Transmission Consideration-Based Electricity Rates Using Optimal Power Flows

By

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P_{gi}, Q_{gi} = active and reactive power generation at bus i,

P_{di}, Q_{di} = active and reactive power demand at bus i,

V_i = voltage at bus i,

Y_{ij} the element ij of admittance matrix of the transmission network,

δ_i = voltage angle at bus i,

θ_{ij} = phase angle of Y_{ij},

N = set of buses in the system,

NG = set of all generator buses,

MC_{pi} = Lagrange multiplier for the active power equation at bus i,

MC_{qi} = Lagrange multiplier for the reactive power equation at bus i,

WC_{pi} = marginal wheeling cost for active power,

WC_{qi} = marginal wheeling cost for reactive power,

C_T = Total operating cost of the system,

C_i = cost function of generating plant at bus i,

k = the assigned price to each unit of reactive and active power,

R_G is a set of all generating buses,

α_i = the fixed fuel cost at generator i,

β_i = the variable fuel cost in proportion to active power at generator bus i,

γ_i = the variable fuel cost in proportion to second order term of active power at bus i,

$kP_i(\gamma_i, \delta_i)$ *and* $kQ_i(\gamma_i, \delta_i)$ are the real and reactive transmission operating costs respectively,

$P_{Gi,\min}$ and $P_{Gi,\max}$ are the minimum and maximum active power output generated at bus i,

$Q_{Gi,\min}$ and $Q_{Gi,\max}$ are the minimum and maximum reactive power output generated at bus i,

X_{ij} = the reactance between line i and j,

$P_{ij}(t)$ = the power flow between bus i and j at time t,

$P_{ij,\min}$ and $P_{ij,\max}$ are minimum and maximum active power flow respectively,

$V_{i,\min}$ and $V_{i,\max}$ are the minimum and maximum voltage levels respectively,

f_1 is the production cost rate, \$/hour at bus 1,

MW_1 is MW injection at bus 1,

f_2 is production cost rate, \$/hour at bus 2,

MW_2 is MW injection at bus 2,

$\frac{\partial f_1}{\partial MW_1}$ and $\frac{\partial f_2}{\partial MW_2}$ represent the spot prices at bus 1 and bus 2 respectively,

$\lambda_{i,\min}$ and $\lambda_{i,\max}$ = Lagrange multiplier for minimum and maximum active power generation limit at bus i,

$\mu_{i,\min}$ and $\mu_{i,\max}$ = Lagrange multiplier for minimum and maximum reactive power generation limit at bus i,

$\eta_{i,\min}$ and $\eta_{i,\max}$ = Lagrange multiplier for minimum and maximum active power flow limit from bus i to bus j and

$\nu_{i,\min}$ and $\nu_{i,\max}$ = Lagrange multiplier for the minimum and maximum voltage level at bus i.

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Abstract

Electricity rate structures has been an area of intense research, undergoing dramatic changes as new and expanded service options are added. Real-time pricing of electricity is one of the options that maximize the economic efficiency of the power system. Under real-time pricing, the electricity prices to the customer follow as closely as technically practical, the real cost of electricity at the time that it is produced and supplied.

One thing which is overlooked at times in determining a rate policy in power supply, is the fact that the cost difference in delivering a killowatt-hour in the same territory is not just because of the cost of generating electricity. The cost difference, primarily lies in transmitting power from its origin to the place of demand. The objective of this work, is specifically to address the incorporation of transmission costs in addition to fuel costs using an Optimal Power Flow (OPF) algorithm in establishing electricity rate structures using real-time pricing under a regulated/deregulated (wheeling) environment. The cost of transmission and the methodology by which it is computed, is therefore, a high priority problem in this thesis. The transmission costs are determined by assigning the same trial price k to every unit of real and reactive power flows respectively in the network. The hourly real-time pricing is the base for the proposed algorithm and the analysis is over a time period of twenty-four hours. Unlike the algorithms which use DC models which capture only the real power rates, a particular feature of the proposed algorithm is that it captures both wheeling or non-wheeling rates of real and reactive power. The algorithm is implemented on standard IEEE test systems varying in size from a 5-bus system to a 57-bus system.

Chapter 1

INTRODUCTION

One of the demands for successful operation of power systems is the provision of service to utility customers with minimum interruption and at the lowest feasible cost. This presents a challenging task for the electric power system engineer, who strives to meet these goals at all times. The problem of providing low-cost electric energy is affected by such items as efficiencies of power-generating equipment, cost of installation, and fuel costs for thermal-electric plants. As the consumption of electric energy has grown, utilities have grown in size to meet the demand to a point where significant savings in operating costs can be achieved with even a fraction of a percent improvements in operating efficiency. This is an important concern for all electric utilities. In general, costs, security, and emissions are all areas of concern in power plant operation, and in practice the system is operated to effect a compromise between the frequently conflicting requirements [1]. However, the discussion in this thesis will be limited to economic considerations.

Economic dispatch started with two or more units committed to take load on a power system whose total capacities exceeded the generation required. The operator, faced the problem of how to divide the active load between the two units such that the total load is served and the total cost is minimized. Specifically, economic dispatch is a computational process whereby the total active generation required is allocated among the available generating units so that the constraints imposed are satisfied and the energy requirements are minimized.

In the late 1950's, the computational problem of power flow made its first appearance. A power flow formulation is characterized by inputs concerning the network under study and injected active power (P) and reactive (Q) powers at the buses. The objective of a load flow computation is to calculate voltages and angles at all buses of the network from which all other quantities can be calculated.

Optimization, was applied to power flow during the 1960's. The objective of an optimal power flow [2-7], is to find a power flow solution which optimizes a performance function such as fuel costs, or network losses, while at the same time enforcing the loading limits imposed by system equipment. A power flow solution is then obtained that is both feasible and achieves a minimum of the objective function. When total fuel cost is minimized, the optimal power flow results in an economic dispatch. The ability to solve the optimal power flow problem is extremely useful for the planning and design of future equipment additions to power systems and also in the pricing of electricity. In this thesis, it is assumed that the fuel cost is a second order polynomial of the power output which takes transmission costs into consideration.

1.2 Deregulation in the Power Industry

Many industries have felt the heavy-hand of deregulation over the past two decades; telephones, airlines, trucking and natural gas to name a few. The electric power industry is no exception. Some of the changes taking place are described by terms such as deregulation, privatization and freedom of market entry. The electric power industry has been moving rapidly away from a half century of regulated, cost-plus monopoly structure to competitive markets, both wholesale and retail [8].

With respect to deregulation, the electric utility industry has been gradually changing from a fully regulated industry to one of partial deregulation. One justification for deregulation, is to level the considerable differences in prices of a kwh (kilowatt-hour) in the same territory. Traditionally, the electric power industry has operated in a vertically regulated environment i.e. one utility is responsible for generation, transmission and distribution. Customers receive one bill from their utility company for their entire service. With the introduction of related regulations allowing competition in power production, deregulation of power utilities has become necessary. This means, instead of relying on regulation to achieve a fair and equitable price to the consumer for electric energy, reliance is placed more and more on market forces, through competition, to provide wholesale energy at the best price. The deregulated power market makes buyers and sellers negotiate their prices through the interconnected power network. Electricity has thus become a commodity. The price of electricity is then determined by the supply and demand in the market, and the fluctuation of the price regulates the supply and demand.

The key questions are, how to formulate rules that will encourage competition that will achieve market efficiency and at the same time; rules that can enforce secure power system operation. One necessary condition for competition to take place is that of power producers being able to reach consumers through the transmission network achieved through open transmission access schemes.

Different approaches have been followed to create open access conditions in interconnected power systems. One line of thinking has viewed third party use of the transmission systems as an isolated transaction between three parties [10], where two users (generally two producers or one producer and a consumer) want to use the transmission system of a third party. The requirement is to determine the cost of such a transaction. This concept is called "wheeling". By definition, wheeling is the transmission of active and reactive power from a seller to a buyer through a transmission network owned by a third party or more. In the UK, for example, the National Grid has become an entity separate from the power generators [11].

The concept of wheeling can act as a foundation for establishing the rates assigned to various services under deregulation. The wheeling rates are the prices charged by the wheeling utility for use of its network. It is anticipated that in future there will be a continuous growth of third-party network users [12]. The end result is that generation owned by the utility will be surpassed by the third-parties which ends up in the "fully deregulated" environment.

Another line of thinking which is gaining increasing popularity under open access conditions, looks at the transmission business as a separate service that provides the conditions for competition and must be treated separately and funded independently, irrespective of the ownership of the wires [10]. The transmission system, provides the capability to transmit power from generators to loads, offering adequate standards of security and quality of supply independent of contracts between producers and consumers. The system is to be paid for by all users (including the user actually owning the transmission network). The challenge here, is to allocate payments to those users. A transmission arrangement must be in place to provide for inputs of energy from generators into the transmission system and deliveries of energy to purchasing utilities to serve their loads. A lot of third-party users are brought into the transactions such as Independent Power Producers (IPPs), Non-Utility Generators (NUGs), Wheeling Transactions (WTs) etc. With this trend, it is crucial for electric utilities to confront the challenging problems that will be created by strong interactions among users of the transmission networks.

A good example of the second alternative is in the USA where the electric power markets are experiencing growth in new participants. Given the varied motivations of these participants, both regulators and incumbent utilities are considering a number of new institutional arrangements designed to ensure open, non-discriminatory, and reliable access [13]. The primary entity created to achieve this goal is the Independent System Operator (ISO). The function of the ISO in the restructured market, has been likened to an air traffic controller: a centralized coordinator for electricity security, emergency, and planning functions i.e. like a separate company that grants access to the transmission system. The ISO principles, recognize that while the ownership of transmission assets will remain with utilities, all operational, pricing, reliability, interconnected operations, and dispute resolution functions should be under the purview of the ISO. This means that the utilities relinquish a significant amount of control, security, and interconnected operations to the ISO. From market perspectives, this centralization of power limits traditional control. From an environmental perspectives, this allows a number of innovative environmental protection measures to be adopted which have the potential for promoting environmental mitigation strategies in an equal, non-discriminatory manner. In both alternatives, the challenge is to define a pricing scheme for transmission services, that provides coherent economic incentives for the business to efficiently operate and expand. Appropriate pricing of transmission services plays a very important role in the process of deregulation. A simple tariff for the transmission service based on average costs is not 'efficient' in this case, since it does not provide economic incentives for efficient operation of the transmission system [14].

Open transmission access will open a long list of issues related to management and operation of the transmission system, and on pricing transmission services. This is the beginning of a competitive marketplace in the electric industry and to achieve the possible benefits, it has to be accompanied by responsible power system operations so as to preserve system reliability. The operation of the transmission and generation networks will be more complex under open transmission access.

Many countries are considering or have already considered deregulating their electric power sectors to allow for competition. The final test of whether deregulation is successful depends on the perception of the population [15]. However, in this thesis, wheeling rates are established using real-time pricing of electricity.

1.3 Thesis Objectives

One of the objectives of the work reported in thesis is to model and study the effects of incorporating transmission costs in establishing real-time electricity rates for the consumers. The second objective is to compare the resulting rates with the outcome of classical optimization for minimum fuel cost at the generating plants. The studies are carried out in two phases i.e.

a. assuming that there are no privately owned generators, thus implying that there is one utility responsible for generation, transmission and distribution,
b. assuming that there is separate ownership of the generation and transmission i.e. under wheeling conditions.

In both cases, the modeling schemes are applied to modified OPF models on standard IEEE test systems. The simulations are implemented using a general purpose software known as MINOS [16] which is based on the augmented Lagrangian technique.

What is overlooked at times is that the cost difference in delivering a kilowatt-hour in the same territory is not just because of the costs of generating electricity. The cost difference primarily lies in transmitting power from its origin to the place of demand. Incorporating transmission costs in establishing electricity rates is, therefore one of the major contribution of this thesis advancing knowledge in the area of power systems engineering.

1.4 Thesis Outline

This thesis consists of seven chapters. Chapter 1 is the introduction and provides a background of economic dispatch, optimal power flow, different ways of achieving "open access" to power systems. The objectives, the organization of the thesis and the work covered are also outlined in this chapter.

Chapter 2 covers the definition of optimal power flow and some of the objective functions prevailing in electric power systems. Also presented is a selection of different algorithms for solving OPF (Optimal Power Flow) problems. In some cases, the algorithms are given in detail.

Chapter 3 describes the concept of electric energy wheeling (transmission services) in power systems. Different types of electric power wheeling are given including the reasons why wheeling occurs. Some of the ways of determining wheeling rates are also discussed together with the impact of wheeling and some technical aspects of wheeling. This chapter concludes with a treatment of the important subject of open transmission access as an extension of the wheeling concept.

The formulation of the optimal power problem is covered in Chapter 4. The formulation, unlike other models includes transmission operating costs based on time-of-use. The equality constraints which characterize the power flow throughout the power system are given including inequality constraints such as power generating limits, transmission limits, voltage limits and phase angle limits. Definitions of real-time pricing of reactive and active power are given together with the corresponding equations. The MINOS Package is described as the base to solve the optimal power flow problem formulated.

Chapter 5 covers computational results testing the proposed methodology. It is assumed that only one utility is responsible for the generation, transmission and distribution. The generators considered in this case are all thermal. The classical formulation using fuel cost at the generating plants to establish the electricity rates is compared with the proposed new formulation.

In Chapter 6, interest is focused on the computational results and discussions under wheeling conditions i.e. assuming that there are now separate ownerships of the generation, transmission and distribution. The results of using classical formulation of fuel cost at the generating plants to establish the wheeling rates are also compared with the proposed methodology which incorporates transmission costs. The generators involved, like those in the previous chapter, are all thermal.

Chapter 7 gives the summary and conclusions drawn from the studies reported in the thesis and some recommendations for future work. A list of references used in this thesis is given after Chapter 7, which is followed by appendices.

Chapter 2

OPTIMAL OPERATION OF ELECTRIC POWER SYSTEMS

Introduction

Emergency conditions may develop in the normal daily operation of a power system. This may be due to forced or random events. In order to maintain an economic and secure state, several options are available to power system operators and planners. Traditionally, a trial and error technique is used to provide sub-optimum corrective strategies for alleviating violations due to planned or unexpected contingencies [17]. This approach does not, in general, guarantee an acceptable optimum solution. The development of an optimal strategy for maintaining power system operation is accomplished by using several optimization approaches. In this chapter a description of the optimal power flow problem and some methods for its solution are given. A review of some of the objective functions used in solving the OPF problem is also given.

2.2 Background

The optimal power flow problem for cost minimization, originated as an extension of the economic dispatch problem. This has been in existence since the early 1920's [18]. At that time, the losses were represented by a loss formula relating the active power losses to the active power generation. However, with the advent of power flows in the 1960's [19,20], losses involved in the process started to be represented by the power flow equations. The present classical OPF has its origin here. Since then, much effort has been spent on obtaining faster, more robust solutions and refinement of different objective functions.

2.3 Formulation Of An OPF Problem

Generally, an Optimal Power Flow (OPF) problem is mathematically formulated as a constrained optimization problem and expressed as follows:

Minimize: $$f(u,x) \tag{2.1}$$

Subject to:

$$g(u,x) = 0\,, \tag{2.2}$$

$$h(u,x) \leq 0\,, \tag{2.3}$$

$$u^{\min} \leq u \leq u^{\max} \quad \text{and} \tag{2.4}$$

$$x^{\min} \leq x \leq x^{\max}\,. \tag{2.5}$$

where:

u is a set of controllable variables in the system,

x is a set of dependent variables and

$f(u,x)$ is the objective function to be minimized.

Equation (2.2) is the set of nodal power balance equations usually referred to as the power flow equations. Equation (2.3), expresses the set of inequality constraints such as line flows. Inequalities depicted by equations (2.4) and (2.5) respectively ensure that the limits on all variables are satisfied. The control variables, vary depending on the objective function being minimized. These control variables can include voltage magnitudes at generator buses, transformer and phase shifter settings, and real power at generator buses. The specified constraints (limits) can include transformer and phase angle regulator settings, voltages at the buses, real and reactive power flows.

2.4 Commonly Used OPF Objective Functions

An OPF program, attempts to calculate the best possible setting for a list of control variables in order to fulfill a desired objective. To name a few, some of the objective functions include minimization of losses, minimization of fuel costs and minimization of added VARs. Sometimes a combination of objective functions maybe formed e.g. minimization of losses as well as VAR additions at the same time. The optimal solution is only valid for the particular system conditions and constraints presented to the OPF program. Some of the objective functions can be classified as follows:

a. minimum cost of operation and

b. minimum active-power transmission losses.

2.5 Minimum Operating Cost

The objective in this case, consists of the total costs of controlled generators and the costs of any controlled interchange transactions. The classical "full OPF" problem is the cost minimization for both active- and reactive- power controls. The important factor in cost for thermal plants minimization, is the knowledge of the fuel-cost curves. This has an effect on overall optimality. The curves, express the heat rate input to the boiler as a function of output real power. The measured heat rate curve can be complex depending on the valve positions of the steam turbines. The major interest lies in the cost of the fuel needed to produce the required power. By knowing the type of fuel used, together with its calorific value and cost, the heat rate curve is translated to a cost curve.

There are different approaches to approximate the cost curves. One method is to represent the cost curve by a convex polynomial or exponential. A second method is to model each intermediate segment by a quadratic function and maintaining overall convexity. The third method is to use an arbitrary number of linear segments and again keeping convexity. The cost curve associated with an MW interchange transaction is normally linearly segmented. One example of an objective whose formulation and solution is closely related to minimum cost is that of minimum emissions [21,22]. A commonly used single heat rate curve with upper and lower limits on generating is usually approximated by a quadratic polynomial of the form:

$$C(P_g) = a + bP_g + cP_g^{\ 2}, \tag{2.6}$$

where:

P_g is the MW (or per unit) output of the generator and a, b, c are constants coefficients.

The majority of the generators in the power system belong to the following classifications:

a. nuclear,

b. hydro and

c. fossil (e.g. coal, oil, and gas).

Nuclear plants are operated at constant output and hydro plants have essentially no operating costs that depend on the amount of generation. The only component, therefore, to be considered for dispatching purposes is the cost of the burned fuels used in the fossil-fired plants.

2.6 Minimum Active Power Transmission Losses

The minimization of active power transmission losses is another important objective function. This minimization of the losses is usually associated with voltage/VAR scheduling. By reducing the circulating VAR's, the voltage profiles will be flatter.

The first attempt to solve the minimum loss OPF is reported in [23]. The objective is to minimize the losses by varying voltage at a time in an iterative cycle and performing several load flow computations. Zollenkoff [24] used transformer taps to minimize losses. Dommel and Tiney [20] minimized losses within a cycle which includes the Newton's power flow computation.

Another approach based on reactive power line flows is given by Hano et al.[25]. Hano used the conjugate gradient technique with the network represented by a linearized set of equations. Shoults and Shen [26] minimized the sum of the squares of the deviations of the controlling parameters (generator voltages, transformer taps, and VAR sources) from a given state using sensitivity factors.

Mamandur and Chenoweth [27], employed linearized sensitivity relationships of the network to establish the objective function for minimizing the active power transmission losses, as well as system performance sensitivities. A dual-programming approach is used to control variables including transformer taps, generator voltages, and VAR sources.

Mansour and Abdel-Rahman [28] used decomposition techniques and express the active power transmission losses in terms of the rectangular components of the bus voltages and use rectangular coordinates to formulate the constraints. El-Sayed et al. [29] applied the quadratic programming to decomposition of the system using a decoupled formulation of the network equations as an extension to the approach of [28].

2.7 Methods for Solving OPF Problems

There are numerous optimization methods that have been proposed over the years to solve optimal power flow problems. Others are refinements of earlier methods. Some of the methods used are as follows:

a. Generalized Reduced Gradient,

b. Newton Raphson Methods,

c. Reduced Gradient Methods,

d. Conjugate Gradient Methods,

e. Hessian Based Methods,

f. LP Methods,

g. QP Methods and

e. Interior-Point Methods.

These methods are briefly either discussed or given in detail in the following sections.

2.7.1 Generalized Reduced Gradient Methods

As an extension of Wolfe's reduced gradient method [30], to the case of non-linear constraints, Abadie and Carpentier [31] introduced the Generalized Reduced Gradient Method (GRG). The GRG method was applied to OPF problems by Peschon [32] and Carpentier [33] in 1973. Since then, a number of researchers [34,35] have used this method to solve optimal power flow problems.

2.7.2 Newton Raphson Methods

The Newton Raphson method has been widely accepted and used to solve problems such as load flows. The Taylor's series expansion for a function of two or more variables is the basis for this method [36].This is a general purpose method used to solve non-linear equations. By assuming that $f(x)$ is a non-linear equation in one dimension; the function $f(x)$ can then be expanded by Taylor's series around the initial estimate of the solution $x^{(0)}$ as follows:

$$f(x^{(0)})+\Delta x^{(0)}\left(\frac{df}{dx}\right)^{(0)}+\frac{1}{2}\left(\Delta x^{(0)}\right)^{(2)}\left(\frac{d^2f}{dx^2}\right)^{(0)}+\dots\dots\dots\dots\dots=0. \tag{2.7}$$

If the error $\Delta x^{(0)}$ is relatively small, the higher order terms can be neglected resulting in a linear function. The linear function f(x) is expressed as :

$$f(x^{(0)}) \quad + \quad \Delta x^{(0)}\left(\frac{df}{dx}\right)^{(0)} \qquad \approx \qquad 0\ . \tag{2.8}$$

Solving Equation (2.8), an approximate value of the error is obtained i.e.

$$\Delta x^{(0)} \quad = \quad \frac{-f(x^{(0)})}{(df\ \ dx)^{(0)}}\ . \tag{2.9}$$

The approximate value of the error is then added to the original estimate $\Delta x^{(0)}$ to obtain an improved solution expressed as follows:

$$x^{(1)}=x^{(0)}+\Delta x^{(0)}=x^{(0)}-\frac{f(x^{(0)})}{(df\ \ dx)^{(0)}}\quad . \tag{2.10}$$

Since the series expansion has been truncated, the value added to the initial estimate does not determine the correct solution. As a result, equation (2.10) is repeated iteratively to obtain a new estimate. This recursion formula necessary for iterative estimates of the root is given by:

$$x^{(n+1)} = x^{(n)} + \Delta x^{(n)} = x^{(n)} - \frac{f(x^{(n)})}{(df / dx)^{(n)}}, \qquad n = 0,1,2,\ldots\ldots\ldots \quad . \tag{2.11}$$

The concept in one dimension can be extended to n-dimensions, where the functions and the variables are represented by vectors. The slope of the function $f'(x)$ is replaced by the Jacobian matrix. In multidimensional problems, Equation (2.8) is transformed to:

$$f(x^{(0)}) + J^{(0)} \Delta x^{(0)} = 0, \tag{2.12}$$

where:

$J^{(0)}$ indicates that the initial estimates $x_i^{(0)}$ have been used to compute the numerical value of the partial derivatives (the Jacobian matrix). Solving Equation (2.12) for the error vector, the following is obtained:

$$\Delta x^{(0)} = \; -\left[J^{(0)}\right]^{-1} f(x^{(0)}) \; . \tag{2.13}$$

This error vector, as in one dimension, is then added to the original estimate to get the new approximate roots of the function $f(x)$. The iterative process is expressed by:

$$x^{(n+1)} = x^{(n)} + \Delta x^{(n)} = x^{(n)} - \left[J^{(0)}\right]^{-1} f(x^{(0)}) . \tag{2.14}$$

The process represented by Equation (2.14) is repeated until the precision index selected is met or till there is no significance change in the correction vector $\Delta x^{(n)}$.

The Newton OPF formulation was implemented in 1984 by Sun et al. [37] and later by Maria et.al [38]. The formulation consists of an augmented Lagrangian given by:

$$F(x) = f(x) + \sum \lambda_i \ g_i \ (x) . \tag{2.15}$$

The first set of derivatives of $F(x)$ with respect to x gives a set of non-linear equations. The equations are then solved simultaneously using the Newton-Raphson method.

2.7.3 Reduced Gradient Methods

Dommel and Tinney [20] were the first to use the reduced gradient method in 1968 to solve an OPF problem. For the reduced gradient method, the variables are divided into unknown "x" i.e. state variables which consists of V (voltage) and δ (phase angle) on P-Q buses, and δ on P-V buses. On P-Q buses, the fixed parameters P, Q and δ on the slack bus are denoted as parameter "p". The control variables "u" consist of voltage magnitudes on generator buses, generator real power P, and transformer tap ratios. This method is summarized as follows:

Assume the non-linear constrained optimization problem to be solved is given by:-

Minimize: $$f_0(u,x) \tag{2.16}$$

Subject to:

$$g(u,x)=0\,, \tag{2.17}$$

$$h(u,x)\le 0\,, \tag{2.18}$$

$$u^{\min} \le u \le u^{\max} \quad \text{and} \tag{2.19}$$

$$x^{\min} \le x \le x^{\max}\,. \tag{2.20}$$

The functional inequality constraints are handled by a penalty function method, i.e. a term is added to the objective function so that it becomes artificially large whenever a functional inequality constraint is violated. The corresponding modified cost function will be as follows:

$$f(x,u) = f_0(x,u) + \sum_i W_i h_i^2(x,u) \,. \tag{2.21}$$

Then $f(x)$ is minimized subject to the load flow equations $g(x,u) = 0$. The augmented Lagrangian function is now expressed as:

$$L(x,u) = f(x,u) + \lambda^T g(x,u) \,, \tag{2.22}$$

where λ^T is the row vector. For the minimum to occur, the necessary conditions are as follows:

$$\frac{\partial L}{\partial x} = \frac{\partial f}{\partial x} + \left[\frac{\partial g}{\partial x}\right]^T \lambda \quad = \quad 0 \,. \tag{2.23}$$

$$\frac{\partial L}{\partial u} = \frac{\partial f}{\partial u} + \left[\frac{\partial g}{\partial u}\right]^T \lambda \quad = \quad 0 \,. \tag{2.24}$$

$$\frac{\partial L}{\partial \lambda} = \quad g(u,x) \,. \tag{2.25}$$

Equation (2.24) represents the power flow solution. Equation (2.22) contains the transpose of the Jacobian which can be solved for λ as follows:

$$\lambda \; = \; -\left\{\left[\frac{\partial g}{dx}\right]^{T}\right\}^{-1}\frac{\partial f}{\partial x}, \tag{2.26}$$

and $\partial L/\partial u$ in Equation (2.24) represents the reduced gradient vector ∇f i.e.

$$\nabla f = \frac{\partial L}{\partial u} = \frac{\partial f}{\partial u} + \left[\frac{\partial g}{\partial u}\right]^{T}\lambda . \tag{2.27}$$

The negative of the gradient is the direction of steepest descent. The target, is to move along this direction of steepest descent from one feasible point to another feasible solution point with lower value for f. A set of feasible control variables u is assumed. Newton's method is used to solve the power flow equations for x. Following this, the Lagrangian multipliers λ, are calculated. The reduced gradient ∇f is next calculated from Equation (2.27) from which a correction in u can be calculated as:

$$\Delta u \; = \; -c\Delta f . \tag{2.28}$$

The control variables are then improved as:

$$u^{new} = u^{old} + \Delta u . \tag{2.29}$$

The procedure is repeated until convergence i.e. until the solution can not be improved further. The choice of the factor "c" is the major drawback in this method. Too small a value assigned to "c", slows down the convergence and too high a value, cause oscillations. Many authors [39,40] refined this method by using the Hessian matrix.

2.7.4 Conjugate Based OPF Methods

In 1982, Burchett et.al [18] improved the Dommel and Tinney algorithm. Instead of using the negative gradient $-\nabla f$ as the direction of steepest descent, the descent direction at adjacent points are linearly combined in a recursive manner as follows:

$$r_k = -\nabla f + \beta_k \, r_{k-1} , \tag{2.30}$$

where:

$\beta_0 = 0$,

k is the iteration count and

r_k is new descent direction at iteration k.

The change in control variable, Δu becomes:

$$\Delta u = cr_k . \tag{2.31}$$

This formulation represents the conjugate direction algorithm.

In Equation (2.30), the scalar value β_k is defined according to the direction search technique used. The two most popular methods used in the estimation are:

a. the Fletcher-Reeves method [41] where

$$\beta_k = \frac{\|\nabla f_k\|^2}{\|\nabla f_{k-1}\|} \qquad \text{and} \tag{2.32}$$

b. the Polak-Ribiere method [42], where

$$\beta_k = \frac{(\nabla f_k - \nabla f_{k-1})^T \nabla f_k}{\|\nabla f_k\|^2} . \tag{2.33}$$

2.7. 5 Hessian Based Methods

The constrained optimization problem is transformed into a sequence of unconstrained problems. Sasson [43] discussed the transformation [44,45]. He used a transformation due to Powell given by:

$$F(x,r,s) = f(x) + \sum_{i=1}^{m} (g_i(x) + s_i)/r_i \,, \qquad g_i(x) \geq 0 \qquad (2.34)$$

where:

s_i and r_i, $i = 1, \ldots\ldots\ldots\ldots m$, are constants. Sasson used the Fletcher-Powell method [46] of minimization. In this method, the Hessian matrix is evaluated indirectly. It is built initially with the identity matrix so that at the optimum point it becomes the Hessian itself.

However, the Fletcher-Powell has some drawbacks. The result was that in 1973 [47] Sasson developed a Hessian load flow with extension to OPF. Here, the Hessian matrix is evaluated directly. The objective function is transformed as before to an unconstrained objective. The objective function becomes:-

$$F(x) = f(x) + \sum r_i g_i(x)^2 . \tag{2.35}$$

All equality constraints including the violated inequality constraints are included in Equation (2.35). The Hessian matrix is sparse and hence reduces storage and computational time.

2.7.6 Linear Programming (LP) Methods

A linear programming method is based on linear/piecewise-linear approximations of objectives and constraints to locate the optimal feasible solution by using an iterative approach to satisfy the non-linear constraints, where each iteration is based on the linear approximation with the optimal search direction [20,48,49,50-53]. Since its inception in the 1940s [54], linear programming has been used in many applications. It has been successfully adapted for optimal power flow. The method has proven to be robust and fast in solving a large subset of optimization problems with linearized relationships.

The original linear-programming algorithm called the simplex method has served industry well. In [55,56] the dual simplex method was applied to OPF problems. In 1981, Romano et al., [57] used the Dantzig-Wolfe decomposition principle [58] to solve the economic dispatch problems of multi-area systems. The solution method uses the revised simplex method.

However, LP methods have their own drawbacks. The methods have inaccuracies caused by linearization and cannot be considered for on-line use especially for power systems with dynamic constraints [59]. Stott et. al. [60] gives a good review of the special features needed to match an LP technique to an OPF problem.

2.7.7 Quadratic Programming (QP) Methods

The quadratic algorithm techniques use the Kuhn-Tucker conditions and the Newton search direction at each iteration, where the non-linear constraints are linearized and the sub-problems are solved [61-63]. Burchett et al., [64] used a sparse implementation of this method. The original problem is defined as simply:

Minimize: $$f(x), \tag{2.36}$$

Subject to:

$$h(x) = b \quad \text{and} \tag{2.37}$$

$$x_{\min} \leq x \leq x_{\max}. \tag{2.38}$$

The above equality constraints involving $h(x)$ and b are the nonlinear power flow mismatch equations. To solve the problem expressed by Equation (2.36), a sequence of simpler subproblems are solved which quadratically converge to the solution of the original problem. The sub-problem takes the following quadratic expression :

Minimize: $$\tilde{g}^T p + \frac{1}{2} p^T Hp \tag{2.39}$$

Subject to:

$$Jp = 0, \tag{2.40}$$

where:

$$p = x - x_k, \tag{2.41}$$

$\tilde{g}$ is the gradient vector of the original objective function f with respect to the variables x,

J is the Jacobian matrix containing derivatives of the original equality constraints,

H is the sparse symmetric Hessian containing second derivatives of the objective function and a linear combination of the constraints functions with respect to x,

x_k is the current point of linearization of the constraints.

The solution to the quadratic program would give the values of p which is then added to the current x.

The Burchett's method was later extended by El-Kady et al., [65], in a study of the Ontario Hydro System for on-line voltage var control. Glavitsch [66] and Contaxis [63], implemented the non-sparse version of the problem. Successive quadratic programming has been used for the reactive power dispatch problem.

