Temperature dependent resistivity in the low-resistance region for diffusive transport in two dimensions

D. J. W. Geldart^{1,2,3} and D. Neilson^{1,2}

¹Dipartimento di Fisica, Università di Camerino, 62032 Camerino, Italy

²School of Physics, University of New South Wales, Sydney 2052, Australia

³Department of Physics, Dalhousie University, Halifax, NS B3H3J5, Canada

(Received 25 August 2004; published 28 December 2004)

The interpretation of the metal-insulator transition phenomena in disordered two-dimensional electron systems in terms of density-dependent scaling variables suggests the existence of a quantum critical point at some critical electron density. However a first principles scaling theory based on renormalization group (RG) methods predicts a strong temperature dependence of the dimensionless resistivity $\mathcal{R}(T)$, even at small $\mathcal{R}(T)$, that is not observed. The observed properties are in fact consistent with a weakly disordered Fermi liquid, and there are no indications of strong temperature dependence induced by scaling. While the RG expansion in a power series in $\mathcal{R}(T)$ has only been evaluated to lowest order, this should be sufficient to describe experiments in the region of very small \mathcal{R} . A further apparent anomaly is a return from metal-like to insulating-like behavior for increasing density. We explain these fundamental discrepancies between the first principles theory and experiment. We find that the $\mathcal{R} \ll 1$ data in the currently attainable temperature range are in a weak scaling regime described by the logarithmic approximation. We independently determine the density dependent prefactor of the logarithm using data for the spin susceptibility and effective mass. We find good agreement between theory and experiment for $\mathcal{R}(T)$ in the diffusive regime. We point out that there are corrections to the leading logarithm approximation that should be observable at still lower temperatures.

DOI: 10.1103/PhysRevB.70.235336 PACS number(s): 73.20.Qt, 71.10.Ca, 71.30.+h

I. WEAK TEMPERATURE DEPENDENCE OF RESISTIVITY AT SMALL ${\cal R}$

Scaling equations based on renormalization group (RG) methods have played an important part in theoretical investigations of the metal-insulator (MI) transition phenomena observed in two-dimensional electron systems (2DES) in high purity semiconductor MOSFETs and heterostructures. A scaling picture is relevant if the observed MI transition anomalies at low temperature turn out to be due to a continuous second order transition. While the bifurcation of the temperature dependent resistivity R(T) observed at critical carrier densities n_c (Refs. 2–5) is a primary feature of the MI transition, properties of R(T) in the very low resistivity region at low T have also remained puzzling. 6

In this paper we consider this low resistivity region, $\mathcal{R} \ll 1$ in the low temperature diffusive regime $k_B T \ll k_B T_\tau = \hbar/\tau_{el}$, where τ_{el} is the Drude elastic scattering time. The dimensionless resistivity \mathcal{R} is proportional to the resistance per square, $\mathcal{R} = (e^2/\pi h)R_\square$. It is the diffusive regime of interacting diffusive modes that is described by RG methods where a scaling picture is expected to apply. The ballistic regime which occurs at larger T/T_τ requires a separate treatment, but this regime is not relevant near the zero temperature limit.

A first principles RG theory based on a perturbation expansion in powers of R has been proposed for the low temperature diffusive regime. This takes into account the long range effect of coupled diffusive modes for the 2DES with Coulomb interactions. The electron-electron (e-e) interactions which can have a very strong effect in two dimensions (2D) were treated nonperturbatively. Only the leading

term (one-loop approximation) of the series expansion in powers of \mathcal{R} for the RG equations has been calculated so far but this should be sufficient for small resistance. The resulting T dependence due to RG scaling in this theory is very strong, and it is even singular for certain physical quantities. However, measurements for small \mathcal{R} show no indications of a T dependence induced by rescaling which would be expected from an RG flow in the experimentally accessible temperature range. On the contrary, experiments at high conductivity can be interpreted in terms of a Fermi liquid that is only weakly perturbed by quantum interference and diffusion corrections.

