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Constraint on nonmetric theories of gravity from supernova 1987A

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The transit time of the neutrinos and photons from supernova 1987A to Earth was the same to better than one part in 10^9. This result provides a strong test of the weak equivalence principle, by showing that neutrinos and photons fall at the same rate in the gravitational field of the Galaxy. We examine a class of nonmetric theories of gravity in which neutrino trajectories follow a path equation, while photons follow a geodesic equation. Such theories can be parametrized in a manner similar to, but more general than, the parametrized-post-Newtonian formalism, and we discuss the constraints on the parameters of such theories imposed by the equality of transit times. The available experimental and observational constraints are not yet sufficient to show that the only allowed nonmetric theories of this class reduce to general relativity in the weak-field limit.

After traveling for 170 000 years, the neutrinos and photons from supernova 1987A arrived nearly simultaneously at Earth. The time delay between detection of the neutrino burst by the Kamioka and Irvine-Michigan-Brookhaven (IMB) detectors and the first detected optical brightening is less than three hours and consistent with the time required for the shock wave from core collapse to propagate to the stellar surface. Thus there is no evidence for any difference in propagation times to within about one part in 10^9.

This observation provides a test of the weak equivalence principle.[1,2] In the parametrized-post-Newtonian (PPN) formalism, the time delay suffered by any massless particle in a weak gravitational potential $U(\mathbf{r})$ is given by (we set $c=1$)

$$\Delta t = -(1+\gamma)\int_0^t U[\mathbf{r}(t')]dt' , \tag{1}$$

where the particle is emitted at time 0 and absorbed at t, its trajectory is $\mathbf{r}(t)$, and γ is a PPN parameter which equals unity in general relativity.

All theories subsumed in the PPN formalism obey the weak equivalence principle and hence the time delay should be the same for photons and neutrinos (so long as neutrinos are massless). The approach taken in Refs. 1 and 2 is to assume that γ is different for photons and neutrinos, say γ_γ and γ_ν, and to regard the limit on the difference in arrival times as providing a limit on $|\gamma_\gamma-\gamma_\nu|$ for a given model of the Galactic potential $U(\mathbf{r})$. The resulting limit on $|\gamma_\gamma-\gamma_\nu|$ is between 0.8% and 0.1% depending on the model of the Galaxy that is used.[1,2]

In this paper we attempt to motivate the use of Eq. (1) with $\gamma_\gamma \neq \gamma_\nu$ by examining a class of theories of gravity in which the weak equivalence principle is not necessarily satisfied. We shall also show that the observations of supernova 1987A yield a new constraint on the properties of these theories.

We examine a framework[3,4] in which photons follow null geodesics defined by the equation

$$\frac{d^2x^a}{d\sigma^2}+\left\{ \begin{matrix} a \\ bc \end{matrix} \right\} \frac{dx^b}{d\sigma}\frac{dx^c}{d\sigma}=0 , \tag{2}$$

where $\{\ \}$ denotes the Christoffel symbol of the metric connection, and the equation of motion of neutrinos is

$$\frac{d^2x^a}{d\lambda^2}+\Gamma^a_{bc}\frac{dx^b}{d\lambda}\frac{dx^c}{d\lambda}=0 , \tag{3}$$

where the Γ^a_{bc} are functions that depend on the gravitational field and transform like a connection. We do not assume that $\Gamma=\{\ \}$. Equation (3) is sometimes referred to as the path equation.

Let us attempt to justify considering Eq. (3). There are many nonmetric theories of gravity in which the geometry is non-Riemannian and test particles are postulated to follow the natural curves in the non-Riemannian manifold: namely, the paths.[4] Moreover, Eq. (3) is quite general if one insists that the equation of motion is an acceleration ($d^2x^a/d\lambda^2$) which depends on powers of the velocity ($dx^a/d\lambda$) of order 2 or less. Demanding that (3) is covariant than implies that Γ transforms like a connection.

We now specialize to a static, spherically symmetric gravitational field, which is an adequate approximation for the case that we are considering. We insist on time-reversal invariance for the equations of motion and that Γ depends only on the Newtonian potential U and its first derivatives. Then it can be shown[3] that Eq. (3) can be written in three-vector notation as

$$\frac{d^2\mathbf{r}}{dt^2}=\frac{1}{r}\frac{dC}{dr}\mathbf{r}+\frac{1}{r}\frac{dB}{dr}v^2\mathbf{r}+\frac{2}{r}\frac{dA}{dr}(\mathbf{r}\cdot\mathbf{v})\frac{d\mathbf{r}}{dt}\ , \tag{4}$$

in a coordinate system (t,r,θ,ϕ), where $t\equiv x^0$ is the coordinate time associated with the static nature of the gravitational field, $\mathbf{v}\equiv d\mathbf{r}/dt$, and A,B,C are arbitrary functions of U that can be expanded as

$$\begin{aligned}2A&=A_1U+O(U^2),\quad B=B_1U+O(U^2)\ ,\\ C&=C_1U+O(U^2)\ .\end{aligned}\tag{5}$$