2.7.8 Interior-Point Methods

Since no approach has received universal acceptance for solving the general OPF problem, many researchers have turned to the area of mathematical programming. Some recent advances in algorithms are changing linear programming and a new class of algorithms are emerging. Changes began in 1984 when Karmarkar [67] published a landmark paper introducing interior-point methods to solve linear programming problems. Interior-point methods are different from the classic simplex method. The easiest way to see the difference is to look at the polytope, a generalized polygon associated with linear-programming problems, which is formed from a problem's constraints [54].

Assuming that the polytope is many sided and in three-dimensional space. At each iteration, the simplex method moves along the polytope's edge toward the vertex of the previous iteration i.e. the simplex algorithm searches the optimal solution by examining extreme points on the boundary of the feasible region. In contrast, the interior-point method begins its iterations strictly from the interior of the polygon. Each iteration marches through the interior, without regard for the polygon edges, toward the lowest cost vertex (optimal solution).

Karmakars's algorithm provoked many researchers to dramatically improve the simplex method. Several papers [53,68-70] have been reviewed in this area. Preliminary research results show that variants of the interior-point method can solve large linear programming problems faster than the simplex algorithm [17,71]. The interior-point method can also be more attractive when the problem size exceeds several thousand variables and constraints [72]. Since there is great interest in this area by many researchers [54,72,73,74], two of these variant methods are discussed in detail below.

In [72], the primal-dual algorithm (a variant of interior-point method) is applied to an optimal reactive dispatch problem. To apply the algorithm the problem is stated generally as:

Minimize: $f(z)$, (2.42)

Subject to:

$$h(z) = 0\,, \tag{2.43}$$

$$z - s_1 = l\,, \tag{2.44}$$

$$z + s_2 = u\,, \tag{2.45}$$

$$s_1 \geq 0 \quad and \quad s_2 \geq 0\,. \tag{2.46}$$

where:

$z \varepsilon R^n$, f and h are continuously differentiable functions,

l and u are vectors corresponding to upper and lower bounds in the variables respectively. Eliminating the inequality constraints in Equation (2.46) by incorporating them into a logarithmic barrier function as follows:

$$\text{Minimize: } \left\{ f(z) - \mu \sum_{j=1}^{n} \log(s_{1_j}) - \mu \sum_{j=1}^{n} \log(s_{2_j}) \right\}, \tag{2.47}$$

Subject to:

$$h(z) = 0\,, \tag{2.48}$$

$$z - s_1 = l \ , \tag{2.49}$$

$$z + s_2 = u \ . \tag{2.50}$$

where:

μ is the barrier parameter, $\mu > 0$.

The first order necessary optimality condition for Equation (2.47) are :

$$\nabla f(z) - J(z)^T \lambda - \pi_1 - \pi_2 \quad = \quad 0 \, , \tag{2.51}$$

$$h(z) \qquad = 0 \, , \tag{2.52}$$

$$z + s_2 - u \qquad = 0 \, , \tag{2.53}$$

$$\mu \ \ e - S_1 \pi_1 = 0 \, , \tag{2.54}$$

$$\mu \ \ e + S_2 \pi_2 = 0 \, , \tag{2.55}$$

where:

$\nabla f(z)$ is the gradient of $f(z)$,

$J(z)$ is the Jacobian of $h(z)$,

$\lambda \ \varepsilon \ \mathrm{R}^m$ are multipliers of constraints (2.48),

$\pi_{1,} \ \pi_2 \ \varepsilon \ \mathrm{R}^n$ are multipliers of constraints (2.49) and (2.50) respectively,

$e \ \varepsilon \ \mathrm{R}^n$, $e = (1,..,1)^T$,

S_1, S_2, are diagonal matrices in R^{nxn} whose diagonal elements are s_{1_i}, s_{2_i}, respectively.

By assuming that the current estimate of the primal and dual variables satisfies:

$$s_1 \geq 0, \quad s_2 \geq 0, \tag{2.56}$$

$$\pi_1 \geq 0, \quad \pi_2 \geq 0, \tag{2.57}$$

$$z - s_1 = l \quad \text{and} \tag{2.58}$$

$$z + s_2 = u\,. \tag{2.59}$$

The Newton equations for generating a search direction ($\Delta z, \quad \Delta s_1, \quad \Delta s_2, \Delta\lambda, \Delta\pi_1, \Delta\pi_2$) are expressed as follows:

$$W(z,\lambda)\Delta z - J^T(z)\Delta\lambda - \Delta\pi_1 - \Delta\pi_2 = -t\,, \tag{2.60}$$

$$J(z)\Delta z = -h(z), \tag{2.61}$$

$$\Delta z - \Delta s_1 = 0 \quad , \tag{2.62}$$

$$\Delta z + \Delta s_2 = 0\,, \tag{2.63}$$

$$-\Pi_1\,\Delta s_1 - S_1\Delta\pi_1 = -v_1 \quad , \tag{2.64}$$

$$\Pi_2 \Delta s_2 + S_2 \Delta \pi_2 = -v_2, \tag{2.65}$$

where:

$$t = \nabla f(z) - J(z)^T \lambda - \pi_1 - \pi_2, \tag{2.66}$$

$$v_1 = \mu e - S_1 \pi_1, \tag{2.67}$$

$$v_2 = \mu e + S_2 \pi_2, \tag{2.68}$$

$$W(z\lambda) = \nabla^2 f(z) - \sum_{i=1}^{m} \lambda_1 \nabla^2 h_i(z), \tag{2.69}$$

$\nabla^2 f(z), \nabla^2 h_i(z), \quad i = 1,\dots m$ are Hessian matrices for $f(z)$,

$h_i(z), \quad i = 1,\dots m$ respectively,

Π_1, Π_2, are diagonal matrices in R^{nxn} whose diagonal elements are π_{1j}, π_{2j}, respectively.

These equations may be solved by defining:

$$\overline{W}(z,\lambda) = W(z,\lambda) + S_1^{-1} \Pi_1 - S_2^{-1} \Pi_2, \tag{2.70}$$

$$\bar{t} = -t + S_1^{-1} v_1 - S_2^{-1} v_2 \tag{2.71}$$

and then solving the following equations:

$$\begin{bmatrix} \overline{W}(z,\lambda) - J^T(z) \\ -J(z) \end{bmatrix} \begin{bmatrix} \Delta z \\ \Delta\lambda \end{bmatrix} = \begin{bmatrix} \bar{t} \\ h(z) \end{bmatrix} \tag{2.72}$$

and finally solving the following equations:

$$\Delta s_1 = \Delta z, \tag{2.73}$$

$$\Delta s_2 = -\Delta z, \tag{2.74}$$

$$S_1 \Delta\pi_1 = v_1 - \prod\nolimits_1 \Delta s_1 \quad \text{and} \tag{2.75}$$

$$S_2 \Delta\pi_1 = -v_2 - \prod\nolimits_2 \Delta s_2 . \tag{2.76}$$

The next step is to compute step-lengths in the primal and dual spaces:

$$\alpha_p = \min\left\{ \min_{\Delta s_{ij} < 0} \frac{s_{1j}}{|\Delta s_{1j}|}, \min_{\Delta s_{2j} < 0} \frac{s_{2j}}{|\Delta s_{2j}|}, 1.0 \right\} \quad \text{and} \tag{2.77}$$

$$\alpha_d = \min\left\{ \min_{\Delta \pi_{ij} < 0} \frac{\pi_{1j}}{|\Delta\pi_{1j}|}, \min_{\Delta \pi_{2j} < 0} \frac{-\pi_{2j}}{|\Delta\pi_{2j}|}, 1.0 \right\} \quad . \tag{2.78}$$

A new approximation to the optimal solution is then determined as follows:

$$z = z + \sigma\, \alpha_p \Delta z \ , \tag{2.79}$$

$$s_1 = s_1 + \sigma\, \alpha_p \Delta s_1 \, , \tag{2.80}$$

$$s_2 = s_2 + \sigma\, \alpha_p \Delta s_2 \, , \tag{2.81}$$

$$\lambda = \lambda + \sigma\, \alpha_d \Delta\lambda \ , \tag{2.82}$$

$$\pi_1 = \pi_1 + \sigma\, \alpha_d \Delta\pi_1 \ , \tag{2.83}$$

$$\pi_2 = \pi_2 + \sigma\, \alpha_d \Delta\pi_2 \, , \tag{2.84}$$

where in this reported case $\sigma = 0.9995$. (2.85)

This value is selected by the user in such a way that the new point does not hit the boundary of the feasible region. Also, the critical point in the primal-dual algorithm is the choice of the barrier parameter μ.

Another variant of the Karmarkar's interior-point method is presented in [74]. This variant is called the quadratic interior-point method and is used to solve an OPF problem. The algorithm is based on improvement of the initial starting point. The algorithm is used to solve the following minimizing problem:

$$P = \frac{1}{2} X^T Q X + a^T X, \tag{2.86}$$

Subject to:

$$b^{\min} \leq AX \leq b^{\max}, \tag{2.87}$$

where:

X is an unknown n-vector,

a is a fixed n-vector,

$b^{\min}$ and $b^{\max}$ are fixed m-vectors respectively,

Q is a symmetric square matrix and

A is an m by n coefficient matrix with $m < n$.

Linear programming is a special case of $Q = 0$. Two new m-vectors S_1 and S_2, called slack variables are introduced to change the inequality in Equation (2.87) into the following equality form:

$$AX - S_1 = b^{\min} \ , \tag{2.88}$$

$$AX + S_2 = b^{\max}, \qquad S_{1,} \ S_2 \geq 0 \, . \tag{2.89}$$

By defining

$$\widetilde{X} = \begin{pmatrix} X \\ S_1 \\ S_2 \end{pmatrix}, \qquad \widetilde{Q} = \begin{pmatrix} Q_{(nxn)} & 0_{(nx2m)} \\ 0_{(2mxn)} & 0_{(2mx2m)} \end{pmatrix}, \tag{2.90}$$

$$\widetilde{a} = \begin{pmatrix} a_{(nx1)} \\ 0_{(2mx1)} \end{pmatrix}, \qquad \widetilde{b} = \begin{pmatrix} b^{\min}_{(mx1)} \\ b^{\max}_{(mx1)} \end{pmatrix}, \tag{2.91}$$

and

$$\widetilde{A} = \begin{pmatrix} A_{(mxn)} & -I_{(mxm)} & 0_{(mxm)} \\ A_{(mxn)} & 0_{(mxm)} & I_{(mxm)} \end{pmatrix} \ , \tag{2.92}$$

where I is the m by m identity matrix, the quadratic optimization presented by Equation (2.86) becomes:

$$P = \frac{1}{2}\widetilde{X}^T\widetilde{Q}\widetilde{X} + \widetilde{a}^T\widetilde{X}, \tag{2.93}$$

Subject to:

$$\widetilde{A}\widetilde{X} = \widetilde{b} , \tag{2.94}$$

$$\widetilde{X}_j \geq 0, \qquad j = n+1,....,n+2m . \tag{2.95}$$

The quadratic optimization problem described in (2.93-2.95) assumes a bounded interior point $\widetilde{X}^0$ or else the problem has no solution or the solution is unbounded. By starting at the initial feasible point $\widetilde{X}^0$, the quadratic algorithm optimization process generates a sequence of feasible interior-points $\widetilde{X}^1, \widetilde{X}^2, ..., \widetilde{X}^k, \widetilde{X}^{k+1}, ...,$ such that

$$P_{k+1} = \frac{1}{2}(\widetilde{X}^{k+1})^T\widetilde{Q}\widetilde{X}^{k+1} + \widetilde{a}^T\widetilde{X}^{k+1} < P_k = \frac{1}{2}(\widetilde{X}^k)^T\widetilde{Q}\widetilde{X}^k + \widetilde{a}^T\widetilde{X}^k . \tag{2.96}$$

The stopping criterion is either based on the relative changes in the objective function at iterations or relative changes in interior feasible solutions in iteration. Mathematically this is expressed as:

$$\left|P_{k+1} - P_k\right| / \max\left\{1, \left|P_k\right|\right\} < \varepsilon \qquad \text{or} \tag{2.97}$$

$$\left|\widetilde{X}^{k+1} - \widetilde{X}^k\right| < \varepsilon \tag{2.98}$$

The algorithm has to calculate the initial starting interior feasible point $\widetilde{X}^0$, i.e. $\widetilde{A}\widetilde{X}^0 = \widetilde{b}$ with $\widetilde{x}_j^0 \geq 0,\ j = n+1,..,n+2m$. The initial feasible point is obtained by using an auxiliary problem [75].

2.7.9 Summary

In this chapter, a number of approaches to the nonlinear program problem have been considered. There is no definite preferred method for solution, but rather a number of possibilities need to be considered in solving a specific problem.

Chapter 3

WHEELING

Introduction

Recent trends in the electric power utility industry have been toward increased deregulation of the services provided by the utilities. Electric power utilities need to know the actual cost of providing separate services in order to make correct economic decisions on the various types of services they should promote. Utilities also need to know such costs in order to make correct engineering decisions on upgrading and expanding their generation, transmission and distribution facilities. One result of the recent trends is that consumers and suppliers may make direct commercial contact across third party wholly owned transmission systems [11,76]. This is known as wheeling.

The purpose of this chapter is to lay the foundation for the studies reported in Chapter 6 and to provide a coherent consistent engineering and economic basis for settling wheeling rates. Deregulation of the power system industry is discussed. Wheeling and transmission open access are both discussed at length as forms of deregulation. The impact of deregulation is presented at the end of this chapter.

3.2 Regulation to Deregulation

Historically, the electric power industries have been operating in a vertically regulated environment i.e. the utilities are integrated as shown by the dashed box in Figure 1. In other words, a single utility, owns and operates all the components necessary for providing service to its customers, from generation, through transmission, down to the distribution facilities required to service the customers in their assigned area. However, with the introduction of related regulations allowing competition in power production, deregulation essentially turns the dashed box in Figure 1 on its side. Distinct industries may arise which specialize in only one area i.e. a GenCo.(generation), a TransCo.(transmission) and a DisCo.(distribution). Competition replaces cooperation. As competition enters the picture, as we will see later; regulators play another key role. Wheeling is one of the special electrical supply options available to transmitting utilities.

3.3 The concept of electric energy wheeling

By definition, wheeling is the transportation of a product from a supplier to a consumer over some facility owned by a third party. In the electric utility industry, it is the transmission of electrical power and reactive power from a seller to a buyer through a transmission network owned by a third party [11,76,77]. At least three parties are involved in a wheeling transaction: a seller, a buyer, and one or more wheeling utilities

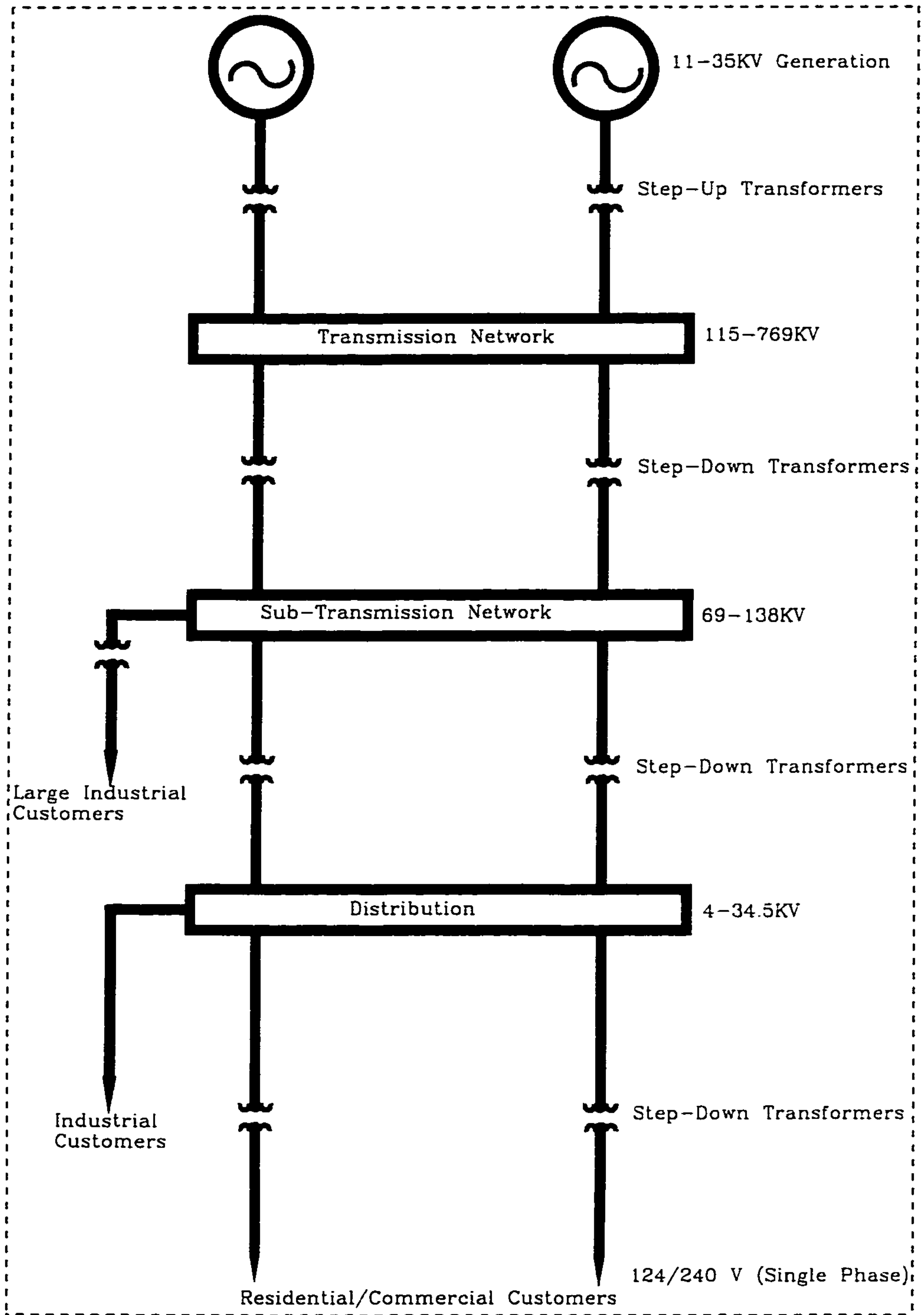

Figure 3.1 A Typical Power System Structure

that transmit the power from the seller to the buyer. The third party charges for the use of its network. These charges are known as wheeling rates. The establishment of wheeling rates is presently the subject of extensive debate [78-81].

Almost every arrangement in which a producer supplies energy to a consumer involves transmission wheeling. Energy cannot be delivered from a remote source without movement over a physical transmission system. Even when a utility is delivering its own generation to its own customers, changes in its own line flows will affect line flows in other utilities to which it is synchronously interconnected. Consumers costs are affected by the matching or mismatching of wheeling costs and revenues if their utility is providing transmission service to others.

Wheeling is not confined to the electric utility industry. Natural gas and oil are delivered from producers to consumers via pipelines, barges or trucks. Mail is wheeled over third party transportation from senders to recipients. Many products are wheeled over provincial and local roads from suppliers to consumers. Long distance phone calls are wheeled by a variety of competing long distance companies from one party to another over extensive physical networks belonging to a third party.

Until the late 1970's, wheeling in North America was not especially important [8] since most utilities delivered power and energy from their own generators to their own consumers over their own transmission and distribution systems. Now, however, many producers and even retail customers seek access to wheel power and energy over other utility transmission systems.

3.4 Why Wheeling Occurs

As already indicated, traditionally, a single utility would service a geographic region. The utility was responsible for providing enough generation to meet all of their customers needs. Even today, this is still the case in a lot of areas. However, after the infamous New York city blackout of 1965 [8,82], utilities began to interconnect. Interconnection yielded many advantages. Reliability was improved since neighboring systems could act as buffer zones and provide additional power during fault conditions. Many companies started to work cooperatively together to avert power failures. Additionally, utilities started to buy and sell power across the interconnections, or tie lines. Frequently, a utility would produce excess power and sell it at a cheaper price than a neighbor could produce it. Thus both companies profited from this arrangement. The system load therefore is met reliably, through the dispatch of the cheapest available generating plant, without undue regard to the plants geographical location. Therefore, transmission inter-connections were made between utilities for reliability purposes, but provided opportunities for utilities to sell and purchase excess capacity and energy.

Recent trends in the electric power utility industry in North America and elsewhere have been toward increased deregulation of the services provided by the utilities. Many utilities jointly owned large power plants, and in many cases wheeling was required for a joint owner to receive its portion of the generation output. Electric power utilities need to know the actual cost of providing separate services in order to make correct economic decisions on the various types of services they should promote or curtail while at the same time fulfilling their service obligations. Utilities also need to know such costs in order to make correct engineering decisions on upgrading and expanding their generation, transmission and distribution facilities.

In the 1970's, the North American electric power industry was building more power plants to meet rapid growth in demand. Coal-fired steam and nuclear plants were the units of choice because of their low generating costs [83]. Unfortunately, plants built in the late 1970's and early 1980's experienced high inflation costs and delays due to safety concerns. This doubled or tripled the cost of building these plants and drove up the cost of electricity. Some utilities who needed capacity within this period became reluctant to risk construction programs. Instead they resorted to wheeling power from utilities which had substantial surpluses of installed generation capacity.

Recently, this has resulted in a competitive market in generation and supply which demands that the transmission network should not place any undue constraints on the operation of generators and their customers, i.e. the transmission system should potentially, allow any generator to supply any customer. The wheeling of power over the transmission network can be regarded as a mechanism for resource integration in electricity supply.

3.5 Types of Wheeling

There are different types of wheeling in the electric utility industry. This depends on the relationships between the wheeling utility and other two parties [84]. The four broad categories are as follows:

(a) utility to utility,

(b) utility to private user ,

(c) private generator to utility and

(d) private generator to private user.

Wheeling can occur between individual buses or areas. Category (a) illustrates area-to-area wheeling i.e. selling and buying utilities cover geographic areas which are interconnected by multiple tie lines to the transmission network of a wheeling utility. Category (b) illustrates area-to-bus wheeling, or another type of area-to-area wheeling, or another type of area-to-area wheeling where the requirements of customer are so small that he is fed only at one bus (this is effectively area-to-bus wheeling). Category (c) illustrates bus-to-area wheeling. Lastly, category (d) illustrates bus-to-bus wheeling, i.e. the seller of power is located at one bus while the buyer is located at a different bus. The equations for category (d) are more complicated and difficult to implement.

3.6 Nature and Duration of Wheeling

Wheeling may be firm, or uninterruptible [8]. The 'native' firm load is the highest level of firmness. This means that the wheeling transaction has the same priority as the "native" load of the utility providing the wheeling. Interruptible wheeling allows the utility providing the service to cease sending for specific reasons e.g. unavailability of surplus or transmission capacity.

Wheeling may be long-term, sometimes involving a contract term as long as ten or twenty years. Long-term wheeling is generally associated with long-term power purchase contracts or remote generation units. The wheeling may also be short-term i.e. from a few hours to a few months. This is generally the case for the exchange of economy energy or other temporary opportunity transactions, and for short emergency circumstances. Long-term wheeling poses a special challenge in determining the capability of the interconnected network. While adequate transmission capacity may be available at the beginning of a long-term transaction, facilities improvements or additions may be necessary that would not otherwise be regarded as soon or at all. This can result in a profound impact on pricing and terms of wheeling.

3.7 Wheeling Rates (Transmission Rates)

Transmission is the link between the generation and the local distribution systems. While there is a wide diversity of opinion on the principle and methods of charging for transmission services, there is a common agreement that transmission pricing should promote economic efficiency and encourage competition in electricity supply. Cost differences in the price of a *kWh* (kilowatt-hour) of electrical energy in the same region primarily lie in transmitting the power from its origin to the place of demand. The challenge here is to define a pricing scheme for transmission services that provides a coherent economic incentives for business to efficiently operate and expand. The methodology by which the cost of wheeling is computed is a high priority problem throughout the power industry due to the growth in transmission facilities.

Many different methodologies have been proposed for transmission charges (wheeling charges)-a payment for using a transmission system. One of the popular methodologies in the USA is probably the $Megawatt - Mile$ ($MW - Mile$) [15] method i.e.

$$\begin{gathered} Cost\ to\ use\ the\ transmission\ system\ for\ a\ transaction = \\ \sum j[MW_j * Length\ in\ Miles_j * Rate_j\ \$ / MW - Mile]. \end{gathered} \tag{3.1}$$

The MW on any circuit for a transaction is determined by network distribution factors which give each transactions breakdown over the entire transmission system. The rates are adjusted seasonally.

Reference [85], suggests that spot pricing can be used as a vehicle for defining wheeling rates. The theory underlying wheeling rates using marginal cost pricing has been introduced in [77]. Reference [86] summarizes further some of the algorithms based on the idea of marginal wheeling pricing. Real-time pricing of wheeling means that electricity prices to the customer follow as closely as technically practical the real cost of electricity at the time it is produced and supplied. In short, an energy marketplace for electric power is established. Spot pricing, does away with concepts such as block rate, demand charges, back-up charges, and so on. Note that despite the unbundling of generation, transmission and distribution, the system still operates as one integrated single entity to deliver energy economically and reliably to the consumer [87].

Modifications of the OPF models are used in [11,76] to calculate real time pricing of real and reactive power wheeling. The models use the variation of fuel cost for generation to estimate the rate structures. As will be seen in Chapter 6, unlike these models, the model proposed in this thesis incorporates in addition to variation of fuel cost for generation, the optimal allocation of transmission system operating costs based on time-of-use pricing. The transmission costs include all variable operating and maintenance costs, that are flow dependent but exclude transmission loss costs. These are included in the demand and nodal balance relationship. Marginal costing is applied to transmission pricing in wheeling. The short-term marginal cost of wheeling between two buses is defined as the difference in the cost of producing an additional megawatt at each bus [88]. Expressed in terms of partial derivatives this is:

$$\text{Marginal cost} = \left[\frac{\partial f_1}{\partial MW_1} - \frac{\partial f_2}{\partial MW_2}\right] \quad , \tag{3.2}$$

where:

f_1 is production cost rate, \$/hour at bus 1,

f_2 is production cost, \$/hour at bus 2,

MW_1 is power injection at bus 1 and

MW_2 is power injection at bus 2.

The quantities $\frac{\partial f_1}{\partial MW_1}$ and $\frac{\partial f_2}{\partial MW_2}$ are defined as the spot prices at bus 1 and bus 2 respectively. These marginal costs are available from OPFs which use partial derivatives to minimize the objective function. If the objective function is the production cost, the partial derivatives of the cost with respect to real power can be obtained for each bus in the system.

The wheeling costs are computed from the spot prices (marginal costs) at the buses where it enters and leaves the wheeling utility. This provides a coherent, consistent engineering and economic basis for settling wheeling rates. The package MINOS [16] is used in this thesis to compute the spot prices at each bus. These prices are then used to establish the marginal wheeling costs.

The real time (spot) pricing functions as a load management tool because it interacts with consumer behavior. In practice, every time power is bought or sold, it must be metered to know how much of the product (MWh) has been delivered. Deregulation has opened the way for a great deal of metering. The most important aspect of metering is that it must record MWh use or production for each hour. There is a need to automatically record and then download the metered data to a central accounting system. Most interesting, is the ultimate possibility of requiring each and every residential and small business load to have a kWh meter that records this hourly usage. With 'retail" wheeling, the metering and accounting functions become enormous.

3.8 Advantages of Real-Time Pricing

There are several advantages suggested in favor of real-time pricing [89] where the price signal is set to reflect the instantaneous cost of production. Among some of the advantages are the following:-

a. as a load management tool, this type of pricing redistributes demand away from expensive production periods to other times when more cost-effective generation can pick some demand and

b. as a consumption rationalization tool, base period consumers which occurs with present day average cost based tariffs. In general, rational consumers will at anytime, pay more/less and consume less/more, depending on how important consumption at any particular time of day/week/ season is to them.

3.9 Open Transmission Access

Open Transmission Access (OTA) is another issue related to evaluation of wheeling. This is another form of deregulation whereby utilities give "open access" to their transmission systems to both suppliers of power and customers. This means, like a sidewalk, anyone who chooses to, may use it. It is a relatively new concept whose economic, regulatory, and implementation structure continues to be adapted to the specific needs of each nation. In broad terms OTA refers to the regulatory construct (e.g., rights, obligations, operational procedures, and economic conditions) enabling two or more parties to use a transmission network, belonging totally or in part to another party or parties, for electric power transfers [90].

Under OTA, consumers are no longer constrained to purchase their power from the regional utility. The consumers can shop around for the best price. Retail customers for example can choose their supplier and wheel the power across the local transmission and distribution providers. In short, "open access" demands that a new generator should be free to operate from any location. This initiative will change the face of how electricity has traditionally been produced and transmitted.

A good example is in the USA, where utilities are being encouraged to create Independent System Operators (ISOs) [83] who are responsible for providing access to the grid in a large geographic region. This includes the transmission system of several interconnected utilities. The ISO and its employees can have no financial interest in the economic performance of any market participant. They adopt and enforce strict conflict-of-interest standards. The transmission owner is not required to give up ownership of transmission, but is required to give up how the system is operated. The ISO provides non-discriminatory, open access transmission and reliability of the system. The ISO principle, removes transmission from the economic control of the wholesale market and is designed to foster a competitive environment. Probably the segment which is more profoundly affected by this deregulation is power generation. Regulators allow the generation business to sell power at market-based rates.

The new structure in California [83,91] is already set up to have generators sell their energy into a market-based pool. Utilities and marketers will then buy all of their energy out of the pool. This market deregulation and lifting of price controls creates a liquid market that allows electricity to be a commodity traded in the market. Power marketers or energy merchants are presently active in the wholesale market. They do not own generation, but instead buy and sell energy and use future markets to manage price volatility. By mid 1997, over 350 power marketers had entered the market in the USA. They have since become the biggest traders of wholesale power. The end result is that competition will be fierce in future. Generators will be competing with power marketers to sell in the wholesale and retail markets. Power marketers will utilize new risk management tools and creative options not offered by utilities [83].

3.9.1 Scheduling Procedures

The scheduling process in California has been designed to provide reasonable time frames so that the ISO can do periodic analyses to ensure reliable transmission system operation and market participants can make operational decisions. The process is also meant to encourage parties to accurately forecast loads and generation and give market participants price security [91]

There are two time frames; namely

a. the day-ahead schedule to be completed approximately 5-10 hours before the beginning of the operating day (12 a.m.) and

b. hour-ahead schedule which occur approximately an hour prior to each hour within the operating day.

For the day-ahead schedule, the ISO uses a computer modeling tool to evaluate whether all the schedules that it receives can be accommodated at the same time on the transmission system. If there is congestion, the ISO informs scheduling parties how, based on modeling results, it would eliminate congestion, if it were to do so, and at what price.

The market participants are free to select another schedule to avoid anticipated congestion fees. The ISO then resolves any remaining congestion by making cost-minimized schedule modifications for generators that have indicated a willingness to be rescheduled. The transmission congestion charges are intended to act as economic signals to induce market participants to build new generation and transmission facilities at appropriate sites.

The “day-ahead’ accepted schedule has become the basis for settlement in the real-time market (hour-ahead schedule). The goal is to move the time at which the last schedule adjustments can be accepted by the ISO to as close to real time as possible.

3.9.2 Impact of Deregulation

In principle, “open access” is a fine idea, but there is a problem in devising a system of charging for the services that is consistent with this principle. Also, the impact on the network is that certain transmission corridors may be forced to carry more power than they are designed for. The result can be a myriad of technical difficulties such as decreased reliability and increased wear and tear on system equipment [82,92].

A good example are the blackouts that affected western USA and Canada in July and August of 1996. Both blackouts were worsened by unscheduled power flows along the

North-South coastal corridors [82]. For example, in August of the same year, California was experiencing a heat wave. Large amounts of electric power were shipped along two parallel tie lines (one AC and one DC) from the north-west, south to the heavily populated areas of California. However, since electric current takes the path with least impedance, not all of the current flowed through the desired corridor. The transmission lines forced to carry extra current began to overload since they were not designed to carry such large amounts of current. As the line overloaded, they began to heat up due to large I^2R (resistive) losses. As the heating up continued, the lines expanded and began to sag. Finally, one line sagged enough to make contact with a tree and thus causing a short circuit to ground. As this line was removed from service, the current was then shunted in greater amounts over the remaining lines causing more overloads [82].