This result is surprising, particularly if the MI transition phenomena are indeed due to a quantum critical point (QCP) accompanied by critical fluctuations. Castellani *et al.* noted that the strong rescaling predicted for the dynamical energy scale may restrict the observability of the RG scaling of \mathcal{R} to temperatures so low that they are not currently accessible. However, this does not address the role of the tuning parameter if the primary bifurcation is due to a QCP with associated critical fluctuations. The tuning parameter in this case is the relative density shift $\delta_n = [(n-n_c)/n_c]$.

In this paper we point out that even if the primary bifurcation is due to a QCP with associated critical fluctuations, the properties in the very low $\mathcal R$ region will still be those of a Fermi liquid with weak disorder even at very low temperature. This result is consistent with the experimental observations of Ref. 12. There are two reasons for this Fermi liquid-like behavior. First, the carrier density for low $\mathcal R$ is far from the primary critical density n_c so the relative density shift δ_n is not small and hence lies well outside the critical regime. The correlation length $\xi \sim \delta_n^{-\nu}$ must therefore remain micro-

scopically small (ν is a critical exponent). Effects of criticality due to any QCP will not be observed at these noncritical densities. No matter how low the temperature is taken, there can be no critical fluctuations and no renormalization of the kind associated with the primary transition.

The second reason for the Fermi liquid-like behavior is the very slow evolution of scaling in the RG equations of Refs. 8–10 when $\mathcal{R} \leq 1$. This has the consequence that the resistivity data for $\mathcal{R} \leq 1$ in the diffusive regime in the currently available temperature range all lie in the range of weak scaling corrections. As described in Sec. III, the leading T dependence of $\mathcal{R}(T)$ in the diffusive regime is then given by the well known $\log T$ term. $^{8-10,14}$ The leading T dependence induced by scaling is similarly weak for all other RG functions. Further evidence of weak scaling in this region is the change of sign with density of the prefactor of the $\log T$ term at the secondary reentrant insulator transition.

II. REENTRANT INSULATOR TRANSITION

A reentrant insulator, that is a second crossover from "metallic" behavior $(\partial R/\partial T>0)$ back to a reentrant "insulator" behavior $(\partial R/\partial T<0)$ as the density increases, was reported for p-GaAs in Ref. 15. The primary MI bifurcation was observed at a critical density p_c . In addition to this primary transition, it was found that when the density was further increased the T dependence of $\mathcal{R}(T)$ reverts from "metallic-like" back to "insulatorlike" behavior.

We first point out that this reentrant phenomenon cannot be accounted for by the two-band model for p-GaAs. Recall that for hole densities $p > 2 \times 10^{11}$ cm⁻² the Fermi energy exceeds the energy where the degenerate hole band splits into two subbands split by spin-orbit coupling. It has been argued that the combination of inelastic intersubband scattering and hole-hole interactions causes the temperature dependence of $\mathcal{R}(T)$ to become metalliclike in behavior and this picture was applied to the onset of metallic behavior with increasing hole density and the magnetoresistance at the primary MI transition. 16,17 Later experiments indicated that the simple two-band model is incomplete and not sufficient to account for all of the results. 18 However, irrespective of the success or otherwise of this two-band model in accounting for phenomena at the primary transition, it is not relevant to the secondary reentrant insulator transition. The two-band model can only predict insulating to metallic behavior with increasing density whereas the reentrant insulator effect goes in the opposite direction, that is, from metallic to insulating with increasing density.

The reentrant insulator phenomenon has also been reported in conductivity data $\sigma = (e^2/h)G$ in Si (see Ref. 19). The T dependence of G was found to be logarithmic, $G = G_0 + C(n)\log T$, with a prefactor C(n) that increased as a function of density. At low densities C(n) was found to be negative corresponding to "metallic" dependence of G on T. However at a second critical density n_{c_2} the C(n) vanished, and for $n > n_{c_2}$, C(n) was positive, corresponding to reentrant "insulator" behavior.