To first order in U we have

$$\frac{d^2\mathbf{r}}{dt^2}=\frac{C_1}{r}\frac{dU}{dr}\mathbf{r}+\frac{B_1}{r}\frac{dU}{dr}v^2\mathbf{r}+\frac{A_1}{r}\frac{dU}{dr}(\mathbf{r}\cdot\mathbf{v})\mathbf{v}\ . \tag{6}$$

If $U=$const the solution of the equation of motion for neutrinos is $\mathbf{r}_0(t)=\mathbf{r}_0+\hat{\mathbf{n}}t$. In computing the time delay to first order in U we may neglect the deflection of the neutrino path; thus the time delay Δt is given by

$$\frac{d^2\Delta t}{dt^2}=-\hat{\mathbf{n}}\frac{d^2\mathbf{r}}{dt^2}\bigg|_{\mathbf{r}_0(t)}\ . \tag{7}$$

Substituting from Eq. (6) and using $v^2=1$, $\mathbf{v}=\hat{\mathbf{n}}$, we obtain

$$\frac{d^2\Delta t}{dt^2}=-(A_1+B_1+C_1)\frac{1}{r}\frac{dU}{dr}\mathbf{r}\cdot\hat{\mathbf{n}}\ . \tag{8}$$

This expression may be integrated using the relation $r\,dr=\mathbf{r}\cdot\hat{\mathbf{n}}\,dt$:

$$\frac{d\Delta t}{dt}\bigg|_0^t=-(A_1+B_1+C_1)U[\mathbf{r}(t)]\,|_0^t\ . \tag{9}$$

The most natural choice for the constant of integration leads to

$$\frac{d\Delta t}{dt}=-(A_1+B_1+C_1)U[\mathbf{r}(t)]\ . \tag{10}$$

Hence the time delay for neutrinos is

$$\Delta t_\nu=-(A_1+B_1+C_1)\int_0^t U[\mathbf{r}(t')]dt'\ , \tag{11}$$

which is equivalent to Eq. (1) except that the coefficient $(A_1+B_1+C_1)$ replaces $(1+\gamma)$.

In the isotropic coordinate system that we employ, the metric can be written in the form

$$g_{00}=f,\quad g_{0\alpha}=0,\quad g_{\alpha\beta}=-g\delta_{\alpha\beta}\ , \tag{12}$$

where greek indices run from 1 to 3, and f and g are arbitrary functions of U which can be expanded in the form

$$f=1+f_1U+O(U^2),\quad g=1+g_1U+O(U^2)\ . \tag{13}$$

It can be shown that the geodesic equation (2) can be written to first order in U in the form (6), but with A_1,B_1,C_1 replaced by a_1,b_1,c_1, where

$$a_1=f_1-g_1,\quad b_1=\tfrac{1}{2}g_1,\quad c_1=-\tfrac{1}{2}f_1\ . \tag{14}$$

Hence the time delay for photons may be written in the form

$$\begin{aligned}\Delta t_\gamma&=(a_1+b_1+c_1)\int_0^t U[\mathbf{r}(t')]dt'\\ &=\tfrac{1}{2}(g_1-f_1)\int_0^t U[\mathbf{r}(t')]dt'\ .\end{aligned}\tag{15}$$

In the usual PPN notation,[5] we have $f_1=2$ and $g_1\equiv-2\gamma$, which yields Eq. (1). Solar system tests (especially radio ranging to the Viking spacecraft) show that γ is equal to its general-relativistic value of unity, to within about 0.2%.

Thus, the observed equality of the arrival times of neutrinos and photons from SN 1987A indicates that

$$A_1+B_1+C_1=a_1+b_1+c_1\ , \tag{16}$$

to an accuracy better than 1%. This provides a new physical constraint on nonmetric theories of gravity; in particular, it shows that $(A_1+B_1+C_1)$ is equal to the value predicted by general relativity ($=2$) to about 1%.

Let us assume for the moment that all bodies composed of fermions (including neutrinos and massive test bodies) are governed by the path equation (3) rather than the geodesic equation (2). Then the usual solar-system tests of the motions of planets and satellites constrain the various parameters in the formalism.[3,5] The Newtonian limit demands that $C_1=-1$, so that the data from supernova 1987A can be regarded as constraining (A_1+B_1). Perihelion precession observations demand that $(A_1-B_1+C_2)$ has the value predicted by general relativity, where C_2 is the coefficient of U^2 in the expansion of $C(U)$. Therefore, we may conclude that (A_1+B_1) and $(A_1-B_1+C_2)$ have the values close to those predicted by general-relativity; however we cannot conclude that A_1 and B_1 separately have their general-relativistic values (4 and -1, respectively).

In conclusion, it is clear that the data from supernova 1987A offer a new constraint on the weak-field limit of plausible nonmetric theories of gravity. However, additional independent constraints are needed to show that the only allowed nonmetric theories reduce to general relativity in this limit.

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