ross (1985a, 1985b). The individual considered in this second case is assumed to make consumption and investment choices continuously while the random processes governing uncertainties follow stochastic processes with normally distributed increments along continuous sample paths. This assumption of normality of increments not only facilitates the link between the static CAPM and its continuous time version but also enhances tractability.

Breeden (1979) reformulated Merton's multibeta intertemporal CAPM (1973) to contain only one measure of risk. The model reduces to simply multiplying the market price of risk by an asset's consumption-beta, i.e. sensitivity of asset returns to aggregate consumption in real terms. This is the consumption oriented capital asset pricing model (CCAPM) which is discussed in the next section.

1.3 The Consumption Capital Asset Pricing Model

In intertemporal portfolio theory it is assumed that individuals choose policies on consumption and investment to maximize expected utility over all feasible consumption paths. It is also assumed that consumers' preferences are time-additive and state independent. If the direct utility function of consumption is state dependent then investors do not choose portfolios which minimize the variance of their consumption flows subject to expected rates of return constraints. Their portfolio choices would

instead be characterized by minimum variance of marginal utilities. Since state - independence is assumed, a typical consumer, k , has as expected lifetime utility

$$E\left[\int u^k(c^k, t)dt\right] \quad (11)$$

In addition, instantaneous utility of consumption is monotonically increasing and strictly concave in consumption i.e. $u_c^k > 0$ and $u_{cc}^k < 0$.

c^k is current consumption of individual k .

u^k is utility of current consumption for individual k .

u_c^k is the first partial derivative of the utility function with respect to consumption

u_{cc}^k is the second partial derivative of the utility function with respect to

consumption

E is the expectations operator.

The objective function of the consumer/investor is

$$\max_{\{c, w\}} \left\{ u^k(c^k, t) + E_t \left[J^k(W^k, s, t) \right] \right\} \quad (1.12)$$

where J^k is an indirect utility function;

$s : S \times 1$ is an S - component vector of state variables describing consumption,

investment and employment opportunities. These include the real riskless interest rate,

expected inflation rates on goods prices, the level of economic activity and expected

productivity of capital.

The first part of (1.12) represents, at time t , utility of current consumption over the next instant in time, while the second part is expected utility over all the following periods. This second part is the indirect utility function for wealth, whereby utility depends positively on the amount of current wealth, W^k , and is also functionally related to investment opportunities.

The standard condition for an optimal policy requiring that marginal utility of consumption should be equal to marginal utility of wealth is obtained by differentiating equation (1.12) with respect to current consumption ,

$$u_c^k [c^k(W^k, s, t), t] = J_w^k(W^k, s, t) \quad (1.13)$$

The optimal portfolio for a risky asset in the case of an intertemporal economy is

$$w^k W^k = T^k \left[V_{aa}^{-1} (\mu - r_f \cdot 1) \right] + V_{aa}^{-1} V_{as} H_s^k \quad (1.14)$$

where
$$T^k = \frac{-\left(\frac{\partial U}{\partial \mu}\right) W^k}{\left[2\left(\frac{\partial U}{\partial \sigma^2}\right)\right]}, \quad H_s^k = -\frac{J_{sw}^k}{J_{ww}^k}$$

w^k is individual k 's $A \times 1$ vector of portfolio weights for risky assets.

W^k is individual k 's initial wealth

T^k : is individual k 's compensating variation in terms of variance, assuming utility is constant i.e. the risk tolerance parameter

V_{aa}^k : $A \times A$ variance-covariance matrix for asset returns

V_{as}^k : $A \times S$ covariance matrix of asset returns with state variables;

$T^k: H_j^k$: is a coefficient vector representing individual k's holdings from the S portfolios used in hedging opportunity sets changes;

$T^k: V_{aa}^{-1}V_{as}$: is a product matrix with columns representing the portfolio of assets most highly correlated in returns with changes in state variable j;

μ : is the mean of returns; r_f : is the risk free rate and $1: A \times 1$ is a vector of ones.

Equations (1.13) and (1.14) constitute the envelope and optimal asset demand conditions in Merton's model. These conditions are used by Breeden to derive the consumption CAPM (CCAPM).

In a continuous time framework, optimal current consumption depends on the consumer's current wealth and a vector of state variables for investment opportunities, s , described above, i.e.

$$c^k = c^k(W^k, s, t) \quad (1.15)$$

Using a first order Taylor series approximation for the stochastic component of consumption gives relations for stochastic movements in consumption and covariances of assets' returns with k's consumption changes as

$$d\tilde{c}^k = c_w^k(d\tilde{W}^k) + c_s^k(d\tilde{s}) \quad (1.16)$$

$$V_{a, c^k} = V_{a, w^k} c_w^k + V_{as} c_s^k \quad (1.17)$$

The individual's direct utility function is used to express his optimal asset demands.

First the envelope condition of equation (1.13) is differentiated implicitly to obtain

$$T = -\frac{J_w}{J_{ww}} = -u_c/u_{cc}c_w = T_c/c_w \quad (1.18)$$

$$H_s = -\frac{J_{sw}}{J_{ww}} = -c_s/c_w, \quad (1.19)$$

for every k . Substituting for T and H_s in asset demand conditions of equation (1.14)

gives

$$w^k W^k = \frac{T_c^k}{c_w^k} \left[V_{aa}^{-1} (\mu - r_f \cdot 1) \right] + V_{aa}^{-1} V_{as} \cdot -c_s^k / c_w^k \quad (1.20)$$

Equation (1.20) is then pre-multiplied by $c_w^k V_{aa}$ and simplified further by exploiting the covariance of assets' returns with consumption equation (1.17) to obtain for every individual, k ,

$$V_{a,ck} = -T_c^k \left[\mu - r_f \cdot 1 \right] \quad (1.21)$$

According to equation (1.21) asset holdings are chosen such that the covariance of optimal consumption with each asset is proportional to expected excess returns on the asset. Then, aggregation of optimality conditions over all individuals yields the same proportionality between excess expected returns and the asset's covariance with aggregate consumption. Thus the consumption beta, β_i , this time is the ratio of covariance of asset returns with proportionate changes in aggregate consumption to the variance of proportionate changes in aggregate consumption. This ratio can be measured as a coefficient from an instrumental variable regression of asset returns on

portfolio returns using per capita consumption as an instrument for portfolio returns.

The aggregated version of equation (1.21) is

$$\mu - r_f = \left[\frac{\beta_c}{\beta_{mc}} \right] (\mu_m - r_f) \quad (1.22)$$

where excess returns per consumption beta is used to eliminate the risk tolerance parameter, thus leading to the CCAPM.

Two important concepts for an understanding of the workings of this model arise, namely risk and the marginal utility of consumption.

To highlight the role of risk in explaining the workings of the model, suppose in the first instance that investors were risk neutral. How would equilibrium be attained in asset markets?

Investors who do not care about the riskness of their asset purchases would then buy in accordance with expected rates of return. Given two assets with unequal returns, they would buy that offering higher returns and sell the asset with lower returns. Increased demand for the asset with higher returns raises its price. At higher prices the asset returns fall since its payoffs are only forthcoming at an increased cost. On the other hand, selling pressure on the asset with lower returns reduces its price, thereby raising its expected rate of return. This is because its payoffs are forthcoming at a reduced cost. Asset prices and expected rates of return adjust until in equilibrium asset returns are

equal. Thus, the model predicts that in a relatively free market and assuming risk neutrality, asset prices would adjust to equate all rates of return.

Now suppose in the second instance that investors / consumers were risk averse. How would the model's prediction change?

Investors who care about the riskiness of assets would have to be compensated by a higher rate of return to venture into riskier investments. The CCAPM exploits the relationship between asset prices, expected returns and the marginal utility of consumption. Marginal utility of consumption represents the value a consumer places on obtaining more funds. With regard to asset prices and marginal utility, Hirshleifer (1970), present a time state - preference model of Arrow (1964) and Debreau (1959) showing that a fair price on an additional share of an asset equals expected marginal utility of its returns. This expected marginal utility of returns is a function of their size, dates of payment and covariances between sizes of payoffs and marginal utilities of consumption at different dates.

When an investor's endowment of wealth is relatively high, his consumption in turn is relatively high. He thus finds unit addition of funds or payoffs to have a relatively low value. Analogously an investor with relatively lower amounts of overall wealth and therefore consumption finds unit additions of funds or payoffs a higher value. An asset with positive consumption betas, that is one with high payoffs when consumption is high with low values of additional funds, is a risky asset according to

the CCAPM. This is also true for assets that give low payoffs when consumption is low. If an asset on the other hand yields higher payoffs when consumption is low and lower payoffs when consumption is high, then it bears negative risk. Thus it is a form of insurance because it generates higher payoffs when the investor places the highest value on additional funds with his consumption being relatively low. The model's prediction in view of the foregoing is that, assuming risk aversion, agents will choose assets earning the highest expected value of returns, whilst taking into account the value they place on additional funds. An adjustment process, akin to that outlined above under risk neutrality, takes place until asset market equilibrium is achieved. This equilibrium holds when for all assets, expected returns, weighted by their respective values of additional funds, are equal. That is

$$E\{(1+r_1).MU\} = E\{(1+r_2).MU\} \quad (1.23)$$

where MU is marginal utility of consumption representing the value placed on additional funds; r_i : $i = 1,2$ are real rates of return on asset i ; and E is the expectations operator.

When this condition is satisfied, higher risk assets will, by and large, pay higher returns. The larger payoffs of relatively risky assets are partly offset by the correspondingly lower values of additional funds. Now it is known that an investor/consumer maximizing expected utility of lifetime consumption typically

smoothe his consumption. His consumption rate would be set to depend on his expected income stream or wealth. Therefore consumption volatility would be of the same order of magnitude as that of wealth, with both volatilities being no greater than the volatility of his income.

Empirical evidence however shows volatility of wealth measured in stock markets to be a number of times larger than volatility of consumption, which in turn equals volatility of income (Mankiw and Shapiro 1986). The CCAPM thus runs aground in its empirical aspects. More importantly for this study, this observation is crucial for the equity premium puzzle, identified by Mehra and Prescott (1985) and for the forward rate puzzle. For the very smooth empirically observed stochastic behavior of consumption, the expected returns on equity (stocks) exceed substantially the returns on riskless assets (bills). That is, measured by the volatility of consumption, the magnitude of risk in the economy is not large enough to account for observed return premia on assets. Theoretical premia generated by pricing models are too small relative to empirical premia. Ability to match the two requires assuming a relative risk aversion coefficient of around 40 percent. Now in the CCAPM, the size of the premium on risky assets is determined by two factors. These are the coefficient of relative risk aversion and the covariance of consumption growth with asset returns. Here, the coefficient of relative risk aversion shows the extent to which a fall in consumption raises the value of additional funds. The covariance of consumption growth with asset returns proxies the

volatility of returns and the strength of the relationship between this volatility and consumption volatility.

Mehra and Prescott (1985) used the historically observed variability of US consumption, in a representative agent framework, to study the covariance of consumption with asset returns. Among the important ingredients of their analysis are the subjective discount factor, β , and the coefficient of relative risk aversion, α . Their choice of plausible values for these parameters was to restrict β to lie between 0 and 1 while α lies between 0 and 10. They considered 90 annual observations of US data from 1889 to 1978 over which the average real rate of return on Standard and Poor's (S&P) Composite Stock Index is 6.98 percent and the average real rate of return on short term government bonds is 0.8 percent. Taking the return on short term government bonds as the risk free rate, this gives an equity premium, the difference between these two rates of return, of 6.18 percent.

Now within the plausible ranges of both β and α the largest equity premium their model generates is 0.35 percent compared to the observed 6.18 percent. In order to obtain the observed 6.18 percent premium, values of α have to be set at around 30 percent and higher. These magnitudes of risk are considered implausible as demonstrated by Abel (1991). Suppose one faced a risky situation whereby one's total wealth could rise or fall by fifty percent, the probability of each outcome being 0.5.

What proportion of wealth would be paid for insurance, given one's coefficient of relative risk aversion? This proportion is given by the formula

$$y = 1 - \left[0.5(1-x)^{1-\alpha} + 0.5(1+x)^{1-\alpha} \right]^{\frac{1}{1-\alpha}} \quad (1.24)$$

where x : is the proportion of wealth gained or lost with a 50-50 chance.

y : is the proportion of wealth paid to avoid risk.

α : is the coefficient of relative risk aversion.

The proportion of wealth paid for insurance ranges from zero percent for the risk neutral to 49 percent for those with a relative risk aversion coefficient of 30 percent. Thus with α set at 30 percent, one would be willing to pay 49 percent of total wealth to insure against a 50 percent loss when the loss has an even chance of occurring. It is on this account that most analysts have viewed the 30 percent coefficient of relative risk aversion as unreasonable. The model therefore fails to account for the empirical equity premium with respect to US data when the parameters are chosen over a feasible range. This is what has come to be known as the equity premium puzzle, which is formally presented in the next section.

1.4 The Equity Premium Puzzle.

This section derives from Kocherlakota (1996) where not only an up-to-date survey of both the risk free rate and the equity premium puzzles can be found but it is also argued that the two puzzles have not yet been satisfactorily resolved. For purposes of this study only the equity premium puzzle will be considered. As discussed by Kocherlakota (1996) and Mehra and Prescott (1985) base their asset returns model on six major assumptions, of which three are qualitative assumptions and the remaining three are technical.

Firstly, they assume that preferences over future random consumption streams are identical for all individuals and are of the form

$$E_t \sum \beta^s \frac{(c_{t+s})^{1-\alpha}}{1-\alpha}, \quad \alpha \geq 0, U(c) = \ln(c) \text{ for } \alpha = 1. \quad (1.25)$$

where E_t : expectation conditional upon information available at time t ;

$\{c_t\}_{t=1}^{\infty}$: random consumption stream;

β : discount factor; and

α : coefficient of risk aversion.

The discount factor β is applied by consumers to discount utility from future consumption. Higher values of this factor imply higher levels of savings. The coefficient of risk aversion, α , is high when consumers being highly averse to risk, mainly prefer a higher level of similarity between consumption in different states of the

world. Larger values of α may also indicate that consumers do not like growth of their consumption profiles.

Secondly, it is assumed that asset markets are frictionless. The financial market is said to be frictionless whenever frictions such as holding costs, taxes or brokerage fees and other transactions costs do not cause the equilibrium to differ from the one achieved under perfect market outcomes. When such an equilibrium holds, investors should not be able to profit from selling bonds and using the earnings so obtained to buy stocks. Consumption profiles are required to satisfy the following two first order conditions

$$E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\alpha} (R_{t+1}^s - R_{t+1}^b) \right\} = 0 \quad (1.26)$$

$$\beta \cdot E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\alpha} R_{t+1}^b \right\} = 1 \quad (1.27)$$

where R_t^s : Gross stock return from t-1 to t.

R_t^b : Gross bond return from t-1 to t.

The two conditions in equations (1.26) and (1.27) provide for equilibrium asset pricing in terms of the neoclassical theory of marginal utility as explained earlier with reference to the workings of the consumption capital asset pricing model.

Thirdly it is assumed that markets are complete. With complete markets traders have access to a large set of assets that facilitates diversification of idiosyncratic risk. This assumption partly facilitates the use of a representative agent framework. Asset

markets are said to be complete when any number of securities that some agents would wish to hold or trade are available for trading. Thus cases where some investors wish to hold a given asset but cannot do so owing to some restrictions are ruled out. Such market incompleteness could be endogenous or postulated (exogenous). Endogenous incompleteness could result, for example, from the inability to make contracts in such a way that contract outcomes are observable by all parties or verifiable by third parties responsible for enforcement of those contracts such as members of the judicial system. The absence of enforcement mechanisms, both private and public, may generally engender market incompleteness.

The assumption of complete markets thus ensures the construction of a representative agent since trading in such markets renders individuals, who might have started out being heterogeneous, marginally homogeneous. The completeness assumption also implies that the first order conditions in equations (1.26) and (1.27) are met for each individual's consumption and also hold for consumption per capita, assuming a constant number of consumers.

The other three important assumptions are technical. (1) Per capita consumption growth is assumed to follow a two state markov process with population means, variances and autocorrelations equivalent to those observed in the US sample data. (2) It is assumed that at time t , realizations of current and past consumption growths constitute the only information available to individuals. (3) It is assumed that there

exists a perfect correlation (coefficient equals 1.0) between the growth rate of total dividends on stocks in the S&P 500 and the growth rate of per capita consumption. In addition, a perfect correlation is assumed between real returns to nominally risk free treasury bills and the returns to a bond which is risk free in real terms.

Given the foregoing assumptions, the first order conditions are used to specify the population means of real returns to S&P 500 and the three-month treasury bills in terms of the preference parameters α and β , which, as stated earlier, are restricted to ranges from 0 to 10 and from 0 to 1 respectively. Under these restrictions they can account for a premium of only 0.35 percent.

The main finding of Mehra and Prescott is that there are major inconsistencies between evidence from US data on consumption plus asset returns and their model of asset returns. Kocherlakota shows that the equity premium is a result of restrictions imposed by Mehra and Prescott on the values that α and β can take. He estimates the population means

$$E\left\{\left(C_{t+1}/C_t\right)^{-\alpha} \left(R_{t+1}^s - R_{t+1}^b\right)\right\} = 0 \quad (1.28)$$

by the corresponding sample means

$$e_{t+1}^s = \beta \left\{ \left(C_{t+1} / C_t \right)^{-\alpha} \left(R_{t+1}^s - R_{t+1}^b \right) \right\} \quad (1.29)$$

and finds that for $\alpha \leq 8.5$, the sample mean of e_t^r is statistically significantly positive, using US data from 1889 to 1978. This finding implies that for such values of the coefficient of relative risk aversion, an investor can realize some gains at the margin through borrowing at treasury bill rates and investing in stocks, which epitomizes the equity premium. Table 1 reports sample means over the period 1959 to 1993. Figures 1 and 2 are plots of growth of per capita consumption and average annual returns on stocks and average real returns over the same period.

Table 1

Sample Means of Annual Returns

α	E	t-stat
0.0	0.11210	4.989
0.50	0.11098	4.989
1.00	0.10987	4.987
1.50	0.10878	4.986
2.00	0.10770	4.985
2.50	0.10664	4.984
3.00	0.10559	4.982
3.50	0.10455	4.980

4.00	0.10353	4.978
4.50	0.10252	4.976
5.00	0.10152	4.974
5.50	0.10054	4.972
6.00	0.099571	4.969
6.50	0.098614	4.966
7.00	0.097669	4.963
7.50	0.096737	4.960
8.00	0.095817	4.957
8.50	0.094909	4.953
9.00	0.094012	4.950
10.00	0.092254	4.942

In this case the mean is significantly greater than zero for $\alpha \leq 10$.