A reentrant effect carries with it an implication that *T* dependent resistivity curves associated with different densi-

ties may cross as the temperature is lowered. Such a crossing of lines does not occur at a conventional second order transition, and the reentrant phenomenon would be very puzzling indeed if it were within the critical range of influence of the presumed QCP. However this is not the case since the tuning parameter $\delta_n \not\leq 1$ when $\mathcal{R} \leq 1$. This is a further indication that the reentrant insulator is outside the primary critical regime and is a distinct additional feature of the disordered 2DES.

III. RG PREDICTIONS AT SMALL $\mathcal R$ AND WEAK SCALING

We start by recalling the RG equation for \mathcal{R} to leading order for purely potential scattering, $^{8-10}$

$$d\mathcal{R}/dy = \alpha(\gamma_2)\mathcal{R}^2,\tag{1}$$

$$\alpha(\gamma_2) = n_v + [1 + ((2n_v)^2 - 1) \times \{1 - ((1 + \gamma_2)/\gamma_2)\log(1 + \gamma_2)\} - \gamma_c].$$
 (2)

 $\Gamma_2 = Z\gamma_2$ is the electron-hole scattering amplitude for the triplet spin state, with Z the dynamical energy scaling function, and $\Gamma_c = Z\gamma_c$ the particle-particle (p-p) scattering amplitude. n_v is the number of valleys. 20 $y = \log \lambda^{-1}$, and rescaling of momentum and energy is specified by integrating over the momentum and energy shells $\lambda k_0^2 < k^2$, $Z\omega/D < k_0^2$, with D the diffusion constant. 13 There are also coupled RG equations for Γ_2 ,

$$d\gamma_2/dy = \mathcal{R}[1/2(1+\gamma_2)^2 + \gamma_c(1+3\gamma_2+2\gamma_2^2)],$$
 (3)

and for Z and Γ_c .^{8–10} A review of the solution to the set of coupled differential equations for the RG scaling functions, with emphasis on the strong scaling limit (large y) is given in Ref. 21. p-p scattering plays no essential role in our discussion so we neglect γ_c .

Although for the 2D case only a single term in $d\mathcal{R}/dy$ has been computed to date, Eq. (1) will nevertheless be valid in the regime of parameters where $\mathcal{R} \! \leq \! 1$ provided the other coupled RG equations also remain valid. This is quite different from the situation in the vicinity of the bifurcation, where $\mathcal{R} \! \sim \! 1$. Reference 22 showed that higher order terms in the series are essential for even a qualitative description in the bifurcation region.

The set of coupled differential equations for the RG parameters is integrated upward in y and solved simultaneously. This leads to a renormalization of the parameters. The energy scale k_BT also renormalizes due to the y dependence of dynamical energy rescaling function energy Z(y). The renormalizations with increasing y can then be interpreted in terms of the physical temperature dependence of RG parameters as T decreases. The observation by Castellani $et\ al$. that the physical temperature at which strong scaling effects can be observed is sharply suppressed by Z(y) (Ref. 13) is particularly important in the strong scaling regime but is less relevant here since experimental results in the diffusive range for $\mathcal{R} \ll 1$ all lie in the weak scaling regime.

The weak scaling regime is defined as having small scaling induced corrections to all of the RG functions. From the right-hand sides of Eqs. (1) and (3) it is clear that when

 $\mathcal{R}(y=0) \leq 1$ the weak scaling regime can extend over a considerable range of y. In this range the leading T dependence of $\mathcal{R}(T)$ is given by replacing $\alpha(\gamma_2(y))$ in Eq. (1) by $\alpha(\gamma_2(y=0))$, integrating, and using Z(0)=1 to set $y=\log(T_0/T)$. This yields the well-known logarithmic correction to the Drude conductivity σ_0 ,

$$\Delta \sigma(T) = \sigma(T) - \sigma_0 = (e^2/\pi h)\alpha(\gamma_2)\log(T/T_\tau). \tag{4}$$

