PHYSICAL REVIEW LETTERS

VOLUME 49 — 20 SEPTEMBER 1982 — NUMBER 12

Schiff's Conjecture on Gravitation

A. Coley
Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada
(Received 14 June 1982)

Considered here is a class of theories of gravity characterized by a set of equations which represent the gravitational and electromagnetic structure of the theories in a spherically symmetric and static gravitational field. If one demands that the weak equivalence principle (WEP) and the principle of universality of gravitational red shift (UGR) be satisfied, it is found that the theories under investigation must be metric. This result lends support to the current version of Schiff's conjecture that WEP + UGR → EEP, where EEP refers to the Einstein equivalence principle.

PACS numbers: 04.20.Cv, 04.50.+h, 04.80.+z

For the last twenty years people have been interested in a conjecture, due to Schiff,[1] concerning the foundations of gravitation theory. In its original form, the conjecture essentially stated that every theory of gravity that satisfies the weak equivalence principle (WEP) and is relativistic necessarily satisfies the Einstein equivalence principle (EEP), and is consequently a metric theory of gravity[2] (i.e., WEP → EEP). More recently the conjecture seems to refer to the least criteria such that EEP is implied (what Will calls the fundamental criteria for viability[3]). In its most current form, due to Will,[3] the conjecture essentially states that every theory of gravity that satisfies WEP and the principle of universality of gravitational red shift (UGR) necessarily satisfies EEP [4,5] (i.e., WEP + UGR → EEP).

Attempts to prove the validity of Schiff's conjecture have been made within the so-called T-H-ϵ-μ formalism.[6,7] Unfortunately, in practice there is only one "nonmetric" degree of freedom in this formalism ($\epsilon\mu = H/T$ is implied from various considerations outside the scope of the conjecture) with the result that the connection between WEP and EEP may not be explored very fully. As a result, the author felt that a more general formalism was needed in order for Schiff's conjecture to be investigated in greater depth.

In some recent work[8] a class of relativistic theories of gravity is considered. These theories are characterized by a set of equations which represent the gravitational and electromagnetic structure of the theories in an external, spherically symmetric and static (SSS) gravitational field. This set of equations consists of the gravitationally generalized Maxwell equations, which, for a source consisting of discrete charged particles, are given by

$$\nabla^2\varphi = \frac{g}{f}\frac{\partial^2\varphi}{\partial t^2} + \mathcal{A}\,\vec{g}\cdot\left(\frac{\partial\vec{A}}{\partial t} + \nabla\varphi\right) - 4\pi\left(\frac{f}{g}\right)^{1/2}\sum_k e_k\,\delta^3(\vec{x} - \vec{x}_k), \tag{1}$$

and

$$\nabla^2\vec{A} = \frac{g}{f}\frac{\partial^2\vec{A}}{\partial t^2} + \frac{f}{g}(\nabla\cdot\vec{A})\nabla\frac{g}{f} + \mathcal{B}(\nabla\times\vec{A})\times\vec{g} - 4\pi\left(\frac{g}{f}\right)^{1/2}\sum_k e_k\frac{d\vec{x}_k}{dt}\,\delta^3(\vec{x} - \vec{x}_k), \tag{2}$$

and the gravitationally generalized Lorentz equations, which are given by

$$\frac{d^2\vec{x}_k}{dt^2} = -\nabla(\gamma) - \left[\nabla(\alpha)\cdot\frac{d\vec{x}_k}{dt}\right]\frac{d\vec{x}_k}{dt} - \left(\frac{d\vec{x}_k}{dt}\right)^2\nabla(\beta) + \frac{e_k}{M_{0k}}L\left[-\frac{1}{g}\left\{\frac{\partial\vec{A}}{\partial t} + \left(\frac{d\vec{x}_k}{dt}\cdot\nabla\right)\vec{A} + \nabla\varphi - \nabla\left(\frac{d\vec{x}_k}{dt}\cdot\vec{A}\right)\right\} + \frac{1}{f}\left\{\left(\frac{\partial\vec{A}}{\partial t} + \nabla\varphi\right)\cdot\frac{d\vec{x}_k}{dt}\,\frac{d\vec{x}_k}{dt}\right\}\right]. \tag{3}$$

In the above $A_a = (-\varphi, \vec{A})$ is the electromagnetic four-potential; $\vec{g} \equiv \nabla U$, where U is the Newtonian gravitational potential; k denotes the kth charged particle, $\vec{x}_k$ its position, $d\vec{x}_k/dt$ its velocity, e_k its charge, and M_{0k} its rest mass; and L is a general scalar function depending on the gravitational field and $d\vec{x}_k/dt$, and in the context of this brief note depends on $\mathcal{P}$ and $\mathcal{Q}$, two arbitrary functions of U. (The motivation and explanation of these equations can be found in Ref. 8. I note that the theories represented in the T-H-ϵ-μ formalism are a special case of this class of theories.)