There have been concerted efforts to resolve the equity premium puzzle. The main approach has been to alter one or other of the three basic assumptions regarding specification of the representative consumer's preferences, absence of market friction and completeness of markets. For instance in search for results from other specifications of consumer preferences, Chang Mo (1990) considered the case of multiplicative separable preferences of the form

Fig 1 Annual Consumption Growth

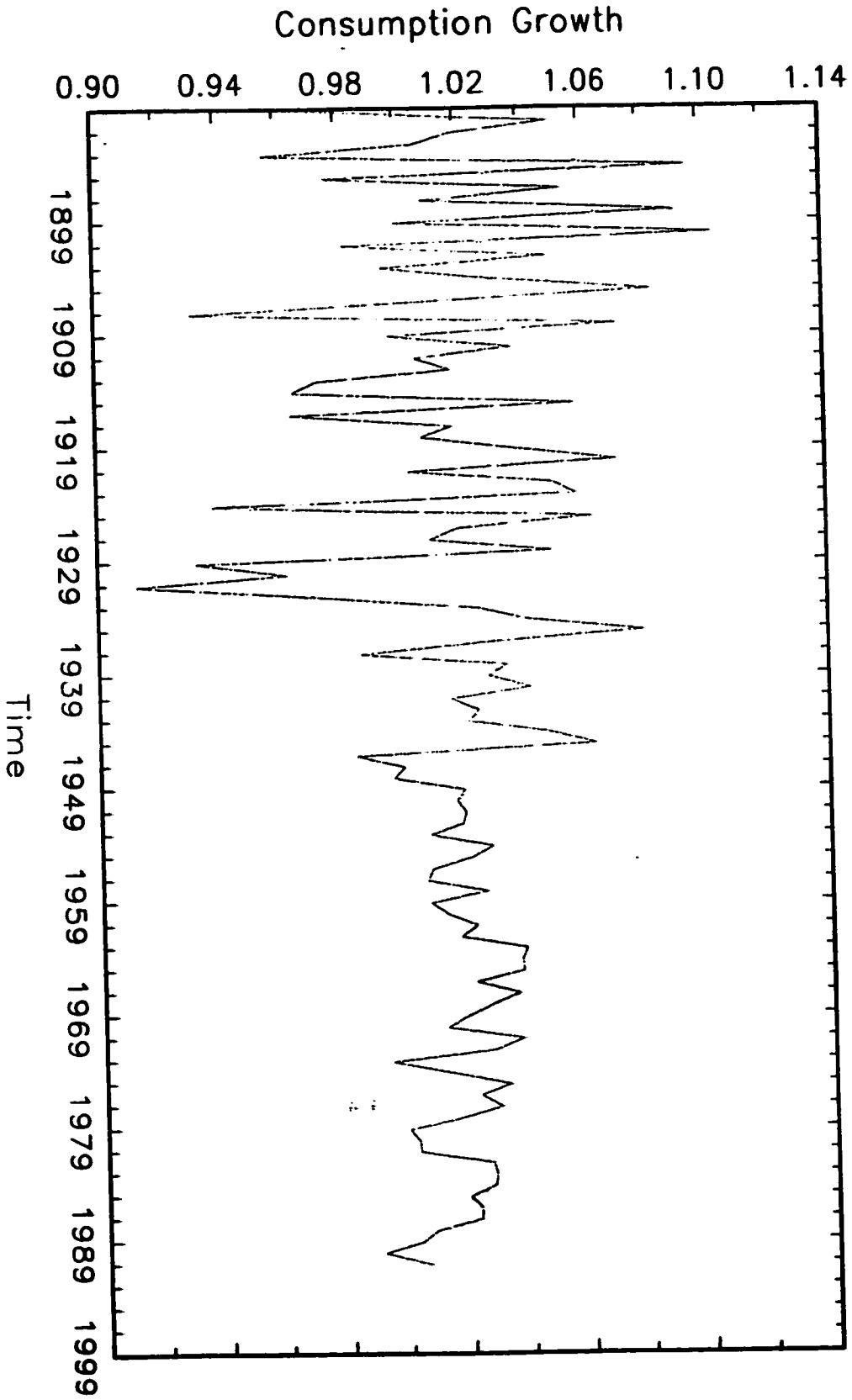
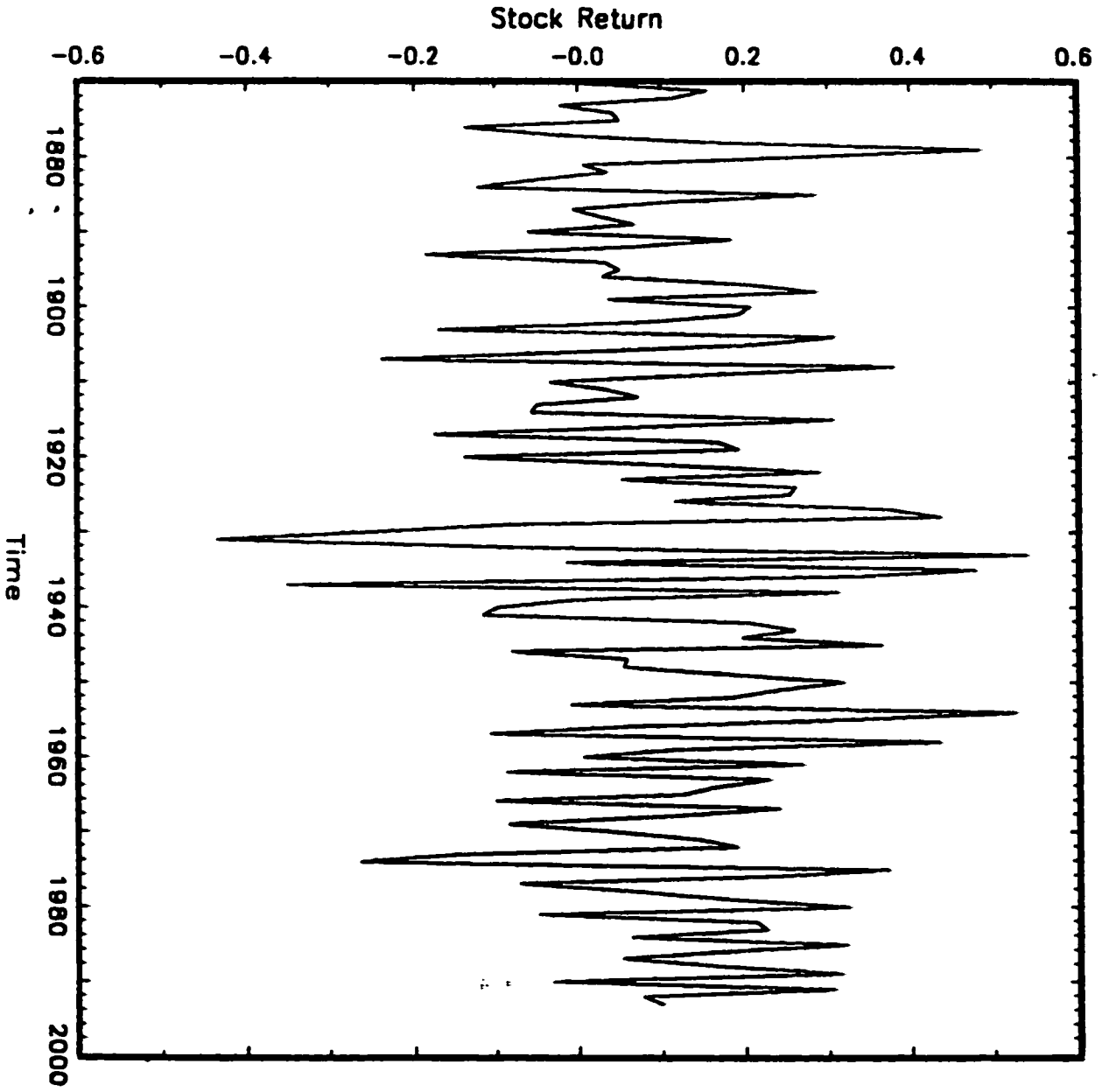


Fig 2
Annual real return to S & P 500



$$E_0 \left[\frac{1}{\gamma} \exp \left[\int_0^{\infty} \gamma \beta^t U(C(t)) dt \right] \right] \quad (1.30)$$

where $U(C(t)) = \ln C(t)$, $\gamma < 1$ is a constant parameter and β , ($0 < \beta < 1$), is a subjective discount factor. This particular specification is notably interesting because Chang was able to obtain a closed form solution to explain the mean and variance of the equity premium. Most other studies report improvements in model performance but without fully accounting for the observed premium. Since this result thus stands out, the analysis will be reviewed in more detail in chapter two.

The main focus of this study however will be to examine the effect of replacing consumption growth with dividend growth as well as growth in earnings per share in a model of foreign exchange determination. All the qualitative assumptions are maintained thereby avoiding any counterfactuals that follow from tinkering with these assumptions. The intent is to assess the potential of an empirical counterpart of such a model to resolve the forward rate puzzle. The puzzle is restated in the next section.

1.5 The Forward Rate Puzzle

It is well documented that world foreign exchange markets are highly liquid and register very large volumes of trade in various currencies. According to Mark (2001) an estimate for the United States of America alone during 1998 for instance, was US\$

105.3 trillion traded while GDP for the same year was US\$ 9.00 trillion. This suggests that the market should be largely efficient in the sense that substantial opportunities for excess profits should not exist over prolonged or significant durations. There is however much evidence that these foreign exchange markets are inefficient. In particular, econometric tests have consistently shown that uncovered interest parity is strongly rejected by the data. Why has this been the case?

There are three main approaches that have featured in efforts to explain this persistent deviation from uncovered interest parity. These are the presence and activities of noise traders, the peso problem and the existence of a time varying risk premium approaches. The noise traders and peso problem approaches are mentioned below in passing so as to concentrate attention on the risk premium interpretation since it is the main subject of this study.

The noise trader approach posits that a proportion of market participants hold irrational beliefs about asset values. According to Black(1986), the real world is so complicated that some traders cannot distinguish false signals from actual news. As a result of being either irrationally exuberant or apathetic they overvalue or undervalue prospective returns on financial investments thereby supporting trading dynamics with deviations from correct asset prices.

The peso problem approach suggests that agents form expectations rationally but do rely on imperfect knowledge about the actual economic environment.

They attach more weight to factors other than the long run determinants or economic fundamentals such as money supply. Indeed Lewis (1989) showed that in the case of the U.S. dollar's appreciation of 1980 to 1985, some of the error implicit in the forward rate was attributable to the fact that investors were slow to learn about an unobservable shift in the US money supply process. Other studies, such as Froot and Frankel (1989) and Frankel and Chinn (1993), have looked at surveys of expectations to assess the extent to which the asset pricing anomalies arise from violations of the rational expectations assumption. The robustness of results from such studies however has been questioned on grounds of survey data inaccuracies.

Under the time varying risk premium approach, studies of the unbiasedness hypothesis, of forward rates not being optimal predictors of future spot rates, it is maintained that even although markets are efficient there may exist varying risk premia over and above market forecasts of future spot rates. As noted by Backus et al (1993), this course of inquiry is yet to produce a satisfactory for the variation in expected returns. Now epitomizing the same weakness is the representative agent intertemporal asset pricing theory's failure to replicate the main quantitative properties of data on risk premia on foreign exchange. The theory is assessed in this area using two complementary procedures. The Generalized Method of Moments is used for standard estimation and is buttressed by model calibration to find out what the data show when

we change what determines the intertemporal marginal rate of substitution for the Canadian dollar and the German Mark.

1.6 Objectives and Plan of the Study.

Against the foregoing background, this study will examine the forward premium in foreign exchange markets assuming time separable preferences with constant relative risk aversion. Specific reference will be made to the implications of a departure from consumption based discount factors for the forward premium on foreign exchange in currency markets. The model is modified by applying earnings and dividend-based discount factors. The main goal is to evaluate this approach of changing discount factors given that it preserves the assumptions of frictionless, complete markets of the canonical model.

To this end, the following questions are addressed:

1. What are the effects of introducing dividend growth as a measure of the intertemporal marginal rate of substitution in modelling speculative returns on foreign exchange using the consumption based capital asset pricing model in the framework of a representative consumer? Here we compare the outcomes to those obtained from using consumption based discount factors. In particular, what is the size of the coefficient of relative risk aversion and how precise are the estimates compared to those of the consumption growth model?

2. **What are the effects of introducing earnings growth as a measure of the intertemporal marginal rate of substitution in modelling speculative returns on foreign exchange using the consumption based capital asset pricing model in the framework of a representative consumer? Here we compare the outcomes to those obtained from using both consumption based and dividend growth discount factors. In particular, what is the size of the coefficient of relative risk aversion and how precise are the estimates compared to those of the consumption growth model and the dividend growth model?**
3. **What are the implications of these model specification changes for the forward premium on exchange rates? How do the moments of the forward premium generated from the model compare with empirical moments?**
4. **What are the implications of these model specification changes for realized foreign exchange profits? How do the moments of realized foreign exchange profits generated from the model compare with empirical moments?**

The remainder of the study proceeds as laid out below.

Chapter 2 reviews the literature on proposed solutions of the equity premium puzzle and the forward rate puzzle with reference to approaches that rely on changing utility function specifications. This leads to chapter 3 which revisits the model of Mark(1985) with time separable preferences over an extended sample period. Modifications to the model involving both dividend growth and earnings growth are evaluated. GMM

estimation results from consumption, dividends and earnings growth models are reported. The model is evaluated again using the calibration methodology in Chapter 4. It is applied to US and Canadian data. Chapter 5 discusses the results, provides conclusions of the study and suggests some issues for further research in this area.

CHAPTER 2

LITERATURE REVIEW

2.1 The Importance of Resolving Asset Pricing Anomalies

A definitive resolution of both the equity premium puzzle and the forward rate puzzle would have important implications for the consumption based capital asset pricing models, introduced in chapter one. Although there do not exist clear and universally acceptable criteria for judging the correctness of models, it is pertinent to require that a model be plausible enough to shed light on some key aspects of the reality to which it purportedly pertains. For instance, the fact that the CCAPM is unable to adequately account for the observed average returns on equity undermines its practical value. There is ongoing debate about investing the Social Security Fund in the US on the stock market. A fruitful contribution from economic analysis cannot be marshaled when the theory misleads as it currently does. The equity premium puzzle implies the existence of a free lunch. In the foreign exchange markets countries with high interest rates experience appreciations of currencies when arbitrage should mean such currencies depreciate.

Furthermore, it is crucial to bring this model into some relief because this same basic model underlies more or less the entire new classical macroeconomics

which, in particular, includes a substantial part of long run growth theory. Partly for these reasons, the literature on possible resolutions of the equity premium puzzle, the forward rate puzzle and related asset pricing modeling issues has been expanding rapidly. For an extensive survey see John Cochrane (2000) on the equity premium; Robert Hodrick (1984), Lewis(1994) and Charles Engel(1996) are good surveys on the forward premium.

The main approaches in the search for a solution have been threefold and consist in altering either one of the three main assumptions made by Mehra and Prescott (1985), namely, the specification of the representative consumer's preferences, market completeness and the absence or presence of market frictions in the economy. This chapter reviews, briefly, these suggested solutions in order to provide a perspective for an assessment of the model that will be specified in subsequent chapters. To this end it is expedient to outline some of the alterations in basic assumptions and the corresponding implications for the equity premium puzzle so far documented in the literature.

2.2 The Consumer's Preferences and CCAPM

The CCAPM does poorly partly because of the excessive smoothness of aggregate consumption series compared to asset returns. If all other assumptions of the model are sustained the magnitude of the equity premium, in general, increases with the marginal rate of substitution. Given the smoothness of consumption, the variability in the marginal rate of substitution derived from power utility functions can increase only by increasing α , the coefficient of relative risk aversion (Hansen and Jagannathan (1991)). One way to render the intertemporal marginal rate of substitution more responsive to relatively small changes in consumption without raising α is to work with a broader class of consumer preferences.

One of the advantages of altering only the consumer's utility function is that the equilibrium framework under which fully fledged optimization takes place is retained, thus preserving the sound microfoundations for subsequent analysis. As stated earlier, in the Mehra and Prescott (1985) model the representative agent is assumed to have the well known power utility function. Attempts to take advantage of a richer variety of utility functions to resolve the equity premium puzzle include, among others, the habit persistence in the utility function, generalized expected preferences, relative consumption effects preferences and multiplicative separable preferences.

2.21 Habit Persistence

With reference to the power utility function it is assumed that marginal utility of consumption for period t is not affected by the level of consumption during period $t-1$. One can argue however that for a consumer whose level of consumption during period $t-1$ was relatively large, the desire to consume much more during period t is enhanced since he is accustomed to the previous period's high consumption level. Thus marginal utility of consumption in period t increases with consumption in period $t-1$. This process of attaining a customary range of consumption is known as habit persistence. Accordingly in the habit persistence model of Ryder and Heal (1973), the consumer's instantaneous utility is a function of both current consumption and the habitual standard of living, a proxy of which is provided by past levels of consumption. Constantides (1990) used this model to show that habit formation has the potential to account for the equity premium puzzle, while implying only modest risk aversion. Backus, Gregory and Telmer (1993) used the habit persistence model to account for forward rates in foreign exchange markets. Boldrin, Chritiano and Fischer (1995) also use habit persistence in modeling

business cycles. Mansoorian (1993,1996) used the model to reconsider the Herberger - Laursen - Metzler effect and examined the general macroeconomic policy implications of habit persistence. The model has thus been used in a variety of contexts and has some empirical support. But why was it envisaged in the first instance that habit persistence might help resolve the equity premium puzzle?

Boldrin et al(1995), pointed out that, according to a classic covariance formula, the conditional covariance between the one-period ahead marginal utility of consumption and the rate of return on equity is negatively related to the equity premium. Thus changes in preference specifications result in changes in the two arguments of the covariance term. Thus introducing habit persistence in the utility function generates two effects referred to in Boldrin et al(1995) as the curvature and capital gains channels.

The curvature channel is based on increases in the spread of the one-period ahead marginal utility of consumption across states of nature. This, other things being constant, increases the equity premium and is the curvature effect being so named because it depends on the degree of curvature in the utility function.

The capital gains channel on the other hand is based on the effects of the consumption- smoothing motive implied by habit persistence on the pattern of asset

demands across states of nature. Consumers are generally looking to buy assets when consumption opportunities are high. They, in turn, seek to sell assets when consumption opportunities are low. Given that in an exchange economy capital is assumed to be constant, equity demand changes lead to pronounced fluctuations in prices of capital across states of nature. In this way large capital gains are realized when consumption is high while small capital gains occur when consumption is low. The overall effect, again other things being constant, is to raise the equity premium. This is the capital gains channel.

More formally, however, Kocherlakota (1996) expounds the link between asset returns and habitual standards of living in a simple model assuming the following utility function

$$E_t \sum_{s=0}^{\infty} \beta^s (C_{t+s} - \lambda C_{t+s-1})^{1-\alpha} / (1-\alpha), \quad \alpha > 1, \beta > 0 \text{ and } \lambda > 0. \quad (2.1)$$

Here, the consumer's instantaneous utility decreases in previous period consumption. The marginal utility in period t is

$$MU_t = (C_t - \lambda C_{t-1})^{-\alpha} - \beta \lambda E_t (C_{t+1} - \lambda C_t)^{-\alpha} \quad (2.2)$$

where the second negative term shows that marginal utility is reduced on future purchases. He provides the first order conditions for the optimal consumption portfolio, which are

$$\beta E_t \left\{ \left(\frac{MU_{t+1}}{MU_t} \right) (R_{t+1}^s - R_{t+1}^b) \right\} = 0 \quad (2.3)$$

and

$$\beta E_t \left\{ \left(\frac{MU_{t+1}}{MU_t} \right) R_{t+1}^b \right\} = 1 \quad (2.4)$$

Both (2.3) and (2.4) yield through the law of iterated expectations

$$\beta E \left\{ \left(\frac{MU_{t+1}}{MU_t} \right) (R_{t+1}^s - R_{t+1}^b) \right\} = 0 \quad (2.5)$$

and

$$\beta E \left(\frac{MU_{t+1}}{MU_t} \right) R_{t+1}^b = 1 \quad (2.6)$$

In (2.5) and (2.6), MU_t partly depends on the consumer's ability to predict consumption growth, thus introducing the dependence of MU_t on information available to the individual (which may not necessarily be observable). It is then assumed that consumption growth from period t to period $t+1$ is not predictable so that the ratio of marginal utilities can be expressed as

$$MU_{t+1}/MU_t = \left\{ (\varphi_{t+1}\varphi_t - \lambda\varphi_t)^{-\alpha} - \lambda\beta(\varphi_{t+1}\varphi_t)^{-\alpha} E(\varphi - \lambda)^{-\alpha} \right\} / \left\{ (\varphi_t - \lambda)^{-\alpha} - \lambda\beta(\varphi_t)^{-\alpha} E(\varphi - \lambda)^{-\alpha} \right\}$$

with $\varphi_t \equiv C_t/C_{t-1}$. (2.7)

The ratio given in (2.7) can then be estimated by using sample means to estimate unconditional expectations. In the subsequent analysis it is argued that while habit persistence helps resolve the risk free rate puzzle, it does not help resolve the equity premium puzzle. It is indeed demonstrated that for the proposed solution of Constantinides both a large value of λ and a low value of α are required. A large value of λ implies large amounts of minimum consumption for the consumer's mere survival. Even if consumers are not required to be averse to wealth, they are still required to be strongly averse to consumption risk.