Equation (4) with $\alpha(\gamma_2)$ given by Eq. (2) at y=0, its value without rescaling, was first given by Finkelstein [see Eq. (5.2) of Ref. 8 or Eq. (2.104) of Ref. 21]. The strong e-e interactions were treated nonperturbatively. Logarithmic corrections to the conductivity in 2D due to e-e interactions, in addition to that due to weak localization, were considered earlier by Altshuler et $al.^{23}$ and by Fukuyama, 24 but only to first order in the e-e interactions. A derivation of Eq. (4) and an interpretation of the relative contributions of particle-hole singlet and triplet spin states was given in Ref. 14 using conventional many-body theory to all orders rather than RG methods. Reference 14 also explained why the lowest order perturbation theory result^{23,24} differed from the correct weak coupling limit of Ref. 8.

It is important to stress that Eq. (4) is only the leading correction to the conductivity at the onset of the diffusive regime. There are further corrections of higher order in \mathcal{R} arising both from higher order terms neglected in Eq. (1) and from the scaling variation with y of $\gamma_2(y)$ and Z(y). However, even though Eq. (4) gives only the leading quantum contribution to the conductivity, this can still provide a good quantitative description of the results of Refs. 15 and 19. The reason is that the starting $\mathcal{R}(y=0) \ll 1$, so the temperature range of validity of weak scaling is sufficiently large to cover the experimental range.

IV. RESULTS AND COMPARISON WITH EXPERIMENT

We now determine γ_2 from the spin susceptibility χ which contains the factor $1 + \gamma_2$, and thereby $\alpha(\gamma_2)$ from Eq. (2). We use a combination of experimental data and theoretical results. Attaccalite *et al.*²⁵ have calculated χ for the pure 2DES with zero layer thickness and no disorder. However, for real samples the effects of both disorder and finite layer thickness must be taken into account.

A. Si

For Si MOSFETs, where significant disorder along the interface is unavoidable, the calculated values of χ/χ_0 (Ref. 25) differ significantly from the values directly measured by different experimental groups. 26,27 The experimental values are in mutual agreement for densities $r_s \leq 6$. χ_0 is the spin susceptibility with the same bare band mass m_b but no Coulomb interactions. Pudalov *et al.* 27 also directly measured the effective mass m^*/m_b from Shubnikov de Haas oscillation data. For $r_s < 5$ it is a good approximation to assume the Dingle temperature T_D , is constant since the change in the resistance over the studied temperature range is small. The resulting values of m^* are consistent with other measurements. At lower densities $r_s > 5$, the parameters m^* and T_D

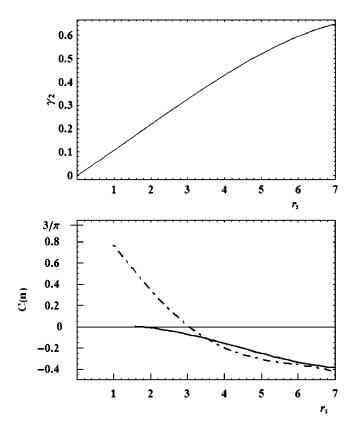


FIG. 1. (a) Calculated γ_2 for Si. (b) Dependence of the prefactor C(n) in Si on density parameter r_s . Solid line, from measurements in Ref. 19. Dashed line, calculated $C(n) \equiv \alpha(\gamma_2)/\pi$.

become progressively more correlated and this introduces some uncertainty in the value of m^* .

In Fig. 1(a) we show the resulting $\gamma_2 = (m_b/m^*)(\chi/\chi_0)$ –1 as a function of the density parameter r_s using the values of m^*/m_b and χ/χ_0 given in Ref. 27. We use throughout the assumption that T_D is a constant. Figure 1(b) compares our calculated prefactor of $\log T$ with the prefactor C(n) taken from Fig. 2 of Ref. 19. The measured C(n) should be compared with our $\alpha(\gamma_2)/\pi$ since $\Delta\sigma$ in Fig. 2 of Ref. 19 is expressed in the form $\Delta\sigma(T) = (e^2/h)C(n)\log T$.