As the above case shows, transmission lines have limits. Transfer capacity is governed by a number of factors, not just physical current carrying capacity. There are limits which are imposed by stability considerations. Too much power across a particular line could result in generators pulling out of synchronism. This is manifested by large oscillations in power. A phenomenon called voltage collapse can also result; the voltage in the system slowly declines until it suddenly drops, with little or no warning. The power system could be pushed to the limits of its capability. Unfortunately, this limit is not usually known ahead of time. One solution might be to add more transmission capacity but then this is not a typically viable option since there is bound to be some resistance from the public.

The centralization of control under one "independent" institution, encompassing multiple ownership systems and control areas, can create a number of opportunities and threats for traditional electric power utilities: not only with regard to security, control and planning issues, but also with regard to environmental protection strategies. The future impact of electric restructuring on environmental protection is unclear at this point unclear [13]. The pressures of competition and reduced regulation could jeopardize decades of effort (and struggle) between environmentalists and utilities to reduce the level of pollutants. Alternatively, the environment may benefit from increased energy efficiency and reduced emissions.

However, open transmission access will in the long run transform the paradigms of system control. It is the ideal, even if actual practice falls short where the sector of the power industry has been legislated and the industry is unbundled [93]. There are considerable differences in how open access is practiced in different countries. Although the independent system operator system (ISOs) is already being practiced in other countries under a variety of names such as National Grid, State or Public Transmission Company, it is not analyzed beyond this point in this thesis.

Chapter 4

FORMULATION OF THE OPTIMAL POWER FLOW PROBLEM

Introduction

As mentioned in Chapter 2, Carpentier [19] introduced a generalized nonlinear programming formulation of the economic dispatch problem in power delivery. This was later named optimal power flow (OPF) by Dommel and Tinney [20]. Since then, OPF computation has received widespread attention. It is of current interest to many power utilities, and has been identified as one of the most important operational tools of the power industry.

OPF can be used to determine the best control settings to accommodate wheeling or no-wheeling options so as to maintain system security. The task of quickly and accurately evaluating the merits of such options is becoming an important function of utility system planners. Also the methodology by which these options are implemented is a high priority due to the growth in transmission facilities, the cost differentials between utility companies and the dramatic growth in non-utility generation capacity.

In this chapter, a description of the formulation and implementation of OPF incorporating transmission costs to establish electricity rates for the consumers is presented. The formulation is then extended to wheeling conditions. The required tools used are presented first and are then followed by the theoretical aspects of the implementation.

4.2 Formulation Tools Used

For successful implementation of the OPF, a set of data and an optimization program are necessary. The set of data used will be described and presented in the corresponding sections. The optimization is carried out using a software package called MINOS [16] (already mentioned in Chapter 1), designed for solving large scale linear or nonlinear optimization problems. The description of the MINOS package will also be presented in this chapter since MINOS has a generalized approach to solve any optimization problem. The capability of this package in handling the optimization power flow problem will be emphasized.

4.3 Data Available for the OPF Implementation

The available data are the standard IEEE test systems. There are four test systems used in this study, namely the 5,14, 30, and 57 bus systems. The generators under consideration in these test systems are all thermal. It is assumed that the fuel-cost curves are available for all committed generating units. The optimization then concerns this particular set of generators. The fuel cost parameters, active power generation limits, line flow limits and transformer tap ratio limits are assumed for the test systems. The reactive generation limits are also assumed. Though the problems solved in this thesis are similar in nature, each has its own unique characteristics.

4.4 Objective Used in the OPF Based Model

The usual economic dispatch problem formulation used in establishing electricity rates is the minimization of fuel cost of operating a power system subject to operational constraints. The objective function used under these conditions is generally expressed as follows:

$$F = \sum_{i \in R_G} (\alpha_i + \beta_i P_{Gi +} \gamma_i P_{Gi}^2) \ , \tag{4.1}$$

where:

P_{Gi} is the active power generation at bus i ,

R_G is a set of all generating buses,

α_i is the fixed fuel cost at generator i ,

β_i is the variable fuel cost in proportion to active power at generator bus i and

γ_i is the variable fuel cost in proportion to second order term of active power at bus i .

The economic dispatch objective depicted in Equation (4.1), has been used as a modified OPF by some researchers [11,94,95,96] to calculate the real time pricing of real and/or reactive power. These models use the variation of fuel cost of generation to estimate rate structures. As already stated in Chapter 1, many times what is overlooked is the fact that cost differences in prices in delivering a kilowatt-hour of electrical energy across the same territory is not just due to the costs of generating the electricity. The cost differences, lie heavily in the costs of transmitting this power from its origin to the place of demand.

With this in mind, unlike other models, the model proposed in this thesis is a modification of the OPF model that incorporates in addition to variation of fuel costs for generation, the optimal allocation of transmission system operating costs based on time-of-use pricing. These costs under non/or wheeling conditions are obtained by assigning the same price k to each unit of reactive and active power flows respectively in the network. For wheeling conditions, the short-term marginal costs of wheeling between two buses is found by the differences in the costs of producing an additional megawatt at each bus as already discussed in Chapter 3. The hourly spot pricing is the base for the proposed algorithm. The analysis spans a total time period of twenty-four hours. Both non/or wheeling rates of real and reactive power are captured in this model unlike the algorithms which use DC [97-99] models which capture real power rates only.

As an optimization problem, the main objective of OPF is to minimize the total cost of generation plus the transmission capacity. In reference [100] the assignment of a price k ignores the reactive power, but in [80], the reactive power is taken into consideration. Both these cases are reported in this thesis. By comparison, the operating cost of producing reactive power is independent of fuel usage. If the price k is assigned to each unit of reactive and real flows injected at each bus i, the objective function of the OPF problem is expressed as:

$$\text{Minimize:} \quad C_T(P_G, V, \delta) = \sum_{i \in R_G} C(P_{Gi}) + \sum_{i \in N_B} k[P_i(V,\delta) + Q_i(V,\delta)], \tag{4.2}$$

where:

$C(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2$ is the operating fuel cost of producing P_{Gi} units of real power at generating plant at bus i, α_i, β_i *and* γ_i are constant parameters as defined in Section 4.3 for generator i and $kP_i(V,\delta)$, $kQ_i(V,\delta)$ are the real and reactive transmission operating costs respectively. This economic dispatch problem expressed by Equation (4.2) is subject to a number of constraints which are discussed in the following section.

4.4.1 The OPF Constraints

There are certain factors which have to be considered in order to come up with a more realistic model applicable to spot-pricing theory. In addition to transmission costs, some of the factors that should be incorporated into the model are quality and technical limitations. All the resulting constraints are considered during one time interval only. Subsequent intervals use the same variable allocations.

Equality Constraints (or Network Constraints)

These are constraints determined by Kirchoff's laws, that characterize the power flow throughout the system. Kirchoff's laws require that real and reactive power flows balance at each bus i throughout the network and are expressed as follows:

$$P_{Gi} - P_{Di} - \sum_{i \in N_B} |V_i||V_j||Y_{ij}| \cos(\delta_i - \delta_j - \theta_{ij}) = 0 . \tag{4.3}$$

$$Q_{Gi} - Q_{Di} - \sum_{i \in N_B} |V_i||V_j||Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij}) = 0 . \tag{4.4}$$

Many power system textbooks [101-104] fully explain the derivation of these expressions of the equality constraints. For a given time t, and at a given bus i, the above equality constraints can be expressed in a simpler form as follows:

$$P_{Di}(t) - P_{Gi}(t) + P_i(t) = 0 \qquad i = 1, \ldots\ldots N_B . \tag{4.5}$$

$$Q_{Di}(t) - Q_{Gi}(t) + P_i(t) = 0 \qquad i = 1,......N_B . \tag{4.6}$$

The corresponding Lagrange multipliers MC_{pi} and MC_{qi} give the marginal cost of supplying real and reactive power at bus i respectively. A given bus may have a generator and/or a load. When one does not exist at a given bus, then the corresponding power variable is set to zero at that bus.

Power Generating Limits

The generating constraints give the maximum and minimum generating capacity, outside of which it is not feasible to generate due to technical or economic reasons. The generating limits are expressed as follows:

$$P_{Gi,\min} \leq P_{Gi} \leq P_{Gi,\max} , \tag{4.7}$$

$$Q_{Gi,\min} \leq Q_{Gi} \leq Q_{Gi,\max} , \tag{4.8}$$

where:

$P_{Gi,\min}$ and $P_{Gi,\max}$ are the minimum and maximum active power output generated at bus i ,

$Q_{Gi,\min}$ and $Q_{Gi,\max}$ are the minimum and maximum reactive power output generated at bus i.

The value of $P_{Gi,\min}$ depends on boiler stability requirements and is not necessarily zero. The reactive power limits on the other hand depend on system excitation limits. The upper generation limit $P_{Gi,\max}$ takes into consideration the power system's operating reserve requirements. The power generating constraints can be regarded as a generation quality of supply component of the hourly spot-price [105].

Transmission Limits

These constraints represent the maximum power a given transmission line is capable of carrying and are usually based on thermal and dynamic stability considerations and these constraints can be expressed as follows:

$$P_{ij,mim} \leq P_{ij}(t) \leq P_{ij,\max} \qquad i, j = 1,........, N_B . \tag{4.9}$$

where, if one assumes an all inductive model of the transmission links

$$P_{ij} \;=\; \frac{V_i V_j}{X_{ij}} \sin(\delta_i - \delta_j) \;, \tag{4.10}$$

X_{ij} is the reactance of the line between nodes i and j,

$P_{ij}(t)$ is the power flow between bus i and j at time t,

$P_{ij,\min}$ and $P_{ij,\max}$ are minimum and maximum active power flow respectively.

Voltage Limits

This is usually a service quality requirement. Thus, to satisfy legal requirements and design limitations, the voltage magnitudes are restricted to lie between specific upper and lower limits expressed as follows:

$$V_{i,\min} \leq V_i \leq V_{i,\max}, \tag{4.11}$$

where:

$V_{i,\min}$ and $V_{i,\max}$ are the minimum and maximum voltage levels respectively. The limits used in this thesis are 0.9 p.u. and 1.10 p.u. respectively. Since voltages are affected primarily by reactive power flows, any voltage limits will show up as a premium on the price assigned to reactive power. Wherever necessary, reactive power sources are used in the system to keep the voltages within the required limits.

Phase Angle Limits

Since the power flow between buses i and j is already constrained as expressed by Equation (4.9), there is no need to constrain the voltage phase angles. For the purpose of the solution approach used in this thesis (Chapters 5 and 6), the voltage phase angles are constrained within $\pm\frac{\pi}{2}$ i.e.

$$-\frac{\pi}{2} \le \delta_i \le \frac{\pi}{2}. \tag{4.12}$$

Since this is not part of the model constraints, this constraint is absent (as will be seen later) in the Lagrangian function. The bounds for the phase angle may be varied depending on the problem under consideration.

However, the upper and lower limits of the constraints that have been presented may change with time. In most applications, the bounds are fixed quantities for a particular network. The work reported in this thesis considers only fixed values of the lower and upper limits of the constraints.

4.5 Solution to the OPF Model

The constrained minimization problem outlined or discussed so far, can be now transformed into an unconstrained minimization problem. In order to achieve this, the power flow constraints are incorporated into the objective function using the method of Lagrange multipliers [106]. This results in a Lagrangian function whose critical point is given by optimal values of the function variables and the Lagrange multipliers.

The Lagrangian function to be minimized over all active power generation levels, P_G, reactive power generation levels, Q_G, voltages, V, and voltage phase angles, δ, is now:

$$
\begin{aligned}
L_f(P_G,Q_G,V,\delta) = \quad & C_T(P_G,V,\delta) \\
& -\sum_{i\in N_B}(MC_{pi})[P_{Gi}-P_{Di}-\sum_{j\in N_B}|V_i||V_j||Y_{ij}|\cos(\delta_i-\delta_j-\theta_{ij})] \\
& -\sum_{i\in N_B}(MC_{qi})[Q_{Gi}-Q_{Di}-\sum_{j\in N_B}|V_i||V_j||Y_{ij}|\sin(\delta_i-\delta_j-\theta_{ij})] \\
& -\sum_{i\in R_G}\lambda_{i,\min}(P_{Gi}-P_{Gi,\min})+\sum_{i\in R_G}\lambda_{i,\max}(P_{Gi}-P_{Gi,\max}) \\
& -\sum_{i\in R_G}\mu_{i,\min}(Q_{Gi}-Q_{Gi,\min})+\sum_{i\in R_G}\mu_{i,\max}(Q_{Gi}-Q_{Gi,\max}) \\
& -\sum_{i\in N_B}\sum_{i\in N_B}^{j\neq i}\eta_{i,\min}(P_{ij}-P_{ij,\min})+\sum_{i\in N_B}\sum_{i\in N_B}^{j\neq i}\eta_{i,\max}(P_{ij}-P_{ij,\max}) \\
& -\sum_{i\in N_B}\nu_{i,\min}(V_i-V_{i,\min})+\sum_{i\in N_B}\nu_{i,\max}(V_i-V_{i,\max}) \ . \qquad (4.13)
\end{aligned}
$$

The set of solutions for the problem expressed by Equation (4.13) is obtained by taking the first derivative of L_f with respect to each variable or Lagrangian multiplier for each time interval. For a global minimum value of L_f to exist, the derivative must be equal to zero This results in the Karush-Kuhn Tucker (KKT) conditions [30,106-110]. These equations are analyzed further in the Appendices.

Starting from a reasonable initial guess, the solution to the above set of equations gives the real-time global optimum values for all of the variables in the Lagrangian Equation (4.13). In this thesis, the initial guess is obtained through trial and error of a number of different initial guesses. In subsequent simulations, the previous solution is used as the initial guess.

4.6 Real-Time Rates (or Spot Pricing)

Real-time pricing represents a radically different approach to marketing electricity. Traditional methods of pricing electricity for the vast majority of customers are based on the average cost of generation, transmission and distribution. These methods do not recognize the variations from day to day or hour to hour even though there are marked differences in instantaneous costs. Spot pricing on the other hand, allows utilities to introduce time and space-differentiated pricing schemes which can be used to unbundle (or separate) the electric services offered to consumers.

Based on marginal costs [95,105], the real-time pricing based electricity rates of real power at a particular time and bus i are determined at the optimal operating point of L_f expressed in the following form:

$MC_{pi} = \dfrac{\partial}{\partial P_{Di}}$ [total cost of supplying electricity to all customers subject to operational constraints]

$$=\frac{\partial L_f}{\partial P_{Di}}. \tag{4.14}$$

Likewise, the real-time price of reactive power based on the marginal cost at bus i will be as follows:

$MC_{qi} = \frac{\partial}{\partial Q_{Di}}$ [total cost of supplying electricity to all customers subject to operational

constraints]

$$=\frac{\partial L_f}{\partial Q_{Di}}. \tag{4.15}$$

Note that these are the Lagrangian multipliers for the Lagrangian Equation (4.13). Further explanation is given in the Appendices. MC_{pi} and MC_{qi} are rates in $\$/MWh$ and $\$/MVARh$ respectively.

4.7 MINOS

MINOS is an optimization program coded in FORTRAN. This package mainly solves optimization problems which have linear and/or nonlinear objective functions which are unconstrained, linearly constrained and/or non-linearly constrained. The technique used by MINOS in finding the optimal solution, depends on the problem under consideration. The optimization problems that can be handled by this software can be classified as follows:

- linear programming,
- linearly constrained optimization with nonlinear objective function,
- unconstrained optimization and
- non-linearly constrained optimization with a nonlinear objective.

Generally, the optimization problem can be mathematically expressed in the following standard form:

Minimize: $$F(x) + c^T x + d^T y, \tag{4.16}$$

Subject to: $$f(x) + A_1 y = b_1, \tag{4.17}$$

$$A_2 x + A_3 y = b_2, \tag{4.18}$$

$$l \leq \begin{pmatrix} x \\ y \end{pmatrix} \leq u, \tag{4.19}$$

where:

c, d, b_1, b_2, l and u are constant vectors,

A_1, A_2 and A_3 are constant matrices,

$F(x)$ is a smooth scalar function,

$f(x)$ is a vector of smooth functions $f_i(x)$,

x are the n_1 nonlinear variables and

y are the n_2 linear variables.

MINOS, ideally needs the first derivatives of $F(x)$ and $f_i(x)$ to be known and supplied by the user. The generality of the expression of the optimization problem given above, allows a number of different problems to be formulated. In the following sections, some of the problems are briefly discussed but more time will be spent on the work under investigation in this thesis.

4.7.1 Linear Programming

By eliminating $F(x)$ and $f(x)$ in the equation set (4.16) to (4.19), the problem becomes one of linear programming. Thus, linear programs are expressed as:

Minimize: $$c^T x \ , \tag{4.20}$$

Subject to: $$A x + Is = 0, \tag{4.21}$$

$$l \leq \begin{pmatrix} x \\ s \end{pmatrix} \leq u \ , \tag{4.22}$$

where:

the elements of x are called structural variables (or column variables),

s is a set of slack variables (also called logical variables),

l and u are the redefined bounds.

MINOS solves such problems (linear programs) by using the primal simplex method [108]. Basically this method partitions the constraints expressed by Equation (4.21) into the form:

$$Bx_S + Nx_N = 0 \ , \tag{4.23}$$

where:

B is the basis matrix which is square and nonsingular,

x_S and x_N are the basic and nonbasic variables respectively.

The simplex method reaches a solution by performing a sequence of iterations, in which one column of B is replaced by one column of N (and vice versa), until no such interchange can be found that will reduce the value of $c^T x$.

4.7.2 Nonlinear Objective Function

If there are nonlinearities only in the objective function $F(x)$, the problem becomes a linearly constrained nonlinear program. MINOS solves such problems by using a Reduced-Gradient Algorithm [110] in conjunction with a Quasi-Newton Algorithm [111]. Details of the implementation are given by Murtagh and Saunders [112]. Unlike the linear programming in the previous section, the constraints represented by Equation (4.6) are partitioned into the form:

$$Bx_B + Sx_S + Nx_N = 0 \ , \tag{4.24}$$

where:

x_S is a set of superbasic variables.

4.7.3 Nonlinearly Constrained Problem

The problem generally expressed by Equation (4.16) to (4.19) may have a nonlinear objective function and nonlinear constraints. Assuming the nonlinear objective function is expressed by $F(x)$ and the nonlinear constraints by vector functions $f(x)$, the problem expressed by Equation (4.16) to (4.19) can therefore be redefined as follows:

Minimize: $F(x)$, (4.25)

Subject to: $f(x) \leq 0$ and (4.26)

$$l \leq x \leq u \ . \tag{4.27}$$

Application methods of nonlinear programming are widespread and the different approaches to tackle the problems are many [107,109,113]. Under such scenarios, MINOS uses a projected Augmented Lagrangian Algorithm [114]. By using a first order Taylor's series approximation, the nonlinear constraints are then linearized. The first two terms of Taylor's series are considered only in this case. The approximation at the k^{th} linearization is generally expressed as:

$$\tilde{f}(x, x_k) = f(x_k) + J(x_k)(x - x_k) \ , \tag{4.28}$$

or in a simpler form:

$$\tilde{f} = f_k + J_k(x - x_k) \ , \tag{4.29}$$

where:

x_k is the estimate of the nonlinear variables at the start of the k[th] major iteration and $J(x)$ is the Jacobian of the constraints function vector $f(x)$ i.e.

$$J(x) \quad = \quad \nabla f(x) = \begin{bmatrix} \frac{\partial f_1}{dx_1} & \frac{\partial f_1}{dx_2} & \cdots & \frac{\partial f_1}{dx_{n1}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{n1}}{dx_1} & \frac{\partial f_{n1}}{dx_2} & \cdots & \frac{\partial f_{n1}}{dx_{n1}} \end{bmatrix}. \tag{4.30}$$

From the above, the modified augmented Lagrangian is given by:

$$L = F(x) - \lambda_k^T (f - \tilde{f}) + \tfrac{1}{2} \rho \ (f - \tilde{f})^T (f - \tilde{f}), \tag{4.31}$$

where:

λ_k is the estimate of λ the Lagrangian multipliers for the nonlinear constants,

ρ is a penalty parameter and the term involving ρ is a modified quadratic penalty function. The subproblem to be solved in this case is expressed as:

$$\text{Minimize } F(x) - \lambda_k^T (f - \tilde{f}) + \tfrac{1}{2} \rho \ (f - \tilde{f})^T (f - \tilde{f}), \tag{4.32}$$

$$\text{Subject to :} \quad \tilde{f}(x) \leq 0 \ \text{ and} \tag{4.33}$$

$$l \leq x \leq u. \tag{4.34}$$

The objective function depicted in Equation (4.32) from Equation (4.31) is called the augmented Lagrangian. This is now a linearly constrained problem. MINOS uses the Reduced Gradient Algorithm to minimize this subproblem expressed by Equation (4.32) subject to the now linear constraints, Equation (4.33). The nature of the solution process can be briefly stated as follows. A sequence of major iterations is performed, each one requiring the solution of a linearly constrained subproblem. The subproblems contain the original linear constraints and bounds, as well as linearized versions of the nonlinear constraints.

The optimal power flow methodology proposed in this thesis to establish the electricity rates falls into this category i.e. a nonlinear constrained problem. The objective is to minimize fuel cost plus transmission cost which is a nonlinear function. The constraints functions are the nonlinear power flow equations. All inequality constraints in P_{Gi}, Q_{Gi}, V and δ can be modeled into the MINOS software to obtain the solution to the problem being proposed in this work. The following sections describe concisely how the implementation was drafted into the MINOS package.

4.7.4 Requirements For Using MINOS

For the optimization application using MINOS, the user is required to supply some data and some FORTRAN coded routines. These routines are then used by MINOS to solve the problem under consideration. The required routines are described in the following:

The MPS file: All the problems require this file. The file should specify and define the following:

- variables names (referred to as COLUMNS),
- names of constraints (referred to as ROWS),
- variable limits,
- constraints limits (if any) and
- initial or starting point.

Reasonable initial values are significant. In the case of successive problems to be solved, it is advisable to use the solution from the previous step as a starting point. The bounds or limits on the variables are not specified in the Rows section as constraints. They are specified separately in the BOUNDS section.

If the default option of JACOBIAN=DENSE is used, any constant non-zero Jacobian elements may be specified in the columns section besides the respective variable. If the alternative option JACOBIAN=SPARSE is selected, all non-zero elements of the Jacobian must be specified in the Columns section. Here, the constant known values can be given their appropriate values and the others should be given a dummy value such as zero. The actual values of these elements will be computed later by user written subroutines for the case of nonlinear problems.

The SPECS file: In this file, several parameters are defined which characterize the type of problem to be solved. Among the parameters to be defined are; tolerances, number of Rows and Columns, etc. Most of these parameters have default values and need not be specified in the SPECS file unless they are to be changed. An example of such parameter describes the manner in which the derivatives of the constraints with respect to the variables should be stored. The key word is JACOBIAN and the parameter is either DENSE or SPARSE as described previously.

The FUNOBJ routine: This is a routine to be supplied by the user. It specifies in FORTRAN code, the problem's objective function. In this thesis, as already stated, the objective is to minimize all fuel costs plus transmission costs in power delivery. This routine also specifies all non-zero gradients of the objective function with respect to each of the objective function variables.

The FUNCON routine: Like the FUNOBJ routine, this is a user supplied routine. It specifies in FORTRAN code, all the problem's constraints and all their non-zero gradients (Jacobian). The gradients can be stored as sparse or dense matrix depending on the solution approach desired.

The MATMOD routine: This is an optional user supplied routine that may be required by MINOS. It is required only when the formulation involves a sequence of closely related problems.

In all these routines, specific formats have to be strictly adhered to when creating them. Any non-zero derivatives, not computed by these subroutines are computed by MINOS using the finite differencing method. On exit, MINOS indicates the nature of exit and creates various output files as desired. It also saves the relevant information in a BASIS file if requested (like a snapshot). In turn, this can be used in a subsequent run of the same or modified problem as a starting BASIS.

4.8 The OPF Problem Modeling with MINOS

A description of the use of MINOS in solving the OPF problem expressed by Equation (4.2) to (4.12) is given in this section. There are a number of variables connected with these equations as already explained. The first phase in MINOS is to allocate the variables of the problem compatible with the MINOS software. This variable matching is significant since wrong variable definition can result in a wrongly defined Jacobian. In turn, a solution may not be found because of the incompatibility. A good starting point is also an additional important thing.

4.8.1 Variables Allocation

The allocation of variables to be used by MINOS depends on which routine the variables are to be used in. There are variables for the objective function routine FUNOBJ and for the constraint function(s) routine FUNCON. The function may or may not be differently defined in the two routines. The same applies to the variables.

4.8.2 The Objective Function

The objective function under investigation is a cost minimization function. The variables involved in this function are P_{Gi}, V and δ. Restating the objective function:

$$C_T = \sum_{i \in N_G} (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) \quad + \quad \sum_{i \in N_B} k[P_i(V,\delta) + Q_i(V,\delta)], \tag{4.35}$$

where:

N_G is a set of all generating buses and

N_B is a set of all load buses in the network.

The cost function C_T consists of fuel costs for the N_G generators and transmission costs in the network. Therefore the variables involved in this objective function are:

- N_G variables for the active generation variables,
- N_B variables for the bus voltages in the transmission costs and
- N_B variables for the bus phase angles in the transmission costs.

This gives $N_T = N_G + 2N_B$ variables for the objective function. These variables are mapped for the MINOS routine FUNOBJ, into an array of N_T variables i.e.

$$\begin{bmatrix} P_{Gi} \\ \vdots \\ P_{GN_g} \\ V_1 \\ \vdots \\ V_{N_B} \\ \delta_1 \\ \vdots \\ \delta_{N_B} \end{bmatrix} \rightarrow \begin{bmatrix} X_1 \\ \vdots \\ X_{N_T} \end{bmatrix} \text{ where } i \in N_G. \tag{4.36}$$

It follows that the derivatives of the objective function with respect to the variables are mapped in terms of X. For example, the derivatives of the total cost function with respect to P_{Gi} variable will be mapped as follows:

$$\frac{\partial C_T}{\partial P_{Gi}} \rightarrow \frac{\partial C_T}{\partial X_i} \text{ where } i \in N_G. \tag{4.37}$$

4.8.3 The Constraint Functions

The constraints functions of the problem are expressed by Equations (4.3) to (4.12). These functions can be summed up as follows:

$$PM_i = P_{Gi} - P_{Di} - P_i(V,\delta) \qquad i = 1,\dots\dots\dots, N_B \ . \tag{4.38}$$

$$QM_i = Q_{Gi} - Q_{Di} - Q_i(V,\delta) \qquad i = 1,\dots\dots\dots, N_B \ . \tag{4.39}$$

The above constraints functions give rise to a total of $2N_B$ constraint functions. These equations and their variables were fully explained in Section 4.4.1. The variables for these constraints can now be categorized as follows:

- the N_G active generation variables,
- the N_G reactive generation variables,
- the N_{SC} reactive generation variables ,
- N_B the bus voltages and
- N_B the bus phase angles

This results in $N_C = 2N_B + N_{SC} + 2N_G$ constraint function variables. Some constants like the voltage angle of the swing bus(es) are still considered as variables. These are then fixed (fixed variables) in the MINOS SPECS file. The variable mapping in this case will be as follows:

$$\begin{bmatrix} P_{G1} \\ \vdots \\ P_{Gi} \\ \vdots \\ P_{GN_B} \\ Q_{G1} \\ \vdots \\ Q_{Gi} \\ \vdots \\ Q_{GN_B} \\ Q_{SC1} \\ \vdots \\ Q_{SCj} \\ \vdots \\ Q_{SCN_B} \\ V_1 \\ \vdots \\ V_{N_B} \\ \delta_1 \\ \vdots \\ \delta_{N_B} \end{bmatrix} \rightarrow \begin{bmatrix} X_1 \\ \vdots \\ X_{N_C} \end{bmatrix} \quad \textit{where} \quad i \in N_G \textit{ and } j \in N_S. \tag{4.40}$$

The constraint functions PM_i and QM_i can similarly be defined for MINOS using the following one to one mapping:

$$\begin{bmatrix} PM_1 \\ \vdots \\ PM_{N_B} \\ QM_1 \\ \vdots \\ QM_{N_B} \end{bmatrix} \rightarrow \begin{bmatrix} F_1 \\ \vdots \\ F_{2N_B} \end{bmatrix}. \tag{4.41}$$

The Jacobian following the above constraint functions with respect to the various variables is now given by:

$$J = \begin{bmatrix} \dfrac{\partial F_1}{\partial X_1} & \cdots & \dfrac{\partial F_1}{\partial X_{N_C}} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial F_{2N_B}}{\partial X_1} & \cdots & \dfrac{\partial F_{2N_B}}{\partial X_{N_C}} \end{bmatrix} . \tag{4.42}$$

Generally, this Jacobian has many zero entries since most of the variables do not appear in most of the equations. Hence, this Jacobian is highly sparse.

4.8.4 Rate Extraction From MINOS

After solving the OPF problem, MINOS outputs the results into a specified file. The rates are stored in the primary work array $Z(*)$ that is used by MINOS. The rates as defined by Equation (4.14) to Equation (4.15) are stored in the array $Z(LPI)$ to $Z(LPI+2NB)$. The variable LPI is the first integer in the $M5LOC$ common block. The active rates MC_{pi} are stored in $Z(LPI)$ to $Z(LPI+NB-1)$ and the reactive power rates MC_{qi} are in a similar stored in $Z(LPI+NB)$ to $Z(LPI+2NB-1)$. These rates are also used to calculate wheeling rates as already discussed in Chapter 3.

Chapter 5

COMPUTATIONAL RESULTS UNDER A REGULATED POWER ENVIRONMENT

Introduction

Some of the simulated results of the proposed OPF algorithm discussed in Chapter 4 are presented in this chapter. The IEEE standard test systems used in the studies are briefly discussed. Some of the data of the test systems are given in the Appendices. The studies are carried out under a regulated environment i.e. under the assumption that all the generation, transmission and distribution are under one utility.

In quite a number of countries, the production, transmission and distribution of electrical power is still under one utility or one supplying authority. Hence, the necessity of the studies reported in this chapter. Under such a regulated environment, there is a need to find an appropriate price policy that leads to the correct recovery of costs of the resources used in supplying electricity.

5.2 Details of the Test Systems

The application of the proposed algorithm was tested on standard IEEE test systems varying in size from 5-buses to 57-buses. The basic characteristics of the selected test systems are given in Table 5.1.

Table 5.1 Characteristics of the four IEEE standard test systems.

Number of buses	5	14	30	57
Number of Lines	7	20	41	78
Number of Thermal Generators	2	2	3	4
Number of Synchronous Condensers	0	3	3	3
Number of Transformers	0	3	4	15

The standard 5-bus system is taken from Stagg and El-Abiad [115]. This system consists of two generators. These two generators are both thermal and are identical. The generators are located on buses 1 and 2 respectively. The fuel cost models for the two generators are assumed to be quadratic and expressed as follows:

$$F_i = 53.60700 + 10.662 P_{gi} + 0.01165 P_{gi}^2 \quad \$/hr, \tag{5.1}$$

where P_{gi} is in MW.

The line parameters of the 5-bus system are given in the Appendices.

The 14-bus system, is an IEEE-AEP standard system [116]. Like the 5-bus system, the 14-bus system has two generators. They are located on buses 1 and 2 respectively. They are also thermal and the fuel cost models are given by:

$$F_i = 50.60700 + 10.662 P_{gi} + 0.01165 P_{gi}^2 \quad \$/hr \tag{5.2}$$

where:

P_{gi} are in MW.

The 30-bus system is also an IEEE-AEP standard system [116]. This system has three identical thermal generators. The three generators are located on buses 1, 2 and 13 respectively. The line parameters are given in the Appendices as well. The fuel cost models are expressed as follows:

$$F_i = 53.60700 + 10.662 P_{gi} + 0.11650 P_{gi}^2 \quad \$/hr \tag{5.3}$$

In the 57-bus system, there are four generators and the fuel costs are assumed to be the same as for the 30-bus system. The generators are located on buses 1, 3, 8 and 12 respectively. The line parameters are given in the Appendices as well.

