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# Coupled-power theory of nonlinear distributed-feedback lasers, yielding reduced longitudinal spatial hole burning

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An analysis of the nonlinear distributed-feedback (DFB) laser action is presented. A set of nonlinear coupled-power equations is derived. A general condition for a nonlinear DFB laser with reduced spatial hole burning (SHB) is obtained. It is shown that the elimination of SHB in DFB lasers can be achieved by introducing properly-chosen variations of the coupling strength along the longitudinal direction; a novel complex-coupling AR-coated DFB laser diode is proposed as a result. It is also shown that the optical nonlinearity must be considered for the design of high-power laser structures with reduced SHB. © *1998 American Institute of Physics.*
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Optical communication systems require single-mode lasers with a narrow linewidth. Quantum-well distributed-feedback (QW DFB) lasers are favorite candidates. There are two possible ways to reduce the linewidth of DFB lasers: the use of a larger coupling coefficient and the introduction of a longer cavity. However, lasers with a larger coupling coefficient and/or a long cavity often become multimode.[1] This deterioration is due to longitudinal spatial hole burning (SHB), i.e., a nonuniform carrier density distribution resulting from a nonuniform optical power density distribution. Special laser structures with reduced SHB have been and are still being intensely investigated,[2–5] especially for high-power lasers.[6,7]

Since the pioneering work of Kogelnik and Shank,[8,9] a great deal of attention has been devoted to the properties of DFB lasers.[10–13] All the work originated from the coupled-wave equations. In this letter, the coupled-power theory instead is employed to study nonlinear DFB lasers. First, a set of coupled-power equations for nonlinear DFB lasers is derived. Second, a general condition for reduced SHB is obtained. This allows one to design novel DFB structures with the SHB even completely eliminated. The physical mechanism behind the properties of such new laser structures is also discussed.

We start with the scalar wave equation for the electric field describing DFB lasers.[9] For nonlinear active media such as GaAs, InGaAsP, etc., the intensity variation of the light field causes a refractive index variation, and this refractive index variation induces, in turn, a light intensity ($I$) variation. Since the nonlinear refractive index $n(I)$ may thus be written as $n+n_2(I)$, the propagation constant $k$ can be determined from

$$k^2=\frac{\omega^2}{c^2}\,\epsilon\left(1+2\,\frac{n_2(I)}{n}\right)-j\omega\mu\sigma, \tag{1}$$

where the same notation for parameters as in Refs. 8 and 9 is used. $n_2(I)$ is the optical nonlinear coefficient as a function of intensity, which is usually dependent on the carrier density, the material gain and the light intensity due to the saturation of both the nonlinearity and the absorption. More detailed discussions about optical nonlinear coefficient and its expression can be found in Refs. 12 and 14. In the following discussions, the amplitudes of the forward and backward propagating waves are denoted as $E^+(z)$ and $E^-(z)$; the index and gain coupling coefficients are denoted as $\kappa_i$ and $\kappa_g$; $\alpha$ denotes the net mode gain per unit length: $\delta_0(\delta_0\approx\beta-\beta_0)$ represents the Bragg deviation. We assume in-phase or antiphase relation between the index and gain coupling coefficients. This corresponds to practical cases. We introduce the power density variables $I_t(z)=E^+(z)E^{+*}(z)+E^-(z)E^{-*}(z)$, $I_d(z)=E^+(z)E^{+*}(z)-E^-(z)E^{-*}(z)$, and $I_c(z)=2E^{-*}(z)E^+(z)\equiv I_{cr}(z)+jI_{ci}(z)$, where $I_t(z)$ describes the total photon power density along the cavity, $I_d(z)$ is the net photon power density flux along the cavity, and $I_c(z)$ is the power density of the mutual interaction along the cavity between the forward and backward waves due to coupling. $I_c(z)$ describes the mutual interaction strength and phase relations between the forward and backward waves. Using the above new power density variables, the nonlinear DFB structure can be described by the following set of coupled-power equations:

$$\frac{dI_d(z)}{dz}=2(\alpha L)I_t(z)\pm 2(\kappa_g L)I_{cr}(z), \tag{2}$$

$$\frac{dI_t(z)}{dz}=2(\alpha L)I_d(z)-2(\kappa_i L)I_{ci}(z), \tag{3}$$

$$\frac{dI_{cr}(z)}{dz}=2(\delta L)I_{ci}(z)\mp 2(\kappa_g L)I_d(z), \tag{4}$$

$$\frac{dI_{ci}(z)}{dz}=-2(\delta L)I_{cr}(z)-2(\kappa_i L)I_t(z). \tag{5}$$

Here all power densities are normalized with respect to $I_s$ which is a normalizing quantity. The cavity longitudinal axis $z$ is normalized with respect to the cavity length $L$ and

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$$(\delta L)=(\delta_0 L)+(\Theta L), \quad (\Theta L)=\frac{(\beta_0 L)n_2(I_t)}{2\eta_0} I_s. \qquad (6)$$