We see that theory and experiment are in good agreement except in the region $r_s \leq 3$. If we follow the alternate assumption of Pudalov *et al.*²⁷ that T_D is proportional to $\mathcal{R}(T)$, then for $r_s > 5$ our C(n) decreases significantly faster with r_s and the agreement is much poorer in this low density region.

While it is not visible on the scale of this figure, the measured C(n) does indeed change sign. ¹⁹ However it only just crosses zero whereas our calculated C(n) continues to grow with decreasing r_s . The high density limit of the theoretical C(n) is $C(r_s \rightarrow 0) = 3/\pi$ for Si with its two valleys. This is quite large and positive. It is not clear why the experimental data at small r_s appear not to extrapolate to this theoretical limit (see Sec. V).

B. p-GaAs

For p-GaAs there are no measurements for χ , and this together with the second hole subband and the significant

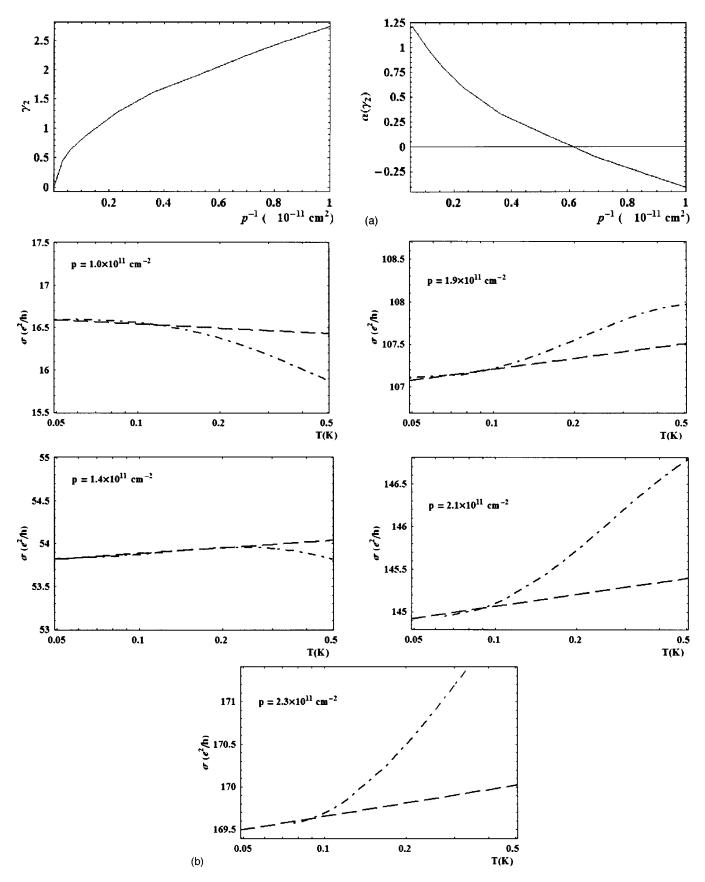


FIG. 2. (a) Calculated γ_2 and $\alpha(\gamma_2)$ as a function of inverse hole density for *p*-GaAs. (b) Conductivity $\sigma(T)$ for *p*-GaAs for different hole densities as indicated. Dashed lines, theoretical predictions for diffusive region $T/T_{\tau} < 1$. Dashed-dotted lines, measurements (from Ref. 15).

anisotropy raise serious difficulties when extracting the data we need. Therefore in this case we use the χ calculated for the disorder free system. ²⁵ That this is a reasonable approximation for the GaAs/AlGaAs system is supported by the recent experiments of Vakili *et al.* ²⁸ for AlAs narrow quantum wells. These give values of χ consistent with Ref. 25. Effects of disorder should be less important for these systems since the dopants are remote from the interface.