5.3 Computational Results

In the following sections, the simulation results of the thermal OPF problem on all the standard test systems are documented and analyzed. For the simulations, it is assumed that the utility is able to set and communicate prices instantly to each customer. The studies are then done in three phases. In the first phase, only the fuel costs at the generating plants are considered for the simulations. This helps to compute the loss in revenue by comparing the rates obtained considering fuel costs only with those obtained when transmission costs are taken into consideration in addition to the fuel costs. For the second phase, the transmission costs involving only the active power (**P**) injected at the respective buses are considered in addition to the fuel costs. Each power flow unit is assigned a trial price k. The final phase, takes into consideration the fuel costs together with the transmission costs involving both the reactive and active power (**P & Q**) injected at the buses. In this final phase, each reactive and active power flow unit is assigned the same trial price k. All these phases, will be discussed in the following subsequent sections. The value of k used in all the simulations reported in this thesis is $\$100/MW$

and $\$100/MVAR$ for active and reactive power respectively. Also for all the simulations in the three phases, a 100-MVA base was used.

In each of the phases, the loading of each bus is considered to vary over a day, which is divided into two time intervals (8hrs and 16hrs). The hourly loads at each bus evolve using a load scaling factor (LSF). The LSF varies from 0.6 to 1.0. Only fully convergent load levels were taken as acceptable. Any violations or failure to converge by any load data set was discarded and a new load set put in place. For the second and third phases for the transmission costs considerations, the same assumed trial prices k per unit for both reactive and real power flows are firstly tested and the best suitable price is then assigned for the simulations reported in this chapter. The demand for the power flows and transmission on the electricity supply system, like the demand for any bundle of economic goods, depends upon the assigned transmission prices. The assignment does not discriminate between participants located at differing parts of the network. The hourly spot price is the base for the rate structure of the electricity supply being proposed.

The parameters of the fuel cost models for all generators where purposely made identical in each of the systems so that the difference in spot price in all the three phases could only be attributed to the transmission costs considerations. Also the total generation in each of the systems for the same time interval should remain constant in all of the three phases. The result is that the total cost at the generation plants is expected to remain constant in all the systems. This then makes it possible to compare the spot prices at the load buses.

Note also that for all the results reported in this chapter, the constraints boundaries at each bus were satisfied. Voltage boundaries obtained were acceptable in the range 1.0 ± 0.1 *pu*. Power generation and power flow boundaries were also found to be within acceptable boundaries.

5.3. 1 First Phase Results

Tables 5.2, 5.3, 5.4 and 5.5 show the real-time prices at different buses in the respective power system networks for the two time intervals (8hrs and 16hrs). These results are obtained considering only the variation of fuel cost at the generating plants thereby corresponding to the first phase of the studies. This is the classical economic dispatch OPF problem.

In the tabulated results for the 5-bus, 14-bus and 30- bus systems, it is generally observed that voltages at heavier loads (LSF 1.0) are slightly lower than at lighter loads (LSF 0.8). At the generator buses i.e. buses 1 & 2 for the 5-bus system, 1 & 2 for the 14-bus system, 1, 2 & 3 for the 30-bus system and 1, 3, 8 and 12 for the 57-bus system, the voltages remain relatively constant in the two time intervals. In the 57-bus system, the voltages at the load buses are distorted by the voltage constraints since some simulation results (not shown in this chapter) at lower LSF than 0.8 (i.e. at LSF 0.7) the voltages do show the same trend observed in the other systems.

In all the systems under investigation, the OPF schedules higher generation during high demand (corresponding to the first interval in each system) than during low demand i.e. in the second interval. Though the difference between the LSF for higher demand and the low demand in all the systems is 0.2, in the 57-bus system the OPF schedules a higher level of capacity to meet the increase than in any of the other systems. In the 5-bus, 14-bus and 30-bus systems, the increase in capacity is closely in the same range.

The real-time prices at the corresponding buses in the two time intervals show the opposite effect to that of voltages i.e. the real-time prices at heavier loads are higher than those at lighter loads. This is true for both real and reactive power rates. This is expected, since at higher demand the OPF schedules higher thermal generation and hence more thermal fuel costs than required during low demand. The differences in the reactive power rates at the corresponding buses for the two time intervals are very small except in the 57-bus system. For the active power rates, the differences in the two time intervals at the corresponding buses are significant.

Comparing the active power rates (MC_{pi}) to the reactive power rates (MC_{qi}), the rates for the latter are relatively small or at times negligible except at bus numbers 25 and 30-34 at LSF 0.8 (first interval) in the 57-bus system where the reactive power rates are more than half the rates for the active power rates. It is also noticeable in all the tables of results that the spot-prices for the reactive power rates at the generation buses are all zeros. This is only true till the reactive power generating ($Q_{gi,min}$ and $Q_{gi,max}$) limits are reached.

The high spot-prices for the reactive power recorded in the 57-bus system, may be due to the voltage constraints at these buses. In such situations, it seems to be unfair to pass the burden of meeting voltage constraints solely to the consumers by charging exorbitant prices for reactive power demand. The utility can resort to installing capacitors to ease out these high costs and charge a fixed amount for the same demand if it works out to be cost effective. These high prices can also provide incentives to all customers to reduce their consumption of reactive power regardless of the power factor. This is one of the advantages of real-time pricing of electricity that it functions as a load management tool since it interacts with and influence the consumer behavior.

Although the generating capacity in each of the system is expressed in per unit, it is still possible to compare the capacities since the power base in all the systems is taken to be the same. It is clear therefore from the results that as the system gets larger, the demand increases as well and hence the generating capacity.

Comparing the losses in the two time intervals for the same network, it is observed that the losses are higher at heavier loads than at the lighter loads. In terms of the losses in the 5-bus, 14-bus and 57-bus systems respectively, it is generally observed that as the system becomes larger, the losses increases as well. On the other hand, in the 30-bus system the losses are slightly lower than those of the 14-bus system.

Table 5.2 Rates for the 5-bus system without transmission cost considerations.

Interval	Bus #	P_G [p.u.]	Q_G [p.u.]	V [p.u.]	MC_{pi} [\$/p.u.MWh]	MC_{qi} [\$/p.u.MVARh]
Time Interval I (8hrs) with LSF 1.0 without transmission costs.	1	0.79638	-0.14502	1.10000	1228.65115	0.00000
	2	0.88498	0.05307	1.09507	1248.39763	0.00000
	3			1.08251	1285.37880	-3.07518
	4			1.08469	1288.94196	-6.15478
	5			1.07113	1297.50150	3.34521
Time Interval II (16hrs) with LSF 0.8 without transmission cost	1	0.63623	-0.13644	1.10000	1192.97737	0.00000
	2	0.70409	-0.07160	1.09551	1208.09297	0.00000
	3			1.08927	1236.22138	-5.80646
	4			1.09149	1238.91440	-8.68054
	5			1.07870	1245.36736	0.42220

For Time Interval # I ; Active Power Loss = 3.136E-02 p.u. MWh

For Time Interval # II; Active Power Loss = 2.032E-02 p.u. MWh

Total Energy Loss in p.u. MWH = 0.5760

Table 5.3 Rates for the 14-bus system without transmission costs

Interval	Bus #	P_G [p.u.]	Q_G [p.u.]	V [p.u.]	MC_{pi} [\$/p.u.MWh]	MC_{qi} [\$/p.u.MVARh]
Time Interval I (8hrs) with LSF 1.0 without transmission costs.	1	1.21663	-.29374	1.10000	1349.67532	0.00000
	2	1.46011	0.19159	1.10000	1377.20409	0.00000
	3		0.24422	1.07031	1477.24874	0.00000
	4			1.07780	1452.63973	-0.89180
	5			1.08516	1435.09560	-2.24451
	6		0.40564	1.05087	1436.78040	0.00000
	7			1.04972	1451.55414	0.00025
	8		0.04679	1.05752	1451.55414	0.00000
	9			1.04367	1450.99296	0.54126
	10			1.03744	1456.80907	5.52535
	11			1.04060	1451.64023	5.22415
	12			1.03577	1461.62330	8.21371
	13			1.03097	1470.34283	13.08558
	14			1.01605	1501.67857	21.47694
Time Interval II (16hrs) & LSF 0.8 without transmission cost	1	0.96939	-0.23701	1.10000	1292.06702	0.00000
	2	1.15758	0.07356	1.09879	1312.76394	0.00000
	3		0.15956	1.07443	1387.53917	0.00000
	4			1.08262	1369.29733	-2.25440
	5			1.08934	1356.15205	-3.59273
	6		0.31993	1.05039	1357.19931	0.00000
	7			1.05175	1368.40948	-1.68995
	8		-0.01040	1.05000	1368.40948	0.00000
	9			1.04974	1367.91384	-2.40157
	10			1.04390	1372.21480	1.64735
	11			1.04436	1368.36441	2.51484
	12			1.03876	1375.78751	6.06245
	13			1.03531	1382.22850	9.28081
	14			1.02590	1405.15482	14.47097

For Time Interval # I ; Active Power Loss = 8.675E-02 p.u. MWh
For Time Interval # II; Active Power Loss = 5.497E-02 p.u. MWh
Total Energy Loss in p.u. MWH = 1.5734

Table 5.4 Rates for the 30-bus system without transmission costs

Interval	**Bus** #	P_G [p.u.]	Q_G [p.u.]	**V** [p.u.]	MC_{pi} [\$/p.u.MWh]	MC_{qi} [\$/p.u.MVARh]
Time Interval I (8hrs) & LSF 1.0 without transmission costs.	1	0.91680	-0.25853	1.10000	1279.81431	0.00000
	2	1.02191	0.15600	1.09809	1304.30597	0.00000
	3			1.09382	1317.82766	0.17409
	4			1.09201	1328.33545	-0.00718
	5		0.29905	1.06886	1396.32673	0.00000
	6			1.08213	1351.36093	1.64779
	7			1.06970	1376.20462	5.49498
	8		0.35366	1.08138	1359.75306	0.00000
	9			1.04830	1364.19031	-0.00024
	10			1.03515	1371.41273	-1.14099
	11		0.07498	1.06297	1364.19031	0.00000
	12			1.05507	1293.63830	4.32928
	13	0.97393	0.41043	1.10000	1293.12612	0.00000
	14			1.04029	1318.26970	11.76488
	15			1.03187	1338.98875	14.33750
	16			1.03722	1337.05024	8.25820
	17			1.02975	1367.55215	5.33347
	18			1.02008	1370.42090	16.21952
	19			1.01661	1383.93456	14.98730
	20			1.02027	1382.32677	11.07086
	21			1.02291	1383.91830	7.91750
	22			1.02354	1382.71414	7.47598
	23			1.02006	1365.10019	18.87685
	24			1.01390	1388.66178	17.54470
	25			1.01056	1389.98432	18.21737
	26			0.99276	1415.93241	35.54660
	27			1.01763	1378.93921	11.04133
	28			1.07850	1359.56409	3.12384
	29			0.99777	1417.30024	21.85059
	30			0.98634	1443.75464	26.28251

Table 5.4 Rates for the 30-bus system without transmission costs (continued)

Interval	**Bus #**	**P_G [p.u.]**	**Q_G [p.u.]**	**V [p.u.]**	**MC_{pi} [\$/p.u.MWh]**	**MC_{qi} [\$/p.u.MVARh]**
Time Interval I (16hrs) & LSF 0.8 without transmission costs.	1	0.73012	-0.20496	1.10000	1236.31818	0.00000
	2	0.80964	0.06355	1.09732	1254.84562	0.00000
	3			1.09592	1265.12292	-1.37790
	4			1.09444	1273.04966	-1.67713
	5		0.20867	1.07304	1324.19050	0.00000
	6			1.08494	1290.37718	0.53606
	7			1.07478	1309.05939	3.37332
	8		0.24195	1.08328	1296.72779	0.00000
	9			1.04655	1300.01905	-0.41431
	10			1.04209	1305.43203	-1.29050
	11		-0.03277	1.04000	1300.01905	0.00000
	12			1.05951	1247.14590	1.85132
	13	0.77585	0.35440	1.10000	1246.97260	0.00000
	14			1.04784	1265.71327	7.10726
	15			1.04115	1281.19688	8.81387
	16			1.04488	1279.73334	4.92040
	17			1.03820	1302.52256	3.26782
	18			1.03135	1304.54278	10.54244
	19			1.02825	1314.57471	9.96475
	20			1.03094	1313.42255	7.26534
	21			1.03273	1314.71061	5.07043
	22			1.03334	1313.80094	4.61675
	23			1.03193	1300.57695	11.55725
	24			1.02706	1317.97338	10.01278
	25			1.02350	1318.84310	11.44501
	26			1.00950	1337.71021	24.00019
	27			1.02845	1310.71520	6.76525
	28			1.08234	1296.51374	1.36129
	29			1.01291	1338.68391	14.36033
	30			1.00396	1357.75873	17.45855

For Time Interval # I ; Active Power Loss = 8.465E-02 p.u. MWh
For Time Interval # II; Active Power Loss = 5.320E-02 p.u. MWh
Total Energy Loss in p.u. MWH = 1.5284

Table 5.5 Rates for the 57-bus system without transmission costs

Interval	Bus #	P_G [p.u]	Q_G [p.u]	V [p.u.]	MC_{pi} [$/p.u.MWh]	MC_{qi} [$/p.u.MVARh]
Time Interval I (8hrs) & LSF 0.8 without transmission costs	1	2.36906	-0.17502	1.10000	1613.45209	0.00000
	2		0.57195	1.09982	1602.20956	0.00000
	3	2.33495	-0.24598	1.10000	1605.57382	0.00000
	4			1.09855	1636.65480	3.03489
	5			1.09590	1685.64100	2.43934
	6		0.26964	1.09818	1700.00170	0.00000
	7			1.09121	1708.77569	22.92994
	8	2.69305	-0.01520	1.10000	1688.29474	0.00000
	9		0.68280	1.10000	1716.16592	0.00000
	10			1.08738	1718.20318	10.99345
	11			1.08677	1722.04711	27.08691
	12	2.74894	0.53004	1.10000	1701.20539	0.00000
	13			1.08979	1706.75128	28.76256
	14			1.08550	1697.36969	42.59506
	15			1.08971	1657.85928	27.36247
	16			1.09577	1688.83789	1.29726
	17			1.09274	1660.26738	3.21201
	18			1.07543	1638.31839	17.14410
	19			1.03438	1750.55099	89.52245
	20			1.01840	1797.84694	129.98128
	21			0.96949	1800.20097	175.11434
	22			0.96745	1808.46046	184.47704
	23			0.96642	1819.39285	194.82682
	24			0.96127	1968.05604	357.69674
	25			0.94671	2074.15610	1139.67693
	26			1.00434	1966.25093	328.08126
	27			1.02436	1827.02951	159.95507
	28			1.03681	1759.94160	96.54006
	29			1.04759	1711.05656	54.35212
	30			0.92791	2301.5542	1423.35870
	31			0.90000	2791.51681	2097.18673
	32			0.90293	2185.17275	1378.85957
	33			0.90100	2195.32412	1383.94388
	34			0.94453	2061.54561	424.81669
	35			0.94920	1992.94539	344.07076
	36			0.95628	1926.93805	283.08522
	37			0.95900	1888.04598	252.83648
	38			0.96891	1789.53758	166.80882
	39			0.95746	1893.48221	253.33094
	40			0.95901	1921.11781	270.13898
	41			0.99057	1744.81135	87.32651
	42			0.94836	1849.31490	178.12143
	43			1.02719	1727.15970	42.12068
	44			0.98432	1751.78395	133.29463
	45			1.02574	1654.42027	57.96080
	46			0.97575	1696.55377	80.61664
	47			0.96723	1738.60484	126.32383
	48			0.96771	1754.53910	135.96249
	49			0.96830	1733.66968	110.79585
	50			0.96470	1761.18696	92.56798
	51			1.00187	1715.28156	15.36210
	52			1.01815	1783.04589	59.91413
	53			1.00743	1811.78505	56.96086
	54			1.01491	1770.70318	36.53556
	55			1.03004	1714.63878	9.11825
	56			0.93152	1904.55822	239.27179
	57			0.92814	1912.71083	250.99176

Table 5.5 Rates for the 57-bus system without transmission costs (continued)

Interval	**Bus #**	**P_G [p.u.]**	**Q_G [p.u.]**	**V [p.u]**	**MC_{pi} [\$/p.u.MWh]**	**Mc_{qi} [\$/p.u.MVARh]**
Time Interval II (16hrs) & LSF 0.6 without transmission costs	1	1.78609	-0.5300	1.10000	1478.78780	0.00000
	2		0.39781	1.09988	1470.09996	0.00000
	3	1.76734	-0.02831	1.10000	1474.45633	0.00000
	4			1.09478	1494.24342	1.10484
	5			1.08380	1525.06763	0.53610
	6		-0.05406	1.08093	1533.94990	0.00000
	7			1.07893	1536.88153	0.29602
	8	1.99373	0.09766	1.08620	1526.75174	0.00004
	9		0.21178	1.08188	1545.60589	0.00002
	10			1.07538	1547.82821	1.19069
	11			1.07657	1544.73755	3.58088
	12	2.03635	0.14973	1.08523	1536.59735	0.00000
	13			1.08302	1535.67183	4.09263
	14			1.08319	1528.46169	7.04323
	15			1.08878	1505.19822	3.59191
	16			1.08708	1528.65032	0.11525
	17			1.09001	1509.80512	1.28024
	18			1.07711	1493.78139	3.29469
	19			1.04262	1547.01038	18.08907
	20			1.02818	1564.77513	27.94218
	21			0.97685	1564.19834	17.79893
	22			0.97462	1565.97401	19.05434
	23			0.97386	1568.62732	18.91038
	24			0.96998	1592.00311	11.27031
	25			0.96730	1594.57708	5.01451
	26			1.01231	1592.37040	11.60955
	27			1.02350	1570.24203	5.50057
	28			1.03140	1551.84358	2.45980
	29			1.03859	1536.22878	1.46584
	30			0.95264	1616.38430	20.02042
	31			0.92996	1645.33665	44.08963
	32			0.92836	1621.65755	47.22493
	33			0.92695	1625.12588	48.96146
	34			0.96128	1620.91035	32.46640
	35			0.96424	1612.58179	29.53383
	36			0.96907	1600.30017	24.83300
	37			0.97026	1590.67220	24.82905
	38			0.97548	1561.29886	19.71690
	39			0.96907	1593.79293	26.11034
	40			0.97175	1601.52561	22.05687
	41			0.99066	1548.00899	16.34183
	42			0.95715	1587.02592	16.95362
	43			1.02023	1545.69454	1.30714
	44			0.98953	1546.18239	12.70867
	45			1.02556	1501.68983	4.91502
	46			0.97560	1527.42319	15.13471
	47			0.97116	1546.70859	23.73866
	48			0.97223	1551.64425	22.13130
	49			0.97012	1547.81274	24.50990
	50			0.96672	1569.25481	26.01795
	51			0.99330	1547.03236	1.21625
	52			1.01590	1584.54278	5.46960
	53			1.00752	1603.95979	5.61934
	54			1.00860	1580.57478	7.75562
	55			1.01561	1545.93413	4.09158
	56			0.94248	1596.69686	32.69924
	57			0.94035	1598.72903	31.10854

For Time Interval # I; Active Power Loss = 0.1396 p.u MWh
For Time Interval # I; Active Power Loss = 7.872E-02 p.u. MWh
Total Energy Loss in p.u. MWH = 2.3763

5.3.2 Second Phase Results

In this section, the results corresponding to the second phase of the investigation are presented i.e. transmission costs involving active power (**P**) injected at the respective buses and is a function of the voltage and the phase angle at the bus. These transmission costs are in addition to the fuel costs discussed in the first phase. Tables 5.6, 5.7, 5.8 and 5.9 show the simulation results on the 5,14, 30 and 57 -bus systems respectively. From the tabulated results, it can be seen that the voltages at the corresponding buses within the same system in the two time intervals at different load levels, still show the same pattern as already discussed in the first phase. This refers to the fact that at low demand the voltages are generally slightly higher than the voltages at high demand except at the generation buses and in the 57-bus system where the voltages at the generation buses are relatively constant and the voltages in the 57-bus system seem to be influenced by the voltage constraints.

Comparing the corresponding results in both phases during the same time interval, it is observed that there is significant increases in the spot-prices for both real and reactive power at the different corresponding load buses. However, the increases in the reactive power rates are not all that pronounced in the smaller systems i.e. 5-bus and 14-bus systems compared to the bigger systems i.e. 30-bus and 57-bus systems. In terms of the active power rates (MC_{pi}) to the reactive power rates (MC_{qi}) in the same time interval, it is seen that the reactive power rates are still relatively small except at bus 25 and 30-34 at

LSF 0.8 in the 57-bus system where the reactive rates are more than half the active power rates. This is similar to the first phase. The spot-prices at the generator buses remain relatively the same to those in the first phase. Also the total spot-prices at these generating buses in each time interval remain the same as well.

Comparing the voltage magnitudes in the same time intervals with those in the first phase, it is observed that the voltage magnitudes remain relatively constant. The total generation capacity in the corresponding time intervals remain the same. The difference in the losses registered in the corresponding time intervals in both phases is negligible. Like in the first phase, the spot-prices for the reactive power rates at the generation buses are all zero. All these factors help to confirm that the increase in the spot-prices at the load buses are the direct result of the transmission costs under consideration.

Since most of power utilities charge only the real power consumption, the various increases at the different buses relative to the results in phase one for all the test systems were computed. The reason being to easily see and appreciate the margin of the increases in the respective spot-prices. These increases are shown in Tables 5.14 through 5.16 in the columns titled "Increase when considering **P** only". Note that these are the load buses only since the spot prices at the generating buses have already been established to be relatively the same as in the first phase and hence negligible differences.

For the 5-bus,14-bus and 30-bus systems, the increases are in single digits except at bus 30 in the 30-bus system; otherwise the range of the increases in these three systems seem to be the same. At a glance, the increases seem to be small but they translate into significant amounts when one considers them in terms of days, weeks, months or years. In the 57-bus system at LSF 0.8 , the increases at some buses rise to two digits. At low demand (LSF 0.6) the increases compare favorably with those obtained in the other systems. In the 5-bus system the lowest increase is at bus 3 in the second time interval and the highest increase is at bus 5 in the first time interval. For the 14-bus system, the lowest increase is at bus 5 in the second interval and highest increase is at bus 14 in the first interval. In the 30-bus system, the lowest increase is at bus 12 in the second interval and highest is at bus 30 in the first time interval. Finally, for the 57-bus system the lowest is at bus 45 in the second time interval and highest is at bus 31 in the first time interval. By comparing the results at LSF 0.8 to those at LSF 0.6 in each of the systems, it can be concluded that higher increases are experienced as the load increases. In turn, the rates become dependent on the demand.

Table 5.6 Rates for the 5-bus system considering transmission costs involving **P** only

Interval	**Bus #**	**P_G [p.u.]**	**Q_G [p.u.]**	**V [p.u.]**	**MC_{pi} [$/p.u.MWh]**	**MC_{qi} [$/p.u.MVARh]**
Time Interval I (8hrs) & LSF 1.0 with transmission costs involving **P** only.	1	0.79306	-0.14507	1.10000	1227.91111	0.00000
	2	0.88825	0.05292	1.09512	1249.11600	0.00000
	3			1.08255	1289.12175	-2.32633
	4			1.08473	1292.96594	-5.65378
	5			1.07118	1302.19724	3.61396
Time Interval II (16hrs) & LSF 0.8 with transmission costs involving **P** only.	1	0.63361	-.13647	1.10000	1192.39211	0.00000
	2	0.70668	-0.07171	1.09555	1208.67092	0.00000
	3			1.08930	1239.16445	-5.29172
	4			1.09152	1242.07680	-8.40468
	5			1.07873	1249.05278	0.45670

For Time Interval # I ; Active Power Loss = 3.130E-02 p.u. MWh

For Time Interval # II; Active Power Loss = 2.029E-02 p.u. MWh

Total Energy Loss in p.u. MWH = 0.5751

Table 5.7 Rates for the 14-bus system with transmission costs involving **P** only

Interval	**Bus #**	P_G **[p.u.]**	Q_G **[p.u.]**	**V [p.u.]**	MC_{pi} **[\$/p.u.MWh]**	MC_{qi} **[\$/p.u.MVARh]**
Time Interval I (8hrs) & LSF 1.0 with transmission costs involving **P** only	1	1.21246	-.29260	1.10000	1348.70227	0.00000
	2	1.46421	0.19029	1.10000	1378.07588	0.00000
	3		0.24426	1.07031	1485.46861	0.00000
	4			1.07780	1459.08144	-0.59627
	5			1.08515	1440.26521	-2.14064
	6		0.40555	1.05084	1442.07150	0.00000
	7			1.04971	1457.91735	0.00027
	8		0.04674	1.05749	1457.91735	0.00000
	9			1.04365	1457.31559	0.58041
	10			1.03742	1463.55979	5.92991
	11			1.04058	1458.01623	5.60680
	12			1.03575	1468.73564	8.81607
	13			1.03095	1478.09359	14.04503
	14			1.01603	1511.72243	23.05170
Time Interval II (16hrs) & LSF 0.8 with transmission costs involving **P** only	1	.966607	-.23705	1.10000	1291.29463	0.00000
	2	1.16084	.07350	1.09883	1313.45892	0.00000
	3		.15957	1.07446	1393.97756	0.00000
	4			1.08265	1374.35538	-2.12493
	5			1.08936	1360.21202	-3.46784
	6		.31990	1.05041	1361.33837	0.00000
	7			1.05176	1373.40030	-1.18817
	8		-.01050	1.05000	1373.40030	0.00000
	9			1.04976	1372.86712	-2.10406
	10			1.04391	1377.49911	1.77509
	11			1.04438	1373.35642	2.70843
	12			1.03878	1381.35182	6.52740
	13			1.03533	1388.28610	9.99273
	14			1.02592	1412.96745	15.58158
For Time Interval # I ; Active Power Loss = 8.666E-02 p.u. MWh For Time Interval # II; Active Power Loss = 5.491E-02 p.u. MWh Total Energy Loss in p.u. MWH = 1.5719						

Table 5.8 Rates for the 30-bus system with transmission costs involving **P** only.

Interval	**Bus** #	P_G [p.u.]	Q_G [p.u.]	V [p.u.]	MC_{pi} [\$/p.u.MWh]	MC_{qi} [\$/p.u.MVARh]
Time	1	0.91294	-0.25851	1.10000	1278.91566	0.00000
Interval	2	1.02559	0.15589	1.09814	1305.16343	0.00000
I (8hrs) &	3			1.09384	1319.79084	0.18625
LSF 1.0	4			1.09205	1331.08807	-0.00667
with	5		0.29906	1.06891	1404.30440	0.00000
transmission	6			1.08217	1355.88410	1.77641
costs	7			1.06974	1382.64016	5.91981
involving **P**	8		0.35371	1.08141	1364.92478	0.00000
only	9			1.04832	1369.70752	-0.00016
	10			1.03517	1377.48964	-1.12992
	11		0.07489	1.06297	1369.70752	0.00000
	12			1.05508	1293.70280	4.66633
	13	0.97404	0.41033	1.10000	1293.15068	0.00000
	14			1.04030	1320.23841	12.67678
	15			1.03189	1342.55960	15.44723
	16			1.03724	1340.47120	8.89756
	17			1.02977	1373.33087	5.74548
	18			1.02010	1376.42167	17.47398
	19			1.01663	1390.97994	16.14603
	20			1.02029	1389.24778	11.92656
	21			1.02293	1390.96189	8.52878
	22			1.02356	1389.66463	8.05311
	23			1.02008	1370.68932	20.33636
	24			1.01392	1396.07163	18.89969
	25			1.01058	1397.49525	19.62213
	26			0.99278	1425.44869	38.29059
	27			1.01766	1385.59578	11.89049
	28			1.07854	1364.72137	3.36602
	29			0.99780	1426.92110	23.53491
	30			0.98637	1455.41964	28.30923

Table 5.8 Rates for the 30-bus system with transmission costs involving **P** only (continued)

Interval	**Bus** #	**P_G** [p.u.]	**Q_G** [p.u.]	**V** [p.u.]	**MC_{pi}** [\$/p.u.MWh]	**MC_{qi}** [\$/p.u.MVARh]
Time Interval II (16hrs) & LSF 0.8 with transmission costs involving **P** only	1	0.72704	-0.20494	1.10000	1235.59937	0.00000
	2	0.81248	0.06349	1.09736	1255.50771	0.00000
	3			1.09594	1266.65176	-1.24915
	4			1.09447	1275.19590	-1.51035
	5		0.20867	1.07307	1330.41843	0.00000
	6			1.08497	1293.90675	0.58026
	7			1.07481	1314.08216	3.64424
	8		0.24200	1.08331	1300.76660	0.00000
	9			1.04657	1304.32508	-0.24754
	10			1.04210	1310.17404	-1.19469
	11		-0.03285	1.04000	1304.32508	0.00000
	12			1.05952	1247.20613	2.00242
	13	0.77605	0.35433	1.10000	1247.01863	0.00000
	14			1.04785	1267.26289	7.68001
	15			1.04117	1283.99027	9.52261
	16			1.04489	1282.41071	5.31640
	17			1.03821	1307.03036	3.52983
	18			1.03136	1309.21049	11.38902
	19			1.02826	1320.04811	10.76440
	20			1.03095	1318.80412	7.84816
	21			1.03274	1320.19650	5.47643
	22			1.03335	1319.21379	4.98637
	23			1.03195	1304.92603	12.48504
	24			1.02708	1323.71929	10.81515
	25			1.02352	1324.65928	12.36027
	26			1.00952	1345.03926	25.92219
	27			1.02847	1315.87974	7.30436
	28			1.08237	1300.53554	1.47122
	29			1.01294	1346.09091	15.50834
	30			1.00398	1366.69507	18.85495

For Time Interval # I ; Active Power Loss = 8.457E-02 p.u. MWh
For Time Interval # II; Active Power Loss = 5.316E-02 p.u. MWh
Total Energy Loss in p.u. MWH = 1.5272

Table 5.9 Rates for the 57-bus system with transmission costs involving **P** only

Interval	Bus #	P_G [p.u.]	Q_G [p.u.]	V [p.u.]	MC_{pi} [\$/p.u.MWh]	MC_{qi} [\$/p.u.MVARh]
Time Interval I (8hrs) & LSF 0.8 with transmission costs involving **P** only	1	2.36021	-0.17431	1.10000	1611.40852	0.00000
	2		0.57201	1.09982	1609.66652	0.00000
	3	2.32452	-0.24288	1.10000	1603.16415	0.00000
	4			1.09852	1637.37464	3.17110
	5			1.09583	1687.55385	2.55713
	6		0.26669	1.09809	1702.62365	0.00000
	7			1.09118	1711.73643	24.11121
	8	2.70033	-0.01537	1.10000	1689.97736	0.00000
	9		0.68237	1.10000	1719.61667	0.00000
	10			1.08739	1721.77490	11.57183
	11			1.08678	1725.89892	28.50662
	12	2.75995	0.52635	1.10000	1703.74889	0.00000
	13			1.08980	1709.79608	30.26477
	14			1.08552	1699.97177	44.82779
	15			1.08972	1658.30645	28.77657
	16			1.09579	1690.83428	1.34593
	17			1.09276	1660.78470	3.37154
	18			1.07541	1638.90176	17.96341
	19			1.03439	1756.22580	94.27049
	20			1.01843	1806.05845	136.89865
	21			0.96949	1808.55459	184.20403
	22			0.96745	1817.22071	194.05462
	23			0.96642	1828.73555	204.92832
	24			0.96127	1985.08110	375.97444
	25			0.94671	2096.56611	1197.18575
	26			1.00433	1983.19633	344.87792
	27			1.02434	1836.49763	168.16915
	28			1.03679	1765.71649	101.49781
	29			1.04757	1714.11367	57.11234
	30			0.92791	2335.75225	1495.34018
	31			0.90000	2850.84759	2203.34137
	32			0.90293	2213.72544	1448.89352
	33			0.90100	2224.44407	1454.26194
	34			0.94454	2083.82957	446.74305
	35			0.94921	2011.65134	361.88444
	36			0.95629	1942.13396	297.75025
	37			0.95901	1901.15239	265.96186
	38			0.96891	1797.28046	175.49647
	39			0.95746	1906.90618	266.50032
	40			0.95902	1936.02384	284.12089
	41			0.99060	1749.84846	91.57073
	42			0.94837	1860.28561	187.40403
	43			1.02720	1731.28010	44.24709
	44			0.98432	1757.46815	140.19153
	45			1.02573	1654.66740	60.81561
	46			0.97576	1699.11081	4.85502
	47			0.96724	1743.55931	132.99210
	48			0.96771	1760.35599	143.10595
	49			0.96831	1738.27125	116.68391
	50			0.96471	1767.38715	97.58635
	51			1.00189	1718.67356	16.12905
	52			1.01813	1790.44194	63.11314
	53			1.00742	1820.91807	60.04039
	54			1.01490	1777.40886	38.53549
	55			1.03004	1718.00853	9.61037
	56			0.93152	1918.55155	251.82426
	57			0.92814	1927.18479	264.11711

Table 5.9 Rates for the 57-bus system with transmission costs involving **P** only (cont'd)

Interval	**Bus #**	**P_G [p.u.]**	**Q_G [p.u.]**	**V [p.u.]**	**MC_{pi} [\$/p.u.MWh]**	**MC_{qi} [\$/p.u.MVARh]**
Time Interval II (16hrs) & LSF 0.6 with transmission costs involving **P** only.	1	1.77961	-0.05375	1.10000	1477.29097	0.00000
	2		0.39785	1.09988	1476.35761	0.00000
	3	1.75987	-0.02828	1.10000	1472.73065	0.00000
	4			1.09482	1494.35685	1.18934
	5			1.08388	1526.41333	0.04868
	6		-0.05442	1.08104	1535.79181	0.00000
	7			1.07907	1538.82761	0.31535
	8	1.99897	0.09784	1.08636	1527.96302	0.00004
	9		0.21156	1.08201	1548.11320	0.00002
	10			1.07551	1550.47262	1.26953
	11			1.07669	1547.26386	3.80878
	12	2.04455	0.14880	1.08536	1538.49073	0.00000
	13			1.08312	1537.65811	4.34856
	14			1.08327	1530.05486	7.48936
	15			1.08883	1505.36230	3.81071
	16			1.08718	1530.14350	0.10888
	17			1.09007	1510.20241	1.34918
	18			1.07714	1494.28423	3.52886
	19			1.04269	1549.89416	19.26711
	20			1.02827	1568.76314	29.77396
	21			0.97693	1568.17355	18.95295
	22			0.97470	1570.04577	20.29252
	23			0.97394	1572.87000	20.13914
	24			0.97009	1597.71256	11.99827
	25			0.96741	1600.45397	5.32448
	26			1.01241	1598.10925	12.36052
	27			1.02362	1574.43891	5.85879
	28			1.03152	1554.79563	2.61978
	29			1.03873	1538.12929	1.56414
	30			0.95275	1623.70956	21.32320
	31			0.93006	1654.59239	46.98477
	32			0.92846	1629.37251	50.32876
	33			0.92705	1633.07038	52.18022
	34			0.96138	1628.57877	34.59087
	35			0.96433	1619.70142	31.46480
	36			0.96916	1606.60879	26.45342
	37			0.97035	1596.35360	26.44939
	38			0.97556	1565.06475	20.99907
	39			0.96916	1599.67842	27.81585
	40			0.97185	1607.90624	23.49411
	41			0.99077	1550.74525	17.43531
	42			0.95726	1592.39229	18.06045
	43			1.02034	1548.28269	1.40352
	44			0.98960	1548.98940	13.52312
	45			1.02561	1501.74330	5.26838
	46			0.97567	1528.95267	16.11749
	47			0.97123	1549.50802	25.29094
	48			0.97231	1554.76163	23.57660
	49			0.97021	1550.61283	26.11940
	50			0.96683	1573.40865	27.73142
	51			0.99344	1549.61069	1.29522
	52			1.01603	1589.62907	5.90669
	53			1.00766	1610.32545	5.98164
	54			1.00873	1585.39005	8.26521
	55			1.01574	1548.45678	4.36401
	56			0.94257	1602.75778	34.84485
	57			0.94043	1604.94453	33.14576

For Time Interval # I; Active Power Loss = 0.1386 p.u. MWh
For Time Interval # I; Active Power Loss = 7.821E-02 p.u. MWh
Total Energy Loss in p.u. MWH = 2.3603

5.3.3 Third Phase Results

In this final section, the simulation results on electricity rates considering transmission costs which involve both reactive (**Q**) and active power (**P**) flows injected at the same buses in the networks so far discussed in phase one and two are presented. The transmission costs are in addition to the fuel cost dealt in phase one. Tables 5.10, 5.11, 5.12 and 5.13 show the simulation results of the real-time prices for both active and reactive power involving the transmission costs in the respective power system networks for the same two time intervals discussed in phase one and two.