2.22 Generalized Expected Utility

Another generalization of preferences was proposed by Epstein and Zin(1989, 1991)whereby they were able to separate the intertemporal elasticity of substitution from the coefficient of relative risk aversion. By changing some of the assumptions of the expected utility theory, non-expected utility theory offers a somewhat different framework for the analysis of decision making at the individual level in the face of risk and uncertainty. In expected utility theory, given a time

additive preference set over a random consumption stream and assuming a temporal utility function with constant relative risk aversion, the intertemporal elasticity of substitution is shown to be equal to the reciprocal of the temporal coefficient of relative risk aversion. Therefore consumption across periods is, in the view of a risk averse consumer, largely complementary. Now by applying non-expected utility theory it is possible to disentangle the two aspects of choice behavior. Non-expected utility theory is surveyed in Karni and Schmeidler (1991) and Epstein (1991), with the latter being particularly more relevant to applications in finance.

In the generalized expected utility formulation the elasticity of intertemporal substitution is given by $1/\rho$ while the parameter α still measures risk aversion. The elimination of a tight relationship between α and ρ is an advantage because individual attitudes on risk and growth are then governed by different parameters.

Kocherlakota (1990b) derives the first order conditions corresponding to the specification in equation (2.1) for an individual investing in stocks and bonds. These are

$$E_t \left\{ U_{t+1}^{\rho-\alpha} \left(C_{t+1}/C_t \right)^{-\rho} (R_{t+1}^s - R_{t+1}^b) \right\} = 0 \quad \text{and} \quad (2.8)$$

$$\beta E_t \left\{ (E_t U_{t+1}^{1-\alpha})^{\alpha-\rho/\alpha} U_{t+1}^{\rho-\alpha} \left(C_{t+1}/C_t \right)^{-\rho} R_{t+1}^b \right\} = 1. \quad (2.9)$$

Since the marginal rate of substitution depends on the unobservable $t+1$ period utility of the consumer, Kocherlakota imposes the assumption of serial independence between consumption growth and all information available to the consumer in period t . By the law of iterated expectations from (2.8) and (2.9),

$$\beta E \left\{ \left(C_{t+1} / C_t \right)^{-\alpha} \left(R_{t+1}^s - R_{t+1}^b \right) \right\} = 0 \quad (2.10)$$

$$\beta E \left[\left(C_{t+1} / C_t \right)^{1-\alpha} \right]^{\frac{\alpha-\rho}{1-\alpha}} E \left\{ \left(C_{t+1} / C_t \right)^{-\alpha} R_{t+1}^b \right\} = 1.$$

(2.11)

As far as the equity premium puzzle is concerned, the use of non-expected utility leads to the first order condition in equation (2.10) which does not change the results in table 1, because for given values of α , marginal utility derived from bond sales to finance investments in stocks does not change with the generalized expected utility or power utility function specifications. One is interested in results whereby the equity premium is large enough while parameters such as α lie within a reasonable range. This particular outcome is not delivered by the application of generalized expected utility.

2.23 Catching Up With The Joneses Preferences

The power utility function attributes utility to individual levels of consumption. In catching up with the Joneses the utility function is specified to include, over and above the utility an individual derives from personal consumption, that utility derived from consumption of the rest of society. The notion that ones utility depends on the consumption of others is traced to Duesenberry (1949) whereas its application to asset pricing puzzles appears in Abel (1990), Nason (1988), Gali (1994), and Hansen and Cochrane (1995). Under this class of preferences the consumer/investor guards against not only a decline of personal consumption but also a decline relative to the societal per capita consumption during the previous year. The “catching up” terminology is motivated by the fact that the consumer cares about lagged aggregate consumption. Kotcherlakota combines the models studied by Abel (1990) and Gali (1994) to examine some of the asset pricing implications of the catching up utility function. The representative agent is taken to have preferences given by

$$E_t = \sum_{s=0}^{\infty} \beta^s c_{t+s}^{1-\alpha} C_{t+s-1}^{\lambda} C_{t+s-1}^r / (1-\alpha), \beta > 0 \text{ and } \alpha > 1 \quad (2.12)$$

where c_{t+s} , is individual consumption in period $t + s$. The consumer’s current utility is a function of his consumption relative to the current average consumer’s utility (consumption) and the average consumer’s utility (consumption during the previous

period. Negative values of γ and λ mean the individual is less happy when the average consumer is relatively better off. Positive values of γ and λ , on the other hand imply that the average consumer is doing relatively well, signifying either patriotism or some sense of symbiotic beliefs on consumption. The representative consumer takes per capita consumption as given during his portfolio selection.

Accordingly, the first order conditions are

$$E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{\gamma-\alpha} \left(\frac{C_t}{C_{t-1}} \right)^\lambda (R_{t+1}^s - R_{t+1}^b) \right\} = 0 \quad (2.13)$$

and

$$\beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{\gamma-\alpha} \left(\frac{C_t}{C_{t-1}} \right)^\lambda R_{t+1}^b \right\} = 1 \quad (2.14)$$

Again by the law of iterated expectations

$$E \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{\gamma-\alpha} \left(\frac{C_t}{C_{t-1}} \right)^\lambda (R_{t+1}^s - R_{t+1}^b) \right\} = 0 \quad (2.15)$$

and

$$\beta E \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{\gamma-\alpha} \left(\frac{C_t}{C_{t-1}} \right)^\lambda R_{t+1}^b \right\} = 1 \quad (2.16)$$

This model is able to satisfy the sample analogs of (2.14) and (2.15) for appropriate parameter values. For example Kocherlakota reports that for $\beta=0.99$, $(\alpha-\gamma)=19.28$ and $\lambda=3.813$ the sample analogs are exactly satisfied, thus resolving the

equity premium puzzle. The author also points out the intuitive argument for resolving the puzzle. A large absolute value of γ implies that the individual's marginal utility of personal consumption is very sensitive to volatility of per capita consumption and hence its stronger negative relationship with stock returns. In this way even if α is reasonably low so that the investor is not that averse to own consumption risk, he may not be that enticed by stocks because of being highly averse to per capita consumption risk. Abel (1990) also reports good results in this respect from a model which nests catching up utility functions, time separable utility functions, and habit persistence utility functions. For the foreign exchange markets to be empirically investigated in chapter four, a version of this model will be estimated. It will be assumed for this purpose that a consumption externality arises from lagged aggregate consumption.

2.24 Multiplicative Separable Preferences

According to Chang Mo (1990), the application of a non-additive utility function can help explain the mean and variance of the equity premium for reasonable values of α , the coefficient of relative risk aversion. It is assumed that a

representative agent chooses consumption and investment to maximize his lifetime expected utility index

$$E_0 \left[\frac{1}{\gamma} \exp \left[\int_0^{\infty} \gamma \beta^t U(C(t)) dt \right] \right] \quad (2.17)$$

where $U(C(t)) = \ln C(t)$, $\gamma < 1$ and $0 < \beta < 1$; β is a subjective discount factor and γ is a constant.

Meyer (1970) examined the lifetime portfolio selection problem in continuous time for multiplicative utility functions. His results did not, however, include an analytical solution except for the limiting case of the additive family of utility functions. Merton (1969) directly tackled the lifetime portfolio selection problem for the additive family of utility functions in continuous time. Pye (1973) obtained the analytic solution for a subset of the multiplicative functions studied by Meyer (1970). Of particular relevance to our study are the coefficient of relative risk aversion and the optimality conditions for consumption.

The coefficient of relative risk aversion is given by

$$\left(-\frac{\partial^2 U}{\partial c_i \partial c_i} \right) \frac{c_i}{\left(\frac{\partial U}{\partial c_i} \right)} = 1 - \gamma \beta^t \quad (2.18)$$

where U is the utility index and C_t is consumption in period t . Considering an investment opportunity set of one risky and one risk free-asset, Chang Mo assumes that returns of the risky asset evolve according to a lognormal process, given by

$$dq = q\alpha dt + q\sigma dz \quad (2.19)$$

where q is the value of the risky asset, α and σ are constants and dz is the Wiener process. The consumer's wealth is allocated among the risky asset, the riskless asset and consumption. The consumer's budget constraint is

$$dW = [w(\alpha - r)W + rW - C]dt + wW\sigma dz. \quad (2.20)$$

W is wealth, w is the proportion of wealth invested in the risky asset and r is the risk free interest rate. The consumer seeks to maximize (2.19) subject to (2.20).

The resulting indirect utility function and Bellman equation are given by

$$J(w(t), t) = \max_{c, w} E_t \left[\frac{1}{\gamma} \exp \left(\int_t^{\infty} \gamma \beta^s \ln C(s) ds \right) \right] \quad (2.21)$$

and

$$0 = \max_w \left[\gamma \beta^t U(C) J - J_t + J_w (w(\alpha - r)W + rW - C) + 1/2 J_{ww} w^2 \sigma^2 W^2 \right] \quad (2.22)$$

The rule for optimal consumption is

$$C^* = (\ln \beta) W, \quad (2.23)$$

while the optimal proportion of wealth allocated to the risky asset is

$$w^*(t) = \frac{(\alpha - r) \ln \beta}{(\ln \beta + \gamma \beta^t) r^2}. \quad (2.24)$$

By substituting (2.23) and (2.24) into (2.20), one arrives at the wealth dynamics representation

$$dW = \left[\frac{(\alpha - r)^2 \ln \beta}{(\ln \beta + \gamma \beta^t)} \sigma^2 + r + \ln \beta \right] W dt + \frac{(\alpha - r) \ln \beta}{(\ln \beta + \gamma \beta^t)} \sigma W dz \quad (2.25)$$

Given Ito's lemma, we have

$$dC^*/C^* = dW/W \quad (2.26)$$

Equation (2.25) implies that the optimal consumption process is

$$dC^*/C^* = \left[\frac{(\alpha - r)^2 \ln \beta}{(\ln \beta + \gamma \beta^t)} \gamma^2 + r + \ln \beta \right] dt + \frac{(\alpha - r) \ln \beta}{(\ln \beta + \gamma \beta^t)} \sigma dz \quad (2.27)$$

This model is solved as noted earlier to yield a closed form solution, which is then used to compute the mean and variance of the risk premium.

2.25 Models with Non-Time Separable Preferences and Durability

Two other models as far as preferences with both habit formation and durability of consumption are concerned are the models of Ferson and Constantinides (1991) and Heaton (1995), whereby these two sources of time non-separability are investigated.

For durable goods the time of acquisition does not necessarily accord directly with the time of consumption, and past consumption may provide utility during the current period so that consumption is rendered durable. Habit formation on the other hand, as discussed earlier, can lead to a situation whereby an increase in previous consumption, holding time-t consumption fixed, causes period utility to fall. There is thus a complementarity in consumption. Dunn and Singleton (1986) estimate a model to this effect. For models considering these two effects, overidentifying restrictions are tested typically on the basis of the Euler equation

$$E_t = \left[\sum_{\tau=0}^{\infty} a_{\tau} \beta^{\tau} (C_{t+\tau})^{\gamma-1} \right] = E_t \left[\left(\sum_{\tau=0}^{\infty} a_{\tau} \beta^{\tau+1} (C_{t+\tau+1})^{\gamma-1} \right) r_{t+1} \right] \quad (2.29).$$

In this case, positive values of the coefficients on the lag operator specifying preferences, is indicative of dominance of durability in accounting for time non-separability while a negative coefficient a , would signify the dominance of habit persistence. Using US monthly consumption from National Income and product Accounts (NIPA), to study securities markets from January, 1959 to December 1985, they accommodate exponential growth by using consumption growth in such a way that their disturbance term in estimation is

$$\varepsilon_{t+2} = \sum_{\tau=0}^1 a_{\tau} \beta^{\tau} (C_{t+\tau} / C_t)^{\gamma-1} - \left[\sum_{\tau=0}^1 a_{\tau} \beta^{\tau+1} (C_{t+\tau+1} / C_t)^{\gamma-1} \right] r_{t+1} \quad (2.30)$$

They obtain a positive coefficient in line with a predominance of durability.

However as noted by Singleton (1997), with respect to the goodness of fit test for the overidentifying restrictions, their small reported P-value of 0.001 shows that this particular route of introducing time non-separability of preferences does not achieve a substantial improvement of the model's fit.

In a similar study on the other hand, Ferson and Constantinides (1991), use quarterly and annual data from 1929 to 1986, and scale their Euler equation somewhat differently, leading to the estimating econometric model

$$\varepsilon_{t+2} = \sum_{\tau=0}^1 a_{\tau} \beta^{\tau} \left(\frac{C_{t+\tau} + a_1 C_{t+\tau-1}}{C_t + a_1 C_{t-1}} \right)^{\tau-1} - \left[\sum_{\tau=0}^1 a_{\tau} \beta^{\tau+1} \left(\frac{C_{t+\tau+1} + a_1 C_{t+1}}{C_t + a_1 C_{t-1}} \right)^{\tau-1} \right] r_{t+1} \quad (2.31)$$

The instrument vector for estimation having as elements, nominal treasury bill returns, nominal term and default premiums, dividend yields and industrial production.

Not only are they able to verify the validity of the overidentifying restrictions but they also find a value of -0.95 for a_2 , thus establishing that over the sample interval they considered using aggregate consumption expenditure on non-durables habit persistence was dominant. The same result holds in the case of consumption

on durable goods with a value of -0.65 for a_2 . This is in contrast to the results in Dunn and Singleton (1986).

In the model of habit persistence and durability of consumption estimated for stock returns in Heaton (1995), the role of long term habit and durability in empirical performance is investigated. In this model, identification of habit formation as well as durability effects calls for higher order polynomials. Because of this the simulated moments method is used to estimate preference parameters. For a finite number of lags in the preference specification, this model can be estimated by the generalised method of moments. The model however quickly becomes overly complicated as the number of lags is increased.

In order to obtain the intertemporal marginal rate of substitution, a numerical approach is used as explained in appendix A of Heaton (1995) to solve for the marginal utility of consumption. The numerical algorithm uses a modified Gerlakin method described in Judd (1991,1998). The relevant expressions needed for marginal utility to be computed are as follows:

$$muc(t) = mus^d(t) + \beta \delta E[muc(t+1)|I_t] \quad (2.32)$$

which states that marginal utility of consumption at time t is the sum of marginal utility accruing from a unit of service flows from the durable good at time t

and the discounted marginal utility from time $t+1$, taking into account the depreciation of the durable good governed by the parameter δ . For stationarity purposes, (2.32) is scaled by a factor of $(C_t)^{-\gamma}$, giving

$$\frac{muc(t)}{(C_t)^{-\gamma}} = \frac{mus^d(t)}{(C_t)^{-\gamma}} + \beta\delta E \left[\frac{muc(t+1)}{(C_{t+1})^{-\gamma}} \frac{C_t}{C_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \middle| I_t \right] \quad (2.33)$$

To incorporate habit formation, the marginal utility of service flows from the durable good is hypothesized to take the form

$$mus^d(t) = (S_t)^{-\gamma} - \alpha(1-\theta)\beta E \left[\sum_{j=0}^{\infty} \beta^j \theta^j (S_{t+1+j})^{-\gamma} \middle| I_t \right] \quad (2.34)$$

where α is the factor of proportionality of the weighted average of past flows of services from durable goods relative to current levels of the same services. Now using the same scaling factor as in (2.33), and defining $m(t)$ as

$$m(t) \equiv E \left[\sum_{j=0}^{\infty} \beta^j \theta^j (S_{t+1+j})^{-\gamma} \middle| I_t \right] \quad (2.35)$$

and letting $m^*(t) \equiv \frac{m(t)}{(C_t)^{-\gamma}}$, we have

$$m^*(t) = E \left[\left(\frac{S_{t+1}}{C_t} \right)^{-\gamma} \middle| I_t \right] + \beta\theta E \left[m^*(t+1) \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \middle| I_t \right] \quad (2.36)$$

From functional equations (2.33) and (2.36), and substitution for marginal utility of services from the durable good in relation (2.34), a solution for marginal

utility of consumption can be obtained and used in turn to solve for an equilibrium price measure. The non-linearities involved in these equations necessitate a numerical approach to obtaining marginal utility of consumption. The state of the economy represented by $X(t)$, constituted by a vector of seven variables, namely $c(t)$, $c(t-1)$, $f(t)$, $f(t-1)$, $f(t-12)$, $s(t)/c(t)$ and $x(t)/c(t)$, where $s(t)$ and $x(t)$ represent the accumulation of service flows from the durable good and the habit stock respectively and are scaled by consumption to induce stationarity while $f(t)$ represents dividends. The solution requires a monte-carlo procedure to solve a seven dimensional integral in order to obtain artificial moments for SMM estimation. The results obtained however are not essentially different from most others. That is, although some improvement is registered in terms of the magnitude of estimates of the risk aversion parameter, the equity premium puzzle is not evidently resolved.

The use of richer specifications of preferences has led to some improvements but not eliminated the mixed results that have become common in this literature. We would like to go back to the form of the utility function used in Mehra and Prescott(1985) but use dividends and earnings to evaluate IMRS.

2.3 Dividend and Earnings Discount Factors

The other notable departure from the canonical model is the use of dividend discount factors. As noted in chapter one, dividend factors have featured before in this context,(Abel, 1988; S.Cecchetti et al., 1990). There may be enough information in dividends to explain variability of stock prices when they are used to evaluate IMRS as noted in Hagiwara and Herce(1997). They also find, in the case of US annual data for the period 1889 to 1994 , a sample standard deviation of 0.126 for real dividend growth whilst that of consumption growth is 0.034. Their sample correlation coefficient between dividend growth and consumption growth is 0.635. They use the GMM and work with a CRRA utility function. Their instruments are lagged values of returns, dividend growth, consumption growth and dividend yield. They estimate and compare two consumption based models to their dividend based counterparts.They run one asset and two asset models for each type of discount factor. They find that for the single asset models, the t-ratios from the dividend model are greater than 2 and higher than those of the consumption based model. They also report that no set of instruments enables them to generate a t-ratio greater than 1.4.

In addition the p-values for the dividend model range from 0.678 to 0.973 while range for the consumption model is 0.207 to 0.267. This clearly demonstrates

that the empirical performance of the dividend growth model is much better than the consumption growth model. Consumption is much smoother than returns and fails to serve as a good measure of IMRS. The difference in empirical performance could also be because dividends are given out only to stock holders. Mankiw et. Al.,(1991) show that consumption of stockholders is a better measure of IMRS. Even then, consumption is plagued by measurement problems due to temporal aggregation. These two concerns can be sidestepped with the use of dividend growth. Also with dividend growth we can preserve the simpler utility functional form. This more so given some of the findings about unconventional preferences in an international setting by Anne Sibert (19..) where there is no substantive gain in explaining risk premia in currency markets. Earnings have not been used as a measure of IMRS in the open economy version of the model. What we do here is argue that since dividends are driven by earnings, we can indeed investigate the potential of these measures to improve on past results. The next section addresses this motivation, citing the derivation in Lucas (1978; 1982).

2.4 Using Dividends and Earnings to measure IMRS in the Lucas Model

Why do we use dividends and earnings in the Lucas model? After all exchange rates are determined by relative outputs or productivities. We do so first

because in the Lucas model, nominal exchange rates are determined by relative money stocks, outputs and preferences, where output equals consumption. But consumption, dividends and earnings are equal in the Lucas economy. It is important to clarify our interpretation here since it is the basis of our main innovation in the empirical analysis of the model.

A critical insight from the Lucas model is that the budget constraint requires that in equilibrium, for the one asset economy, $consumption = earnings(output) = dividends$. That is the exogenous variable used to price the one asset may be interpreted as consumption, dividends or earnings. In fact, the appropriate interpretation is earnings since whatever the asset produces is paid out as a dividend, which must be consumed because it is perishable. The earnings of the asset determine dividends, which in turn govern consumption! The Lucas model uses a general equilibrium framework to derive the asset pricing function. The condition that $consumption = earnings(output) = dividends$ is essentially a general equilibrium argument.