The upper and lower signs in the above equations represent in-phase and antiphase coupling, respectively. Equations (2)–(5) represent general coupled-power equations for DFB lasers and are a set of nonlinear equations due to the existence of optical nonlinearity. These equations describe the power exchange between the forward and backward waves along the cavity due to coupling. Equation (2) [similar results as in Eq. (2) were obtained by Kapon *et al.*[15]] describes the rate of the net power density flux change for each mode resulting from (a) the total power density change experienced by each mode due to the gain/loss along the cavity and (b) the real part of the power density change of the mutual interaction between the forward and backward waves due to gain coupling (in general, due to gain and index coupling) for the cases of in-phase and antiphase coupling. Equation (3) describes the rate of the total power density change for each mode resulting from (a) the net power density flux change experienced by each mode due to the gain/loss and (b) the imaginary part of the power density change of the mutual interaction between the forward and backward waves due to index coupling. Equations (4) and (5) describe the rate of the power density change of the mutual interaction resulting from detuning, coupling, and optical nonlinearity.

We have derived the coupled-power equations used for modeling of nonlinear DFB lasers in terms of four real power density variables. These equations show that the coupling coefficient plays an important role in describing the amount of the power transfer between the two contradirectional waves. The existence of gain (or index) coupling may change the net photon power density flux (or the total power density) along the cavity via the power density of the mutual interaction between the forward and backward waves. Equations (2)–(5) also show that the optical nonlinearity plays the same role as the Bragg deviation: it may affect the mutual interaction between the contradirectional waves and then change the net power density flux and the total power density of each mode.

Generally, the boundary conditions for nonlinear DFB lasers could be: (1) a monochromatic wave near the Bragg frequency incident on one end of the amplifying medium or (2) perfect AR coating of the facets. As an example of applying the coupled-power equations, a general condition for reducing the longitudinal SHB in nonlinear DFB lasers is proposed in the next section.

We note that a direct result of Eq. (3) is that the SHB can be completely eliminated if the following condition is satisfied:

$$(\alpha L)I_d(z)=(\kappa_i L)I_{ci}(z). \qquad (7)$$

This is a general condition eliminating the SHB irrespective of the boundary conditions. One can apply it to design laser structures with a uniform power distribution along the cavity. Let us demonstrate how to design a coupling profile for a reduced SHB in a laser with a perfect AR coating of the facets, and show the corresponding relations among the power densities.

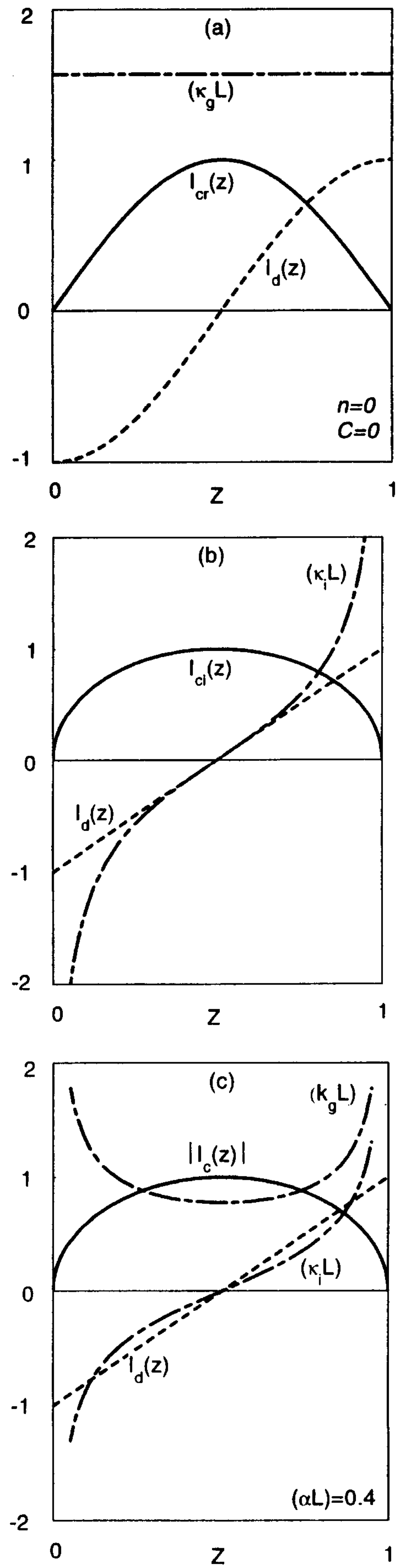


FIG. 1. (a) The dependence of $\kappa_g L$, $I_d(z)$ and $I_{cr}(z)$ on $z$ for the case a; (b) the dependence of $\kappa_i L$, $I_d(z)$ and $I_{ci}(z)$ on $z$ for the case b; and (c) the dependence of $\kappa_g L$, $\kappa_i L$, $I_d(z)$ and $I_c(z)$ on $z$ for the case c, where $|I_c(z)|=\sqrt{I_{cr}^2(z)+I_{ci}^2(z)}$.