From the tables of results, the voltages in the two time intervals for the 5-bus, 14-bus and 30-bus system still show a slight increase at heavier loads than at lighter loads. In the 57-bus system, this relationship of voltages is distorted by the voltage constraints. Also comparing the voltage magnitudes in the same time interval with those in phase one and two, it is observed that they are still relatively the same especially at the generating buses. The total generating capacity in all the system is still the same as in previous phases and this means in turn the total cost at the generating buses is the same as well.

In all systems considered, it can be observed that there are now remarkable increases in spot-prices particularly in the active power rates at the load buses. This suggests that the flow of reactive power has a great influence on the active power rates. It also helps to establish the fact that utilities can help their consumers by installing capacitors at some buses just like what was observed with reactive power in phase two to avoid high rates of electricity. Like in the first and second phase, the spot-prices at the generation buses are still zero. This is still true only if the power generating limits are not reached. Comparing the reactive power rates (MC_{qi}) to the active power rates (MC_{pi}), in the same time interval, the reactive rates are still very small except for the 57-bus system at the same buses as in phase two where the reactive power rates are more than half those of active power. The differences in the losses registered in the corresponding time intervals in all the three phases for the same network are negligible.

Also, to help easily see and appreciate the increase in the active power rates, like in phase two, the increases were computed. Tables 5.14 through 5.16 show the increases in the columns titled "Increase when considering **P & Q**". These increases are calculated relative to the results obtained in phase one. Like in the second phase, the increases in the active power rates were the only ones calculated for the same reason given in phase two.

It can be seen from the tabulated results that the increases in the active power rates are generally now in double digits with bus 5 in the first interval and bus 3 in the second interval registering the highest increase and lowest increase respectively for the 5-bus system. In the 14-bus system, the highest rates are at bus 14 in the first interval and the lowest increases are at bus 5 in the second interval. For the 30-bus system, the highest increase is registered at bus 30 in the first interval and lowest at bus 12. These results show the same pattern as in the second phase. Finally, in the 57-bus system, the highest is at bus 31 in the first interval and lowest has now moved from bus 45 in phase two to bus 4 but still in the second interval.

In conclusion, from the utility point of view, these transmission costs calculations which have been presented in phase two and this phase, are worth looking into especially the combination of the transmission costs with the concept of spot-pricing based electricity rate structure which seem to guarantee the recovery of capital. Though the studies reported in this chapter were restricted to two time intervals for simplicity, other simulated results with different LSF for each hour still show the pattern discussed in this chapter.

Table 5.10 Rates for the 5-bus system considering transmission costs involving **P & Q.**

Interval	**Bus** #	**P_G** [p.u.]	**Q_G** [p.u.]	**V** [p.u.]	**MC_{pi}** [\$p.u.MWh]	**MC_{qi}** [\$/p.u.MVARh]
Time Interval I (8hrs) , LSF 1.0 with transmission costs involving **P & Q**	1	0.78039	-.22713	1.10000	1225.09055	0.00000
	2	0.90081	0.13140	1.09905	1251.91356	0.00000
	3			1.08571	1300.43524	-0.80573
	4			1.08813	1305.00048	-4.33168
	5			1.07504	1316.63872	6.18244
Time Interval II (16hrs), LSF 0.8 with transmission costs involving **P&Q** only.	1	0.62305	-.22073	1.10000	1190.04149	0.00000
	2	0.71721	0.00914	1.09956	1211.01459	0.00000
	3			1.09250	1248.39279	-4.36245
	4			1.09496	1251.92549	-7.62510
	5			1.08264	1260.65289	2.39470

For Time Interval # I ; Active Power Loss = 3.120E-02 p.u. MWh

For Time Interval # II; Active Power Loss = 2.0259E-02 p.u. MWh

Total Energy Loss in p.u. MWH = 0.5737

Table 5.11 Rates for the 14-bus system with transmission costs involving **P** **&Q.**

Interval	**Bus #**	P_G **[p.u.]**	Q_G **[p.u]**	**V [p.u.]**	MC_{pi} **[\$/p.u.MWh]**	MC_{qi} **[\$/p.u.MVARh]**
Time Interval I (8hrs), LSF 1.0 with transmission costs involving **P** & **Q**	1	1.19864	-.29922	1.10000	1345.48276	0.00000
	2	1.47780	0.14049	1.10000	1380.97183	0.00000
	3		0.25946	1.07312	1515.48824	0.00000
	4			1.08053	1482.76705	-0.22908
	5			1.08729	1459.67431	-1.34217
	6		0.40555	1.05447	1478.31578	0.00000
	7			1.05710	1492.57214	2.64426
	8		0.08499	1.07108	1492.57206	0.00000
	9			1.04976	1498.15919	5.47866
	10			1.04314	1505.22167	10.87728
	11			1.04529	1497.78619	8.44681
	12			1.03962	1509.07275	10.66741
	13			1.03501	1519.39986	16.56007
	14			1.02132	1557.78014	29.04306
Time Interval II (16hrs), LSF 0.8 with transmission costs involving **P** & **Q**	1	0.95442	-0.26517	1.10000	1288.58078	0.00000
	2	1.17240	0.06591	1.10000	1315.92130	0.00000
	3		0.17582	1.07781	1417.57424	0.00000
	4			1.08518	1392.97361	-1.61444
	5			1.09139	1375.48077	-2.88139
	6		0.31540	1.05263	1389.82815	0.00000
	7			1.05669	1400.80528	0.32462
	8		0.01031	1.05841	1400.80521	0.00000
	9			1.05386	1405.20811	1.32008
	10			1.04771	1410.44639	5.70798
	11			1.04741	1404.73236	4.96693
	12			1.04116	1413.05417	7.96880
	13			1.03785	1420.76472	11.95338
	14			1.02937	1449.18628	20.26204
For Time Interval # I ; Active Power Loss = 8.644E-02 p.u. MWh For Time Interval # II; Active Power Loss = 5.482E-02 p.u. MWh Total Energy Loss in p.u. MWH = 1.5687						

Table 5.12 Rates for the 30-bus system with transmission costs involving **P &Q.**

Interval	**Bus** #	P_G [p.u.]	Q_G [p.u.]	**V** [p.u.]	MC_{pi} [$/p.u.MWh]	MC_{qi} [$/p.u.MVARh]
Time Interval I (8hrs), LSF 1.0 with transmission costs involving **P & Q**	1	0.92343	-0.31734	1.10000	1281.35564	0.00000
	2	1.06132	0.13656	1.10000	1313.48642	0.00000
	3			1.09642	1335.18948	1.94287
	4			1.09538	1349.94032	1.88271
	5		0.31681	1.07408	1439.12000	0.00000
	6			1.08707	1379.77695	3.20395
	7			1.07480	1413.11068	7.86030
	8		0.39180	1.08761	1390.99671	0.00000
	9			1.05667	1397.43268	3.94066
	10			1.04224	1407.42300	6.20014
	11		0.10788	1.07749	1397.43259	0.00000
	12			1.05789	1306.23028	12.25223
	13	0.92785	0.38284	1.10000	1282.38797	0.00000
	14			1.04364	1338.77230	21.97082
	15			1.03590	1363.49987	24.90915
	16			1.04200	1361.73075	17.49617
	17			1.03621	1401.73104	14.26529
	18			1.02530	1403.87116	27.47584
	19			1.02250	1421.38800	26.07372
	20			1.02647	1419.77395	21.28130
	21			1.02990	1423.13773	17.65640
	22			1.03047	1421.66795	17.15023
	23			1.02507	1397.71600	31.06141
	24			1.02009	1429.39894	30.05389
	25			1.01666	1432.53010	29.95576
	26			0.9997	1463.74692	50.79967
	27			1.02357	1420.07195	20.49868
	28			1.08382	1390.33135	5.26412
	29			1.00385	1467.36335	33.79852
	30			0.99248	1499.95264	39.24867

Table 5.12 Rates for the 30-bus system with transmission costs involving **P & Q** (continued)

Interval	**Bus #**	P_G **[p.u.]**	Q_G **[p.u.]**	**V [p.u.]**	MC_{pi} **[$/p.u.MWh]**	MC_{qi} **[$/p.u.MVARh]**
Time Interval II (16hrs) , LSF 0.8 with transmission costs involving **P & Q**	1	0.73450	-0.28422	1.10000	1237.33526	0.00000
	2	0.84110	0.05785	1.10000	1262.17612	0.00000
	3			1.09929	1278.78145	-0.62706
	4			1.09879	1290.08771	-0.89161
	5		0.22258	1.07886	1357.66962	0.00000
	6			1.09101	1312.71859	0.97007
	7			1.08079	1337.97332	4.66426
	8		0.25802	1.08994	1321.28276	0.00000
	9			1.06039	1326.08367	1.43857
	10			1.05277	1333.58447	2.28816
	11		0.03955	1.06810	1326.08358	0.00000
	12			1.06339	1257.13937	7.90566
	13	0.74016	0.32046	1.10000	1238.65723	0.00000
	14			1.05255	1281.78244	14.75676
	15			1.04676	1300.36713	16.22060
	16			1.05173	1299.14010	10.93031
	17			1.04778	1329.22096	8.27211
	18			1.03883	1330.49347	17.87767
	19			1.03682	1343.55618	16.84966
	20			1.04004	1342.45894	13.34129
	21			1.04314	1345.19675	10.44840
	22			1.04363	1344.09668	9.97262
	23			1.03894	1325.94912	19.76670
	24			1.03586	1349.56936	17.99916
	25			1.03154	1351.78617	19.45600
	26			1.01765	1374.43794	34.52682
	27			1.03593	1342.66509	13.44177
	28			1.08870	1320.66441	2.31829
	29			1.02052	1377.14267	22.78656
	30			1.01163	1400.64034	26.59657

For Time Interval # = I ; Active Power Loss = 8.459E-02 p.u. MWh
For Time Interval # = II; Active Power Loss = 5.336E-02 p.u. MWh
Total Energy Loss in MWH = 1.5305

Table 5.13 Rates for the 57-bus system with transmission costs involving **P & Q.**

Interval	**Bus #**	**P_G [p.u.]**	**Q_G [p.u.]**	**V [p.u.]**	**MC_{pi} [$/p.u.MWh]**	**MC_{qi} [$/p.u.MVARh]**
Time Interval I (8hrs), LSF 0.8 with transmission costs involving **P&Q**	1	2.32623	-0.17786	1.10000	1603.53660	0.00000
	2		0.58129	1.10000	1611.78473	0.00000
	3	2.29527	-0.23663	1.10000	1596.40647	0.00000
	4			1.09846	1638.01089	3.99288
	5			1.09566	1693.63190	3.12092
	6		0.25846	1.09785	1710.63034	0.00000
	7			1.09108	1720.94776	25.65339
	8	2.71752	-0.01535	1.10000	1693.93007	0.00000
	9		0.68108	1.10000	1732.50522	0.00000
	10			1.08742	1736.42436	13.49582
	11			1.08680	1738.76595	30.53203
	12	2.80281	0.51247	1.10000	1713.62895	0.00000
	13			1.08983	1720.18122	32.55314
	14			1.08557	1708.86434	48.13035
	15			1.08976	1660.98186	31.06835
	16			1.09585	1699.20203	1.94416
	17			1.09283	1663.52525	4.52990
	18			1.07533	1649.10508	23.46537
	19			1.03443	1774.06865	102.12412
	20			1.01853	1826.05056	145.72678
	21			0.96947	1830.79363	192.94476
	22			0.96745	1839.55884	202.84253
	23			0.96642	1851.48890	213.79652
	24			0.96125	2011.25286	385.07169
	25			0.94671	2139.85015	1212.16379
	26			1.00428	2008.62735	353.45999
	27			1.02427	1856.98037	173.99888
	28			1.03672	1782.84725	106.16513
	29			1.04750	1728.74229	60.79741
	30			0.92791	2381.87108	1511.08391
	31			0.90000	2899.35913	2218.43509
	32			0.90294	2259.26438	1465.72370
	33			0.90101	2270.43229	1471.31724
	34			0.94457	2115.44124	459.09240
	35			0.94925	2041.93617	373.71976
	36			0.95632	1970.69360	308.83420
	37			0.95903	1928.00654	276.51137
	38			0.96892	1818.83411	184.13950
	39			0.95749	1934.15602	277.18600
	40			0.95906	1965.15871	295.23556
	41			0.99067	1774.26456	99.43066
	42			0.94842	1889.96295	198.77261
	43			1.02724	1747.24352	47.98070
	44			0.98432	1775.90350	147.74210
	45			1.02571	1664.15662	65.39224
	46			0.97578	1712.09340	90.31543
	47			0.96725	1762.21577	140.78311
	48			0.96773	1779.94097	151.11917
	49			0.96836	1757.19111	124.21463
	50			0.96477	1789.22569	105.38393
	51			1.00196	1736.15787	19.63351
	52			1.01808	1812.45612	69.25252
	53			1.00737	1846.08516	66.98742
	54			1.01488	1798.38073	43.24038
	55			1.03004	1733.45168	11.46462
	56			0.93153	1947.96611	263.28366
	57			0.92815	1957.91705	275.92318

Table 5.13 Rates for the 57-bus system with transmission costs involving **P & Q** (cont'd)

Interval	**Bus #**	**P_G [p.u.]**	**Q_G [p.u.]**	**V [p.u.]**	**MC_{pi} [\$/p.u.MWh]**	**MC_{qi} [\$/p.u.MVARh]**
Time Interval II (16hrs), LSF 0.6 with transmission costs involving **P & Q**	1	1.75117	-0.15280	1.10000	1470.81444	0.00000
	2		0.40402	1.10000	1477.89945	0.00000
	3	1.73382	-0.16234	1.10000	1466.71172	0.00000
	4			1.09735	1494.72349	2.64823
	5			1.09035	1531.89368	1.55448
	6		-0.04974	1.08932	1542.78497	0.00000
	7			1.09035	1546.82845	0.58803
	8	2.01822	0.21640	1.09993	1532.40923	3.02275
	9		0.23941	1.09317	1558.29999	1.70778
	10			1.08467	1561.80470	1.64187
	11			1.08544	1557.12312	4.18120
	12	2.07846	0.20318	1.09346	1546.40132	0.00000
	13			1.08995	1545.51559	5.53436
	14			1.08860	1536.54693	9.58757
	15			1.09193	1507.00325	5.47673
	16			1.09304	1536.62005	0.51569
	17			1.09316	1512.14024	2.17183
	18			1.08004	1502.72643	3.73790
	19			1.04684	1563.04575	26.27663
	20			1.03319	1582.96815	36.25220
	21			0.98267	1584.21995	25.53281
	22			0.98064	1586.06291	26.56930
	23			0.97999	1589.19852	26.41402
	24			0.97789	1617.00526	17.54955
	25			0.97541	1631.67806	17.11910
	26			1.02080	1616.86533	17.82597
	27			1.03349	1589.98276	8.22447
	28			1.04191	1568.29463	3.78422
	29			1.04941	1550.14572	1.32397
	30			0.96072	1656.84235	34.43457
	31			0.93782	1689.88901	62.05000
	32			0.93562	1660.91689	64.88324
	33			0.93422	1664.83848	66.84677
	34			0.96800	1650.83832	42.75353
	35			0.97086	1640.99744	39.20766
	36			0.97562	1626.76132	33.71028
	37			0.97665	1615.32265	33.50618
	38			0.98135	1580.46315	27.19824
	39			0.97550	1618.86692	34.91064
	40			0.97842	1628.47884	30.62750
	41			0.99843	1569.24691	13.57092
	42			0.96450	1613.62437	24.71352
	43			1.02855	1560.48453	0.01782
	44			0.99476	1562.19464	19.23630
	45			1.02942	1508.57536	1.01999
	46			0.98078	1538.31150	20.12157
	47			0.97676	1562.76420	30.97905
	48			0.97799	1568.73389	29.30403
	49			0.97640	1564.21779	31.04941
	50			0.97387	1589.04770	32.28434
	51			1.00170	1563.05056	0.19851
	52			1.02705	1605.87875	7.36870
	53			1.01881	1628.39057	7.84855
	54			1.01964	1600.76836	9.15746
	55			1.02636	1560.42564	3.91755
	56			0.94925	1623.38609	42.39180
	57			0.94689	1626.44948	41.21115

For Time Interval # I; Active Power Loss = 0.1354 p.u. MWh
For Time Interval # I; Active Power Loss = 7.687E-02 p.u. MWh
Total Energy Loss in p.u. MWH = 2.3134

Table 5.14 Increases in real power rates considering transmission costs in the 5 and 14-bus systems respectively.

System	Interval	Load Bus #	Increase when considering P only: MC_{pi} [$/p.u.MWh]	Increase when considering P& Q: MC_{pi} [$/p.u.MWh]
5-bus system	Time Interval I (8hrs) with LSF 1.0	3	3.74295	15.0544
		4	4.02398	16.05852
		5	4.69574	19.13722
5-bus system	Time interval II (16hrs) with LSF 0.8	3	2.94307	12.17141
		4	3.1624	13.01109
		5	3.68542	15.28553
14-bus system	Time Interval I (8hrs) with LSF 1.0	3	8.21987	38.2395
		4	6.44171	30.12732
		5	5.16961	24.57871
		6	5.2911	41.53538
		7	6.36321	41.0180
		8	6.36321	41.01792
		9	6.32263	47.16623
		10	6.75072	48.4126
		11	6.37600	46.14596
		12	7.11234	47.44945
		13	7.75076	49.05703
		14	10.04386	56.10157
14-bus system	Time Interval II (16hrs) with LSF 0.8	3	6.43839	30.03507
		4	5.05805	23.67628
		5	4.05997	19.32872
		6	4.13906	32.62884
		7	4.99082	32.3958
		8	4.99082	32.39573
		9	4.95328	37.29427
		10	5.28431	38.23159
		11	4.99201	36.36795
		12	5.56431	37.26666
		13	6.0576	38.53622
		14	7.81263	44.03146

Table 5.15 Increases in real power rates considering transmission costs in the 30- bus system.

System	**Interval**	**Load Bus #**	**Increase when considering P only: MC_{pi} [$/p.u.MWh]**	**Increase when considering P & Q: MC_{pi} [$/p.u.Mwh]**
30-bus system	Time Interval I (8hrs) with LSF 1.0	3	1.96318	17.36182
		4	2.75262	21.60487
		5	7.97767	42.79327
		6	4.52317	28.41602
		7	6.43554	36.90606
		8	5.17172	31.24365
		9	5.51721	33.24237
		10	6.07691	36.01027
		11	5.51721	33.24228
		12	0.06450	12.59198
		14	1.96871	20.50260
		15	3.57085	24.51112
		16	3.42096	24.68051
		17	5.77872	34.17889
		18	6.00077	33.45026
		19	7.04538	37.45344
		20	6.92101	37.44718
		21	7.04359	39.21943
		22	6.95049	38.95381
		23	5.58913	32.61581
		24	7.40985	40.73716
		25	7.51093	42.54578
		26	9.51628	47.81451
		27	6.65657	41.13274
		28	5.15728	30.76726
		29	9.62086	50.06311
		30	11.665	56.19800

Table 5.15 Increases in real power rates considering transmission costs in the 30- bus system (continued).

System	**Interval**	**Load Bus #**	**Increase when considering P only: MC_{pi} [\$/p.u.MWh]**	**Increase when considering P & Q: MC_{pi} [\$/p.u.MWh]**
30-bus system	Time Interval II (16hrs) with LSF 0.8	3	1.52884	13.65853
		4	2.14624	17.03805
		5	6.22793	33.47912
		6	3.52957	22.34141
		7	5.02277	28.91393
		8	4.03881	24.55497
		9	4.30603	26.06462
		10	4.74201	28.15244
		11	4.30603	26.06453
		12	0.06023	9.99347
		14	1.54962	16.06917
		15	2.79339	19.17025
		16	2.67737	19.40676
		17	4.5078	26.69840
		18	4.66771	25.95069
		19	5.47370	28.98147
		20	5.38157	29.03639
		21	5.48589	30.48614
		22	5.41285	30.29574
		23	4.34908	25.37217
		24	5.74591	31.59598
		25	5.81618	32.94307
		26	7.32905	36.72773
		27	5.16454	31.94989
		28	4.02180	24.15067
		29	7.40700	38.45876
		30	8.93634	42.88161

Table 5.16 Increases in real power rates considering transmission costs in the 57- bus system.

System	**Interval**	**Load Bus #**	**Increase when considering P only: MC_{pi} [$/p.u.MWh]**	**Increase when considering P & Q: MC_{pi} [$/p.u.MWh]**
57-bus system	Time Interval I (8hrs) with LSF 0.8	2	7.45696	9.57517
		4	0.71984	1.35609
		5	1.91285	7.99090
		6	2.62195	10.62864
		7	2.96074	12.17207
		9	3.45075	16.33930
		10	3.57172	18.22118
		11	3.85181	16.71884
		13	3.04480	13.42994
		14	2.60208	11.49465
		15	0.44717	3.122580
		16	1.99639	10.36414
		17	0.51732	3.25787
		18	0.58337	10.78669
		19	5.67481	23.51766
		20	8.21151	28.20362
		21	8.35362	30.59266
		22	8.76025	31.09838
		23	9.34270	32.09605
		24	17.02506	43.19682
		25	22.41001	65.69405
		26	16.94540	42.37642
		27	9.46812	29.95086
		28	5.77489	22.90565
		29	3.05711	17.68573
		30	34.19805	80.31688
		31	59.33078	107.84232
		32	28.55269	74.09163
		33	29.11995	75.10817
		34	22.28396	53.89563
		35	18.70595	48.99078
		36	15.19591	43.75555
		37	13.10641	39.96056
		38	7.74288	29.29653
		39	13.42393	40.67381
		40	14.90603	44.04090
		41	5.03711	29.45321
		42	10.97071	40.64805
		43	4.12040	20.08382
		44	5.68420	24.11955
		45	0.24713	9.73635
		46	2.55704	15.53963
		47	4.95447	23.61093
		48	5.81689	25.40187
		49	4.60157	23.52143
		50	6.20019	28.03873
		51	3.39200	20.87631
		52	7.39605	29.41023
		53	9.13302	34.30011
		54	6.70568	27.67755
		55	3.36975	18.81290
		56	13.99333	43.40789
		57	14.47396	45.20622

Table 5.16 Increases in power rates considering transmission costs in the 57- bus system (continued)

System	Interval	Load Bus #	Increase when considering P only: MC_{pi} [\$/p.u.MWh]	Increase when considering P & Q: MC_{pi} [\$/p.u.MWh]
57-bus system	Time Interval II (16hrs) with LSF 0.6	2	6.25765	7.79949
		4	0.11343	0.48007
		5	1.34570	6.82605
		6	1.84191	8.83507
		7	1.94608	9.94692
		9	2.50731	12.69410
		10	2.64441	13.97649
		11	2.52631	12.38557
		13	1.98628	9.84376
		14	1.59317	8.08524
		15	0.16408	1.80503
		16	1.49318	7.96973
		17	0.39729	2.33512
		18	0.50284	8.94504
		19	2.88378	16.04537
		20	3.98801	18.19302
		21	3.97521	20.02161
		22	4.07176	20.08890
		23	4.24268	20.57120
		24	5.70945	25.00215
		25	5.87689	37.10098
		26	5.73885	24.49493
		27	4.19688	19.74073
		28	2.95205	16.45105
		29	1.90051	13.91694
		30	7.32526	40.45805
		31	9.25574	44.55236
		32	7.71496	39.25934
		33	7.94450	39.71260
		34	7.66842	29.92797
		35	7.11963	28.41565
		36	6.30862	26.46115
		37	5.68140	24.65045
		38	3.76589	19.16429
		39	5.88549	25.07399
		40	6.38063	26.95323
		41	2.73626	21.23792
		42	5.36637	26.59845
		43	2.58815	14.78999
		44	2.80701	16.01225
		45	0.05347	6.88553
		46	1.52948	10.88831
		47	2.79943	16.65561
		48	3.11738	17.08964
		49	2.80090	16.40505
		50	4.15384	19.79289
		51	2.57833	16.01820
		52	5.08629	21.33597
		53	6.36566	24.43078
		54	4.81527	20.19358
		55	2.52265	14.49051
		56	6.06092	26.68923
		57	6.21550	27.72045

Chapter 6

COMPUTATIONAL RESULTS UNDER A DEREGULATED POWER ENVIRONMENT

Introduction

The concepts of transmission access and wheeling were discussed at length in Chapter 3 of this thesis. In this chapter, some simulation results under a deregulated environment are presented i.e. under wheeling conditions which assume that there is separate ownership of generation, transmission and distribution. The four test systems reported in Chapter 5, namely the 5-bus, 14-bus, 30-bus and 57-bus systems are the same test systems used in the studies reported in this chapter. Therefore, there will be no further discussion on the data of the systems since they have already been covered. The intention here, is to concentrate on the wheeling charges communicated to the users of the network which belongs to the third party.

Deregulation, as already pointed out in Chapter 3, is moving forward at a rapid pace in many countries as a means to improve electric markets through the use of competitive forces. Although wheeling has been in practice in North America for some years, there is still intense research in determining the wheeling charges since most of the methods do not necessarily guarantee the recovery of an appropriate share of the imbedded capital investment in facilities used for wheeling. The primarily objective of this chapter is therefore to evaluate the proposed algorithm used in Chapter 5 in establishing wheeling charges.

6.1 Computational Results

In the following sections, some of the simulated results under wheeling conditions are documented and analyzed. Like in Chapter 5, it is assumed that the wheeling charges are communicated instantly to the users of the network. The studies are then done hourly for 24 hours and are categorized into three phases. In the first phase, the fuel costs at the generation plant from which power is to be wheeled are used to establish the wheeling rates in the network. The power to be wheeled in the 5-bus system is assumed to be from the generating plant located at bus 2. This location is the same for the 14-bus and 30-bus systems respectively. In the 57-bus system, this is done with respect to the generating plant located at bus 8.

The classical way to establish the wheeling charges, is to use the fuel cost of the generating plant whose power is to be wheeled. These studies are done in the first phase. In the second phase, the wheeling charges incorporate in addition to the fuel cost of phase one, transmission cost which involve only the active power flow (**P**) injected at the entrance and exit buses respectively. Like in Chapter 5, each active power flow unit is assigned a trial price k. The third and final phase involves in addition to the fuel cost of phase one, transmission cost which considers both the active and reactive power flow (**P & Q**) injected at the entrance and exit buses respectively. Each of the reactive and active power flow units is assigned the same trial price k.

In all the phases, the loading at each of the exit buses varies over a day. The wheeling charges are recorded hourly and the loading at the exit buses evolve using a load scaling factor (LSF). The LSF varies from 0.3 to 1.1. Like in Chapter 5, only fully convergent load levels were taken as acceptable. The same procedure reported in Chapter 5 for establishing the trial prices is followed in all the cases reported in this chapter. The selection does not have an intention of recovering a specific amount of capital. Finally, as will be noticed later, the results of the same system for all the phases, will be displayed near each other so that they can be easily compared.

6.1.1 First Phase Results

In this phase, the results of the classical way of establishing the wheeling rates will be analyzed i.e. using only the fuel cost at the generating plants whose power is to be wheeled. These studies are useful in comparing with the wheeling results using the algorithm already proposed and discussed in Chapter 5. Table 6.1 shows the wheeling rates of the active and reactive power in the 5-bus system with respect to bus 2 without transmission cost consideration. For the 14-bus system, the phase one results are shown in Tables 6.4 and 6.7 respectively. For the 30-bus system, the phase one results are shown in Tables 6.10 and 6.13 respectively. Finally, for the 57-bus system, the phase one results are shown in Tables 6.16 and 6.19.

Though the studies were done hourly for a day, the results shown are only when the LSF is varying and not repeating. For example, in the 5-bus system, the results shown are for 16 hours. For the remaining 8 hours, it is assumed that the LSF takes the combination of the results already shown in the tables of results. This applies also to all the other systems under investigation. Both for the 14-bus and 30-bus systems, the results shown are for 14 hours. For the 57-bus system, it is only for 11 hours. By plotting the LSF against the number of hours, the hourly variation of the load at each of the load buses is obtained. This is shown in Figures 6.2 through 6.3 for the 5-bus, 14-bus and 57-bus systems respectively. For the 30-bus system, the hourly load variation is assumed to be the same as for the 14-bus system.