Furthermore, risk as far as households are concerned, boils down to how an asset's value fluctuates in relation to items that they value such as consumption, dividends and indeed earnings.

In chapter three, we use GMM to estimate the three models, for the case of the CRRA preferences, where consumption, dividends and earnings are used in turn to measure the intertemporal marginal rate of substitution.

Chapter 3

MODEL AND METHODOLOGY

3.1 Introduction

Two main theoretical approaches have been used to study the relationship between expected future spot foreign exchange rates and forward rates. There is the macroeconomic approach of portfolio balance models of for example Dornbusch (1982), Frankel (1979) and Henderson (1984). There is on the other hand the representative agent approach where an individual investor's optimizing behavior is usually in an infinite horizon setting. Some models in this class are Lucas (1982) Stulz (1984) and Svenson (1983). The basic prediction that emerges from both approaches is that there are significant deviations between the expected spot rate and the forward rate.

The second approach of the representative agent with time additive preferences has also been used to investigate the variability of forward premia with predominantly disappointing results. The theory is unable to account for observed variability. Variations and extensions of the same theory to account for the equity premium are good candidates in

trying to account for the variability of forward premia. In this regard, as mentioned earlier, several studies have incorporated representative agent preferences exhibiting habit persistence.

Backus, Gregory and Telmer (1993) estimate and simulate a model along these lines in which they consider exchange rates of the US dollar against the five major currencies of Canada, France, Germany, Japan and Britain. They find that habit persistence raises the standard deviation of their equilibrium price measure relative to its estimated lower bound while it raises that of expected returns from currency speculation to approximately half its estimate from the sample. Without habit persistence they report that the theory is able to account for less than one percent of each of these standard deviations. However having provided these results the theory was found to remain at variance with the data in two main respects.

The derived standard deviation of the short rate turned out to be at least two orders of magnitude larger than that in the data. In addition the autocorrelation of the forward premium was slightly negative in the theoretical economies is strongly positive in the data. In view of these outstanding weaknesses they recommend three possibilities of

extending the theory. The first is to introduce for more complex versions of intertemporal preference relations . In this direction examples in the literature include Bekaert (1991) , Ferson and Constantinides (1991) and Heaton (1993, 1995). These studies feature the simultaneous presence of durability and habit formation. The second is to allow for the possibility of unobservable and occasional changes in the stochastic process of the economy. They make reference to Lewis (1989), where it is argued that nearly half of the predicted returns from speculation in the US dollar versus the UK pound and the Deutsche mark during the early 1980s can be attributed to the learning process subsequent to regime shifts. It is not clear as to whether the same analysis is able to account for the same returns for other periods across the remaining currencies. The third is to explore models other than the representative agent model of asset pricing. Two examples are the overlapping generations approach of Hakkio and Sibert (1991) and models with borrowing constraints and incomplete markets Lucas (1990), Marcet and Singleton (1991) and Telmer (1993).

In this study it is the first suggested direction that is adopted to investigate the potential of using dividend growth and earnings growth instead of the consumption based discount factor in the asset pricing model with a view to accounting for the forward premium in the forward foreign exchange market. We use the two country formulation as in Backus et al (1993) for instance, following a small open economy formulation in the spirit of Mark (1985). Whereas the dividend discount factor has been shown, as indicated in chapter one , to improve on the model's performance in the case of stock prices, this has not been investigated for the foreign exchange determination model. The earnings model has not been studied with respect to both stock markets and foreign exchange markets. It is investigated here using the GMM estimation strategy as well as the calibration methodology, both of which are elaborated upon below.

Risk aversion implies that agents may not require equivalent rates of return on foreign and domestic assets. This is a possible basis for the existence of a foreign exchange risk premium. It has been emphasized in the literature however that the mere fact that an asset is denominated in a foreign currency is not

sufficient for investors to receive a reward in form of a premium (Frenkel 1979a). This type of risk encountered by just holding foreign exchange can be diversified.

Indeed in most contemporary models of asset returns, investors receive a risk premium whenever there is covariation between returns and a particular benchmark that renders the risk undiversifiable. Popular benchmarks are the return on the market portfolio (beta) and the aggregate consumption marginal rate of substitution. The risk premium on foreign exchange thus is partly determined by the comparative riskiness of domestic and foreign nominal assets. The strand of the literature followed here models the foreign exchange premium on the basis of optimizing behavior.

In the case of a model with uncertainty and intertemporal maximization of utility, such a pricing kernel is the intertemporal marginal rate of substitution.

Of interest in this study are tests of the asset pricing model euler equations and the light the corresponding results may shed on the best determinants of the intertemporal marginal rate of substitution of the investors. The tests will bring out the difference between consumption based and dividend based stochastic discount factors. In addition, an earnings based discount factor will be compared to both consumption and dividend discount factors in direct tests.

Regarding previous studies, the first direct test was that of Mark (1985). He assumed a constant relative risk aversion type of utility function and estimated

the Euler equation using the GMM to test for overidentifying restrictions. His model estimates provide two interesting results. The first is that the coefficient of relative risk aversion is substantially larger than what is generally considered plausible in the literature. The second result is the rejection of the model.

His model is extended in Hodrick (1989b) to include additional currencies, namely the Belgian franc, French franc and Swiss franc. For U.S. data, Hodrick's estimate of the risk aversion parameter is 60.9, higher compared to most of Mark's estimates which were above had 50.9 as the largest value. In this extended model however, the overidentifying restrictions are not rejected.

In a related study, Modjtahedi (1991) extends the model of rates with maturities of one-, three- and six months for the Japanese yen, the Canadian dollar and pound sterling. The sample period is July 1973 to July 1988. He also uses the U.S. consumption expenditures on non-durables and non-durables plus services to measure consumption. The model is rejected even more strongly than that of Mark (1985) even although he finds forward instruments as Mark found.

In contrast to the above mentioned models, Kaminsky and Peruga (1990) use a GARCH type model and register equally disappointing results.

Some summary statistics on returns from currency speculation, rates of currency depreciation and forward premia for the US dollar versus the British Pound sterling, the Deutsche mark, the Canadian dollar and the French franc are given below by way of verification. We then present the equilibrium asset pricing framework following Lucas (1978,1982) In particular, we specify the exchange rate determination model in Lucas (1982), thereby motivating the derivation of pricing relations that specify the implied forward premium. Then we carry out GMM tests of the euler equations, first to revisit the paper of Mark (1985) applying the GMM on our data sample for the consumption based model. We conclude the chapter with a presentation of euler equation tests for the dividend growth and earnings growth models. In the next chapter, the model is calibrated to US and Canadian data, to complement the results obtained from GMM estimation.

3.2 Exchange Rate Data

The exchange rates used are relative to the U.S. dollar for each country, taking the quotations for the last Friday of each month.

The data frequency is monthly over the interval August 1979 to December 1986.

Sample Autocorrelation Function (SACF) and Sample Partial Autocorrelation Function (SPACF) of Returns from Currency Speculation.

Table 3.1

Returns from the German Mark				
K	r_k	s.e. (r_k)	Q(st)	pr_k
1	0.559	0.118	23.5	0.559
2	0.242	0.150	27.9	-0.104
3	0.276	0.156	33.8	0.272
4	0.274	0.162	39.7	0.0146
5	0.251	0.169	44.7	0.123
6	0.188	0.174	47.6	-0.0406
7	0.205	0.176	51.0	0.134
8	0.129	0.180	52.4	-0.143
9	-0.013	0.181	52.4	-0.088
10	-0.066	0.181	52.8	-0.108
11	-0.129	0.181	54.2	-0.144
12	-0.003	0.183	54.2	0.206

Table 3.2**Returns from the French Frank**

K	r_k	$se(r_k)$	Q(st)	pr_k
1	0.562	0.118	23.7	0.562
2	0.245	0.151	28.2	-0.103
3	0.276	0.156	34.1	0.268
4	0.275	0.163	40.1	0.020
5	0.185	0.169	42.8	0.012
6	0.076	0.172	43.3	-0.080
7	0.151	0.172	45.1	0.172
8	0.110	0.174	46.1	-0.140
9	-0.004	0.175	46.1	-0.011
10	-0.029	0.175	46.2	-0.053
11	-0.099	0.175	47.1	-0.140
12	0.047	0.176	47.3	0.275

Table3.3**Returns on CS**

K	r_k	$se(r_k)$	Q(st)	pr_k
1	0.350	0.118	9.21	0.350
2	-0.109	0.132	10.1	-0.264
3	-0.025	0.133	10.2	0.142
4	-0.077	0.133	10.6	-0.187
5	-0.109	0.133	11.6	0.014
6	-0.229	0.135	15.8	-0.290
7	-0.107	0.140	16.8	0.137
8	0.086	0.141	17.4	-0.048
9	0.022	0.142	17.4	0.011
10	0.112	0.142	18.5	0.127
11	0.212	0.143	22.4	0.097
12	-0.040	0.147	22.6	-0.222

Table 3.4**Returns on the Pound Sterling**

K	r_k	$se(r_k)$	Q(st)	pr_k
1	0.584	0.118	25.6	0.584
2	0.236	0.153	29.8	-0.159
3	0.244	0.158	34.4	0.277
4	0.225	0.163	38.4	-0.052
5	0.069	0.167	38.7	-0.086
6	-0.117	0.168	39.8	-0.191
7	-0.030	0.169	39.9	0.198
8	0.010	0.169	39.9	-0.131
9	-0.084	0.169	40.5	0.019
10	-0.120	0.169	41.7	-0.058
11	-0.085	0.171	42.4	0.001
12	0.041	0.171	42.5	0.124

Table 3.5**Depreciation of the German Mark**

K	r_k	$se(r_k)$	Q(st)	pr_k
1	0.046	0.118	0.161	0.046
2	0.162	0.118	2.15	0.160
3	0.065	0.121	2.47	0.052
4	0.158	0.122	4.44	0.132
5	0.103	0.124	5.29	0.081
6	0.059	0.126	5.57	0.001
7	0.163	0.126	7.76	0.128
8	0.073	0.129	8.19	0.033
9	0.001	0.129	8.20	-0.061
10	-0.031	0.129	8.28	-0.076
11	-0.030	0.130	8.37	-0.077
12	-0.104	0.130	9.33	-0.139

Table 3.6**Depreciation of the French Frank**

K	r_k	$sc(r_k)$	Q(st)	pr_k
1	0.031	0.118	0.074	0.031
2	0.145	0.118	1.67	0.144
3	0.081	0.120	2.17	0.074
4	0.215	0.121	5.79	0.196
5	0.078	0.126	6.27	0.055
6	0.002	0.127	6.27	-0.061
7	0.170	0.127	8.63	0.133
8	0.071	0.130	9.05	0.029
9	0.010	0.131	9.06	-0.052
10	0.038	0.131	9.19	-0.021
11	-0.036	0.131	9.30	-0.101
12	-0.045	0.131	9.47	-0.095

Table 3.7**Depreciation of the Canadian Dollar**

K	r_k	$se(r_k)$	Q(st)	pr_k
1	-0.128	0.118	1.23	-0.128
2	-0.133	0.120	2.58	-0.152
3	0.062	0.122	2.87	0.023
4	-0.104	0.122	3.72	-0.116
5	0.062	0.123	4.02	0.046
6	-0.186	0.124	6.81	-0.216
7	-0.063	0.128	7.14	-0.099
8	0.117	0.128	8.27	0.016
9	-0.036	0.130	8.38	-0.021
10	-0.028	0.130	8.45	-0.063
11	0.276	0.130	15.1	0.278
12	-0.114	0.138	16.3	-0.090

Table 3.8**Depreciation of the Pound Sterling**

K	r_k	$se(r_k)$	Q(st)	pr_k
1	-0.098	0.118	0.726	-0.098
2	0.114	0.119	1.72	0.106
3	0.053	0.121	1.93	0.075
4	0.114	0.121	2.95	0.116
5	0.065	0.122	3.28	0.076
6	-0.181	0.123	5.92	-0.204
7	0.026	0.126	5.97	-0.048
8	0.084	0.126	6.55	0.110
9	-0.023	0.127	6.60	0.013
10	-0.036	0.127	6.71	-0.018
11	-0.031	0.127	6.79	-0.027
12	0.012	0.128	6.80	-0.052

Table 3.9**Forward Premium on the German Mark**

K	r_k	$se(r_k)$	Q(st)	pr_k
1	0.060	0.118	0.269	0.060
2	0.191	0.118	3.05	0.188
3	0.183	0.122	5.65	0.170
4	0.143	0.126	7.25	0.102
5	0.171	0.128	9.57	0.111
6	0.027	0.132	9.63	-0.049
7	0.201	0.132	12.9	0.126
8	0.039	0.136	13.1	-0.020
9	-0.030	0.136	13.1	-0.116
10	-0.062	0.136	13.5	-0.144
11	-0.053	0.136	13.7	-0.080
12	-0.082	0.137	14.3	-0.090

Table 3.10**Forward Premium on the French Frank**

K	r_k	$se(r_k)$	Q(st)	pr_k
1	0.061	0.118	0.277	0.061
2	0.140	0.118	1.76	0.137
3	0.177	0.121	4.18	0.165
4	0.148	0.124	5.90	0.121
5	0.099	0.127	6.68	0.051
6	-0.084	0.128	7.25	-0.158
7	0.145	0.128	8.97	0.094
8	0.004	0.131	8.97	-0.016
9	-0.048	0.131	9.17	-0.059
10	0.001	0.131	9.17	0.003
11	-0.093	0.131	9.93	-0.102
12	-0.044	0.132	10.1	-0.056

Table 3.11**Forward Premium on the Canadian Dollar**

K	r_k	$se(r_k)$	Q(st)	pr_k
1	-0.085	0.118	0.543	-0.085
2	-0.040	0.119	0.662	-0.047
3	0.044	0.119	0.809	0.037
4	0.068	0.119	1.17	-0.064
5	0.008	0.120	1.18	0.000
6	-0.209	0.120	4.70	-0.218
7	-0.006	0.125	4.70	-0.039
8	0.045	0.125	4.87	0.016
9	-0.011	0.125	4.88	0.010
10	-0.024	0.125	4.92	-0.052
11	0.200	0.125	8.40	0.198
12	-0.109	0.129	9.45	-0.134

Table 3.12**Forward Premium on the Pound Sterling**

K	r_k	$se(r_k)$	Q(st)	pr_k
1	0.050	0.118	0.190	0.050
2	0.252	0.118	5.01	0.250
3	0.175	0.125	7.38	0.164
4	0.098	0.129	8.13	0.031
5	0.108	0.130	9.06	0.027
6	-0.179	0.131	11.7	-0.262
7	0.042	0.134	11.8	-0.016
8	0.005	0.135	11.8	0.093
9	-0.138	0.135	13.4	-0.090
10	-0.041	0.137	13.6	-0.035
11	-0.105	0.137	14.5	-0.042
12	-0.041	0.138	14.7	-0.051

The aim here was to check for violations of the assumption of covariance stationarity, but from the overall results there is no overwhelming difficulty from this aspect.

In spite of facilitating the ranking of risk premia in general, the CAPM is not a general equilibrium model. In this model the level of the premium itself is not determined since the market price of risk is not determined. Among other problems the model is based on relatively restrictive assumptions. In particular all investors are assumed to have the same beliefs and both lending and borrowing can take place at the same riskless rate. The model's mean-variance basis also implies a representation of uncertainty by density functions which depend on their means and variances.

Important contributions in terms of projecting financial theory in a general equilibrium framework, alluded to in chapter one, are found in Lucas (1978,1982). We specify here the Lucas model of 1982, which will be tested using GMM and calibrated to the Canadian and US exchange rate data.

3.3 The Model in a Barter Economy

Suppose the world economy consists of two countries where agents possess identical amounts of wealth and utility functions so that their respective populations can be normalized to unity. Firms are represented by endowment streams that yield a homogeneous perishable good. Thus labor and capital inputs are equal to zero. These firms are the proverbial fruit trees in the model. The exogenously given outputs are x_t for the home country and y_t for the foreign country.

Outputs evolve according to a stochastic process known to agents in both countries. Specifically,

$$x_t = g_t x_{t-1} \quad (3.1)$$

$$y_t = g_t^* y_{t-1} \quad (3.2)$$

where g_t and g_t^* are random gross rates of change governing the evolution of output. Every firm issues a perfectly divisible share that is freely traded on a stock market. Agents depend entirely on dividends which are paid out from the combined output of all firms. The ex-

dividend value of each domestic firm is e_t , while that of each foreign firm is e_t^* . The numeraire good is domestic output, x_t , with q_t representing the price of foreign output, y_t , in terms of x_t .

Consumption for the home country agent is c_{xt} of the domestic good and c_{yt} of the foreign country good. The foreign country agent's consumption on the other hand is c_{xt}^* of the domestic good and c_{yt}^* of the foreign country good. Both agents are assumed to hold shares in domestic and foreign firms. The domestic agent's shares in the local firm are ω_{xt} while those held in the foreign firm are ω_{yt} . The foreign agent holds ω_{xt}^* shares in the domestic firm and ω_{yt}^* in the foreign firm.

On coming into period t , the domestic agent's total wealth, W_t , is the sum of the with-dividend values of shares held in local and domestic firms. Thus,

$$W_t = \omega_{x,t-1}(x_t + e_t) + \omega_{y,t-1}(q_t y_t + e_t^*) \quad (3.3)$$

The individual's wealth is then re-allocated to new shares and consumption for the current period, so that

$$W_t = e_t \omega_{xt} + e_t^* \omega_{yt} + c_{xt} + c_{yt} \quad (3.4)$$

Setting equation (3.3) equal to equation (3.4) leads to the agent's consolidated budget equation, which is

$$c_{xt} + q_t c_{yt} + e_t \omega_t + e_t^* \omega_{yt} = \omega_{x,t-1} (x_t + e_t) + \omega_{y,t-1} (q_t y_t + e_t^*) \quad (3.5)$$

The utility function for the current period is $U(c_{xt}, c_{yt})$ and the subjective discount factor is β , with $0 < \beta < 1$. The domestic agent's problem is to maximize expected lifetime utility,

$$E_t \left(\sum_{i=0}^{\infty} \beta^i U(c_{x,t+i}, c_{y,t+i}) \right) \quad (3.6)$$

subject to the consolidated budget constraint (3.5). The agent accomplishes this by choosing appropriate paths for consumption as well as share purchases over time. In order to obtain an unconstrained version of this problem, the expression for c_{xt} from equation (3.5) is substituted into the objective function, (3.6). Doing this gives

$$U \left[\left(\omega_{x,t-1} (x_t + e_t) + \omega_{y,t-1} (q_t y_t + e_t^*) - e_t \omega_{xt} - e_t^* \omega_{yt} - q_t y_t c_{yt}, c_{yt} \right) + \right]$$

$$E_t \left[\beta U \left(\omega_{xt} (x_{t+1} + e_{t+1}) + \omega_{yt} (q_{t+1} y_{t+1} + e_{t+1}^*) - e_{t+1} \omega_{x,t+1} - e_{t+1}^* \omega_{y,t+1} - q_{t+1} c_{y,t+1}, c_{y,t+1} \right) + \dots \right] \quad (3.7)$$

The derivatives of equation (3.7) with respect to c_y, ω_x, ω_y provide the euler equations to the maximization problem. The three respective euler equations are

$$q_t u_1(c_x, c_y) = u_2(c_x, c_y) \quad (3.8)$$

$$e_t u_1(c_x, c_y) = \beta E_t \left[u_1(c_{x+1}, c_{y+1}) (x_{t+1} + e_{t+1}) \right] \quad (3.9)$$

$$e_t^* u_1(c_x, c_y) = \beta E_t \left[u_1(c_{x+1}, c_{y+1}) (q_{t+1} y_{t+1} + e_{t+1}^*) \right] \quad (3.10)$$

where u_1 denotes the first partial derivative of the objective function with respect to each of the above three variables, denoting marginal utilities of x and y consumption. These equations represent the underlying conditions for an optimal allocation of consumption and purchases of local and foreign shares for the domestic agent. Any deviations from them will not be utility improving.