Case a. Gain coupling ($\kappa_i L = 0$). Equation (7) means that the threshold gain $(\alpha L)_{\text{th}} = 0$. Therefore one can easily obtain the following result for power densities and the gain-coupling profile:

$$I_d(z) = -\cos[2(n+l)\pi z],$$
$$I_{cr}(z) = \frac{\sin[2(n+1)\pi z]}{\sqrt{1+C^2}}, \quad I_{ci}(z) = C I_{cr}(z),$$
$$(\kappa_g L) = \frac{2n+1}{2}\pi\sqrt{1+C^2}, \tag{8}$$
$$(\delta_0 L) + (\Theta L) = -\frac{2n+1}{2}\pi C \frac{\cos[2(n+1)\pi z]}{\sin[2(n+1)\pi z]},$$

where $n = 0,1,2,...$, and $C$ is an arbitrary constant which denotes the ratio of $I_{ci}(z)$ to $I_{cr}(z)$.

Case b. Index coupling ($\kappa_g L = 0$) with $\Theta L = 0$. For this case, one can easily obtain $I_d(z) = 2z - 1$ and $(\alpha L)_{\text{th}} = 1$ from Eq. (2). Then the power density of the mutual interaction and the index-coupling profile can be found by using Eqs. (7), and (5) as follows:

$$I_{cr}(z) = 0, \quad I_{ci}(z) = 2\sqrt{z(1-z)},$$
$$(\kappa_i L) = -\frac{1-2z}{2\sqrt{z(1-z)}}. \tag{9}$$

Case c. Complex coupling ($\kappa_g L \neq 0$ and $\kappa_i L \neq 0$) with $\Theta L = 0$. For this case, one can find that only if $I_d(z) = 2z - 1$ and $0 < (\alpha L)_{\text{th}} \leqslant 1$, the results for power densities and the complex-coupling profiles follow:

$$I_{cr}^2(z) = 4[1-(\alpha L)]z(1-z), \quad I_{ci}^2(z) = 4(\alpha L)z(1-z), \tag{10}$$

$$\pm(\kappa_g L) = \frac{\sqrt{1-(\alpha L)}}{2\sqrt{z(1-z)}}, \quad (\kappa_i L) = \frac{\sqrt{(\alpha L)}(2z-1)}{2\sqrt{z(1-z)}}, \tag{11}$$

where the threshold gain $(\alpha L)_{\text{th}}$ depends on the complex-coupling profile.

For simplicity, the optical nonlinear effect is considered only for the case a. The result of Eq. (8) shows that the optical nonlinear effect must be considered in the design of high-power laser structures with reduced SHB. The detail discussions will be presented in a full paper.

Figures 1(a)–1(c) show the distribution of $I_d(z)$,$I_c(z)$, and the coupling profile, respectively. For all three cases, $I_c(z)$ reaches a maximum at $I_d(1/2) = 0$, while it reduces to a minimum as $I_d(z)$ increases to its maxima at $z = 0,1$. One can thus conclude that a uniform power density results from the power exchange between the net photon power density flux and the power density of the mutual interaction. In fact, we can easily prove that $I_d^2(z) + I_{cr}^2(z) + I_{ci}^2(z) = 1$ for all the above cases. This is a general relation of the power exchange between the net photon power density flux and the power density of the mutual interaction necessary for elimination of the SHB in DFB lasers with a perfect AR coating of the facets.

Another property of an AR-coated DFB laser with reduced SHB is that the phase difference between the forward and backward waves is locked along the cavity. The phase difference is zero, $\pi/2$ and $\arcsin\sqrt{(\alpha L)_{\text{th}}}$ for the above cases a, b, and c, respectively.

Finally, we should point out that the result for the case b has been reported in Refs. 4 and 5; however, the present analysis is simpler, provides more insight, and the physical mechanism behind the reduced SHB is easier to understand. The results for the cases a and c are reported here for the first time. They propose a gain-coupled or a complex-coupled AR-coated nonlinear DFB laser diode with reduced SHB, respectively. We also note that the result of Eq. (8) reduces to that found in Refs. 4 and 5 if the optical nonlinearity is negligible.

A set of coupled-power equations for nonlinear DFB laser diodes operating near the Bragg frequency has been derived. It is shown that these equations lead in a straightforward way to a general condition for nonlinear DFB lasers with eliminated SHB. For perfectly AR-coated facets, a design of coupling structures has been demonstrated based on this general condition. It is shown that the elimination of SHB in DFB lasers could be achieved by introducing certain variations of the coupling strength along the cavity. Some coupling grating structures have been proposed for gain, index and complex coupling, respectively. For a DFB laser with these structures, it is shown that a uniform power distribution results from the power exchange between the net photon power density flux and the power density of the mutual interaction of the forward and backward waves.

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