From all the tables of results for phase one, it is clear that as the LSF becomes bigger, the wheeling rates increase as well. This is to be expected, since higher LSF are an indication of higher power flow. In the 5-bus system, the highest wheeling charges (rates) for the active power are always registered at bus 5 and lowest at bus 3. For the reactive power wheeling rates in the 5-bus system, the highest rates are at bus 5 and lowest are at bus 4. In the 14-bus system, the highest wheeling charges for active power is at bus 14 and lowest at bus 9. For the reactive power wheeling rates in this system, the highest rates are always at bus 14 and lowest are at bus 8 when considering the magnitudes only. The reactive power wheeling rates are zero at bus 8 in the 14-bus system, because there is a reactive power supply source at this bus. This means there is no transmission of reactive power to this bus and hence no wheeling charges.

In the 30-bus system, the highest wheeling active power rates are always registered at bus 30 and lowest at bus 14. For the reactive power wheeling rates in this system, the highest rates are always at bus 26 and lowest at bus 27. Finally, in the 57-bus system, the highest wheeling active power rates are always at bus 32 and lowest at bus 14. For the wheeling reactive power rates in the 57-bus system, the highest rates are recorded at bus 32 and for the lowest, they are at bus 6. At bus 6, there is a reactive power supply source just like in the 14-bus system. At bus 32 in the 57-bus system, there is a big hike of the rates at the LSF 0.8 which might be attributed to voltage limits since at all other lower load scaling factors there is no such sudden change. Actually this change is noticeable on all the buses of all the systems at this particular LSF i.e. 0.8. Comparing the wheeling charges for active power (WC_{pi}) to that of reactive power (WC_{qi}) for all the results of phase one, it is generally observed that at lower LSF (0.3-0.55), in the 5-bus and 57-bus systems the two different charges in terms of magnitudes, compare favorably. At higher LSF (0.6-1.1)in the same two systems, the wheeling charges for the reactive power becomes relatively small to those of active power. It can therefore be stated that, in the 5-bus and 57-bus systems, the rise of the wheeling rates for active power is much faster than that for reactive power as the LSF rises. In the 14 and 30-bus systems, the wheeling rates for active power (WC_{qi}) are relatively smaller to the active wheeling rates (WC_{pi}) at all the loading scaling factors with the exception at bus 14 in the 30-bus system where both the active and reactive wheeling rates compare favorably at all the load scaling factors.

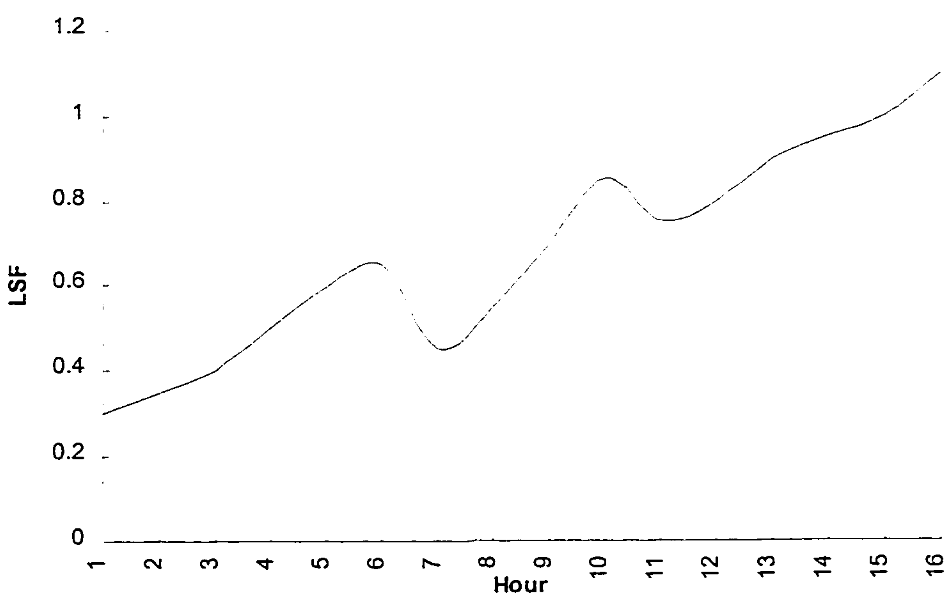

Figure 6.1 Variation of the hourly load at each bus in the 5-bus system

Table 6.1 Wheeling rates with respect to bus 2 in the 5-bus system without considering transmission costs.

Hr #	LSF	WC_{pi} with respect to bus 2 [$/p.u.MWh]			WC_{qi} with respect to bus 2 [$/p.u.MVARh]		
		Bus 3	**Bus 4**	**Bus 5**	**Bus 3**	**Bus 4**	**Bus 5**
1	0.3	9.41196	10.28423	12.44376	-10.74139	-13.16175	-4.86143
2	0.35	11.05155	12.07690	14.63488	-10.43469	-12.91990	-4.48041
3	0.4	12.73432	13.91286	16.88608	-10.12014	-12.67508	-4.08208
4	0.5	16.17849	17.66793	21.51301	-9.47852	-12.18675	-3.24399
5	0.6	20.03545	21.93879	26.50892	-8.09401	-10.77635	-2.02306
6	0.65	21.98906	24.08254	29.10351	-7.56119	-10.29024	-1.45358
7	0.45	14.44328	15.77652	19.17868	-9.80140	-12.43066	-3.67002
8	0.55	18.09155	19.79497	23.96352	-8.71596	-11.38503	-2.61467
9	0.7	23.98862	26.27704	31.76189	-7.00279	-9.77935	-0.85675
10	0.85	30.27010	33.17291	40.13074	-5.16708	-8.09120	1.10601
11	0.75	26.03482	28.52307	34.48515	-6.41812	-9.24302	-0.23177
12	0.8	28.12841	30.82143	37.27439	-5.80646	-8.68054	0.42220
13	0.9	32.46062	35.57834	43.05533	-4.49925	-7.47429	1.82052
14	0.95	34.70073	38.03854	46.04932	-3.80220	-6.82906	2.56662
15	1.0	36.99117	40.55433	49.11387	-3.07518	-6.15478	3.34521
16	1.1	41.72614	45.75611	55.45946	-1.52803	-4.71595	5.00360

Table 6.2 Wheeling rates with respect to bus 2 in the 5-bus system considering **P** only for the transmission costs .

Hr #	LSF	WC_{pi} with respect to bus 2 [$/p.u.MWh]			WC_{qi} with respect to bus 2 [$/p.u.MVARh]		
		Bus 3	**Bus 4**	**Bus 5**	**Bus 3**	**Bus 4**	**Bus 5**
1	0.3	10.27621	11.22939	13.58113	-11.70937	-14.34842	-5.29819
2	0.35	12.05172	13.16711	15.94860	-11.36911	-14.07672	-4.88152
3	0.4	13.87729	15.15847	18.38932	-11.01878	-13.80039	-4.44441
4	0.5	17.60701	19.22403	23.39685	-10.30616	-13.25059	-3.52699
5	0.6	21.77480	23.83832	28.79128	-8.79082	-11.70353	-2.19729
6	0.65	23.88269	26.15091	31.58898	-8.20714	-11.16865	-1.57792
7	0.45	15.72904	17.17743	20.87197	-10.66444	-13.52498	-3.99298
8	0.55	19.66880	21.51428	26.04132	-9.49173	-12.39755	-2.84989
9	0.7	26.03796	28.51583	34.45257	-7.59655	-10.60769	-0.92969
10	0.85	32.79532	35.93265	43.44985	-5.59599	-8.76119	1.19661
11	0.75	28.24062	30.93388	37.38314	-6.95832	-10.01994	-0.25177
12	0.8	30.49353	33.40588	40.38186	-6.29172	-9.40468	0.45670
13	0.9	35.14744	38.51502	46.58828	-4.87037	-8.08872	1.96884
14	0.95	37.55066	41.15384	49.79568	-4.11408	-7.38650	2.77431
15	1.0	40.00575	43.84994	53.08124	-3.32633	-6.65378	3.61396
16	1.1	45.91305	50.30800	61.10019	-1.83625	-4.80203	5.74122

Table 6.3 Wheeling rates with respect to bus 2 in the 5-bus system considering **P &Q** for the transmission costs.

Hr #	**LSF**	**WC_{pi} with respect to bus 2 [\$/p.u.MWh]**			**WC_{qi} with respect to bus 2 [\$/p.u.MVARh]**		
		Bus 3	**Bus 4**	**Bus 5**	**Bus 3**	**Bus 4**	**Bus 5**
1	0.3	11.98527	12.78469	16.70075	-15.23748	-19.12081	-6.46882
2	0.35	14.14681	15.13072	19.63835	-14.79884	-18.76662	-5.93820
3	0.4	16.33443	17.50405	22.61905	-14.35678	-18.41470	-5.39180
4	0.5	20.78807	22.33271	28.71098	-13.46256	-17.71818	-4.25069
5	0.6	26.42340	28.74239	35.50667	-9.50834	-13.12745	-1.72116
6	0.65	29.52827	32.32233	39.09607	-6.85868	-9.98421	-0.14240
7	0.45	18.54817	19.90471	25.64315	-13.91134	-18.06518	-4.82938
8	0.55	23.36609	25.21509	31.9751	-12.12231	-16.23496	-3.26980
9	0.7	32.12907	35.17866	42.55015	-5.94254	-9.07984	0.71820
10	0.85	40.08222	43.86464	53.29433	-3.52407	-6.85093	3.28583
11	0.75	34.72741	38.01586	46.05732	-5.16834	-8.36781	1.53914
12	0.8	37.3782	40.9109	49.6383	-4.36245	-7.62510	2.39470
13	0.9	42.84028	46.87797	57.02665	-2.65238	6.04452	4.21349
14	0.95	45.65315	49.95176	60.83649	-1.74655	-5.20504	5.17869
15	1.0	48.52168	53.08692	64.72516	-0.80573	-4.33168	6.18244
16	1.1	54.42898	59.54498	72.74411	1.8435	-2.47993	8.30970

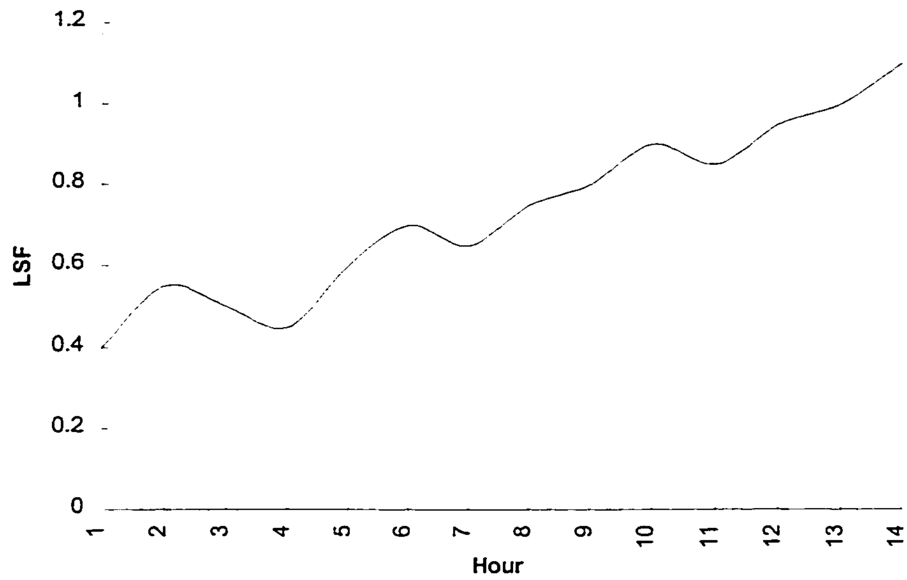

Figure 6.2 Variation of the hourly load at each bus in the 14-bus system

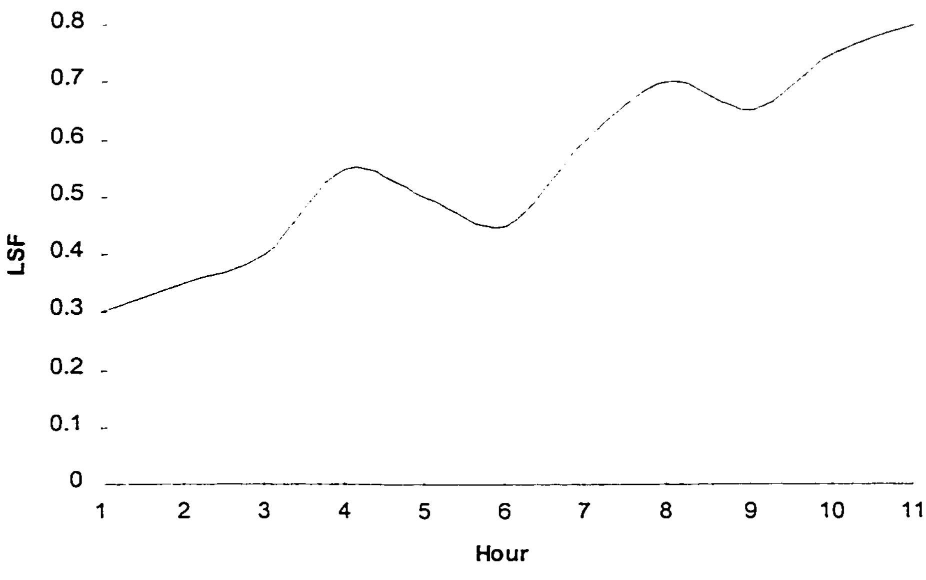

Figure 6.3 Variation of the hourly load at each bus in the 57-bus system

Table 6.4 Wheeling rates of active power with respect to bus 2 in the 14-bus system without transmission costs consideration .

		WC_{pi} with respect to bus 2 [$/p.u.MWh]						
Hr #	**LSF**	**Bus 8**	**Bus 9**	**Bus 10**	**Bus 11**	**Bus 12**	**Bus 13**	**Bus 14**
1	0.4	24.01061	23.74538	25.50243	23.74417	26.74357	29.47228	39.0086
2	0.55	35.15915	34.79799	37.43245	34.98951	39.53646	43.51552	57.51479
3	0.5	31.44278	31.11628	33.45284	31.25912	35.2834	38.82515	51.25974
4	0.45	27.84686	27.5565	29.60643	27.65226	31.17190	34.29371	45.23152
5	0.6	38.99873	38.60481	41.54867	38.84642	43.93391	48.36805	64.00228
6	0.7	47.05867	46.60693	50.20500	46.95189	53.17313	58.57256	77.69732
7	0.65	42.96429	42.54001	45.80500	42.83285	48.47853	53.38599	70.72788
8	0.75	51.28476	50.80904	54.75236	51.20668	58.02054	63.93100	84.91637
9	0.8	55.64554	55.14990	59.45086	55.60047	63.02357	69.46456	92.39088
10	0.9	64.76947	64.24830	69.30298	64.81320	73.50102	81.05519	108.09452
11	0.85	60.14071	59.62969	64.30106	60.13551	68.18384	75.17331	100.11855
12	0.95	69.53798	69.01077	74.45858	69.62826	78.96813	87.10822	116.32604
13	1.0	74.35005	73.78887	79.60498	74.43614	84.41921	93.13874	124.47448
14	1.1	84.45471	83.81501	90.39077	84.54712	95.85922	105.75982	141.44745

Table 6.5 Wheeling rates of active power with respect to bus 2 in the 14-bus system considering **P** only for the transmission costs

		WC_{pi} with respect to bus 2 [$/p.u.MWh]						
Hr #	**LSF**	**Bus 8**	**Bus 9**	**Bus 10**	**Bus 11**	**Bus 12**	**Bus 13**	**Bus 14**
1	0.4	26.03952	25.75187	27.65741	25.75056	29.00342	31.96270	42.30485
2	0.55	38.04031	37.65003	40.49972	37.85976	42.77881	47.08140	62.21847
3	0.5	34.05111	33.69795	36.22776	33.85490	38.21257	42.04586	55.50356
4	0.45	30.18558	29.87121	32.09280	29.97703	33.79188	37.17386	49.02285
5	0.6	42.1560	41.73071	44.91219	41.99467	47.49346	52.28378	69.17331
6	0.7	50.77827	50.29145	54.17306	50.66701	57.37932	63.20215	83.82599
7	0.65	46.40102	45.94337	49.46874	46.26270	52.35935	57.65627	76.37387
8	0.75	55.29071	54.77851	59.02892	55.21084	62.55622	68.9247	91.53557
9	0.8	59.94138	59.40820	64.04019	59.89750	67.89290	74.82718	99.50853
10	0.9	69.65549	69.09582	74.53061	69.70758	79.04996	87.16966	116.23203
11	0.85	64.73029	64.18106	69.20778	64.72938	73.39108	80.90991	107.74326
12	0.95	74.72397	74.15831	80.01115	74.82645	84.86206	93.60470	124.98349
13	1.0	79.84147	79.23971	85.48391	79.94035	90.65976	100.01771	133.64655
14	1.1	90.55429	89.86936	96.91845	90.66061	102.78820	113.39817	151.63951

Table 6.6 Wheeling rates of active power with respect to bus 2 in the 14-bus system considering **P &Q** for the transmission costs

		WC_{pi} with respect to bus 2 [$/p.u.MWh]						
Hr #	**LSF**	**Bus 8**	**Bus 9**	**Bus 10**	**Bus 11**	**Bus 12**	**Bus 13**	**Bus 14**
1	0.4	37.94593	40.14400	42.20933	39.21353	42.2203	45.64483	58.01724
2	0.55	55.02596	58.03168	61.23434	57.38400	62.33375	67.20294	84.95497
3	0.5	49.51499	52.25183	55.07771	51.55993	55.87630	60.23324	76.07887
4	0.45	44.08153	46.55324	49.01312	45.80844	49.50722	53.37580	67.40414
5	0.6	60.69884	63.98018	67.57250	63.37539	68.97420	74.37492	94.11951
6	0.7	72.48758	76.32284	80.71704	75.78487	82.71172	89.22900	113.15814
7	0.65	66.51826	70.07512	74.06299	69.50574	75.76320	81.71356	103.51132
8	0.75	78.60841	82.72567	87.53670	82.21900	89.83409	96.93807	123.07482
9	0.8	84.88391	89.28681	94.52509	88.81106	97.13287	104.84342	133.26498
10	0.9	97.91322	102.89976	109.02766	102.47903	112.25403	121.23383	154.47228
11	0.85	91.31823	96.01114	101.68877	95.56412	104.60572	112.94152	143.73029
12	0.95	104.67265	109.95715	116.54643	109.56146	120.08581	129.72958	165.49768
13	1.0	111.60023	117.18736	124.24984	116.81436	128.10092	138.42803	176.80831
14	1.1	125.96523	132.16728	140.20462	131.83242	144.68726	156.43883	200.28704

6.1.2 Second Phase Results

In this section, the results of the wheeling rates when transmission cost is considered in addition to the fuel cost of phase one are presented. The transmission cost under consideration involves only the active power (**P**) injected at the entrance and exit buses. The classical way of obtaining wheeling charges reported in phase one will at least recover the utility's incremental operating costs, and will in fact usually make a profit. There is no guarantee that the profit will be sufficient to recover an appropriate share of the imbedded capital investment in facilities used for wheeling [94]. Hence the necessity of revenue reconciliation discussed in this second phase of our studies.

Tables 6.2, 6.5, 6.11, and 6.17 show the second phase results for the active power wheeling rates in the 5-bus, 14-bus, 30-bus and 57 -bus systems respectively. Table 6.2 also shows the reactive power wheeling rates for the 5-bus system. The results of the reactive power wheeling rates in the other systems are shown in Tables 6.8, 6.14 and 6.20 for the 14-bus, 30-bus and 57-bus systems respectively. Notice that these results are all taken at the same buses, same load scaling factors (LSFs) and hourly as in phase one can Therefore, the results of the two phases can be compared.

Comparing all the tabulated results of this second phase to those of phase one, it is clear that there is an increase in wheeling rates at all the buses. This is true for both active and reactive power wheeling rates when considering the magnitudes only. Generally, the increase becomes more pronounced as the LSF gets bigger. This shows that the real time wheeling rates trajectories are closely related to demand, except that the percentage change in rates is greater than the corresponding percentage change in demand.

In the 5-bus system, the highest rates are at bus 5 as in phase one for both active and reactive wheeling rates. The lowest rates are distributed as in phase one as well i.e. lowest active wheeling rates at bus 3 and bus 4 for reactive power wheeling rates. For the 14-bus system, the highest rates for both the active and reactive wheeling rates are still at bus 14. The lowest real power wheeling rates are always at bus 9 and bus 8 for reactive power wheeling rates. This is the same distribution as in phase one. In the 30-bus system, the highest active wheeling rates are always at bus 30 and lowest at bus 14, whereas for the reactive power wheeling rates, the highest are at bus 26 and lowest at bus 27. This is the same distribution as in phase. Finally, in the 57-bus system, for both active and reactive wheeling rates, the highest is always recorded at bus 32. The lowest in the 57-bus system are at bus 14 for the active power wheeling rates and bus 6 for the reactive power wheeling rates respectively. This is also the same pattern as in phase one.

It is important to see therefore, that the increases are distributed evenly as in phase one. Where there is a reactive power supply source, like in the 14-bus system at bus 8 and in the 57-bus system at bus 6 respectively, the wheeling charges are still zero as in phase one.

Comparing the reactive power wheeling rates (WC_{qi}) to those of the active power (WC_{pi}), in the 5-bus system, the two different rates compare favorably at lower LSF(0.3-

0.45) than at higher LSF(0.5-1.1). At higher LSF in this system, the reactive wheeling rates becomes relatively small. In the 14-bus and 30-bus systems, the reactive power wheeling rates are generally smaller than the active power wheeling rates at all the LSFs. In the 57-bus system, the two charges compare favorably at all the LSFs. This is true if results of bus 6 where there is a reactive power supply source are disregarded. The results in the 5-bus and 57-bus systems suggest that the reactive power wheeling rates are not necessary negligibly or smaller when compared to the active power wheeling charges.

Table 6.7 Wheeling rates of reactive power with respect to bus 2 in the 14-bus system without transmission costs considerations.

		WC_{qi} with respect to bus 2 [$/p.u.MVARh]						
Hr #	**LSF**	**Bus 8**	**Bus 9**	**Bus 10**	**Bus 11**	**Bus 12**	**Bus 13**	**Bus 14**
1	0.4	0.00000	-7.18644	-4.70001	-1.96534	2.54669	3.08067	3.26369
2	0.55	0.00000	-5.48661	-2.47461	-0.39696	3.73113	5.18341	7.04580
3	0.5	0.00000	-5.90314	-3.09371	-0.85398	3.31699	4.47281	5.80020
4	0.45	0.00000	-6.26138	3.65228	-1.27347	2.919558	3.79634	4.62738
5	0.6	0.00000	-5.00825	-1.79169	0.09936	4.16228	5.92908	8.36713
6	0.7	0.00000	-3.85132	-0.22080	1.21727	5.07676	7.52935	11.24912
7	0.65	0.00000	-4.46441	-1.04159	0.63681	4.61074	6.71076	9.76721
8	0.75	0.00000	-3.16505	0.67428	1.84263	5.56059	8.38574	12.81598
9	0.8	0.00000	-2.40157	1.64735	2.51484	6.06245	9.28081	14.47097
10	0.9	0.00000	-0.51542	3.94003	4.06250	7.10834	11.19161	18.07941
11	0.85	0.00000	-1.52845	2.72707	3.24994	6.57943	10.21595	16.22300
12	0.95	0.00000	0.47181	5.13936	4.86970	7.65885	12.19549	19.98648
13	1.0	0.00000	0.54126	5.52535	5.22415	8.21371	13.08558	21.47694
14	1.1	0.00000	0.82109	6.44414	6.02677	9.33437	14.92345	24.57210

Table 6.8 Wheeling rates of reactive power with respect to bus 2 in the 14-bus system considering **P** only for the transmission costs

		WC_{qi} with respect to bus 2 [$/p.u.MVARh]						
Hr #	**LSF**	**Bus 8**	**Bus 9**	**Bus 10**	**Bus 11**	**Bus 12**	**Bus 13**	**Bus 14**
1	0.4	0.00000	-7.79370	-5.09716	-2.13141	2.76188	3.34099	3.53948
2	0.55	0.00000	-5.93185	-2.67484	-0.42862	4.03492	5.60558	7.62013
3	0.5	0.00000	-6.38832	-3.34753	-0.92374	3.59040	4.84161	6.27891
4	0.45	0.00000	-6.78261	-3.95597	-1.37916	3.16322	4.11328	5.01411
5	0.6	0.00000	-5.40954	-1.93450	0.10804	4.49708	6.40614	9.04085
6	0.7	0.00000	-4.15209	-0.23674	1.31358	5.47543	8.12075	12.13323
7	0.65	0.00000	-4.81759	-1.12301	0.68813	4.97717	7.24422	10.54416
8	0.75	0.00000	-3.40898	0.72801	1.98634	5.99210	9.03663	13.81128
9	0.8	0.00000	-2.58406	1.77509	2.70843	6.52740	9.99273	15.58158
10	0.9	0.00000	-0.55669	4.23283	4.36545	7.64148	12.03028	19.43406
11	0.85	0.00000	-1.64674	2.93156	3.49494	7.07863	10.99040	17.45271
12	0.95	0.00000	0.50450	5.51798	5.22916	8.22672	13.09903	21.46719
13	1.0	0.00000	0.58041	5.92991	5.60680	8.81607	14.04503	23.05170
14	1.1	0.00000	0.87929	6.90548	6.45839	10.00379	15.99345	26.33397

Table 6.9 Wheeling rates of reactive power with respect to bus 2 in the 14-bus system considering both **P & Q** for the transmission costs

		WC_{qi} with respect to bus 2 [\$/p.u.MVARh]						
Hr #	**LSF**	**Bus 8**	**Bus 9**	**Bus 10**	**Bus 11**	**Bus 12**	**Bus 13**	**Bus 14**
1	0.4	0.00000	-6.93327	-4.10463	-1.55053	3.46368	4.15191	5.33129
2	0.55	0.00000	-3.14837	0.10178	1.16193	5.00920	6.93856	10.83647
3	0.5	0.00000	-4.01729	-0.96938	0.43817	4.47212	6.02457	9.11915
4	0.45	0.00000	-4.95004	-2.08429	-0.30570	3.95611	5.13530	7.43141
5	0.6	0.00000	-2.19118	1.26035	1.93737	5.56809	7.89686	12.65411
6	0.7	0.00000	-0.50385	3.40442	3.40136	6.73150	9.85895	16.32005
7	0.65	0.00000	-1.35312	2.32302	2.66255	6.14055	8.86221	14.45829
8	0.75	0.00000	0.38693	4.53250	4.16939	7.34092	10.88929	18.25431
9	0.8	0.00000	1.32008	5.70798	4.96693	7.96880	11.95338	20.26204
10	0.9	0.00000	3.31286	8.19785	6.64840	9.28130	14.18810	24.50213
11	0.85	0.00000	2.29514	6.92941	5.79312	8.61565	13.05301	22.34446
12	0.95	0.00000	4.37406	9.51404	7.53313	9.96519	15.35688	26.73481
13	1.0	0.00000	5.47866	10.87728	8.44681	10.66741	16.56007	29.04306
14	1.1	0.00000	7.82048	13.74940	10.36277	12.12514	19.06727	33.88588

Table 6.10 Wheeling rates of active power with respect to bus 2 in the 30-bus system without transmission costs consideration

		WC_{pi} with respect to bus 2 [\$/p.u.MWh]						
Hr #	**LSF**	**Bus 14**	**Bus 15**	**Bus 18**	**Bus 26**	**Bus 27**	**Bus 29**	**Bus 30**
1	0.4	4.7922	11.59653	21.69612	35.44257	24.23417	35.89127	43.68569
2	0.55	7.21834	16.97418	31.53803	51.75812	35.33959	52.41264	63.90792
3	0.5	6.35650	15.06899	28.05741	46.01784	31.41192	46.61115	56.82076
4	0.45	5.51122	13.26796	24.80414	40.61535	27.73457	41.12996	50.10718
5	0.6	7.90638	18.73202	34.92439	57.51721	39.16109	58.23703	71.11263
6	0.7	9.36364	22.44201	42.07966	69.73251	47.25061	70.58078	86.40797
7	0.65	8.62919	20.56273	38.44615	63.5146	43.14173	64.29891	78.61517
8	0.75	10.10978	24.37109	45.82867	76.17949	51.49179	77.09097	94.50303
9	0.8	10.86765	26.35126	49.69716	82.86459	55.86958	83.83829	102.91311
10	0.9	12.40202	30.43489	57.71954	96.84111	64.99959	98.40994	120.59321
11	0.85	11.63664	28.38327	53.68873	89.7969	60.38802	90.83158	111.65123
12	0.95	13.17761	32.53422	61.8602	104.11446	69.7465	105.36255	129.85440
13	1.0	13.96373	34.68278	66.11493	111.62644	74.63324	112.99427	139.44867
14	1.1	15.56732	39.13084	74.97652	127.39356	84.84012	129.03055	159.68317

Table 6.11 Wheeling rates of active power with respect to bus 2 in the 30-bus system considering **P** only for transmission cost.

		WC_{pi} with respect to bus 2 [$/p.u.MWh]						
Hr #	**LSF**	**Bus 14**	**Bus 15**	**Bus 18**	**Bus 26**	**Bus 27**	**Bus 29**	**Bus 30**
1	0.4	5.21566	12.60960	23.58397	38.52092	26.34242	39.00852	47.47760
2	0.55	7.80556	18.38148	34.17036	56.09487	38.28697	56.80538	69.27399
3	0.5	6.89932	16.36935	30.48306	49.97678	34.12703	50.61572	61.69150
4	0.45	5.98917	14.41196	26.93822	44.10579	30.12073	44.66454	54.4114
5	0.6	8.57136	20.30091	37.84421	62.32138	42.43731	63.10152	77.04913
6	0.7	10.13948	24.28883	45.53352	75.44835	51.12995	76.36610	93.48628
7	0.65	9.34950	22.26979	41.63086	68.76986	46.7171	69.61915	85.11609
8	0.75	10.94141	26.35935	49.55616	82.36580	55.68015	83.35109	102.17231
9	0.8	11.75518	28.48256	53.70278	89.53155	60.37203	90.5832	111.18736
10	0.9	13.40210	32.85546	62.28948	104.49075	70.14263	105.71674	130.11164
11	0.85	12.58061	30.65965	57.97754	96.95544	65.21012	98.07199	120.54542
12	0.95	14.23282	35.10045	66.71493	112.26408	75.21516	113.60870	140.01044
13	1.0	15.07498	37.39617	71.25824	120.28526	80.43235	121.75767	150.25621
14	1.1	16.79168	42.14328	80.70880	137.09941	91.31508	138.85877	171.83727

Table 6.12 Wheeling rates of active power with respect to bus 2 in the 30-bus system considering **P & Q** for transmission cost.