In this class of theories there are nine arbitrary functions of the gravitational field (or, more precisely, functions of the Newtonian gravitational potential U): f, g, α, β, γ, $\mathcal{A}$, $\mathcal{B}$, $\mathcal{P}$, and $\mathcal{Q}$. Metric theories of gravity are a special case of the above, in which the metric g_{ab} is in isotropic form with $\mathrm{diag}[g_{ab}] = (f, -g, -g, -g)$, and

$$\tfrac{1}{2}\alpha' = \mathcal{B} = -\mathcal{A} = \tfrac{1}{2}\{g'/g - f'/f\},$$
$$\beta' = -\tfrac{1}{2}g'/g, \quad \gamma' = \tfrac{1}{2}f'/g, \quad \mathcal{P} = \tfrac{1}{2}f'/f, \quad \mathcal{Q} = 0, \tag{4}$$

where the prime denotes differentiation with respect to U. Consequently, we can regard α, β, γ, $\mathcal{A}$, $\mathcal{B}$, $\mathcal{P}$, and $\mathcal{Q}$ as "nonmetric" degrees of freedom. We recall that for metric theories of gravity, in which relations (4) are obeyed, the EEP is satisfied since all laws of physics reduce to their special relativistic forms in freely falling coordinate frames.

In order to investigate the WEP we calculate the center-of-mass acceleration of a composite test body consisting of electromagnetically interacting charged particles. This calculation proceeds as follows.[6,8] We solve Eqs. (1) and (2) for the electromagnetic potentials, and substitute these solutions into Eq. (3) to obtain an expression for the acceleration of the kth particle entirely in terms of particle coordinates. We then define a center of mass for the test body and obtain an expression for the instantaneous acceleration of the test body $\vec{A}_{\mathrm{c.m.}}$ (certain virial relations are used to simplify this expression). The above equations are not solved exactly but perturbatively in terms of two small quantities; a typical squared velocity v^2 and the size of the test body s (which is appropriate if we wish to compare this analysis with present-day Eötvös experiments). Consequently, the expression $\vec{A}_{\mathrm{c.m.}}$ is in powers of v^2 and s. [Recall that L is a function of two variables $\mathcal{P}$ and $\mathcal{Q}$; it turns out that $\mathcal{P}$ and $\mathcal{Q}$ are the arbitrary multiplying factors of the $O(s)$ and $O(sv^2)$ terms in the expansion of L.]

$\vec{A}_{\mathrm{c.m.}}$ is found to consist of the usual Newtonian-type acceleration and various terms involving the body's electromagnetic structure. These latter terms appear with certain combinations of the arbitrary functions of the gravitational field multiplying structure-dependent accelerations. If the class of theories under investigation is to obey the WEP, $\vec{A}_{\mathrm{c.m.}}$ cannot contain any such structure-dependent terms. Consequently all the multiplying factors must be zero. These conditions can be shown to reduce to the following set of relations[8]:

$$\frac{\alpha'}{2} = \mathcal{B} = -\mathcal{A} = \frac{1}{2}\left\{\frac{(\lambda g)'}{(\lambda g)} - \frac{(\lambda f)'}{(\lambda f)}\right\},$$
$$\beta' = -\tfrac{1}{2}(\lambda g)'/(\lambda g), \quad \gamma' = \tfrac{1}{2}(\lambda f)'/(\lambda g), \tag{5}$$
$$\mathcal{P} = \tfrac{1}{2}(f\lambda^{-1})'/(f\lambda^{-1}), \quad \mathcal{Q} = 0,$$

where λ is a scalar field depending on U and defind by $\lambda'/2\lambda = g\gamma'/f - \tfrac{1}{2}f'/f$. Relations (5) are precisely those for the class of theories to take on a "metric" form with respect to λg_{ab}. [Note that $\mathcal{P}$ contributes to the gravitationally generalized Lorentz equations in the form $(f\lambda^{-1})^{1/2}g^{-1} \equiv (\lambda f)^{1/2}(\lambda g)^{-1}$.]