The foreign country agent similarly selects optimal paths for consumption and share purchases. The agent seeks to maximize

$$E_t \left[\sum_{i=0}^{\infty} \beta^i U(c_{xt+i}^*, c_{yt+i}^*) \right] \quad (3.11)$$

subject to

$$c_{xt}^* + q_t c_{yt}^* + e_t^* \omega_{xt}^* + e_t^* \omega_{yt}^* = \omega_{x,t-1}^* (x_t + e_t) + \omega_{y,t-1}^* (q_t y_t + e_t^*) \quad (3.12)$$

The foreign country agent's first order conditions, by the same procedure are

$$q_t u_1(c_{xt}^*, c_{yt}^*) = u_2(c_{xt}^*, c_{yt}^*) \quad (3.13)$$

$$e_t^* u_1(c_{xt}^*, c_{yt}^*) = \beta E_t \left[u_1(c_{x,t+1}^*, c_{y,t+1}^*) (x_{t+1} + e_{t+1}) \right] \quad (3.14)$$

$$e_t^* u_1(c_{xt}^*, c_{yt}^*) = \beta E_t \left[u_1(c_{x,t+1}^*, c_{y,t+1}^*) (q_{t+1} y_{t+1} + e_{t+1}^*) \right] \quad (3.15)$$

Since it is assumed that output is exhausted at the end of the period, there are also adding up constraints that have to be satisfied for both output and shareholdings. In particular,

$$c_{xt}^* + c_{xt}^* = 1 \quad (3.16)$$

$$c_{yt}^* + c_{yt}^* = 1 \quad (3.17)$$

$$\omega_{xt}^* + \omega_{xt}^* = 1 \quad (3.18)$$

$$\omega_x + \omega_y^* = 1 \quad (3.19)$$

Equations (3.1) to (3.19) specify a dynamic stochastic model of a barter economy. It is expedient, in solving the model, to recast it as a social planner's problem. The social planner's solution here is one of a static but competitive general equilibrium. In this case, attributes of the consumption good regarding time of availability and states of the world have to be specified. This in the end helps to identify the entire range of probable future outcomes so that the very act of defining a good in essence defines a system of markets. The trades carried out in contingency goods can be undertaken at time t and implemented over time.

In solving the model, the domestic and foreign agents are assigned weights of ϕ and $1 - \phi$ respectively, delineating their shares of total wealth and hence importance in the world economy. Having assigned these weights, the objective then is to assign their endowments in such a way that the utility of each agent is maximized. That is, formally, to maximize

$$E_t \left[\sum_{i=0}^{\infty} \beta^i \left(\phi U(c_{x+i}, c_{y+i}) + (1 - \phi) U(c_{x+i}^*, c_{y+i}^*) \right) \right] \quad (3.20)$$

subject to

$$c_x + c_x^* = 1 \quad (3.21)$$

$$c_y + c_y^* = 1 \quad (3.22).$$

Given that the goods are assumed to be perishable here, the same problem can be re-stated as one of maximizing

$$\phi u_1(c_x + c_y) + (1 - \phi) u_1(c_x^* + c_y^*) \quad (3.23)$$

subject to the same constraint (3.22). The corresponding first order conditions are

$$\phi u_1(c_x + c_y) = (1 - \phi) u_1(c_x^* + c_y^*) \quad (3.24)$$

$$\phi u_2(c_x, c_y) = (1 - \phi) u_2(c_x^*, c_y^*) \quad (3.25)$$

Now, apart from the relative shares of wealth, ϕ can be chosen so as to reflect the relative size of the country.

Assuming that the two countries are equally economically big, implies $\phi = \frac{1}{2}$. The quantities allocated by the social planner are equally split between the two agents so that

$$c_{xt} = c_{xt}^* = \frac{x_t}{2}, \text{ and}$$

$$c_{yt} = c_{yt}^* = \frac{x_{yt}}{2}$$

The model is solved as an Arrow-Debrue equilibrium.

Following the determination of Pareto optimal quantities, a price vector that supports such quantities is determined in turn. Share purchases on the other hand reflect, by their distribution, the particular insurance scheme at work. Given that it is somewhat difficult for either agent to insure “worldwide” risk when outputs in both countries are low, it is expedient to consider a risk pooling equilibrium approach. In this case,

$$\omega_{xt} = \omega_{xt}^* = \omega_{yt} = \omega_{yt}^* = \frac{1}{2},$$

, i.e., each agent holds fifty percent shares in domestic and foreign firms.

3.4 The model with CRRA utility function.

If the utility function is defined over a Cobb-Douglas index of both goods, $c_x^\theta c_y^{1-\theta}$, then

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \quad (3.26).$$

The marginal utilities are now

$$u_1(c_x, c_y) = \frac{\theta C_t^{1-\gamma}}{c_x} \quad (3.27)$$

and

$$u_2(c_x, c_y) = \frac{(1-\theta)C_t^{1-\gamma}}{c_y} \quad (3.28).$$

The equilibrium real exchange rate is defined by the first order conditions corresponding to this specification. These are

$$q_t = \frac{1-\theta}{\theta} \frac{x_t}{y_t} \quad (3.29)$$

$$\frac{e_t}{x_t} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} \left(1 + \frac{e_{t+1}}{x_{t+1}} \right) \right] \quad (3.30)$$

$$\frac{e_t}{q_t y_t} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} \left(1 + \frac{e_{t+1}}{q_{t+1} y_{t+1}} \right) \right] \quad (3.31).$$

Equation (3.29) gives the real exchange rate as a comparative function of outputs while equation (3.30) and (3.31) represent dividend-price ratios in a stochastic difference equation format.

3.5 The Model in a single currency world.

Introduction of money into macroeconomic models is not a generally settled issue in the literature. There however two main approaches that have been followed. One is to introduce money directly into the utility function (Sidrauski). The second one, used in the Lucas model, is to impose a cash in advance constraint on all agents. No one can buy goods outside of using money. The specifics of the transactions technology underlying the cash in advance are laid out in Mark(2000), covering five steps. At the beginning of period t ,

(i) quantities of both outputs x_t and y_t are revealed

(ii) the process, λ_t , governing the way money stocks evolve is also revealed. In both the home and foreign countries this is an exogenous stochastic process. The money stock, M_t , changes according to

$$M_t = \lambda M_{t-1} \quad (3.32).$$

Each agent gets $\frac{\Delta M}{2}$, which is half of the increments in the money stock.

(iii) a competitive, centralized stock market opens where agents determine the allocation of their wealth between stocks and cash and the market closes.

(iv) a mall opens where decentralized trading occurs.

Households now split into shopper- seller pairs. Shoppers use their proceeds from the prior stock market trading to buy goods, x_t, y_t . The workers sell all their country specific endowments x_t and y_t in the stores. The goods market shuts down.

(v) All proceeds are then issued to shareholders as nominal dividends.

States of the world depend on the exogenous growth rates of domestic and foreign levels of output. All possible states are revealed prior to trading.

3.6 The domestic agent's problem.

This is formalized by considering the agent's total wealth as well as allocations among stock market purchases and desired cash. In particular, total wealth, W , is given by

$$W_t = \frac{P_{t-1}(\omega_{x-1}x_{t-1} + \omega_{y-1}q_{t-1}y_{t-1})}{P_t} + \omega_{x-1}e_t + \omega_{y-1}e_t^* + \frac{\Delta M_t}{2P_t} \quad (3.33).$$

Allocations by the agent imply

$$W_t = \frac{m_t}{P_t} + \omega_x e_t + \omega_y e_t^* \quad (3.34).$$

The agents cash requirements for recurrent consumption is

$$M_t = P_t(c_x + q_t c_y) \quad (3.35)$$

Substituting equation (3.35) into (3.34) gives

$$W_t = \frac{P_{t-1}(c_x + q_t c_y)}{P_t} + \omega_x e_t + \omega_y e_t^* \quad (3.36)$$

Setting equation (3.36) equal to (3.33) we have

$$\begin{aligned} \frac{P_{t-1}(c_x + q_t c_y)}{P_t} + \omega_x e_t + \omega_y e_t^* &= \frac{P_{t-1}}{P_t} (\omega_{x-1} x_{t-1} + \omega_{y-1} q_{t-1} y_{t-1}) \\ + \omega_{x-1} e_t + \omega_{y-1} e_t^* + \frac{\Delta M_t}{2P_t} & \end{aligned} \quad (3.37).$$

This can be re-arranged so that

$$\begin{aligned} c_x + q_t c_y + \omega_x e_t + \omega_y e_t^* &= \frac{P_{t-1}}{P_t} (\omega_{x-1} x_{t-1} + \omega_{y-1} q_{t-1} y_{t-1}) \\ + \frac{\Delta M_t}{P_t} + \omega_{x-1} e_t + \omega_{y-1} e_t^* & \end{aligned} \quad (3.38)$$

Subject to this budget constraint, the agent's problem is to maximize

$$E_t \left\{ \sum_{i=0}^{\infty} \beta^i U(c_{x+i}, c_{y+i}) \right\} \quad (3.39)$$

Using the budget constraint, equation (3.38), the quantities consumed of x and y are given by

$$\begin{aligned} c_x &= \frac{P_{t-1}}{P_t} (\omega_{x-1} x_{t-1} + \omega_{y-1} q_{t-1} y_{t-1}) + \frac{\Delta M_t}{P_t} + \omega_{x-1} e_t + \omega_{y-1} e_t^* \\ &\quad - q_t c_y - \omega_x e_t - \omega_y e_t^* \end{aligned} \quad (3.40)$$

and

$$c_y = \frac{1}{q_t} \left(\frac{P_{t-1}}{P_t} (\omega_{x-1} x_{t-1} + \omega_{y-1} q_{t-1} y_{t-1}) + \frac{\Delta M_t}{2P_t} + \omega_{x-1} e_t + \omega_{y-1} e_t^* - c_x - \omega_x e_t - \omega_y e_t^* \right) \quad (3.41)$$

Converting the problem into its unconstrained version , the resulting euler equations are

$$q_t u_1(c_x, c_y) = u_2(c_x, c_y) \quad (3.42)$$

$$e_t u_1(c_x + c_y) = \beta E_t \left(u_1(c_{x+1}) \left(\frac{P_t}{P_{t+1}} x_t + e_{t+1} \right) \right) \quad (3.43)$$

$$e_t^* u_1(c_x, c_y) = \beta E_t \left\{ u_1(c_{x+1}, c_{y+1}) \frac{P_t}{P_{t+1}} q_t y_t + e_{t+1}^* \right\} \quad (3.44)$$

Similarly for the foreign agent's cash requirement,

$$M_t^* = P_t (c_x^* + q_t c_y^*) \quad (3.45)$$

The corresponding budget constraint is

$$c^{\cdot}_x + q_t c^{\cdot}_y + \omega^{\cdot}_x e_t + \omega^{\cdot}_y e_t = \frac{P_{t-1}}{P_t} (\omega^{\cdot}_{x-1} x_{t-1} + \omega^{\cdot}_{y-1} q_{t-1} y_{t-1}) + \frac{\Delta M_t}{2P_t} + \omega^{\cdot}_{x-1} e_t + \omega^{\cdot}_{y-1} e_t \quad (3.46)$$

The objective then is to maximize expected utility,

$$E_t \left\{ \sum_{i=0}^{\infty} \beta^i U(c^{\cdot}_{x+i}, c^{\cdot}_{y+i}) \right\} \quad (3.47)$$

subject to (3.46). The euler equations here are analogous to the ones in the case of the domestic agent and are

$$q_t u_1(c^{\cdot}_x, c^{\cdot}_y) = u_2(c^{\cdot}_x, c^{\cdot}_y) \quad (3.48)$$

$$e_t u_1(c^{\cdot}_x, c^{\cdot}_y) = \beta E_t \left\{ u_1(c^{\cdot}_{x+1}, c^{\cdot}_{y+1}) \left(\frac{P_t}{P_{t+1}} x_t + e_{t+1} \right) \right\} \quad (3.49)$$

$$e_t u_1(c^{\cdot}_x, c^{\cdot}_y) = \beta E_t \left\{ u_1(c^{\cdot}_{x+1}, c^{\cdot}_{y+1}) \frac{P_t}{P_{t+1}} q_t y_t + e_{t+1} \right\} \quad (3.50)$$

The revised adding up constraints, including total money

transfers are

$$M_t = m_t + m_t^{\cdot} \quad (3.51)$$

$$x_t = c_x + c_x^{\cdot} \quad (3.52)$$

$$y_t = c_y + c_y^{\cdot} \quad (3.53)$$

$$1 = \omega_x + \omega_y^{\cdot} \quad (3.54)$$

$$1 = \omega_x + \omega_x^* \quad (3.55)$$

Now, aggregating cash in advance constraints over both agents leads to

$$M_t = P_t(x_t + q_t y_t) \quad (3.56)$$

As seen before for the barter economy, the resulting risk-pooling equilibrium on assuming that $\phi = \frac{1}{2}$ is given by

$$\omega_x = \omega_x^* = \omega_y = \omega_y^* = \frac{1}{2} \quad (3.57)$$

$$c_x = c_x^* = \frac{x_t}{2} \quad (3.58)$$

and

$$c_y = c_y^* = \frac{x_{y_t}}{2} \quad (3.59)$$

Given a constant relative risk aversion type of utility function

$$U(c_x, c_y) = \frac{c_t^{1-\gamma}}{1-\gamma}, \text{ relative output levels determine the real}$$

exchange rate q_t :

$$q_t = \frac{(1-\theta) x_t}{\theta y_t} \quad (3.60)$$

From equations (3.56) and (3.60) we have,

$$M_t = P_t \left(x_t + \left(\frac{1-\theta}{\theta} \right) \frac{x_t}{y_t} \cdot y_t \right) \quad (3.61)$$

$$M_t = P_t \left(x_t + \left(\frac{1-\theta}{\theta} \right) x_t \right) \quad (3.62)$$

$$M_t = P_t \frac{x_t}{\theta} \quad (3.63)$$

$$P_t = \frac{x_t}{\theta M_t} \quad (3.64)$$

$$P_{t+1} = \frac{x_{t+1}}{\theta M_{t+1}} \quad (3.65)$$

$$\frac{P_t}{P_{t+1}} = \frac{\left(\frac{x_{t+1}}{\theta M_{t+1}} \right)}{\frac{x_t}{\theta M_t}} \quad (3.66)$$

$$\frac{P_t}{P_{t+1}} = \frac{M_t}{M_{t+1}} \cdot \frac{x_{t+1}}{x_t} \quad (3.67)$$

Equation (3.67) is the expression for the inverse inflation rate which in this case of an economy with one currency will have an impact on the value of nominal dividends during subsequent periods. Accordingly, the formulas for equity prices will reflect this effect as an inflation premium. Thus equity prices are now derived by combining both the euler equations with equation (3.67), and are given by

$$\frac{e_t}{x_t} = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} \left(\frac{M_t}{M_{t+1}} + \frac{e_{t+1}}{x_{t+1}} \right) \right] \quad (3.68)$$

$$\frac{e_t}{q_t y_t} = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} \left(\frac{M_t}{M_{t+1}} + \frac{e_{t+1}}{q_{t+1} y_{t+1}} \right) \right] \quad (3.69)$$

The same procedure for determining the exchange rate and equity pricing is repeated in the next section whereby two currencies are incorporated in the model. This enables one to obtain analytically, the expressions for exchange rates and forward rates that are implied by the theory. In this way, when calibrating the model some form of comparison can be made between the statistical properties of these simulated rates and the properties of sample exchange rate data for Canada and the USA.

3.7 The Model with two currencies.

In presenting this part of the model, the transactions technology is re-specified so that the home good can only be bought with domestic currency and the foreign good can only be

bought in foreign currency. The domestic currency to be considered here will be the US dollar, while the foreign currency will be the Canadian dollar. The model will be calibrated to US and Canadian consumption and money growth data in the first instance. Then in line with the procedure pursued using the GMM methodology, the same model will be calibrated to US and Canadian earnings data to evaluate the role of earnings growth in its empirical performance.

The assumptions on transactions now require that domestic dividends be paid out in US dollars while foreign dividends are paid out in Canadian dollars only. Either agent however can obtain foreign currency needed for consumption by trading on the stock market. In specifying the model with two currencies, the same outline as presented in Mark(2000) is followed with an identical notation. P_t is the US dollar price of the home good, x , while P_t^* is the Canadian dollar price of the foreign good, y . The US money stock is M while N represents the Canadian money stock. The evolution of these money stocks is given by

$$M_t = \lambda M_{t-1}$$

$$N_t = \lambda^* N_{t-1},$$

where, λ^* and λ are exogenous random rates of growth of N and M.

Agents now face foreign exchange risk, which they can hedge against by holding future claims on both US and Canadian dollars. Suppose r_t is the price of a claim on future US dollars expressed in terms of x_t , the US good. Suppose also that r_t^* is the price of future claims on Canadian dollars and that outstanding claims with respect to each of these currencies are perfectly divisible. Let ψ_{Mt}, ψ_{Nt} represent claims held by the home country agent on US dollars and Canadian dollars, respectively. The corresponding claims held by the foreign country agent on US dollars and Canadian dollars are represented by ψ_{Mt}^*, ψ_{Nt}^* . S_t is the exchange rate.

Their initial endowments are $\psi_{Mt} = 1, \psi_{Nt} = 0, \psi_{Mt}^* = 0, \psi_{Nt}^* = 1$. Total wealth in period t, W_t , can be subdivided into nominal dividend payments from previous period equity shares, the market value

equivalent of shares held, current period monetary transfers and outstanding monetary transfer claims. Thus

$$\begin{aligned}
 W_t = & \frac{P_{t-1}}{P_t} \omega_{x-1} x_{t-1} + S_t \frac{P_{t-1}^*}{P_t} \omega_{y-1} y_{t-1} + \frac{\psi_{M_{t-1}} \Delta M_t}{P_t} + \frac{\psi_{N_{t-1}} S_t \Delta N_t^*}{P_t} \\
 & + \omega_{x-1} e_t + \omega_{y-1} e_t^* + \psi_{M_{t-1}} r_t + \psi_{N_{t-1}} r_t^* \quad (3.70)
 \end{aligned}$$

As was the case with the single currency model, trades on the exchanges generate a pattern of allocations of wealth for each agent. Some of the wealth is devoted to equity, some to other claims on future monetary transfers and the rest to the purchase of consumption goods. This decomposition can be expressed as

$$W_t = \omega_x e_t + \omega_y e_t^* + \psi_{M_t} r_t + \psi_{N_t} r_t^* + \frac{m_t}{P_t} + \frac{n_t S_t}{P_t} \quad (3.71)$$

Since current values of outputs and money stocks are revealed prior to trading, both agents are presumed to obtain their exact requirements of US and Canadian dollars for current period

transactions. Given also that cash in advance constraints are binding in equilibrium, it is the case that

$$m_t = P_t c_x \quad (3.72)$$

$$n_t = P_t^* c_y \quad (3.73)$$

These constraints, (3.72) and (3.73), can be used to substitute out the agents' money holdings from equation (3.70) expressing it as

$$W_t = \omega_x e_t + \omega_y e_t^* + \psi_M r_t + \psi_N r_t^* + \frac{P_t c_x}{P_t} + P_t^* c_y \frac{S_t}{P_t} \quad (3.74)$$

which further simplifies to

$$W_t = c_x + \frac{S_t P_t^* c_y}{P_t} + \omega_x e_t + \omega_y e_t^* + \psi_M r_t + \psi_N r_t^* \quad (3.75)$$

Equation (3.75) says that overall wealth is the sum of goods possessed, total equity held and total money transfers. As done before, equating this equation to equation (3.70) generates the consolidated budget constraint for the domestic agent. The agent's objective is to maximize expected utility subject to this consolidated budget constraint. The agent's problem formally therefore is to maximize

$$E_t \left\{ \sum_{i=0}^{\infty} \beta^i U(c_{x+i}, c_{y+i}) \right\} \quad (3.76)$$

such that

$$\begin{aligned} c_x + \frac{S_t P_t^* c_y}{P_t} + \omega_x e_t + \omega_y e_t^* + \psi_M r_t + \psi_N r_t^* \\ = \frac{P_{t-1}}{P_t} \omega_{x-1} x_{t-1} + S_t \frac{P_{t-1}^*}{P_t} \omega_{y-1} y_{t-1} + \frac{\psi_{M-1} \Delta M_t}{P_t} + \frac{\psi_{N-1} S_t \Delta N_t^*}{P_t} + \omega_{x-1} e_t \\ + \omega_{y-1} e_t^* + \psi_{x-1} r_t + \psi_{y-1} r_t^* \end{aligned} \quad (3.77)$$

The euler equations derived from taking derivatives with respect to $c_y, \omega_x, \omega_y, \psi_M, \psi_N$, in that order are

$$\frac{S_t P_t^*}{P_t} u_1(c_x, c_y) = u_2(c_x, c_y) \quad (3.78)$$

$$e_t u_1(c_x, c_y) = \beta E_t \left(u_1(c_{x+1}, c_{y+1}) \left(\frac{P_t}{P_{t+1}} x_t + e_{t+1} \right) \right) \quad (3.79)$$

$$e_t^* u_1(c_x, c_y) = \beta E_t \left(u_1(c_{x+1}, c_{y+1}) \left(\frac{S_{t+1} P_t^*}{P_{t+1}} y_t + e_{t+1}^* \right) \right) \quad (3.80)$$

$$r_t u_1(c_x, c_y) = \beta E_t \left(u_1(c_{x+1}, c_{y+1}) \left(\frac{\Delta M_{t+1}}{P_{t+1}} + r_{t+1} \right) \right) \quad (3.81)$$

$$r_t^* u_1(c_x, c_y) = \beta E_t \left(u_1(c_{x+1}, c_{y+1}) \left(\frac{\Delta N_{t+1} S_{t+1}}{P_{t+1}} + r_{t+1}^* \right) \right) \quad (3.82)$$

The foreign country agent's problem similarly is to maximize expected utility subject to a symmetric consolidated budget constraint. The implied euler equations correspond directly to those of the domestic agent as specified above. The binding cash in advance constraints for the foreign agent in turn are

$$m_t^* = P_t c_x^* \quad (3.83)$$

$$n_t^* = P_t c_y^* \quad (3.84)$$

both of them holding in equilibrium.