		WC_{pi} with respect to bus 2 [$/p.u.MWh]						
Hr #	**LSF**	**Bus 14**	**Bus 15**	**Bus 18**	**Bus 26**	**Bus 27**	**Bus 29**	**Bus 30**
1	0.4	8.10109	16.69588	30.35716	49.30039	35.65467	50.33066	60.14250
2	0.55	12.24672	24.32095	43.69118	71.12930	51.33679	72.73557	87.14111
3	0.5	10.86148	21.74882	39.16683	63.66892	45.99993	65.07504	77.88635
4	0.45	9.47957	19.20755	34.72427	56.39649	40.77522	57.61051	68.89151
5	0.6	13.63597	26.92407	48.29625	78.77960	56.78609	80.59533	96.66161
6	0.7	16.46048	32.25716	57.78372	94.69876	68.06066	96.96486	116.56402
7	0.65	15.03607	29.56527	52.98987	86.6307	62.35561	88.66569	106.46231
8	0.75	18.24821	35.41174	63.18485	103.52896	74.41943	106.01979	127.50019
9	0.8	19.60632	38.19101	68.31735	68.31735	80.48897	114.96655	138.46422
10	0.9	22.39519	43.95605	79.03336	130.62528	93.15894	133.77340	161.60380
11	0.85	20.98905	41.03922	73.60002	121.29264	86.73443	124.21626	149.82991
12	0.95	23.81912	46.93833	84.6183	140.26911	99.7673	143.64866	173.8018
13	1.0	25.28588	50.01345	90.38474	150.26050	106.58553	153.87693	186.46622
14	1.1	28.34016	56.43231	102.47338	171.31316	120.86422	175.43014	213.25520

Table 6.13 Wheeling rates of reactive power with respect to bus 2 in the 30-bus system without transmission costs consideration

		WC_{qi} with respect to bus 2 [$/p.u.MVARh]						
Hr #	**LSF**	**Bus 14**	**Bus 15**	**Bus 18**	**Bus 26**	**Bus 27**	**Bus 29**	**Bus 30**
1	0.4	1.78253	1.01675	0.83532	6.06407	0.88346	3.83175	5.02520
2	0.55	3.47822	3.49906	4.15695	12.19618	3.03340	7.46417	9.26215
3	0.5	2.36386	2.37922	2.88262	10.14538	2.40832	6.31894	7.90448
4	0.45	2.14840	1.70052	1.79238	7.94435	1.49577	4.91291	6.29722
5	0.6	3.99126	4.37872	5.24265	14.30518	3.67646	8.67152	10.70042
6	0.7	5.36891	6.43975	7.71981	18.88080	5.09994	11.32015	13.85182
7	0.65	4.59553	5.37228	6.44045	16.52876	4.36000	9.95000	12.22279
8	0.75	6.20491	7.58543	9.08547	21.36865	5.90027	12.78809	15.59456
9	0.8	7.10726	8.81387	10.54244	24.00019	6.76525	14.36033	17.45855
10	0.9	9.22556	11.44852	13.40227	29.56347	8.73679	17.85929	21.58974
11	0.85	8.07939	10.12941	12.09553	26.78328	7.69895	16.04336	19.45138
12	0.95	10.45347	12.84932	14.77352	32.48142	9.84977	19.79305	23.86440
13	1.0	11.76488	14.33750	16.21952	35.54660	11.04133	21.85059	26.28251
14	1.1	14.65380	17.59181	19.34804	42.14500	13.67807	26.36443	31.58091

Table 6.14 Wheeling rates of reactive power with respect to bus 2 in the 30-bus system considering **P** only for transmission cost.

		WC_{qi} with respect to bus 2 [$/p.u.MVARh]						
Hr #	**LSF**	**Bus 14**	**Bus 15**	**Bus 18**	**Bus 26**	**Bus 27**	**Bus 29**	**Bus 30**
1	0.4	1.93684	1.10492	0.90794	6.58912	0.96004	4.16352	5.46027
2	0.55	3.49668	3.74778	4.47019	13.22817	3.30016	8.10626	10.05656
3	0.5	2.74401	2.68271	3.15403	10.93152	2.51605	6.75847	8.47854
4	0.45	2.33263	1.84502	1.94340	8.62127	1.61953	5.32963	6.83263
5	0.6	4.20690	4.74510	5.68030	15.49465	3.98024	9.39118	11.58899
6	0.7	5.81032	6.96787	8.35179	20.42139	5.51380	12.24216	14.98064
7	0.65	4.97719	5.81727	6.97284	17.89019	4.71703	10.76803	13.22826
8	0.75	6.71000	8.20144	9.82215	23.09603	6.37480	13.82002	16.85361
9	0.8	7.68001	9.52261	11.38902	25.92219	7.30436	15.50834	18.85495
10	0.9	9.95470	12.35169	14.45843	31.88775	9.42079	19.26120	23.28520
11	0.85	8.72426	10.93625	13.05778	28.90851	8.30710	17.31429	20.99301
12	0.95	11.27165	13.85338	15.92690	35.01181	10.61407	21.33267	25.72147
13	1.0	12.67678	15.44723	17.47398	38.29059	11.89049	23.53491	28.30923
14	1.1	15.76795	18.92790	20.81687	45.33990	14.71156	28.36043	33.97267

Table 6.15 Wheeling rates of reactive power with respect to bus 2 in the 30-bus system considering **P & Q** for transmission cost.

		WC_{qi} with respect to bus 2 [$/p.u.MVARh]						
Hr #	**LSF**	**Bus 14**	**Bus 15**	**Bus 18**	**Bus 26**	**Bus 27**	**Bus 29**	**Bus 30**
1	0.4	5.52466	4.05027	3.44398	8.91428	1.70891	5.42113	6.92332
2	0.55	8.30999	7.91593	8.20218	17.51034	5.60681	11.15986	13.41257
3	0.5	7.35434	6.58184	6.55940	14.52249	4.23916	9.14703	11.13628
4	0.45	6.42561	5.29330	4.97353	11.65901	2.94140	7.23617	8.97548
5	0.6	9.28926	9.29314	9.90035	20.62472	7.04511	13.27696	15.80734
6	0.7	11.35159	12.21202	13.49813	27.28703	10.18339	17.88041	21.01180
7	0.65	10.30577	10.72705	11.66694	23.88170	8.56839	15.51437	18.33740
8	0.75	13.21206	14.34643	15.78281	30.96634	11.95496	20.44291	23.89968
9	0.8	14.75676	16.22060	17.87767	34.52682	13.44177	22.78656	26.59657
10	0.9	18.15522	20.32789	22.43882	42.25569	16.75147	27.96293	32.54509
11	0.85	16.41247	18.22349	20.10656	38.29991	15.05079	25.30285	29.48772
12	0.95	20.00075	22.54913	24.88761	46.41304	18.56003	30.78606	35.78938
13	1.0	21.97082	24.90915	27.47584	50.79967	20.49868	33.79852	39.24867
14	1.1	26.24214	30.01198	33.03069	60.25016	24.73684	40.37597	46.80338

Table 6.16 Wheeling rates of active power with respect to bus 8 in the 57-bus system without transmission cost considerations.

		WC_{pi} with respect to bus 2 [$/p.u.MWh]						
Hr #	**LSF**	**Bus 6**	**Bus 14**	**Bus 20**	**Bus 32**	**Bus 48**	**Bus 55**	**Bus 57**
1	0.3	2.91794	0.45552	14.57205	34.86131	9.54079	7.90444	27.33441
2	0.35	3.53113	0.59770	17.78754	42.81156	11.64513	9.54323	33.40943
3	0.4	4.18374	0.76381	21.26075	51.49883	13.91807	11.27674	39.98694
4	0.55	6.12988	5.29324	39.05347	94.54514	28.39987	19.82902	72.80561
5	0.5	5.02643	1.27607	26.92388	43.71813	18.32017	15.36709	47.95771
6	0.45	4.87348	0.97135	25.01652	60.9998	16.38581	13.10752	47.11445
7	0.6	7.19816	1.70994	38.02338	94.90581	24.89251	9.18239	71.97728
8	0.7	8.94811	2.36880	48.35021	122.92755	31.65218	23.72646	91.91069
9	0.65	8.05298	2.02265	43.00969	108.31026	28.15665	21.40474	81.57971
10	0.75	9.64173	2.56909	56.08991	156.13522	36.25292	25.8178	108.12117
11	0.8	11.70696	9.07495	109.5522	496.87801	66.24436	26.34404	224.41609

Table 6.17 Wheeling rates of active power with respect to bus 8 in the 57-bus system considering **P** only for transmission cost.

		WC_{pi} with respect to bus 2 [\$/p.u.MWh]						
Hr #	**LSF**	**Bus 6**	**Bus 14**	**Bus 20**	**Bus 32**	**Bus 48**	**Bus 55**	**Bus 57**
1	0.3	3.21123	0.60752	15.81681	37.66852	10.39454	8.53824	29.56339
2	0.35	3.87755	0.78242	19.26575	12.65922	12.65922	10.28722	36.05851
3	0.4	4.59317	0.98277	23.00926	55.70051	15.11061	12.12578	43.14885
4	0.55	6.95530	1.76717	35.87072	88.41760	23.562530	18.25492	67.55342
5	0.5	6.12350	1.47529	31.26872	76.47658	20.54096	16.51549	58.78504
6	0.45	5.35513	1.20493	26.98962	65.5234	17.72018	14.07047	50.65071
7	0.6	7.82879	2.09184	40.80012	101.40950	26.79861	20.49377	76.98151
8	0.7	9.70027	2.84544	51.71472	130.94251	33.96167	25.27149	97.99496
9	0.65	8.74386	2.45079	46.07501	115.54856	30.26086	22.83253	87.11293
10	0.75	10.45377	3.09825	59.80323	165.26422	38.79412	27.47678	114.8687
11	0.8	12.64629	9.99441	116.08109	523.74808	70.37863	28.03117	237.20743

Table 6.18 Wheeling rates of active power with respect to bus 8 in the 57-bus system considering **P & Q** for transmission cost.

		WC_{pi} with respect to bus 2 [$/p.u.MWh]						
Hr #	**LSF**	**Bus 6**	**Bus 14**	**Bus 20**	**Bus 32**	**Bus 48**	**Bus 55**	**Bus 57**
1	0.3	3.92184	1.84906	20.08981	50.40389	15.03541	12.50420	38.05618
2	0.35	5.1223	1.85281	24.27862	60.84169	17.79523	14.71208	45.63502
3	0.4	6.08719	2.33271	28.97795	72.74258	21.18792	17.25981	54.21051
4	0.55	9.16355	4.03123	45.03617	113.70649	32.67344	25.53145	83.708170
5	0.5	8.09087	3.42624	39.29801	98.93766	28.61980	22.67040	73.22288
6	0.45	7.06602	2.85912	33.97804	85.42035	24.79734	19.91250	63.39946
7	0.6	10.28475	4.68180	50.99084	129.64882	36.96580	28.50301	94.91853
8	0.7	12.6505	6.08488	64.18992	165.5376	46.30992	34.74397	119.70581
9	0.65	11.44279	5.35970	57.38493	146.87645	41.50016	31.57492	106.88805
10	0.75	13.90289	6.86270	71.41774	185.72963	51.40002	38.02919	133.36995
11	0.8	16.70027	14.93427	132.12049	565.33431	86.0109	39.52161	263.98698

Table 6.19 Wheeling rates of reactive power with respect to bus 8 in the 57-bus system without transmission costs consideration

		WC_{qi} with respect to bus 8 [$/p.u.MVARh]						
Hr #	**LSF**	**Bus 6**	**Bus 14**	**Bus 20**	**Bus 32**	**Bus 48**	**Bus 55**	**Bus 57**
1	0.3	0.00000	0.56501	11.01589	12.11386	8.12457	3.58849	6.18411
2	0.35	0.00000	1.37943	13.24442	16.46253	9.93385	3.66463	9.35377
3	0.4	0.00000	2.29004	15.68835	21.33360	11.93227	3.74297	12.87595
4	0.55	0.00000	27.91742	42.69228	61.29605	42.96286	2.89333	47.29398
5	0.5	0.00000	9.58279	21.01872	62.65007	17.27164	3.41961	14.11561
6	0.45	0.00000	3.35374	18.38529	26.81825	14.17944	3.82430	16.81893
7	0.6	0.00000	7.04323	27.94218	47.22494	22.13131	4.09159	31.10854
8	0.7	0.00000	10.19010	35.82038	65.08320	28.80266	4.29812	43.22617
9	0.65	0.00000	8.54542	31.71749	55.66718	25.31900	4.19145	36.88165
10	0.75	0.00000	13.60518	44.52684	146.33488	38.13900	4.57149	60.76522
11	0.8	0.00000	42.59506	129.98128	1378.85957	135.96249	9.11825	250.99176

Table 6.20 Wheeling rates of reactive power with respect to bus 8 in the 57-bus system considering **P** only for transmission cost.

		WC_{qi} with respect to bus 8 [$/p.u.MVARh]						
Hr #	**LSF**	**Bus 6**	**Bus 14**	**Bus 20**	**Bus 32**	**Bus 48**	**Bus 55**	**Bus 57**
1	0.3	0.00000	0.60465	11.86913	13.05114	8.75225	3.86791	6.66002
2	0.35	0.00000	1.47771	14.24062	17.70029	10.67894	3.94205	10.05392
3	0.4	0.00000	2.50816	16.96063	23.93916	12.89879	4.02964	13.97368
4	0.55	0.00000	6.04352	26.11879	42.32939	20.50669	4.27041	27.59886
5	0.5	0.00000	4.72706	22.75676	35.14116	17.69983	4.18221	22.55296
6	0.45	0.00000	3.47906	19.70414	28.66669	15.09604	4.11529	17.93292
7	0.6	0.00000	7.48936	29.77396	50.32877	23.57660	4.36402	33.14577
8	0.7	0.00000	10.80353	38.05145	69.15163	30.58928	4.57108	45.91873
9	0.65	0.00000	9.07303	33.74373	59.23449	26.93006	4.46393	39.23668
10	0.75	0.0000	14.33533	47.04323	152.12062	40.20292	4.84800	63.99153
11	0.8	0.00000	44.82779	136.89865	1448.89352	143.10595	9.61037	264.11711

Table 6.21 Wheeling rates of reactive power with respect to bus 8 in the 57-bus system considering **P & Q** for transmission cost.

		WC_{qi} with respect to bus 8 [$/p.u.MVARh]						
Hr #	**LSF**	**Bus 6**	**Bus 14**	**Bus 20**	**Bus 32**	**Bus 48**	**Bus 55**	**Bus 57**
1	0.3	0.00000	2.45535	12.93483	19.90019	12.08386	4.18810	11.05736
2	0.35	0.00000	2.77228	16.81081	25.34801	13.92321	4.54625	14.61871
3	0.4	0.00000	3.95891	20.10453	31.93272	16.57417	4.73299	19.14301
4	0.55	0.00000	8.23870	31.15184	56.02265	25.95557	5.35521	35.39969
5	0.5	0.00000	6.68062	27.18520	47.15818	22.56120	5.13992	29.48316
6	0.45	0.00000	5.25715	23.50293	39.17135	19.44136	4.93305	24.07759
7	0.6	0.00000	9.93235	35.46886	65.78528	29.63627	5.55063	41.85396
8	0.7	0.00000	13.80070	45.16037	88.56190	37.98869	6.06071	56.62488
9	0.65	0.00000	11.78856	40.13601	76.60086	33.64496	5.81321	48.92003
10	0.75	0.00000	15.98317	50.58444	101.82026	42.69590	6.31440	65.03481
11	0.8	0.00000	48.13035	145.72678	1465.72370	151.11917	11.46462	275.92318

6.1.3 Third Phase Results

This is the final phase of the studies. Here, the results of incorporating transmission cost which involves both reactive (**Q**) and active (**P**) power injected at the entrance and exit buses in addition to the fuel cost covered in phase one are presented. Tables 6.3, 6.6, 6.12 and 6.18 show the results of the third phase for the active wheeling rates in the 5-bus, 14-bus, 30-bus and 57-bus systems respectively. Table 6.3 includes the reactive wheeling rate results for the 5-bus system. The reactive results for the other systems are shown in Tables 6.9, 6.15 and 6.21 for the 14-bus, 30-bus and 57-bus systems respectively. These results are also taken from the same buses, same LSF and hourly as in phase one and two.

Comparing the results of this phase to those of phase one, there is now a remarkable increase in rates at the buses. These increases are much higher than those of phase two. This suggests that the flow of reactive power has great influence on real power pricing and wheeling. This is a very important consideration which is overlooked by transmission pricing which relies on DC-OPF methods. The implications of these results for cost-benefit analysis in reactive compensation investment studies are also significant.

In the 5-bus system, the highest rates are at bus 5 for both reactive and active power. This is in agreement with the other phases. For the lowest rates in the 5-bus system, they are always at bus 3 for active power wheeling rates and at bus 4 for reactive wheeling rates. This is the same as in the other phases. In the 14-bus system, the highest rates are at bus 14 for both reactive and active power wheeling rates. This is also the same as in other phases. The lowest active wheeling rate in the 14-bus system, unlike phase two which was at bus 9, is now at bus 8. As for the lowest reactive power wheeling rates in the same system, they are still at bus 8 when comparing magnitudes only. The reactive power wheeling rates at this particular bus i.e. 8 are all zero as in phases one and two because as already stated, there is a reactive power supply source at this bus. In the 30-bus system, the highest rates are at bus 30 for the active power wheeling rates and bus 26 for the reactive power wheeling rates. For the lowest rates in the 30-bus system, bus 14 shows the lowest active power wheeling rates at all the LSFs and bus 27 shows the lowest reactive power wheeling rates. This distribution for both active and reactive wheeling rates is the same as in the other phases. Finally, in the 57-bus system, the highest both in active and reactive power wheeling rates are at bus 32. The lowest wheeling rates are at bus 14 for active power and bus 6 for reactive power. This pattern of active and reactive power wheeling rates, is the same as in the other phases.

Comparing the reactive power wheeling rates (WC_{qi}) to those of active power (WC_{pi}), in the 5-bus system, just like in the other phases, the two rates compare favorably at lower LSF (0.3-0.45). At higher LSF, the reactive power wheeling rates become relatively small. These comparisons are done in terms of magnitudes only. In the 14-bus and 30-bus systems, the reactive power wheeling rates are always relatively small at all the LSF with the exception of the rates at bus 14 in the 30-bus system where the two different rates compare favorably at all LSFs. Finally, disregarding the zero reactive power wheeling rates at bus 6 in the 57-bus system, it can be seen that for the other buses in this system, the two different rates compare favorably at all LSFs.

In conclusion, the results under a deregulated environment are in agreement with those reported under a regulated environment in Chapter 5 in terms of the increases experienced when transmission costs are incorporated in the model. These increases are dependent on the choice of the trial prices selected. The selection in turn becomes important in terms of control of recovery of the utility's capital and profit.

Chapter 7

CONCLUSIONS AND RECOMMENDATIONS

The modeling and some results of studies on effects of incorporating transmission costs in establishing real-time electricity rates for the consumers have been presented in this thesis under the following scenarios:-

(a) assuming that all generation, transmission and distribution of electricity are under one utility i.e. under a regulated environment and

(b) assuming that there is separate ownership of generation and transmission of electricity i.e. under a deregulated environment.

The results obtained in each of the two different scenarios, were compared to the classical way of establishing the electricity rates. Four IEEE standard systems, namely the 5-bus, 14-bus, 30-bus and 57-bus systems were used in the implementation of the proposed algorithm. The interest of improving methods of charging transmission charges will always remain since there is now an increase in the power industry of institutionally separate utilities for power generation and transmission. Secondly, the cost difference in delivering electricity to consumers in the same region mainly lies in transmission charges.

The intention in this chapter is to present the conclusions reached from the simulations and analysis. The chapter ends by recommending future research work.

7.1 Research Conclusions

In this work, transmission costs were successfully incorporated in establishing electricity rates communicated to the consumers. The following are some of the conclusions of this work:

- The electrical transmission of electric power both from a seller to a buyer through a transmission network owned by a third party has cost implications. The costs incurred are of interest to the owners of transmission systems in view of the increased interest in deregulation of the electricity industry.
- The assignment of a price to each unit being transmitted present a useful tool in the hands of the utility involved in transmission of electricity to the consumers. The use of the fuel costs curves of the generating plants to establish electricity rates does present problems in the sense that there is no guarantee that the utility will recover all its capital [94] whether under wheeling or non-wheeling conditions. The studies done in this thesis do show that by incorporating transmission costs, there is always room for the guarantee of profit provided the right assignment of price to each unit is done.

- Most utilities charge real power consumption. The studies presented in this thesis showed in stages the impact of considering active power (**P**), both active and reactive power flows (**P&Q**) on the active power rates besides considering the fuel costs at the generating plants only. From the case studies, both under regulated and deregulated conditions, the transmission of reactive power shows that it has greater influence rather than real power on the active power rates at each of the load buses. This result is important because it shows the weakness of many of the DC-OPF methods that are used to establish the active power rates. Under DC load flows, the influence of reactive power is not captured. This shows the importance of using AC-OPF methods.
- From the third conclusion, the implications of these results for cost-benefit analysis in reactive compensation investment is significant. This can provide a useful guide line of what savings can be made by installing reactive compensation equipment.
- Although the production and transmission costs of real power under a regulated environment are much higher than those of reactive power, the wheeling costs of real and reactive power can compare favorably as has been illustrated in the 57-bus system and at low LSF (0.3-0.45) in the other systems.
- When system loading is heavy (as noticed in the 57-bus system at LSF 0.8) the flow of reactive power can push the rates of both reactive and real power under wheeling or non-wheeling conditions to very high levels. Under such scenarios, if it proves to be cost effective, the utility can resort to installation of reactive power source supply

like capacitors to easy out the voltage conditions and charge fixed amount to the consumers. This is also an important consideration which is overlooked by all transmission pricing which relies on DC-OPF methods.

- The modification of Optimal Power Flows (OPFs), can effectively be used to compute hourly, wheeling or non-wheeling charges which involve transmission costs. This can be accomplished by setting prices of real and reactive power at each bus at a particular time equal to the marginal costs of supplying real and reactive power, respectively, at that bus and that time. OPFs model both the generation and transmission systems and for a particular snap-shot in time can yield extremely accurate information on such quantities as short term marginal wheeling/non-wheeling costs.
- The proposed algorithm presented in this thesis, makes it possible to charge the customers for real-time reactive power consumption. Some studies done in [95] on real-time pricing of reactive power, have shown to perform better than the power factor penalty scheme in terms of providing incentives to all customers to reduce their consumption of reactive power irrespective of their power factors. This shows that pricing of reactive power should develop simultaneously with that of active power for maximum economic efficiency and smooth operation of the electricity marketplace. With this in mind, the proposed algorithm in this work can easily be used to establish

real-time rates of reactive power incorporating transmission cost at the different load buses as demonstrated.

- The rates for both real and reactive power under wheeling or non-wheeling conditions increase as power demand increases. This was demonstrated in the case studies by comparing results at low LSF(0.3-0.45) with those at higher LSF(0.5-1.1). For practical implementation, the real and reactive power demands become dependent on the prices being communicated to the consumers every hour at the different locations.

7.2 Recommendations

Though the proposed algorithm works well in establishing rates for both active and reactive power supply, there is still room for improvement on the work described in this thesis. Some of the improvements and recommendations are as follows:

- When it comes to assignment of the price k the algorithm reported in this thesis, used the same k for all the lines. In case of practical application of this algorithm, the characteristics of each of the line in the network becomes very useful and in such cases use of a different value of k for each line is recommended. For example, the line with higher maintenance costs can be allocated a higher value of k as well. This will definitely increase the assurance of making a profit for the utility involved in the transmission of power.

- Not much has been covered in this work on the effect of the voltage limits with the rates reported in this thesis. More studies should be done in this area so as to establish ranges of voltage limits acceptable before the installation of reactive power source come into play.
- The proposed algorithm (modification of OPF) assumes that there is no variation of the load once the LSF has been assigned within that hour. This is the conventional way of optimal load flow studies. In the real world, the nature of some of the loads are such that they are dependent on the voltage and frequency of the system. Some studies with loads varying in function of these factors are recommended.
- The studies done in this work under a deregulated environment assumes that the transmission lines are owned by one third party. With some modifications, it will be interesting to do some studies when the wheel has to pass through several intermediate utilities or even when there is an open access to the transmission lines by several competitive power suppliers. Further studies are therefore recommended using the proposed algorithm under these scenarios.

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Appendix - A

DERIVATIVES FOR THE OPF PROBLEM

In this appendix, some of the first derivatives used in the study reported in this thesis are discussed. As already discussed in Chapter 4, the nonlinear optimization package MINOS requires the user to supply subroutines to evaluate the objective function (i.e. Equation 4.2) and its first derivatives (FUNOBJ). In addition, the first derivatives of the constraints functions (FUNCON) should be provided by the user.

Restating the objective function and the constraints involved as follows:

Minimize: $$C_T(P_G, V, \delta) = \sum_{i \in R_G} C_i(P_{Gi}) + \sum_{i \in N_B} k[P_i(V,\delta) + Q_i(V,\delta)] \ , \tag{A.1}$$

where:

$C_i(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2$ is the operating fuel cost of producing P_{Gi} units of real power at generating plant at bus i,

α_i, β_i *and* γ_i are constant parameters as defined in Section 4.4 for the generator i,

$kP_i(V,\delta)$, $kQ_i(V,\delta)$ is the real and reactive transmission operating costs respectively and

N_B *and* R_G are the number of buses in the system and generator buses respectively.

The minimization is subject to equality constraints expressed by the network equations:

$$PM_i = P_{Gi} - P_{Di} - P_i(V,\delta) = 0\,. \qquad \text{A.2}$$

$$QM_i = Q_{Gi} - Q_{Di} - Q_i(V,\delta) = 0\,. \qquad \text{A.3}$$

where the active $\{P_i(V,\delta)\}$ and reactive $\{Q_i(V,\delta)\}$ power injection can be further expressed in terms of the bus voltage and angles as follows:

$$P_i(V,\delta) = V_i \sum_{j=1}^{N_B} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})\,. \qquad \text{A.4}$$

$$Q_i(V,\delta) = V_i \sum_{j=1}^{N_B} V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})\,. \qquad \text{A.5}$$

The optimization also requires meeting inequality constraints imposed on P_{Gi}, Q_{Gi}, V_i, δ_i and P_{ij} as follows:

$$P_{Gi\min} \le P_{Gi} \le P_{Gi\max} \qquad i = 1,\ldots\ldots\ldots\ldots, N_G\,, \qquad \text{A.6}$$

$$Q_{Gi\min} \le Q_{Gi} \le Q_{Gi\max} \qquad i = 1,\ldots\ldots\ldots\ldots, M_B\,, \qquad \text{A.7}$$

$$V_{i\min} \le V_i \le V_{i\max} \qquad i = 1,\ldots\ldots\ldots\ldots\ldots, N_B, \qquad \text{A.8}$$

$$P_{ij\min} \le P_{ij} \le P_{ij\max} \qquad i, j = 1,\ldots\ldots\ldots\ldots\ldots, N_B, \qquad \text{A.9}$$

where $P_{ij} = \dfrac{V_i V_j}{X_{ij}} \sin(\delta_i - \delta_j)$ and X_{ij} is the impedance between line i and j.

From Equation A.1, the problem variables are the active power generations $(P_{Gi},\ \ i = 1,\ldots\ldots, N_G)$, the reactive power generations $(Q_{Gi},\ \ i = 1,\ldots, M_B)$, the voltage magnitude $(V_i,\ \ i = 1,\ldots\ldots N_B)$ and the voltage angle $(\delta_i,\ \ 1,\ldots\ldots, N_B)$.

First Phase Objective Derivatives: For the first phase of the studies, the variable involved is only the power generation since we are not considering transmission costs. Hence:

$$\frac{\partial C_T}{\partial P_{Gi}} = \beta_i + 2\gamma_i P_{Gi}. \qquad \text{A.10}$$

Second Phase Objective Derivatives: For this second phase the variables involved include the power generation, voltage generation and voltage phase angle. The necessary derivatives to evaluate the objective function will be as follows:

$$\frac{\partial C_T}{\partial P_{Gi}} = \beta_i + 2\gamma_i P_{Gi}, \qquad \text{A.11}$$

$$\frac{\partial C_T}{\partial V_j} = k\sum_{i}^{N_B} \frac{\partial P_i(V,\delta)}{\partial V_j} \qquad \text{and} \qquad \text{A.12}$$

$$\frac{\partial C_T}{\partial \delta_j} = k\sum_{i}^{N_B} \frac{\partial P_i(V,\delta)}{\partial \delta_j}. \qquad \text{A.13}$$

The following expressions are required to evaluate the terms in Equations A.12 and A.13

$$\frac{\partial P_i(V,\delta)}{\partial V_j} = V_i Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \qquad \text{for} \quad i \neq j, \qquad \text{A.14}$$

$$\frac{\partial P_i(V,\delta)}{\partial V_j} = \frac{P_i(V,\delta) + V_i^2 Y_{ii} \cos(\theta_{ii})}{V_j} \qquad \text{for} \quad i = j, \qquad \text{A.15}$$

$$\frac{\partial P_i(V,\delta)}{\partial \delta_j} = V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \qquad \text{for} \quad i \neq j \quad \text{and} \qquad \text{A.16}$$

$$\frac{\partial P_i(V,\delta)}{\partial V_j} = -Q_i(V,\delta) - V_i^2 Y_{ii} \sin(\theta_{ii}) \qquad \text{for} \quad i = j. \qquad \text{A.17}$$

Third Phase Objective Derivatives: For this third phase, the variables involved are the same as those in the second phase, namely the active power generation, the voltage and the voltage angle at each of the buses. The derivatives to evaluate the objective function in this case will be as follows:

$$\frac{\partial C_T}{\partial P_{Gi}} = \beta_i + 2\gamma_i P_{Gi}, \tag{A.18}$$

$$\frac{\partial C_T}{\partial V_j} = \frac{\partial P_i(V,\delta)}{\partial V_j} + \frac{\partial Q_i(V,\delta)}{\partial V_j} \quad \text{and} \tag{A.19}$$

$$\frac{\partial C_T}{\partial \delta_j} = \frac{\partial P_i(V,\delta)}{\partial \delta_j} + \frac{\partial Q_i(V,\delta)}{\partial \delta_j}. \tag{A.20}$$

The following terms are required to evaluate the terms involving the reactive power in Equations A.19 and A.20 in addition to the terms involving active power already defined in phase two.

$$\frac{\partial Q_i(V,\delta)}{\partial V_j} = V_i Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \quad \text{for} \quad i \neq j, \tag{A.21}$$

$$\frac{\partial Q_i(V,\delta)}{\partial V_j} = \frac{Q_i(V,\delta) - V_i^2 Y_{ii} \sin(\theta_{ii})}{V_j} \quad \text{for} \quad i = j, \tag{A.22}$$

$$\frac{\partial Q_i(V,\delta)}{\partial \delta_j} = -V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \qquad \text{for} \quad i \neq j \quad \text{and} \qquad \text{A.23}$$

$$\frac{\partial Q_i(V,\delta)}{\partial V_j} = P_i(V,\delta) - V_i^2 Y_{ii} \cos(\theta_{ii}) \qquad \text{for} \quad i = j. \qquad \text{A.24}$$

Derivatives of the constraints: The variables involved here are the active generation(P_G), reactive generation (Q_G), voltage(V) and voltage phase angle(δ). The gradients for the active power balanced equations will be as follows:

$$\frac{\partial PM_j}{\partial P_{Gi}} = -1.0 \qquad j = 1,\ldots\ldots, N_B;\ i = 1,\ldots\ldots, N_B \qquad \textit{for} \quad i = j, \qquad \text{A.25}$$

$$\frac{\partial PM_j}{\partial P_{Gi}} = 0.0 \qquad \textit{for} \quad i \neq j, \qquad \text{A.26}$$

$$\frac{\partial PM_j}{\partial Q_{Gi}} = 0.0 \qquad j = 1,\ldots\ldots, N_B;\ i = 1,\ldots\ldots, M_B \qquad \textit{for} \quad \forall\ i,j, \qquad \text{A.27}$$

$$\frac{\partial PM_j}{\partial V_i} = \frac{\partial P_j(V,\delta)}{\partial V_i} \qquad j = 1,\ldots\ldots, N_B;\ i = 1,\ldots\ldots, N_B \qquad \text{A.28}$$

$$\frac{\partial PM_j}{\partial V_i} = V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \qquad \textit{for} \quad i \neq j, \qquad \text{A.29}$$

$$\frac{\partial PM_j}{\partial V_i} = \frac{P_j(V,\delta) + V_j^2 Y_{jj} \cos(\theta_{jj})}{V_j} \qquad \textit{for} \quad i = j \qquad \text{A.30}$$

$$\frac{\partial PM_j}{\partial \delta_i} = \frac{\partial P_j(V,\delta)}{\partial \delta_i} \qquad j = 1,\dots\dots, N_B;\ i = 1,\dots\dots, N_B, \qquad \text{A.31}$$

$$\frac{\partial PM_j}{\partial \delta_i} = V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \qquad \textit{for} \quad i \neq j \quad \text{and} \qquad \text{A.32}$$

$$\frac{\partial PM_j}{\partial \delta_j} = -Q_i(V,\delta) - V_i^2 Y_{ii} \sin(\theta_{ii}) \qquad \textit{for} \quad i = j. \qquad \text{A.33}$$

The gradients for the reactive power balanced equations will be as follows:

$$\frac{\partial QM_j}{\partial P_{Gi}} = 0.0 \qquad j = 1,\dots\dots, N_B;\ i = 1,\dots\dots, N_G \qquad \textit{for} \quad \forall i, j, \qquad \text{A.34}$$

$$\frac{\partial QM_j}{\partial Q_{Gi}} = -1.0 \qquad j = 1,\dots\dots, N_B;\ i = 1,\dots\dots, M_B \qquad \textit{for} \quad i = j, \qquad \text{A.35}$$

$$\frac{\partial QM_j}{\partial Q_{Gi}} = 0.0 \qquad \textit{for} \quad i \neq j, \qquad \text{A.36}$$

$$\frac{\partial QM_j}{\partial V_i} = \frac{\partial Q_j(V,\delta)}{\partial V_i} \qquad j = 1,\dots\dots, N_B;\ i = 1,\dots\dots, N_B, \qquad \text{A.37}$$

$$\frac{\partial QM_j}{\partial V_i} = V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \qquad \textit{for} \quad i \neq j, \qquad \text{A.38}$$

$$\frac{\partial QM_j}{\partial V_i} = \frac{Q_j(V,\delta) - V_j^2 Y_{ij} \sin(\theta_{jj})}{V_j} \qquad \textit{for} \quad i = j, \qquad \text{A.39}$$

$$\frac{\partial QM_j}{\partial \delta_i} = \frac{\partial Q_j(V,\delta)}{\partial \delta_i} \qquad j = 1,....., N_B;\; i = 1,...., N_B, \qquad \text{A.40}$$

$$\frac{\partial QM_j}{\partial \delta_i} = -V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \qquad for \quad i \neq j \quad \text{and} \qquad \text{A.41}$$

$$\frac{\partial QM_j}{\partial \delta_j} = P_j(V,\delta) - V_j^2 Y_{jj} \cos(\theta_{jj}) \qquad for \quad i = j\,. \qquad \text{A.42}$$

For the transmission (line flow) constraints the derivatives will be as follows:

$for \quad i = j$

$$\frac{\partial P_{ij}}{\partial P_{Gi}} = \frac{\partial P_{ij}}{\partial Q_{Gi}} = \frac{\partial P_{ij}}{\partial V_i} = \frac{\partial P_{ij}}{\partial \delta_i} = 0.0 \qquad j = 1,....., N_B;\; i = 1,...., N_B\,. \qquad \text{A.43}$$

$for \quad i \neq j$

$$\frac{\partial P_{ij}}{\partial P_{Gi}} = \frac{\partial P_{ij}}{\partial Q_{Gi}} = 0.0 \qquad j = 1,....., N_B;\; i = 1,...., N_B\;, \qquad \text{A.44}$$

$$\frac{\partial P_{ij}}{\partial V_i} = \frac{V_j}{X_{ij}} \sin(\delta_j - \delta_i) \qquad \text{and} \qquad \text{A.45}$$

$$\frac{\partial P_{ij}}{\partial \delta_i} = \frac{V_i V_j}{X_{ij}} \cos(\delta_j - \delta_i)\,. \qquad \text{A.46}$$

Appendix -B

RESEARCH DATA

In this appendix, the data used in the study reported in this thesis will be given. All the values are in per unit using 100-MVA base. The price assignment k used in the simulations is $\$100/MW$.