If λg_{ab} is the "physical" metric we have then shown that WEP → EEP; but, in general, this is not the case. In general, g_{ab} is a physical quantity occurring in the laws of physics. We have then shown that the laws of gravitation and electromagnetism must take on a "metric" form with respect to a tensor conformally related to g_{ab} (at least in the SSS idealization) if the WEP is to be satisfied. If we assume that the measuring process is governed by the equation $ds^2 = g_{ab}dx^a dx^b$, the above analysis then shows us that the class of theories under investigation must be essentially characterized by an equation governing test-particle motion depending on λg_{ab} and an equation governing measurements (and therefore clock rates) depending on g_{ab}. (This is precisely the structure of the so-called two-metric theories of gravity.) EEP is only satisfied if λ is constant.

Further analysis within the framework of the principle of UGR does indeed show that λ must be constant. This analysis takes the following form: (a) Solar-system experiments involving test-particle motion and clock measurements show that λ is constant (at least within the parametrized post-Newtonian approximation[8]); (b) if the theory is to yield consistent predictions for the gravitational red shift, which are independent of the nature of the clocks used, λ must be constant.[3,8] Consequently, we find that WEP + UGR $\rightarrow$ EEP.

The following conclusions can be drawn from the analysis. Although the WEP severely constrains the possible form of any theory of gravity, in general it does not imply EEP (thus disproving Schiff's original formulation of the conjecture). We do find, however, that for the class of theories under investigation WEP + UGR $\rightarrow$ EEP. Consequently the analysis supports Will's current version of Schiff's conjecture.

[1]L. I. Schiff, Am. J. Phys. 28, 340 (1960).

[2]For definitions of these terms, see K. S. Thorne, D. L. Lee, and A. P. Lightman, Phys. Rev. D 7, 3563 (1973), or C. M. Will, in *General Relativity*, edited by S. W. Hawking and W. Israel (Cambridge Univ. Press, London, 1979).

[3]Will, Ref. 2.

[4]In another version of the conjecture W.-T. Ni, Phys. Rev. Lett. 38, 301 (1976), postulates the need for the added restriction that for a given initial rotation state, the subsequent rotation state of a polarized test body must be independent of the test body's internal composition.

[5]The two principles, WEP and UGR, are not completely independent; a violation of one may imply a violation of the other—see K. Nordtvedt, Jr., Phys. Rev. D 11, 245 (1975).

[6]A. P. Lightman and D. L. Lee, Phys. Rev. D 8, 364 (1973); M. P. Haugen and C. M. Will, Phys. Rev. D 15, 2711 (1977).

[7]C. M. Will, Phys. Rev. D 10, 2330 (1974).

[8]To be published.

Evidence for Two-Nucleon Processes in $A(p_{\mathrm{pol}},\pi^-)A+1$

W. W. Jacobs, T. G. Throwe, S. E. Vigdor, M. C. Green, J. R. Hall, H. O. Meyer, and W. K. Pitts
Indiana University Cyclotron Facility, Bloomington, Indiana 47405

and

M. Dillig
Institut fur Theoretische Physik, Universität Erlangen-Nürnberg, D-8520 Erlangen, West Germany

(Received 24 May 1982)

Possible signatures of two-nucleon pion production processes in reactions $A(p_{\mathrm{pol}},\pi^-)A+1$ near threshold are identified: a dependence of the analyzing power on the total angular momentum, and a simple scaling of the cross section with subshell occupancy for the struck target neutron. Measurements for $^{12,13,14}C(p_{\mathrm{pol}},\pi^-)$ exhibit these expected features, supporting the view that the fundamental $NN\rightarrow NN\pi$ processes dominate in nuclear pion production.

PACS numbers: 25.40.Rb, 24.70.+s, 27.20.+n

Nuclear pion production, specifically reactions of the type $A(p,\pi)A+1$, is the object of continued study, not only because of intrinsic interest in understanding an unusual, high-momentum-transfer reaction, but also because of its expected close relationship to more general aspects of meson-nucleon interactions in the nucleus. To date, however, there are few clear systematic trends apparent in the available data, and it remains uncertain which, if any, reaction mechanism dominates near-threshold pion production. Recent progress, both theoretical and experimental, has been reviewed by several authors.[1,2]

In the currently favored, so-called "two-nucle-