Adding up constraints with reference to initial endowments, output and money stocks are

$$\psi_{M_t} + \psi_{M_t}^* = 1 \quad (3.85)$$

$$\psi_{N_t} + \psi_{N_t}^* = 1 \quad (3.86)$$

$$c_x + c_x^* = x_t \quad (3.87)$$

$$c_y + c_y^* = y_t \quad (3.88)$$

$$m_t + m_t^* = M_t \quad (3.89)$$

$$n_t + n_t^* = N_t \quad (3.90)$$

The combined constraints (3.83) through (3.90) yield quantity equations for each country :

$$M_t = P_t x_t \Rightarrow P_t = \frac{M_t}{x_t}$$

$$N_t = P_t^* y_t \Rightarrow P_t^* = \frac{N_t}{y_t}$$

These unit velocity quantity equations provide expressions for prices which can be used to eliminate the endogenous price levels from the euler equations. this then means the main determinants of exchange rates are the relative money stocks, quantities of output and the preference specification for the agents. Assuming again that the weight on each country is $\frac{1}{2}$, equilibrium is achieved when

$$\omega_x = \omega_x^* = \omega_y = \omega_y^* = \psi_{M_t} = \psi_{M_t}^* = \psi_{N_t} = \psi_{N_t}^* = \frac{1}{2} \quad (3.91)$$

The equilibrium output allocations are

$$c_x = c_x^* = \frac{x_t}{2}$$

$$c_{yt} = c_{yt}^* = \frac{y_t}{2}$$

To obtain an expression for the nominal exchange we use equation (3.78) to express the real exchange rate as

$$\frac{u_2(c_{xt}, c_{yt})}{u_1(c_{xt}, c_{yt})} = \frac{S_t P_t^*}{P_t} = \frac{S_t N_t x_t}{M_t y_t} \quad (3.92)$$

so that the nominal exchange rate is

$$S_t = \frac{u_2(c_{xt}, c_{yt}) M_t y_t}{u_1(c_{xt}, c_{yt}) N_t x_t} \quad (3.93)$$

With a CRRA type utility function used before, the real

exchange rate in equilibrium is $q_t = \left(\frac{1-\theta}{\theta}\right) \left(\frac{x_t}{y_t}\right)$. This implies

that the nominal exchange rate is

$$S_t = \frac{(1-\theta) M_t}{\theta N_t} \quad (3.94)$$

The corresponding euler equations for the model are

$$\frac{e_t}{x_t} = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} \left(\frac{M_t}{M_{t+1}} + \frac{e_{t+1}}{x_{t+1}} \right) \right] \quad (3.95)$$

$$\frac{e_t^*}{q_t y_t} = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} \left(\frac{N_t}{N_{t+1}} + \frac{e_{t+1}^*}{q_{t+1} y_{t+1}} \right) \right] \quad (3.96)$$

$$\frac{r_t}{x_t} = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} \left(\frac{\Delta M_{t+1}}{M_{t+1}} + \frac{r_{t+1}}{x_{t+1}} \right) \right] \quad (3.97)$$

$$\frac{r_t^*}{x_t} = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} \left(\frac{(1-\theta) \Delta N_{t+1}}{\theta N_{t+1}} + \frac{r_{t+1}^*}{x_{t+1}} \right) \right] \quad (3.98)$$

Prices here are computed the fact that they are not being traded notwithstanding because the computed outcomes represent shadow prices at which the public keeps such instruments in zero net supply. In the same vein, in the absence of explicit forward markets one can still compute equilibrium forward exchange rates.

In the present case, consider a date- t US dollar price of a one period nominal discount bond, b_t , which pays one dollar at the beginning of $t+1$. Let b_t^* denote the corresponding Canadian bond that also pays one Canadian dollar at the beginning of period $t+1$. If covered interest parity hold, then the forward exchange rate is given by

$$F_t = S_t \frac{b_t^*}{b_t} \quad (3.99)$$

where the bond prices in equilibrium are

$$b_t = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} \frac{M_t}{M_{t+1}} \right] \quad (3.100)$$

and

$$b_i^* = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} \frac{N_t}{N_{t+1}} \right] \quad (3.111)$$

The risk premium is a proportion of the conditional covariance of currency speculation profits and the intertemporal marginal rate of substitution of money, given that the latter is expected to be positive. The theory underlying the derivation of the risk premium can be evaluated among other ways by considering tests of the overidentifying restrictions involved. To this end, a popular and largely successful methodology in empirical finance is the Generalized Method of Moments (GMM). The fundamentals of GMM are reviewed below, before using it to estimate pricing relations in the currency markets.

3.8 Overview of GMM

Given a vector of observable variables say x_{t+1} , and a model specifying $m_{t+1} = m(\theta, x_{t+1})$, the GMM as developed by Hansen (1982) can be used to obtain estimates of θ and test the underlying model. Most financial asset pricing models for example generally

imply that the product of any gross asset return $R_{i,t+1}$, and some market wide random variables has a constant conditional expectation, that is

$$E_t \{ m_{t+1} R_{g,t+1} \} = 1 \quad \text{for all } i. \quad (3.112)$$

$R_{g,t+1}$ is the gross asset return. The expectation is typically conditional upon information available at time t , itself a subset of market information for instance a vector of instruments comprising publicly available information observable to the econometrician.

As seen earlier, m_{t+1} is the intertemporal marginal rate of substitution alternatively referred to as an equivalent martingale measure, a stochastic discount factor or a Radon-Nicodym derivative. It is a discount factor in the sense that it can be used to compute the present value of a future payoff. Considering an asset i whose market value at time $t+1$ is given by

$$R_{i,t+1} = \frac{X_{i,t+1}}{P_{it}}. \quad \text{Therefore equation (3.112) can be expressed as}$$

$$P_{it} = E_t(m_{t+1}, x_{i,t+1}) \quad (3.113).$$

This equation indicates that the expected value of the product of the future payoff and the stochastic discount factor yields the present value of the future payoff. Equation (xvi) can be used to define an error term $\varepsilon_{i,t+1}$, such that $E_t(\varepsilon_{i,t+1}) = 0$. That is, the error term is

$$\varepsilon_{i,t+1} = m(\theta, x_{t+1})R_{i,t+1} - 1 \quad (3.114).$$

For N assets over T periods, the error terms in (3.114) form a $T \times N$ matrix, the typical row of which is $\varepsilon_{i,t+1}$.

The model implies, by the law of iterated expectations, that for any information set I_t , $E(\varepsilon_{i,t+1} | I_t) = 0$, and consequently $E(\varepsilon_{i,t+1} I_t) = 0$, a condition to the effect that $\varepsilon_{i,t+1}$ is orthogonal to I_t , hence its being called an orthogonality condition. GMM tests of asset pricing models are typically based on orthogonality conditions such as this one. An important assumption underlying these empirical tests is that of rational expectations. It is assumed that all variables subjected to the expectations operator are considered as mathematical conditional expectations terms. Thus one is able to obtain expressions for $E(\cdot | I_t)$ and $E(\cdot)$ by treating the expected values as mathematical conditional expectations. On account of the rational expectations presumption,

there is no relationship between the information on which expectations are conditioned and the difference between actual realizations and model expectations. In equation (3.18) using any information at time t , the error term ought not be predictably different from zero. Any variations in returns then that is predictable on the basis of instruments I_t can be eliminated. This being done by multiplying the respective returns through by an appropriate stochastic discount factor.

GMM Procedure

Consider N assets (equations) and L instruments and define an $N \times L$ matrix G of sample mean orthogonality conditions ,

$$G = (\varepsilon^z) \quad (3.115).$$

Z is a $T \times L$ matrix of observed instruments , the typical is a subset of instruments.

Let $g = \text{vec}(G)$. Here, $\text{vec}(G)$ partitions G into row vectors of length L each; ($L = (\underline{h}_1, \underline{h}_2, \dots, \underline{h}_N)$). The h s are then stacked into a vector, g , whose length is equal to the number of orthogonality conditions, NL . To obtain GMM estimates $\hat{\theta}$, of θ , one searches for

parameter values which make g as close to zero as possible through minimizing a quadratic form, with a weighting matrix which attaches weights to respective orthogonality conditions, that is

$$g'Wg, \quad W_{NL \times NL}.$$

For a random N vector,

$$\varepsilon_{t+1}(\theta) = R_{i,t+1}m(\theta, x_{t+1}) - 1 \quad \text{we therefore have,}$$

$$\hat{g}(\theta) = T^{-1} \sum_t (u_t(\theta) \otimes Z_{t-1}).$$

Let $\hat{\theta}$, be the parameter values that minimize

$$J_T = \hat{g}(\theta)'B\hat{g}$$

B is an $NL \times NL$ positive definite matrix that could possibly depend on the sample. The minimized value J_T , will possess a weighted chi-square distribution, which Jagannathan and Wang (1993) show can be used to test the hypothesis that (xiv) is valid. It is also shown by Hansen (1982) that estimators of θ that $g'Wg$ for any fixed W are asymptotically consistent. If a consistent estimate of the inverse of the covariance matrix of the orthogonality conditions is selected as the

weighting matrix, the resulting estimators are also asymptotically efficient among the class of estimators obtained by minimizing $g'Wg$ for fixed W .

The theory can accordingly be evaluated by using the first order conditions to estimate the representative agent's preference parameters by the GMM. The extent to which the theory and the data conform is gauged from the J_t statistic. This approach is fairly standard in the finance literature and was used in the Mark(1985) study to investigate currency speculation returns.

In that study the parameterization of the utility function adopted is the constant relative risk aversion type

$$U(C) = \delta C^{1-\gamma} / (1-\gamma) \quad , \gamma \geq 0. \quad (3.116)$$

δ is some arbitrary constant while γ is a parameter for relative risk aversion. Performing the necessary optimization exercise leads

to the following theoretical restrictions

$$E\left(c_{t+1}^{\gamma} p_{t+1} \Pi_{j,t+1} | I_t\right) = 0, \quad j = 1, 2, \dots, M. \quad (3.117)$$

$c_{t+1} = C_t / C_{t+1}$ and is thus the consumption ratio.

$p_{t+1} = P_t / P_{t+1}$ and is thus the price ratio while

$\Pi_{j,t+1} = (S_{j,t+1} - F_{j,t}) / S_{j,t}$ is the speculative return on currency j traded one period forward at time t.

According to equation (3.117), although economic agents have at their disposal an information set that they are motivated to acquire and which is publicly available, they cannot forecast the product of the speculative return on currency j and the intertemporal marginal rate of substitution of money between t and t+1 on the basis of such an information set. The supposition that this product is uncorrelated with each and every element in the information set constitutes the candidate hypothesis for empirical tests.

This model was tested by Mark(1985,2000) using monthly data on the Canadian dollar, the Netherlands guilder, the German mark and the UK pound exchange rates versus the US dollar. The model was estimated jointly across currencies with seasonally adjusted US aggregate consumption data on nondurables and services as well as

with services only. The sample interval was March 1973 to July 1983. The selection of instruments used was broadly based on the relevance of each variable in the agents' solution of their forecasting problem. That is a variable has to have a rationale for being ranked in the quest to predict profits from currency speculation. In view of this, the instruments were the current consumption ratio along with each of speculative returns to each currency and then with respective forward premiums per currency.

The justification for including the consumption ratio among instruments is given as the fact that agents are engaged after all in predicting returns from investing in foreign currency denominated assets in terms of the consumption good. In light of this, the current consumption ratio itself may constitute useful information and is therefore a plausible instrument. On the other hand, the idea that past profits can also contribute to the prediction of speculative profits explains their inclusion as instruments. Lastly, forward premiums are used as instruments because the forward premium can be decomposed into two parts. The risk premium and the expected rate of change of

the exchange rate itself implying that the forward premium can provide indicative information regarding speculative returns.

There are k lags used, where $k = 0, 1, 2$, for each set. Corresponding to each of the lag lengths are 20, 36 and 52 orthogonality conditions respectively, used to estimate γ , the coefficient of relative risk aversion and then the J_T statistic is used to test the model's overidentifying restrictions. The results of this estimation obtained by Mark are reproduced in Table 3.1 below. The point estimates of γ are relatively large with also somewhat big standard errors and are therefore generally imprecise. Nonetheless, in terms of tests of overidentifying restrictions, the model is not rejected strongly because judging by the goodness of fit test only in the particular case of zero lags whilst forward premiums serve as instruments is there evidence against the model. That is using the J statistic, there are no significant between the theory and the data.

We estimate the same model for two sampling intervals in the case of the French frank, the German mark, the U K pound and the Canadian dollar versus the US dollar. The first estimation period is the interval from September 1979 to June 1983, which is within the

interval covered in Mark (1985). The second period is from September 1979 to December 1986. The instruments in our case are also lagged currency speculation returns and forward premiums of respective currencies on the same grounds as those spelt out by Mark and reviewed above. Three different lag specifications are estimated, namely 0, 1 and 2 lags. Aggregate consumption expenditures on non-durables plus services and aggregate expenditure on services only are used to derive per capita real consumption for the consumer which are then used in turn for all the three cases.

Estimation results when Consumption ratios and Forward Premia are used as

Consumption	Lag	Instruments				
		γ	$se(\gamma)$	Tmin(J)	d. f.	P-value
NDS	0	158.98	16.15	5.30	23	0.989
	1	137.80	28.14	6.74	23	0.990
	2	176.29	56.80	6.75	43	0.979
ND	0	48.90	33.95	2.24	23	0.967
	1	176.05	19.34	5.42	23	0.968
	2	186.11	22.41	4.41	43	0.970

Estimation results when Consumption Ratios and Speculative Profits are used as

		Instruments				
Consumption	Lag	γ	se(γ)	Tmin(J)	d. f.	P-value
NDS	0	219.36	22.64	5.12	17	0.997
	1	215.81	22.27	6.69	33	0.989
	2	212.48	5.30	6.68	33	0.959
ND	0	68.32	10.56	2.21	17	0.997
	1	271.27	69.28	5.41	33	0.988
	2	269.27	66.15	5.39	33	0.977

In the model with quarterly data , considering the German Mark , Japanese Yen and UK Pound versus US dollar exchange rates , we found the following estimates from GMM estimation.

Consumption Growth Model: Forward Premium As Instruments

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	50.457	76.625	7.640	0.177
1	49.848	63.856	6.982	0.222
2	49.714	61.877	4.926	0.425

Consumption Growth Model Speculative Profits as Instruments

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	48.245	92.052	11.884	0.036
1	40.239	59.364	11.565	0.041
2	19.317	40.243	12.279	0.031
3	38.924	41.648	11.651	0.040

Dividend Growth Model: Forward Premium As Instruments.

Lags	γ	S.E.	J-Stat	Marg. Sign.
0	26.634	30.031	8.518	0.130
1	18.165	21.461	9.092	0.105
2	9.266	13.769	7.623	0.178
3	9.573	12.194	7.028	0.219

Dividend growth Model: Speculative Profits as Instruments

Lags	γ	S.E.	J-Stat.	Marg. Sign.
0	35.565	33.190	9.447	0.092
1	30.157	12.553	9.579	0.088
2	30.221	10.149	9.281	0.098
3	29.095	17.232	9.923	0.077

Earnings Growth Model: Speculative Profits as Instruments

Lags	γ	Standard error	J-Statistic	Marginal Significance
0	4.6045	76.160	4.5462	0.47372
1	13.3163	9.341	3.7590	0.5846
2	12.3025	9.740	2.0867	0.83702
3	8.2482	11.648	2.7649	0.7362
4	7.0582	10.498	2.1177	0.8336

Earnings Growth Model: Forward Premium as Instruments

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	4.3045	76.160	4.5462	0.4737
1	0.000	26.283	2.7311	0.7414
2	0.000	31.853	1.6916	0.890
3	0.8527	24.937	1.2394	0.9410
4	6.6094	16.171	1.7357	0.8844

In terms of the size of the relative risk aversion parameter, the earnings model is better than the dividend growth model. The dividend growth model is also somewhat better than the consumption based model.

In the Appendix, results are reported for more currencies. These are the Swiss Franc, French Franc, Belgian Franc, Norwegian Kroner, Swedish Krona, Danish Krone, Austrian Schilling, Australian Dollar, Canadian dollar, Italian Lira and Spanish Peseta.

In the next chapter, we look at a calibration of the model to US and Canadian data over the same sample period. We outline the steps for determining the state vector in the model specified above, state the processes governing growth of output and money stocks and set initial values for the time preference and risk aversion parameters and compare model implied moments to their observed sample counterparts.

Chapter 4

CALIBRATING THE LUCAS MODEL

4.1 Introduction

In the preceding chapter, the dividend growth model was estimated and found to improve slightly on the performance of the consumption based model in terms of both the size and precision of the relative risk aversion parameter. Moreover, the earnings growth model did not only give better results than the dividend growth model but substantially outperformed the consumption-based model. In estimating the model by the GMM, the fact that consumption is much more smooth relative to the larger variability of speculative returns rendered the risk estimates implausibly large with large standard errors.

In this chapter we look at a complementary strategy of evaluating the model by way of calibration. Calibrating a model means that one simulates data on endogenous variables and compares such simulated data to sample data. The model is said to fare better if it can replicate moments of the sample data. In our case the moments involved will aim to capture volatility and persistence in the exchange rate data, particularly the depreciation rate and forward premium. We will also conduct a regression of the gross rate of depreciation on the forward premium using sample data and compare the

resulting estimates to those obtained from the same regression using simulated data. We do this using consumption data first and repeat the exercise with earnings data. We outline the calibration steps in section 4.2, specify the state vector and stochastic processes governing consumption and money stocks in section 4.3, describe the computation of the transition matrix in section 4.4 and report model simulation results in section 4.5. section 4.6 provides results from calibrating the model to earnings data and concludes.

4.2 Steps to Calibrate The Lucas Model

As stated earlier, in the calibration approach we compare measures that are implied by, and as such computed from, the model with measures from the real world. Some of the parameters required for such a comparison have to be taken either as given or adapted from previous empirical studies.