B.1 Data File Format Supplied by the User

Some of the details of the user supplied files are given in the Appendixes. In this section, a general description of the format to read the data is given. The first record gives the general network data. The format is as follows:

NB	NL	NS	NG	NC	NSHUNT	NT

where:

NB = Number of buses,

NL = Number of lines,

NS = Number of thermal generators,

NG = Total number of generators,

NSHUNT = Number of shunts and

NC = Number of Synchronous condensers.

The second set of data is coding of each bus. The codes are:

1 = Thermal generator,

3 = Synchronous condenser and

4 = Load bus.

The third data set is for the transmission lines expressed as follows:

LINE	LSB	LEB	SEZ	SHY	TAP

where:

LINE = Line serial number,

LSB = Line starting bus,

LEB = Line ending bus,

SEZ = Series impedance $(R + jX)$,

SHY = Shunt admittance $(Y + jB)$ and

TAP = Tap ratio of the transformer.

The fourth data set is the shunt devices

LSB = Line series number and

SHY = Shunt admittance.

In the fifth data set the number of hours in each time interval, load scaling factor (LSF) or level of the power demand and the assigned price k is read. The sixth data set pertains to power demand. The seventh data set refers to the thermal fuel cost parameters. The eighth data pertains to the real power generation limits, reactive power limits for each generator and synchronous condenser, voltage limits for each bus and the phase angle limits at each bus.

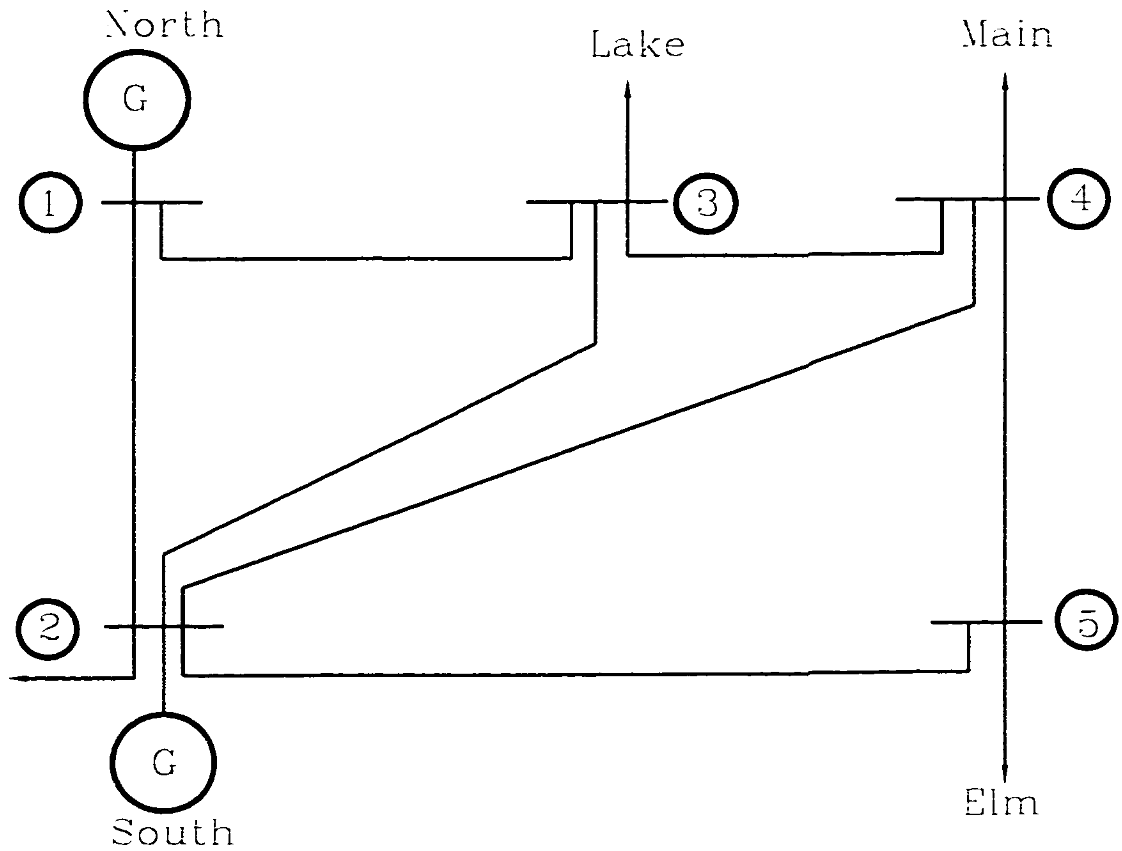

Figure B.1 Diagram of the 5-bus system.

Input data for the standard IEEE 5-bus system

```
 5     7     2     2     0     1     2
 1     1
 2     1
 3     4
 4     4
 5     4
 1     1     2   0.02000   0.06000   0.00000   0.03000
 2     1     3   0.08000   0.24000   0.00000   0.02500
 3     2     3   0.06000   0.18000   0.00000   0.02000
 4     2     4   0.06000   0.18000   0.00000   0.02000
 5     2     5   0.04000   0.12000   0.00000   0.01500
 6     3     4   0.01000   0.03000   0.00000   0.01000
 7     4     5   0.08000   0.24000   0.00000   0.02500
 4   0.00000   0.19000
16.0  8.0
 0.9  1.1
 100.0
 1        0.0        0.0
 2       20.0       10.7
 3       45.0       15.0
 4       40.0        5.0
 5       60.0       10.0
 1  52.02200  10.51250    0.111380E-1
 2  52.02200  10.51250    0.111380E-1
```

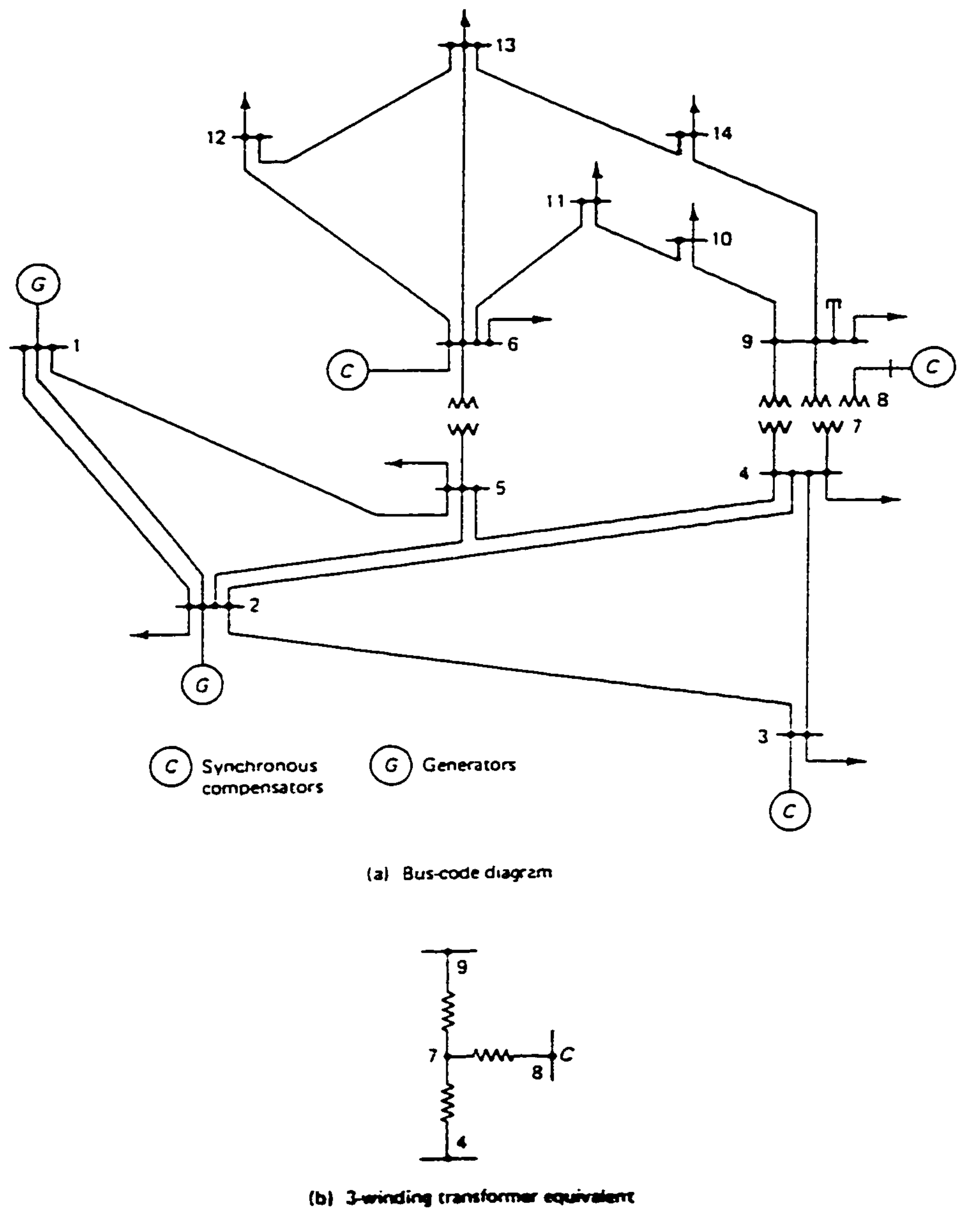

Figure B.2 Diagram of the 14-bus system

Input data for the standard IEEE 14-bus system

```
14   20    2    2    3    1    2
 1    1
 2    1
 3    3
 4    4
 5    4
 6    3
 7    4
 8    3
 9    4
10    4
11    4
12    4
13    4
14    4
 1    1    2   0.01938   0.05917   0.00000   0.02640
 2    2    3   0.04699   0.19797   0.00000   0.02190
 3    2    4   0.05811   0.17632   0.00000   0.01870
 4    1    5   0.05403   0.22304   0.00000   0.02460
 5    2    5   0.05695   0.17388   0.00000   0.01700
 6    3    4   0.06701   0.17103   0.00000   0.01730
 7    4    5   0.01335   0.04211   0.00000   0.00640
 8    5    6   0.00000   0.25202   0.00000   0.00000   0.93200
 9    4    7   0.00000   0.20912   0.00000   0.00000   0.97800
10    7    8   0.00000   0.17615   0.00000   0.00000
11    4    9   0.00000   0.55618   0.00000   0.00000   0.96900
12    7    9   0.00000   0.11001   0.00000   0.00000
13    9   10   0.03181   0.08450   0.00000   0.00000
14    6   11   0.09498   0.19890   0.00000   0.00000
15    6   12   0.12291   0.25581   0.00000   0.00000
16    6   13   0.06615   0.13027   0.00000   0.00000
17    9   14   0.19711   0.27038   0.00000   0.00000
18   10   11   0.08205   0.19207   0.00000   0.00000
19   12   13   0.22092   0.19988   0.00000   0.00000
20   13   14   0.17093   0.34802   0.00000   0.00000
 9   0.00000  0.19000
 1.0  1.0
 0.45 0.4
 100.0
 1       0.0       0.0
 2      21.7      12.7
```

3	94.2	19.0	
4	47.8	-3.9	
5	7.6	1.6	
6	11.2	7.5	
7	0.0	0.0	
8	0.0	0.0	
9	29.5	16.6	
10	9.0	5.8	
11	3.5	1.8	
12	6.1	1.6	
13	13.5	5.8	
14	14.9	5.0	
1	50.60700	10.66200	0.11650E-1
2	50.60700	10.66200	0.10650E-1

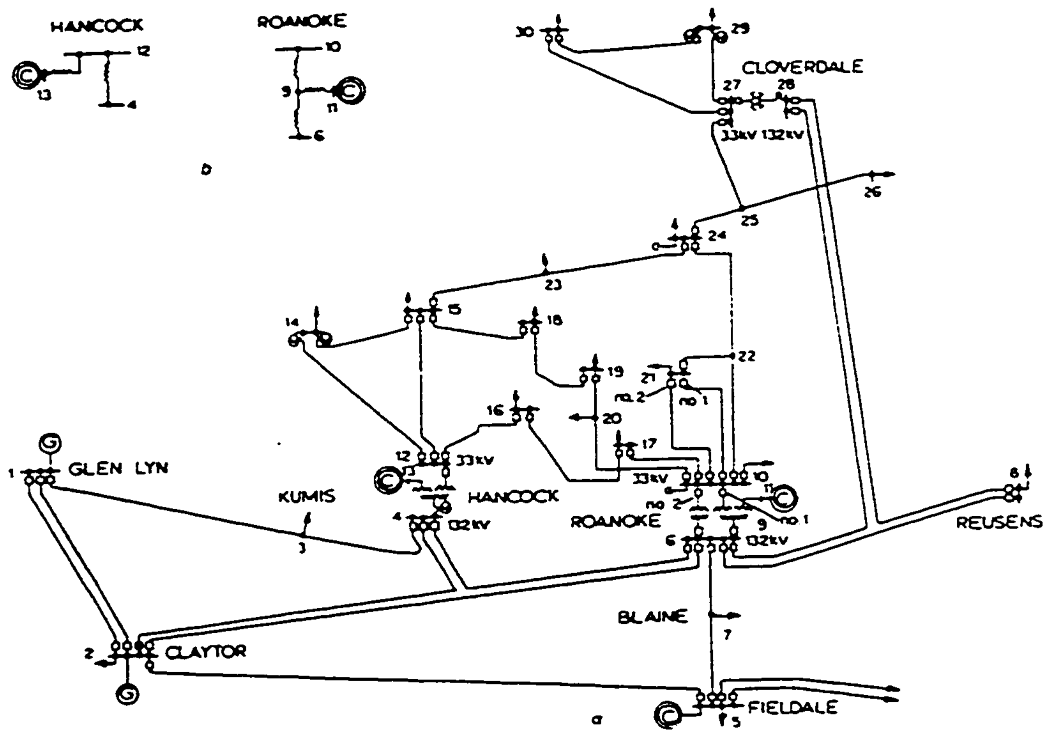

Fig. 7
AEP 30 bus test system

Ⓒ synchronous compensators

Ⓖ generators

a Bus-code diagram
b 3-winding transformer equivalents

Figure B.3 Diagram of the 30-bus system

Input data for the standard IEEE 30-bus system

30	41	3	3	3	2	2
1	1					
2	1					
3	4					
4	4					
5	3					
6	4					
7	4					
8	3					
9	4					
10	4					
11	3					
12	4					
13	1					
14	4					
15	4					
16	4					
17	4					
18	4					
19	4					
20	4					
21	4					
22	4					
23	4					
24	4					
25	4					
26	4					
27	4					
28	4					
29	4					
30	4					
1	1	2	0.01920	0.05750	0.00000	0.02640
2	1	3	0.04520	0.13520	0.00000	0.02040
3	2	4	0.05700	0.17370	0.00000	0.01840
4	3	4	0.01320	0.03790	0.00000	0.00420
5	2	5	0.04720	0.19830	0.00000	0.02090
6	2	6	0.05810	0.17630	0.00000	0.01870
7	4	6	0.01190	0.04140	0.00000	0.00450
8	5	7	0.04600	0.11600	0.00000	0.01020
9	6	7	0.02670	0.08200	0.00000	0.00850
10	6	8	0.01200	0.04200	0.00000	0.00450

```
11    6    9   0.00000   0.20800   0.00000   0.00000   .97800
12    6   10   0.00000   0.55600   0.00000   0.00000   .96900
13    9   11   0.00000   0.20800   0.00000   0.00000
14    9   10   0.00000   0.11000   0.00000   0.00000
15    4   12   0.00000   0.25600   0.00000   0.00000   .93200
16   12   13   0.00000   0.14000   0.00000   0.00000
17   12   14   0.12310   0.25590   0.00000   0.00000
18   12   15   0.06620   0.13040   0.00000   0.00000
19   12   16   0.09450   0.19870   0.00000   0.00000
20   14   15   0.22100   0.19970   0.00000   0.00000
21   16   17   0.08240   0.19230   0.00000   0.00000
22   15   18   0.10730   0.21850   0.00000   0.00000
23   18   19   0.06390   0.12920   0.00000   0.00000
24   19   20   0.03400   0.06800   0.00000   0.00000
25   10   20   0.09360   0.20900   0.00000   0.00000
26   10   17   0.03240   0.08450   0.00000   0.00000
27   10   21   0.03480   0.07490   0.00000   0.00000
28   10   22   0.07270   0.14990   0.00000   0.00000
29   21   22   0.01160   0.02360   0.00000   0.00000
30   15   23   0.10000   0.20200   0.00000   0.00000
31   22   24   0.11500   0.17900   0.00000   0.00000
32   23   24   0.13200   0.27000   0.00000   0.00000
33   24   25   0.18850   0.32920   0.00000   0.00000
34   25   26   0.25440   0.38000   0.00000   0.00000
35   25   27   0.10930   0.20870   0.00000   0.00000
36   28   27   0.00000   0.39600   0.00000   0.00000   .96800
37   27   29   0.21980   0.41530   0.00000   0.00000
38   27   30   0.32020   0.60270   0.00000   0.00000
39   29   30   0.23990   0.45330   0.00000   0.00000
40    8   28   0.06360   0.20000   0.00000   0.02140
41    6   28   0.01690   0.05990   0.00000   0.00650
10   0.00000   0.19000
24   0.00000   0.04300
 1.0  1.0
 1.1  1.0
 100.0
 1       0.0       0.0
 2      21.7      12.7
 3       2.4       1.2
 4       7.6       1.6
 5      94.2      19.0
 6       0.0       0.0
 7      22.3      10.9
```

8	30.0	30.0	
9	0.0	0.0	
10	5.8	2.0	
11	0.0	0.0	
12	11.2	7.5	
13	0.0	0.0	
14	6.2	1.6	
15	8.2	2.5	
16	3.5	1.8	
17	9.0	5.8	
18	3.2	0.9	
19	9.5	3.4	
20	2.2	0.7	
21	17.5	11.2	
22	0.0	0.0	
23	3.2	1.6	
24	8.7	6.7	
25	0.0	0.0	
26	3.5	2.3	
27	0.0	0.0	
28	0.0	0.0	
29	2.4	0.9	
30	10.5	1.9	
1	53.60700	10.66200	0.11650E-1
2	53.60700	10.66200	0.11650E-1
13	53.60700	10.66200	0.11650E-1

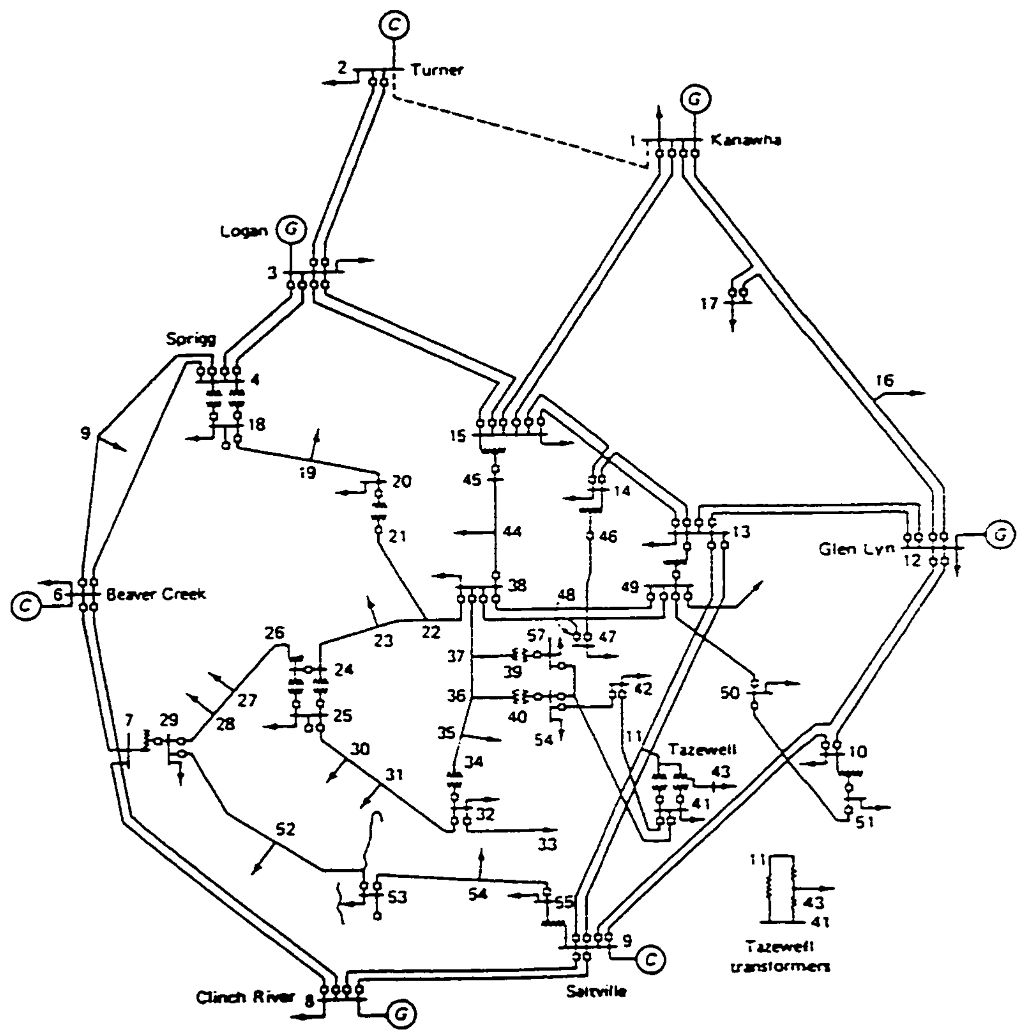

Figure B.4 Diagram of the 57-bus system

Input data for the standard IEEE 57-bus system

```
57    78    4    4    3    3    2
 1     1
 2     3
 3     1
 4     4
 5     4
 6     3
 7     4
 8     1
 9     3
10     4
11     4
12     1
13     4
14     4
15     4
16     4
17     4
18     4
19     4
20     4
21     4
22     4
23     4
24     4
25     4
26     4
27     4
28     4
29     4
30     4
31     4
32     4
33     4
34     4
35     4
36     4
37     4
38     4
39     4
40     4
```

41	4						
42	4						
43	4						
44	4						
45	4						
46	4						
47	4						
48	4						
49	4						
50	4						
51	4						
52	4						
53	4						
54	4						
55	4						
56	4						
57	4						
1	1	2	0.00830	0.02800	0.00000	0.06450	
2	2	3	0.02980	0.08500	0.00000	0.04090	
3	3	4	0.01120	0.03660	0.00000	0.01900	
4	4	5	0.06250	0.13200	0.00000	0.01290	
5	4	6	0.04300	0.14800	0.00000	0.01740	
6	6	7	0.02000	0.10200	0.00000	0.01380	
7	6	8	0.03390	0.17300	0.00000	0.02350	
8	8	9	0.00990	0.05050	0.00000	0.02740	
9	9	10	0.03690	0.16790	0.00000	0.02200	
10	9	11	0.02580	0.08480	0.00000	0.01090	
11	9	12	0.06480	0.29500	0.00000	0.03860	
12	9	13	0.04810	0.15800	0.00000	0.02030	
13	13	14	0.01320	0.04340	0.00000	0.00550	
14	13	15	0.02690	0.08690	0.00000	0.01150	
15	1	15	0.01780	0.09100	0.00000	0.04940	
16	1	16	0.04540	0.20600	0.00000	0.02730	
17	1	17	0.02380	0.10800	0.00000	0.01430	
18	3	15	0.01620	0.05300	0.00000	0.02720	
19	4	18	0.00000	0.24230	0.00000	0.00000	.97800
20	5	6	0.03020	0.06110	0.00000	0.00620	
21	7	8	0.01390	0.07120	0.00000	0.00970	
22	10	12	0.02770	0.12620	0.00000	0.01640	
23	11	13	0.02230	0.07320	0.00000	0.00940	
24	12	13	0.01780	0.05800	0.00000	0.03020	
25	12	16	0.01800	0.08130	0.00000	0.01080	
26	12	17	0.03970	0.17900	0.00000	0.02380	

27	14	15	0.01710	0.05470	0.00000	0.00740	
28	18	19	0.46100	0.68500	0.00000	0.00000	
29	19	20	0.28300	0.43400	0.00000	0.00000	
30	21	20	0.00000	0.77670	0.00000	0.00000	1.04300
31	21	22	0.07360	0.11700	0.00000	0.00000	
32	22	23	0.00990	0.01520	0.00000	0.00000	
33	23	24	0.16600	0.25600	0.00000	0.00420	
34	24	25	0.00000	0.60276	0.00000	0.00000	1.00000
35	24	26	0.00000	0.04730	0.00000	0.00000	1.04300
36	26	27	0.16500	0.25400	0.00000	0.00000	
37	27	28	0.06180	0.09540	0.00000	0.00000	
38	28	29	0.04180	0.05870	0.00000	0.00000	
39	7	29	0.00000	0.06480	0.00000	0.00000	.96700
40	25	30	0.13500	0.20200	0.00000	0.00000	
41	30	31	0.32600	0.49700	0.00000	0.00000	
42	31	32	0.50700	0.75500	0.00000	0.00000	
43	32	33	0.03920	0.03600	0.00000	0.00000	
44	34	32	0.00000	0.95300	0.00000	0.00000	.97500
45	34	35	0.05200	0.07800	0.00000	0.00160	
46	35	36	0.04300	0.05370	0.00000	0.00080	
47	36	37	0.02900	0.03660	0.00000	0.00000	
48	37	38	0.06510	0.10090	0.00000	0.00100	
49	37	39	0.02390	0.03790	0.00000	0.00000	
50	36	40	0.03000	0.04660	0.00000	0.00000	
51	22	38	0.01920	0.02950	0.00000	0.00100	
52	11	41	0.00000	0.74900	0.00000	0.00000	.95500
53	41	42	0.20700	0.35200	0.00000	0.00000	
54	41	43	0.00000	0.41200	0.00000	0.00000	
55	38	44	0.02890	0.05850	0.00000	0.00100	
56	15	45	0.00000	0.10420	0.00000	0.00000	.95500
57	14	46	0.00000	0.07350	0.00000	0.00000	.90000
58	46	47	0.02300	0.06800	0.00000	0.00160	
59	47	48	0.01820	0.02330	0.00000	0.00000	
60	48	49	0.08340	0.12900	0.00000	0.00240	
61	49	50	0.08010	0.12800	0.00000	0.00000	
62	50	51	0.13860	0.22000	0.00000	0.00000	
63	10	51	0.00000	0.07120	0.00000	0.00000	.93000
64	13	49	0.00000	0.19100	0.00000	0.00000	.89500
65	29	52	0.14420	0.18700	0.00000	0.00000	
66	52	53	0.07620	0.09840	0.00000	0.00000	
67	53	54	0.18780	0.23200	0.00000	0.00000	
68	54	55	0.17320	0.22650	0.00000	0.00000	
69	11	43	0.00000	0.15300	0.00000	0.00000	.95800

```
70   44   45   0.06240   0.12420   0.00000   0.00200
71   40   56   0.00000   0.19500   0.00000   0.00000   .95800
72   56   41   0.55300   0.54900   0.00000   0.00000
73   56   42   0.21250   0.35400   0.00000   0.00000
74   39   57   0.00000   0.35500   0.00000   0.00000   .97500
75   57   56   0.17400   0.26000   0.00000   0.00000
76   38   49   0.11500   0.17700   0.00000   0.00300
77   38   48   0.03120   0.04820   0.00000   0.00000
78    9   55   0.00000   0.12050   0.00000   0.00000   .94000
18   0.00000   0.10000
25   0.00000   0.05900
53   0.00000   0.06300
1.0  1.0
0.6  0.8
100.0
 1        55.0        17.0
 2         3.0        88.0
 3        41.0        21.0
 4         0.0         0.0
 5        13.0         4.0
 6        75.0         2.0
 7         0.0         0.0
 8       150.0        22.0
 9       121.0        26.0
10         5.0         2.0
11         0.0         0.0
12       377.0        24.0
13        18.0         2.3
14        10.5         5.3
15        22.0         5.0
16        43.0         3.0
17        42.0         8.0
18        27.2         9.8
19         3.3         0.6
20         2.3         1.0
21         0.0         0.0
22         0.0         0.0
23         6.3         2.1
24         0.0         0.0
25         6.3         3.2
26         0.0         0.0
27         9.3         0.5
28         4.6         2.3
```

29	17.0	2.6	
30	3.6	1.8	
31	5.8	2.9	
32	1.6	0.8	
33	3.8	1.9	
34	0.0	0.0	
35	6.0	3.0	
36	0.0	0.0	
37	0.0	0.0	
38	14.0	7.0	
39	0.0	0.0	
40	0.0	0.0	
41	6.3	3.0	
42	7.1	4.4	
43	2.0	1.0	
44	12.0	1.8	
45	0.0	0.0	
46	0.0	0.0	
47	29.7	11.6	
48	0.0	0.0	
49	18.0	8.5	
50	21.0	10.5	
51	18.0	5.3	
52	4.9	2.2	
53	20.0	10.0	
54	4.1	1.4	
55	6.8	3.4	
56	7.6	2.2	
57	6.7	2.0	
1	50.60700	10.66200	0.11550E-1
3	50.60700	10.66200	0.11550E-1
8	50.60700	10.66200	0.11550E-1
12	50.60700	10.66200	0.11550E-1