Figure 2(a) shows the resulting γ_2 and $\alpha(\gamma_2)$ as functions of r_s . We have used an effective mass to band mass ratio of $m^*/m_b=1.2$, representing the effect of e-e interactions. We compare in Fig. 2(b) the solution of Eq. (4) using the calculated $\alpha(\gamma_2)$ with the $\sigma(T)$ and $\Delta\sigma(T)$ measured in GaAs by Hamilton *et al.* ¹⁵ The starting σ_0 is sensitive to the properties of each sample and so we have used the σ_0 values given in Ref. 15. Following Ref. 15 we have used a band mass $m_b=0.3$ when relating r_s to the hole density p. This procedure gives the crossover back to insulating behavior at the observed density, and in addition there is good agreement between the theoretical and experimental temperature dependences of the conductivity in the diffusive regime which is at small T/T_{τ} . Approximate values of T_{τ} are identified from experiment by the crossover from ballistic to diffusive behavior.7

The temperature range $0.2 \text{ K} \lesssim T \lesssim 1 \text{ K}$ lies outside the diffusive region on which we are focused, and the experimental curves exhibit a steeper gradient than that determined from Eq. (4). This is due at least in part to a broad crossover from diffusive to ballistic behavior at higher T. We note that for experimentally reported temperatures below 0.1 K, the carrier T can be higher than the lattice T.

V. CONCLUSION AND DISCUSSION

There is good agreement between theory and experiment for Si for $r_s > 3$ but a discrepancy appears at higher densities. We have established that the discrepancy is not removed by including the Cooper channel γ_c in the determination of $\alpha(\gamma_2)$. However, part of it may be accounted for by slight changes in the T dependence of Eq. (4) since a more complete treatment of the weak localization correction to the conductivity yields the logarithmic factor $\log[(\tau_\phi^{-1} + \tau_{el}^{-1})/\tau_\phi^{-1}]$. The additional parameter τ_ϕ is the phase relaxation rate. ²⁹ Equation (4) is the low T limit of this more

complete expression. The density dependence of τ_{ϕ} introduces density dependence into the logarithm, and this affects any experimental determination of C(n). In addition, the use of only a single "cutoff" parameter τ_{el} in Eq. (4) may be insufficient since different e-e scattering amplitudes should strictly have different cutoffs.³⁰ These modifications may substantially reduce the discrepancy between experiment and current theory in the case of Si at high densities $r_s < 3$ that appears to be systematic.

The logarithmic behavior at low T reported in Ref. 19 and also in Ref. 15 is experimental evidence for weak rescaling for small \mathcal{R} . The resulting T dependence in the regime $\mathcal{R} \leq 1$ is weak for a combination of two reasons. First any strong T dependence due to a possible QCP will not be observed because the values of the tuning parameter, δ_n , lie outside the critical regime of the primary bifurcation. Second the T dependence predicted by the RG Eqs. (1)–(3) is weak because the currently available resistivity data for $\mathcal{R} \leq 1$ in the diffusive regime are all in the range of weak scaling corrections to all RG functions. The immediate consequence is that the resistance data should be describable by the wellknown log T with a density dependent prefactor. We determined the prefactor from independent considerations. The resulting low temperature diffusive behavior predicted by Eq. (4) is quantitatively consistent with both Refs. 15 and 19, and accounts for the reentrant metal to insulator phenomenon in GaAs and Si. This verifies the consistency of the weak scaling regime.

Finally, although Eq. (4) with the appropriate $\alpha(\gamma_2)$ is a good approximation in the currently accessible diffusive temperature range, there are nevertheless additional scaling corrections and these must be included when the temperature becomes still lower. In particular the log T behavior of the leading logarithmic approximation does not continue to the limit of zero temperature and so provides only limited information on the ground state itself. Experimental detection of these higher order corrections will lead to new insight into the very low T properties of the 2DES.