Some of the measurements that have been long established have come to be regarded as stylized facts. These are taken as given and combined with the observed data to generate measures of endogenous variables according to the model. A comparison of these model data and sample data is said to be good if the model data matches sample data very well.

The Steps are as Follows:

Step I.

The particular measures to be used have to be determined. These are obtained from sample data. They would include for instance, means, variances and autocorrelations in an observed series. Their model simulated counterpart series are used to estimate the same measures for comparison.

Step II

Acceptable estimates of deep parameters are assigned on the basis of well established results in the literature. In the model under consideration here, for example, we will assign values to the coefficient of risk aversion and the discount factor. Where there are processes describing the motion of variables, these are also specified.

Step III

The model is simulated to generate values of the endogenous variables implied by the model solution, for particular assignments of the deep parameters. This is analogous to generating predicted values after estimating a parametric model.

Step IV

The extent to which the simulated data in step 3 is determined. This is analogous to inference in the parametric estimation case. Here we look at some of the statistical properties of the simulated series relative to those of the sample numbers. This particular step is not without controversy. Some contend that by generating numbers that are close to the observed numbers, one is only proving something about the model. The object of research however, it is argued, is the real world. Here it will suffice to say that the methodology is only being used to complement what has been inferred from the results built using the more standard GMM approach. It is also helpful in identifying potential areas of improvement in modeling exchange rates.

In the spirit of the above steps, we now describe the processes and notation to be used in calibrating the model at hand.

The model will be evaluated on the basis of four measures, namely the forward premium, the gross rate of depreciation, the realized forward profit from currency speculation and the estimated slope coefficient obtained from a regression of the gross rate of depreciation on the forward premium. For statistical properties, we consider the standard deviation and autocorrelation coefficient for each of the series. This facilitates a comparison in terms of the volatility and serial dependence of the theoretical and actual series.

4.3 Processes for Evolution of output and Money stocks.

The state vector comprises exogenously given growth rates of output and the money stock for each country. Suppose

In the Lucas model, there is no production and hence no labor nor capital accumulation. This helps to sidestep issues relating to parameter estimates of representative production functions. Instead we have to specify the stochastic processes for the evolution of domestic and foreign output levels. $\tilde{\lambda}$.

Suppose the state vector is given by $\tilde{\phi} = (\tilde{\lambda}, \tilde{\lambda}^*, \tilde{g}, \tilde{g}^*)$ where $\tilde{\lambda}, \tilde{\lambda}^*$ and \tilde{g}, \tilde{g}^* are the respective growth rates of money stocks and output. The state vector is governed by a finite state Markov chain with stationary probabilities. There are two possible states of the world for every element in the state vector. States of the world are identified relative to the average growth rate of each element. When the rate of growth of a variable is less than its average growth rate, a bad state of the world exists. On the other hand, when the variable's growth rate is above its average there exists a good state of the world.

Let λ_1, λ_2 represent high growth and low growth states of money stock respectively while g_1, g_2 similarly represent high and low growth states for output. As before, an asterisk on a variable means it refers to the foreign country. There are therefore sixteen possible states of the world. These are

$$\begin{aligned}
 \phi_1 &= (\lambda_1, \lambda_1^*, g_1, g_1^*), & \phi_2 &= (\lambda_1, \lambda_1^*, g_1, g_2^*) \\
 \phi_3 &= (\lambda_1, \lambda_1^*, g_2, g_1^*), & \phi_4 &= (\lambda_1, \lambda_1^*, g_2, g_2^*) \\
 \phi_5 &= (\lambda_1, \lambda_2^*, g_1, g_1^*) & \phi_6 &= (\lambda_1, \lambda_2^*, g_1, g_2^*) \\
 \phi_7 &= (\lambda_1, \lambda_2^*, g_2, g_1^*) & \phi_8 &= (\lambda_1, \lambda_2^*, g_2, g_2^*) \\
 \phi_9 &= (\lambda_2, \lambda_1^*, g_1, g_1^*) & \phi_{10} &= (\lambda_2, \lambda_1^*, g_1, g_2^*) \\
 \phi_{11} &= (\lambda_2, \lambda_1^*, g_2, g_1^*) & \phi_{12} &= (\lambda_2, \lambda_1^*, g_2, g_2^*) \\
 \phi_{13} &= (\lambda_2, \lambda_2^*, g_1, g_1^*) & \phi_{14} &= (\lambda_2, \lambda_2^*, g_1, g_2^*) \\
 \phi_{15} &= (\lambda_2, \lambda_2^*, g_2, g_1^*) & \phi_{16} &= (\lambda_2, \lambda_2^*, g_2, g_2^*)
 \end{aligned}$$

Let P denote the transition matrix, which is 16 by 16. the probability of moving from state i to state j is $p_{ij} = P[\bar{\phi}_{t+1} = \phi_j | \bar{\phi}_t = \phi_i]$.

Bond prices depend partly on the state of the world. Let these state contingent prices can therefore be written as

$$b_t = \beta E_t \left\{ (g_{t+1}^o g_{t+1}^{*(1-\theta)})^{(1-\tau)} \right\} / \lambda_{t+1} \quad (4.1)$$

$$b_i^* = \beta E_t \left\{ (g_{t+1}^\theta g_{t+1}^{*(1-\theta)})^{1-\gamma} \right\} / \lambda_{t+1}^* \quad (4.2)$$

$$\text{Let } G = \left[(g^\theta g^{*(1-\theta)})^{1-\gamma} \right] / \lambda \quad (4.3)$$

$$G^* = \left[(g^\theta g^{*(1-\theta)})^{1-\gamma} \right] / \lambda^* \quad (4.4)$$

and $d = \frac{\lambda}{\lambda^*}$ represent the gross rate of depreciation of the domestic

currency. From chapter 3 above, the spot exchange rate is given by

$$S_t = \frac{1-\theta}{\theta} \frac{M_t}{N_t}. \quad (4.5)$$

Therefore when the state of the world is θ_k , the spot exchange rate is

$$S_t = \frac{1-\theta}{\theta} d_k. \quad (4.6)$$

The domestic bond price in this case is

$$b_k = \beta \sum_{i=1}^{16} p_{k,i} G_i \quad (4.7)$$

while the foreign bond price is given by

$$b_k^* = \beta \sum_{i=1}^{16} p_{k,i} G_i^* \quad (4.8)$$

The expected gross change in the nominal exchange rate is

$$\sum_{i=1}^{16} p_{k,i} d_i \quad (4.9).$$

The risk premium, contingent upon state of the world k is

$$rp_k = \sum_{i=1}^{16} p_{k,i} d_i - \frac{\left(\sum_{i=1}^{16} p_{k,i} G_i^* \right)}{\left(\sum_{i=1}^{16} p_{k,i} G_i \right)} \quad (4.10)$$

The range of possible values of G, g^*, d to be computed in the calibration exercise is given below:

$$G_1 = \left[(g_1^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad G_1^\circ = \left[(g_1^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad d_1 = \lambda_1 / \lambda_1^\circ$$

$$G_2 = \left[(g_1^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad G_2^\circ = \left[(g_1^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad d_2 = \lambda_1 / \lambda_1^\circ$$

$$G_3 = \left[(g_2^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad G_3^\circ = \left[(g_2^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad d_3 = \lambda_1 / \lambda_1^\circ$$

$$G_4 = \left[(g_2^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad G_4^\circ = \left[(g_2^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad d_4 = \lambda_1 / \lambda_1^\circ$$

$$G_5 = \left[(g_1^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad G_5^\circ = \left[(g_1^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad d_5 = \lambda_1 / \lambda_2^\circ$$

$$G_6 = \left[(g_1^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad G_6^\circ = \left[(g_1^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad d_6 = \lambda_1 / \lambda_2^\circ$$

$$G_7 = \left[(g_2^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad G_7^\circ = \left[(g_2^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad d_7 = \lambda_1 / \lambda_2^\circ$$

$$G_8 = \left[(g_2^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad G_8^\circ = \left[(g_2^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_1 \quad d_8 = \lambda_1 / \lambda_2^\circ$$

$$G_9 = \left[(g_1^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad G_9^\circ = \left[(g_1^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad d_9 = \lambda_2 / \lambda_1^\circ$$

$$G_{10} = \left[(g_1^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad G_{10}^\circ = \left[(g_1^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad d_{10} = \lambda_2 / \lambda_1^\circ$$

$$G_{11} = \left[(g_2^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad G_{11}^\circ = \left[(g_2^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad d_{11} = \lambda_2 / \lambda_1^\circ$$

$$G_{12} = \left[(g_2^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad G_{12}^\circ = \left[(g_2^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad d_{12} = \lambda_2 / \lambda_1^\circ$$

$$G_{13} = \left[(g_1^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad G_{13}^\circ = \left[(g_1^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad d_{13} = \lambda_2 / \lambda_2^\circ$$

$$G_{14} = \left[(g_1^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad G_{14}^\circ = \left[(g_1^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad d_{14} = \lambda_2 / \lambda_2^\circ$$

$$G_{15} = \left[(g_2^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad G_{15}^\circ = \left[(g_2^\theta g_1^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad d_{15} = \lambda_2 / \lambda_2^\circ$$

$$G_{16} = \left[(g_2^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad G_{16}^\circ = \left[(g_2^\theta g_2^{\alpha(1-\theta)})^{1-r} \right] / \lambda_2 \quad d_{16} = \lambda_2 / \lambda_2^\circ$$

4.4 Estimating the Transition Matrix.

The relevant transition probabilities can be estimated by GMM or SMM. However, as pointed out in Mark(2000) this does not typically work well, especially in cases like ours where the sample is relatively small. So we follow here the method of counting relative frequencies of transition events. We consider the average growth of a variable calculated from the sample data.

A variable is then characterized as being high-growth when it is above this sample mean. When its growth rate is below its sample mean it is characterized as being low-growth. The high growth rate in the model then is simply the average of high growth rates identified in the sample. In the same way we assign low-growth rates on the basis of the average of rates found to be less than the sample mean.

By proceeding in this fashion, we assign high growth states $\lambda_1, \lambda_1^*, g_1, g_1^*$ to the mean from the sample and similarly $\lambda_2, \lambda_2^*, g_2, g_2^*$ to the low growth states sample mean. Doing this with per capita consumption money data for Canada and the US, with the US as the home country, generates the following estimates:

Average US Consumption Growth good state	: 1.009
Average US Consumption Growth bad state	: 0.998
Average Canadian Consumption Growth good state	: 1.011
Average Canadian Consumption Growth bad state	: 0.997
Average US Money Growth good state	: 1.010
Average US Money Growth bad state	: 0.990
Average Canadian Money Growth good state	: 1.012
Average Canadian Money Growth bad state	: 0.987

Having determined these growth rates the data are classified into ϕ states depending on whether or not they lie below or above the mean. The relative frequency of transitions from state ϕ_j to state ϕ_k in the sample data gives the transition probabilities p_{jk} . The estimated probabilities for the consumption and money growth model are :

Model Simulation

To simulate the model we assume the weight of each country to be 50 percent setting $\theta = 1/2$. The discount factor, β , is set to 0.99 while the risk aversion parameter, γ , is set to a value of 10. Using the initial vector drawn from the initial probability vector, v , a sequence of T realizations is generated for the risk premium, the forward premium and the gross change in the exchange rate. The rule for determining the initial state, given u_t an identically, independently distributed uniform random variable on domain $[0,1]$, is

$$\phi_1, \text{ if } u_t < v_1$$

$$\phi_2, \text{ if } v_1 < u_t < \sum_{j=1}^2 v_j$$

.

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$$\phi_{16}, \text{ if } \sum_{j=1}^{15} v_j < u_t < 1$$

To determine states for the other observations that follow,

$$\phi_1, if, u_t < p_{k1}$$

$$\phi_2, if, p_{k1} < u_t < \sum_{j=1}^2 p_{kj}$$

.

.

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.

$$\phi_{16}, if, \sum_{j=1}^{15} p_{kj} < u_t < 1.$$

4.5 Results for the Consumption based model.

The outcomes of the above exercise are presented in two ways, the first being graphical and the second reporting several estimated statistics. Using the approach spelt out in the foregoing section, we generate 97 values of the gross rate of depreciation, the forward premium and plot them on the same graph, Figure 4.1A. In addition, we simulate the same number of values for the predicted forward payoff and the realized payoffs which are plotted in Figure 4.1 B.

The first graph does not bring out very clearly the standard problem of finding an apparent negative correlation, which would imply that uncovered interest parity does not hold. Using data on Germany and US, with US as the

home country and the same scheme as presented here, Mark(2000) comes up with an even more pronounced reflection of this anomaly. We reproduce the diagram for reference. The second graph however vividly shows that the predicted risk premium is way too small to explain, let alone mimic, the data.

The second part of the results involves the regression of the gross depreciation rate on the risk premium. This is done using 10,000 data points generated from the model. Using the same data, we compute the standard errors of the gross rate of depreciation, the forward premium and the gross rate of depreciation to capture the volatility of the simulated series. These we compare with the same measure of volatility for the sample data. The table below reports the results for both the model and actual data for comparison.

Table 4.1

Measured and Implied Moments: US - Canada Data

	Slope	Volatility			Autocorrelation		
		S_{t+1}/S_t	F_t/S_t	$(F_t - S_{t+1})/S_t$	S_{t+1}/S_t	F_t/S_t	$(F_t - S_{t+1})/S_t$
Data	-0.0019	0.0214	0.0202	0.00782	0.0397	0.1224	0.0225
Model	-1.0143	0.0137	0.0289	0.00604	0.0178	0.0855	0.0628

The model captures the forward rate puzzle in that the slope coefficient bears a negative sign and is more or less equal to one. It is however much less than unity in the data, a magnitude that is a standard finding. With respect to the measures of volatility, the model does not match the volatility in the data. The same deviation from empirical moments is exhibited by the results with reference to autocorrelation. There is much less persistence in the gross rate of depreciation and the forward premium from the simulated data than in the sample, while the observed realized profits display less persistence than implied profits.

4.6 Results For the Earnings Based Model

In the previous section we used consumption to compute the probability transition matrix. This is partly because in the Lucas model consumption and output are the same and can serve the same purpose. In the same model prices depend on utility, which in turn depends on consumption. One can therefore use consumption data to estimate the transition matrix. In the same vein, since consumption risk is related to future productivity which in turn depends on investment, we can use earnings which investors consider

in gauging what the future portends. The corresponding growth states for the earnings based case estimated in the same manner as before are as follows:

Average US Earnings Growth good state: 1.056

Average US Earnings Growth bad state: 0.974

Average Canada Earnings Growth good state: 1.122

Average Canada Earnings Growth bad state: 0.885

Average US Money Growth good state: 1.010

Average US Money Growth bad state: 0.990

Average Canada Money Growth good state: 1.012

Average Canada Money Growth bad state: 0.987

The transition probability matrix is as given below.

We simulate 10,000 observations once again and estimate the slope coefficient and compute the same measures as before for volatility and persistence for depreciation, the forward premium and speculative profits. The results are as follows. The graph shows in panel B that there is still very little that can be explained in the data by the model as presented through event counting and calibration. There is though an improvement relative to the consumption based model.

Table 4.2

Measured and Implied Moments: US - Canada Data

	Slope	Volatility			Autocorrelation		
		S_{t+1}/S_t	F_t/S_t	$(F_t - S_{t+1})/S_t$	S_{t+1}/S_t	F_t/S_t	$(F_t - S_{t+1})/S_t$
Data	-0.0019	0.0214	0.0202	0.00782	0.0397	0.1224	0.0225
Model	-1.0299	0.0142	0.0086	0.03319	0.3993	0.0676	0.0599

The numerical results again confirm the forward rate puzzle in that the slope coefficient is negative. However there is no close match between the

model's measures of volatility and persistence and those of the sample under consideration.

In conclusion we note that although the model is able to predict that the forward rate is a biased predictor of the future spot rate because of the existence of a risk premium, it is unable to generate a large enough magnitude of such a premium to provide a substantive explanation of the data.

CHAPTER 5

SUMMARY AND CONCLUSION

In this concluding chapter, we begin by summarizing the previous steps of the study, restate the rationale for expecting earnings growth on the one hand and dividend growth on the other hand to impact the empirical performance of the Lucas asset pricing model in foreign exchange markets, interpret the main findings obtained and lastly comment on other avenues that might be considered to improve on what we found.

In Chapter one, the equity premium puzzle of Mehra and Prescott was introduced as connoting the failure of asset pricing models to account for the historical observed premium averaging 6.18 percent in US data over the period 1889 to 1978. Also in chapter one, the forward rate puzzle was introduced. In this case it was noted that to date there is a persistent failure of forward exchange rates to forecast future spot rates. Moreover, regressions of the spot rate on the forward premium provide overwhelming evidence that predictions of changes in the spot rate by the forward rate bear the wrong sign. In this market as the in the stock market, an implausible magnitude of risk aversion is required in order to explain the data in the empirical counterpart of the theory.

Then the basic models of asset price determination developed by Hicks(1946), Makorwitz (1959), Tobin (1958), Sharpe (1964), Lintner (1965) and Mossin (1969) with their extensions to the Capital Asset Pricing Model (CAPM) were outlined, highlighting CAPM's main attributes, namely mean variance efficiency and the β coefficient. The main developments leading to the consumption based variant of this model were outlined and ,by using risk and marginal utility of consumption, the principal workings of such a model were explained.

Having thus laid out the basic models, their prediction of the risk premium for plausible parameter values was restated as being less than adequate based on the Mehra - Prescott(1995) study. In addition , following Kotcherlakota(1996), the equity premium puzzle was reviewed, indicating that efforts to resolve it the have been centered on three main approaches. These involve changing either the specification of the representative consumer's preferences that are adopted or assuming a presence of market frictions of some kind or altering the assumption on completeness of markets or a combination of one or other of these scenarios. The corresponding efforts to solve the related puzzle in currency markets were also outlined, these also being mainly based on three approaches. The peso problem approach, the

noise traders approach and the time varying risk premium approach. It is the third line of inquiry that was pursued in the study.

With the foregoing background laid out, the main focus and objectives of this study were stated. That is, in considering currency markets, the model of asset pricing whereby dividend and earnings based discount were to be investigated. In particular, it was deemed to be of interest to compare and contrast the performance of such models with one where the discount factor is based on aggregate consumption growth. In addition to this estimation by GMM, we proposed to use the calibration methodology to evaluate the model.

In chapter two, a more detailed survey of proposed resolutions of interest motivated mainly by invoking unconventional preference specifications was provided.

Given that the poor performance of the consumption CAPM is partly attributable to the rather excessive smoothness of aggregate consumption series relative to the typically more pronounced volatility of asset return series, a broader class of consumer preferences recommends itself in confronting the puzzle. Thus studies conducted in this spirit and their results were outlined noting among others, the studies by Constantinides (1990), Mansoorian (1993, 1995), Boldrin et al (1995), Backus, Telmer and Gregory

(1993) and Kotcherlakota (1996), all of which introduce habit persistence in one form or the other.

Secondly, the generalized expected utility formulation of Epstein and Zin (1989,1991) was also sketched. It was observed that with regard to both habit persistence and generalized expected utility, satisfactory results have not been obtained for a plausible range of the coefficient of relative risk aversion despite impressive improvements in accounting for the average equity premium.

Thirdly, the catching up with the Joneses preference specification of Duesenberry (1949), Abel (1990) , John Nason (1988), Gali (1994) and Hansen and Cochrane (1995) was presented. Here it was noted that some friction may arise in the consumption series because consumers do not only guard against a fall in their personal consumption but also a decline in relation to societal per capita consumption. As explained above, relatively good results are also obtained in this case but without a fully fledged resolution of the equity premium puzzle.

Fourthly, Multiplicative separable preferences studied in continuous time by Chang Mo (1990) were considered. The solution presented in this case was a closed form solution involving an application of Ito's lemma. But this particular approach was mentioned mainly for the record since it by its

very nature was not amenable to empirical testing like the other studies considered alongside it.

From these changes in preference specifications was also motivated one of the efforts to address the forward rate puzzle. This involved investigating the potential of habit persistence to improve on the extent to which the model can explain the realized forward profits and observed risk premium. The Peso problem and the Noise Traders Varying Risk premium approaches

In chapter three, the representative agent models applied to linkages between expected future spot foreign exchange rates and forward exchange rates was outlined. Starting with a presentation of some time series properties of the data, specifically the autocorrelation and partial autocorrelation functions, the discussion presented the models of Lucas (1978, 1982) which ultimately consist in projecting financial theory in a general equilibrium framework. Equilibrium prices were then derived with a view to facilitating empirical estimation through tests of overidentifying restrictions pertaining to asset pricing model. The generalized method of moments estimator, was then outlined and subsequently used to estimate the standard model with time separable preferences in the same fashion as Mark (1985,2000). Firstly ,the sample period covered by Mark(2000) was used to try and confirm his

results. Then estimates for the current sample were provided. In both cases, three lag specifications were used, 0, 1 and 2 lags of instruments. The instruments, the selection rationale of which was also underscored, were lagged currency speculation returns and lagged forward premiums of the respective currencies i.e. the Pound Sterling, Canadian Dollar, French Frank and the German Mark.

Overall, the results do not reject the model, but the estimates of the risk aversion parameter were on the higher side considering what is typically plausible to most analysts. Then the same model was estimated using the dividend growth discount factor in place of the consumption discount factor. The results registered an improvement relative to those of the consumption growth model but did not eliminate the anomaly. The next model run in GMM was based on earnings in defining the discount factor. In terms of empirical performance, this model was better than the previous two. The estimates of the risk aversion coefficient were much more plausible even although measured with some degree of imprecision.

In chapter four, the model was calibrated to US and Canadian data. We used event counting to estimate transition probabilities and considered the model where real exchange rates are driven by relative consumption growth and contrasted this with a model based on relative earnings. In so doing, it

was assumed that the coefficient of relative risk aversion is 10 while the time preference parameter was set at 0.99. The model was able to capture the forward rate puzzle and predict the existence of a time varying risk premium. The size of the premium however was inadequate in explaining the premium observed in the data. This did not change substantially when the earnings ratios were used in the calibration exercise.

Interpretation of Results.

At the outset, the sample autocorrelations and partial autocorrelations reported in tables 3- were obtained, given the assumption of covariance stationarity in testing procedures. The first estimates for the period within the sample of Mark(1985), using time separable preferences as specified in that study, we found that although there is no overwhelming evidence against the theory, two notable aspects arise. First, the results obtained by Mark were not reproduced. Secondly, the results we obtain include larger and thus less plausible values of the risk aversion parameter, in

some of the instances having bigger standard errors . Extending the estimation of the time separable preferences model to our full sample did not change the results in any substantial way. The same model from Mark(2000) was also run using the same discount factors using quarterly data . The results in this case are much closer to his results.

Having noted these issues however, it has to be recognized that studies in this area have repeatedly turned up poor performance by the theory. A similar experience has been shared with respect to exchange rate studies in the strand of research that does not retain the assumption of market efficiency

Conclusion.

The main phenomena that we tried to exploit in this study to currency speculative returns were the replacement of the consumption based discount factor with discount factors based on dividend growth and earnings growth. As sated earlier, while dividend discount factors have been used in the literature on stcok markets in this area, they had not

been applied to foreign exchange markets. Also earnings have not been applied to both stock markets and foreign exchange markets.

The standard explanation of excess returns accruing to a risky asset above the riskless interest rate in finance theory is that such returns are the product of the amount of risk and the price of risk. In the consumption based strand of models, the amount of stock market risk for example as measured by covariance of consumption growth with excess returns would be multiplied by the price of risk as measured by the coefficient of relative risk aversion. What the excessive smoothness of consumption does is to render the covariance component so low that an unreasonably high risk aversion parameter is required in order to explain expected excess returns. The introduction of other discount factors aims to introduce some friction from the dividend and earnings growth side to explain excess returns. In the Lucas model, consumption dividends and earnings are the same. Accordingly we estimated and calibrated the model using consumption, dividends and earnings. We do find substantial gains in the GMM approach by introducing these measures of the stochastic discount factor. In the calibration methodology, the gains are less apparent.

By using different discount factors, it has been possible to throw more light on the difficult issue of explaining the origination of the risk that drives expected returns not only analytically and thus from a theoretical perspective, but also concretely with an appropriate measurement and calibration procedure. Elsewhere in macroeconomics, this approach has also brought business cycle models into some relief with regard to their ability to explain patterns of equity premia.

The preferred way to resolve the model should be to do it in such a way as to avoid a counterfactual while improving on the performance of the model. This would have the advantage of not stepping out of the original framework assumed in discovering the puzzle by say introducing incomplete markets or market frictions as some have done and yet accounting for forward premia.

Appendix To Chapter 3

The results from Tables 3.15 to 3.20 are for the Deutschemark, Pound Sterling and Japanese Yen versus US dollar exchange rates. Results using three currencies at a time for other countries are presented below. They include the Italian Lira, Australian Dollar, French Franc, Belgian Franc, Swiss Franc, Swedish Krona, Canadian Dollar, Danish Krone, Austrian Schilling, Norwegian Kroner and Spanish Peseta. The exchange rates are the amount of US dollars per unit of respective foreign currencies.

Results for the Italian Lira, Australian Dollar and French Franc US dollar exchange rates.

Table 3.21
Consumption Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	17.4930	58.723	8.95005	0.11107
1	67.3043	33.448	6.67051	0.24632
2	64.3788	25.472	5.92176	0.31391
3	67.7814	17.539	4.71570	0.45155

Table 3.22
Consumption Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	49.5696	61.086	11.20798	0.04741
1	48.5013	43.827	10.64357	0.05892
2	36.8111	-	-	-
3	28.1579	44.625	11.92467	0.03583

Table 3.23
Dividend Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	26.4293	38.505	7.45288	0.18908
1	23.6611	15.526	8.03971	0.15406
2	30.7880	23.024	7.70135	0.17348
3	24.5212	11.032	7.95934	0.15849

Table 3.24
Dividend Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	32.8253	33.055	9.07464	0.10612
1	28.8310	19.217	9.52918	0.08973
2	33.2745	37.504	9.27384	0.09863
3	28.3574	13.650	9.93629	0.07706

Table 3.25
Earnings Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	6.5431	13.540	4.75325	0.44673
1	11.5620	8.970	5.26117	0.38484
2	12.5233	7.419	5.36523	0.37295
3	9.8490	15.197	6.03776	0.30256

Table 3.26
Earnings Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	14.8419	5.896	7.12997	0.21115
1	16.0687	6.611	9.09365	0.10539
2	11.5401	6.102	7.13940	0.21048
3	13.3900	4.880	8.90207	0.11303

Results for the Swiss Franc, Belgian Franc and Swedish Krona US dollar exchange rates.

Table 3.27
Consumption Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	24.4155	26.051	10.87632	0.05389
1	27.5173	28.351	9.38444	0.09468
2	29.0052	29.910	9.94411	0.07683
3	21.8891	26.620	10.52888	0.06156

Table 3.28
Consumption Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	25.0277	27.396	13.33540	0.02043
1	42.1124	29.797	12.55915	0.02788
2	47.0643	57.529	12.31728	0.03069
3	41.9325	27.026	12.80236	0.02530

Table 3.29
Dividend Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	37.2550	12.925	7.52029	0.18473
1	31.0425	10.421	7.99962	0.15626
2	35.6098	11.400	7.28554	0.20026
3	36.0605	12.539	7.74038	0.17114

Table 3.30
Dividend Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	31.1820	9.102	10.54135	0.06127
1	30.2964	8.942	10.72486	0.05712
2	33.3231	21.537	10.15350	0.07100
3	32.3275	38.635	10.25405	0.06835

Table 3.31
Earnings Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	16.7682	8.689	10.07682	0.07309
1	15.1557	8.631	8.38325	0.13634
2	15.4121	7.498	7.85061	0.16466
3	18.7266	6.822	10.41585	0.06427

Table 3.32
Earnings Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	13.8923	10.813	8.70661	0.12135
1	10.7254	49.339	8.44047	0.13357
2	12.8142	7.806	9.01191	0.10859
3	15.4739	7.462	10.62906	0.05925

**Results for the Canadian Dollar, Danish Krone and Austrian Schilling
 US dollar exchange rates.**

Table 3.33
Consumption Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	37.1274	33.395	11.11390	0.04917
1	48.3238	33.356	10.42706	0.06400
2	40.3522	36.558	10.86994	0.05402
3	37.3787	27.796	10.75539	0.05645

Table 3.34**Consumption Growth Model: Speculative Profits as Instruments.**

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	37.0313	39.158	13.33202	0.02046
1	36.6176	82.412	13.13516	0.02214
2	37.3876	52.949	13.33392	0.02044
3	44.3180	31.364	12.85594	0.02477

Table 3.35**Dividend Growth Model: Forward Premia as Instruments.**

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	33.3828	24.362	10.1641	0.07072
1	35.5423	21.542	9.8300	0.0802
2	34.7478	22.162	9.51701	0.09014
3	34.9336	18.822	9.93812	0.07701

Table 3.36**Dividend Growth Model: Speculative Profits as Instruments.**

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	33.0886	35.373	10.99857	0.05141
1	33.1788	17.498	10.94231	0.05254
2	34.2792	15.731	10.80338	0.05542
3	35.5337	21.605	10.49541	0.06236

Table 3.37
Earnings Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	17.3244	7.783	11.17529	0.04801
1	13.9388	11.016	9.55076	0.08901
2	15.5238	10.615	11.02347	0.05092
3	17.4867	7.114	10.73391	0.05692

Table 3.38
Earnings Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	17.4554	3.704	11.24366	0.04676
1	14.5427	9.727	10.45385	0.06335
2	16.0225	5.180	9.77883	0.08175
3	15.0793	8.012	10.73391	0.05685

**Results for the Norwegian Kroner, Spanish Pesetta and German Mark
US dollar exchange rates.**

Table 3.39

Consumption Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	47.5573	31.528	10.32792	0.06646
1	44.4284	32.882	8.90297	0.11300
2	52.2521	26.196	8.90948	0.11273
3	52.1216	26.696	8.69229	0.12199

Table 3.40

Consumption Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	40.4516	23.831	12.66053	0.02678
1	49.0487	39.117	12.14236	0.03289
2	33.9042	83.118	12.79053	0.02542
3	40.5314	--	--	--

Table 3.41

Dividend Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	32.8890	21.985	9.98100	0.07578
1	30.4031	24.447	9.38246	0.09475
2	27.4141	19.044	9.77676	0.08181
3	27.1810	16.016	9.34183	0.09618

Table 3.42
Dividend Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	35.0463	46.727	10.02127	0.07463
1	30.3752	16.896	10.84727	0.05449
2	33.7901	12.409	10.30164	0.06713
3	33.3837	21.056	10.61411	0.05959

Table 3.43
Earnings Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
1	14.2870	12.324	8.19692	0.14571
2	11.7514	19.765	7.42080	0.19118
3	15.6679	7.795	7.38704	0.19341
4	11.7496	10.838	6.56658	0.25492

Table 3.44
Earnings Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	8.46890 *	30.218	6.869600	0.23052
1	13.9875	14.438	11.00224	0.05134
2	15.3087	6.4540	10.32544	0.06652
3	13.1666	16.513	10.74885	0.05660

There is a subtle trap in the implementation of the GMM procedure that partly explains the typically large standard errors obtained in estimation exercises like the one we have carried out above. The other potential source of large standard errors is the presence of small sample bias, as noted in Mark(1985). This problem has not been addressed in our procedures here. The exception to the common reporting of large standard errors is to be found in Hansen and Singleton(1982,1983), Dunn and Singleton(1983) and Rotenberg(1984). Both their estimates of the relative risk aversion parameter and the corresponding standard errors are not large compared to Mark(1985,2001) for instance.

In implementation, the GMM standard procedure minimizes $g_{\tau}(b)' W g_{\tau}(b)$, whilst the first order condition $\partial g_{\tau} / \partial b W g_{\tau}$ is satisfied. The minimization is achieved however by setting $D = \partial g_{\tau} / \partial b = 0$ rather than setting $g_{\tau} = 0$. The matrix D turns up as a null matrix. Having confirmed this in our procedure, we corrected for it using the method suggested by Rui Ribero(See John Cochrane (2001). On correcting for this, the following results were registered. They show overall that the earnings based model yields smaller values of gamma, the relative risk aversion parameter, and overall the model is supported more strongly for most of the lags.

Results for the Italian Lira, Australian Dollar and French Franc versus US dollar exchange rates.

Table 3.45
Consumption Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	13.1885	0.447	5.00954	0.41472
1	15.8679	0.595	2.82534	0.72689
2	19.9076	0.478	4.38218	0.49580
3	16.1751	0.482	4.30779	0.50600

Table 3.46
Consumption Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	28.1958	0.466	4.61483	0.46467
1	38.3125	0.476	4.41087	
2	24.2261	0.380	6.92117	0.22657
3	31.4434	0.374	7.13616	0.21071

Table 3.47
Dividend Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	33.7224	0.769	1.68918	0.89026
1	34.7739	0.865	1.33568	0.93122
2	32.0775	0.886	1.27300	0.93769
3	25.6949	0.438	5.21855	0.38979

Table 3.48
Dividend Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	36.4030	0.616	2.63606	0.75588
1	32.1140	0.524	3.63631	0.60287
2	29.5294	0.621	2.59113	0.76271
3	25.8126	0.539	3.43981	0.63251

Table 3.49
Earnings Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	15.7450	0.508	3.87563	0.56746
1	16.8979	0.563	3.15598	0.67595
2	16.4425	0.596	2.81241	0.72888
3	8.1197	1.125	0.79021	0.97765

Table 3.50
Earnings Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	18.4321	0.384	6.78433	0.23718
1	15.7466	0.558	3.21393	0.66704
2	15.2086	0.513	3.80205	0.57825
3	12.1413	0.636	2.47143	0.78079

Results for the Swiss Franc, Belgian Franc and Swedish Krona US dollar exchange rates.

Table 3.51
Consumption Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	21.9313	0.350	8.15260	0.14802
1	19.8764	0.351	8.09427	0.15112
2	22.5598	0.371	7.27981	0.20065
3	27.1661	0.355	7.91267	0.16111

Table 3.52
Consumption Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	24.2170	0.181	5.80704	0.32545
1	37.7271	0.341	8.15512	0.14789
2	40.5807	0.453	7.42217	0.19109
3	35.1748	0.281	5.97988	0.30818

Table 3.53
Dividend Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	34.5474	0.551	3.29292	0.65493
1	31.7642	0.476	4.41449	0.49141
2	29.9354	0.467	4.57623	0.46975
3	34.6009	0.551	3.29889	0.65041

Table 3.54
Dividend Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	33.7198	0.234	3.63585	0.60294
1	33.1842	0.353	5.50239	0.35768
2	33.2824	0.581	4.71884	0.45115
3	30.4716	0.686	4.76113	0.44572

Table 3.55
Earnings Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	17.3034	0.567	3.11316	0.68255
1	18.3321	0.381	6.88612	0.22925
2	15.1281	0.532	3.53070	0.61875
3	18.1421	0.485	4.25355	0.51352

Table 3.56
Earnings Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	14.3604	0.588	2.89624	0.71598
1	11.4159	0.725	1.90006	0.86279
2	12.3233	0.969	1.06438	0.95721
3	13.9752	0.508	3.86869	0.56847

Results for the Canadian Dollar, Danish Krone and Austrian Schilling versus US dollar exchange rates.

Table 3.57
Consumption Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	27.9359	0.423	5.58450	0.34877
1	30.3306	0.368	7.39579	0.19283
2	24.6030	0.394	6.45464	0.26446
3	30.5264	0.378	6.99848	0.22075

Table 3.58
Consumption Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	-	-	-	-
1	33.4030	0.395	6.42262	0.26724
2	29.1513	0.375	7.11276	0.21239
3	31.0587	0.352	8.06048	0.15293

Table 3.59
Dividend Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	31.4076	0.555	3.24877	0.66169
1	34.3500	0.514	3.78920	0.58014
2	34.1296	0.582	2.95222	0.70735
3	33.3731	0.462	4.68234	0.45586

Table 3.60
Dividend Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	32.5276	0.443	5.09344	0.40458
1	32.4942	0.414	5.83248	0.32286
2	33.3208	0.404	6.12610	0.29414
3	32.7899	0.463	4.66002	0.45876

Table 3.61
Earnings Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	16.4678	-	-	-
1	15.4797	0.623	2.57268	0.76551
2	16.6841	0.743	2.72510	0.74228
3	16.1807	0.445	5.04429	0.41050

Table 3.62
Earnings Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	18.2231	0.667	2.24541	0.81425
1	14.0155	0.743	1.81140	0.87458
2	17.9056	0.386	6.70825	0.24326
3	17.5854	0.491	4.15186	0.52776

Results for the Norwegian Kroner, Spanish Pesetta and German Mark US dollar exchange rates.

Table 3.63
Consumption Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	31.6307	0.383	6.82899	0.23367
1	30.7060	0.369	7.32776	0.19738
2	31.5243	0.373	7.19964	0.20621
3	35.5350	0.437	5.22773	0.38872

Table 3.64**Consumption Growth Model: Speculative Profits as Instruments.**

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	25.4858	0.243	9.89557	0.07825
1	31.2551	0.563	4.98505	0.41771
2	27.9039	0.163	8.79386	0.11757
3	39.1674	0.366	6.82909	0.23366

Table 3.65**Dividend Growth Model: Forward Premia as Instruments.**

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	32.3977	0.630	2.52345	0.77296
1	28.6019	0.537	3.46232	0.62910
2	28.8988	0.636	2.47051	0.78093
3	28.4181	0.504	3.93940	0.55817

Table 3.66**Dividend Growth Model: Speculative Profits as Instruments.**

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	33.8646	-	-	-
1	32.4512	0.413	5.32312	0.37773
2	31.4446	0.343	6.10636	0.29601
3	30.3850	0.505	4.79215	0.44177

Table 3.67
Earnings Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
1	17.5874	0.500	4.00711	0.54839
2	16.2531	0.657	2.31676	0.80380
3	11.7464	1.301	0.59070	0.98842
4	13.2318	0.526	3.75040	0.58588

Table 3.68
Earnings Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	10.4889	0.698	1.06920	0.95679
1	13.4945	0.508	3.00021	0.69995
2	17.2638	0.178	2.95849	0.70639
3	13.3841	0.579	4.48299	0.48216

Results for the Pound Stirling, Deustchemark and the Yen versus US dollar exchange rates.

Table 3.69
Consumption Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	23.2118 *	0.878	1.29623	0.93532
1	17.9396	0.628	2.53559	0.77113
2	18.3392	0.680	2.16133	0.82640
3	23.3139	1.023	0.95487	0.96612

Table 3.70
Consumption Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	28.9737	0.890	6.80591	0.23548
1	34.3676	0.406	4.38093	0.49597
2	19.1160	0.435	5.59435	0.34771
3	32.6385	0.270	8.12766	0.14974

Table 3.71
Dividend Growth Model: Forward Premia as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	30.7593	1.077	0.86136	0.97296
1	23.2241	0.700	2.03841	0.84380
2	23.8658	0.563	3.15017	0.67685
3	24.8509	0.579	2.97970	0.70312

Table 3.72
Dividend Growth Model: Speculative Profits as Instruments.

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	38.0454	1.017	1.17951	0.94683
1	32.0480	0.366	6.02263	0.30402
2	28.3207	0.386	6.25833	0.28189
3	24.8509	0.606	2.15284	0.82762

Table 3.73**Earnings Growth Model: Forward Premia as Instruments.**

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	-	-	-	-
1	4.4609	2.416	0.21715	0.99892
2	7.3622	1.440	0.48229	0.99276
3	8.1609	0.672	2.21313	0.81894

Table 3.74**Earnings Growth Model: Speculative Profits as Instruments.**

Lags	γ	Standard Error	J-Statistic	Marginal Significance
0	4.7314	2.997	0.28502	0.99792
1	7.9398	0.705	0.78121	0.97822
2	7.7637	0.348	1.69985	0.88892
3	10.6585	0.592	2.89263	0.71653

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