ACKNOWLEDGMENTS

This work is supported by the Natural Sciences and Engineering Research Council of Canada and an Australian Research Council Grant.

¹S. L. Sondhi, S. M. Girvin, J. P. Carini, and D. Shahar, Rev. Mod. Phys. **69**, 315 (1997).

²S. V. Kravchenko, W. E. Mason, G. E. Bowker, J. E. Furneaux, V. M. Pudalov, and M. D'Iorio, Phys. Rev. B 51, 7038 (1995); S. V. Kravchenko, D. Simonian, M. P. Sarachik, W. Mason, and J. E. Furneaux, Phys. Rev. Lett. 77, 4938 (1996).

³D. Simonian, S. V. Kravchenko, M. P. Sarachik, and V. M. Pudalov, Phys. Rev. Lett. **79**, 2304 (1997).

⁴P. T. Coleridge, R. L. Williams, Y. Feng, and P. Zawadzki, Phys. Rev. B **56**, R12 764 (1997).

⁵M. Y. Simmons, A. R. Hamilton, M. Pepper, E. H. Linfield, P. D. Rose, D. A. Ritchie, A. K. Savchenko, and T. G. Griffiths, Phys. Rev. Lett. 80, 1292 (1998); Y. Y. Proskuryakov, A. K. Savchenko, S. S. Safonov, M. Pepper, M. Y. Simmons, D. A. Ritchie, A. G. Pogosov, and Z. D. Kvon, Phys. Status Solidi B 230, 89 (2002).

⁶E. Abrahams, Physica E (Amsterdam) **3**, 69 (1998).

⁷G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001).

⁸A. M. Finkelstein, Zh. Eksp. Teor. Fiz. **84c**, 168 (1983) [Sov.

- Phys. JETP 57, 97 (1983)].
- ⁹C. Castellani, C. Di Castro, P. A. Lee, and M. Ma, Phys. Rev. B 30, 527 (1984).
- ¹⁰ A. M. Finkelstein, Z. Phys. B: Condens. Matter **56**, 189 (1984).
- ¹¹E. Abrahams, S. V. Kravchenko, and M. P. Sarachik, Rev. Mod. Phys. **73**, 251 (2001).
- ¹² Y. Y. Proskuryakov, A. K. Savchenko, S. S. Safonov, M. Pepper, M. Y. Simmons, D. A. Ritchie, A. G. Pogosov, and Z. D. Kvon, Phys. Status Solidi B 230, 89 (2002).
- ¹³C. Castellani, C. DiCastro, and P. A. Lee, Phys. Rev. B 57, R9381 (1998).
- ¹⁴B. L. Altshuler and A. G. Aronov, Solid State Commun. 46, 429 (1983).
- ¹⁵ A. R. Hamilton, M. Y. Simmons, M. Pepper, E. H. Linfield, P. D. Rose, and D. A. Ritchie, Phys. Rev. Lett. **82**, 1542 (1999).
- ¹⁶S. S. Murzin, S. I. Dorozhkin, G. Landwehr, and A. C. Gossard, JETP Lett. **67**, 113 (1998).
- ¹⁷ Y. Yaish, O. Prus, E. Buchstab, S. Shapira, G. Ben Yoseph, U. Sivan, and A. Stern, Phys. Rev. Lett. **84**, 4954 (2000); Y. Yaish, O. Prus, E. Buchstab, G. Ben Yoseph, U. Sivan, I. Ussishkin, and A. Stern, cond-mat/0109469 (unpublished).
- ¹⁸S. J. Papadakis, E. P. De Poortere, H. C. Manoharan, J. B. Yau, M. Shayegan, and S. A. Lyon, Phys. Rev. B 65, 245312 (2002).

- ¹⁹ V. M. Pudalov, G. Brunthaler, A. Prinz, and G. Bauer, JETP Lett. 68, 534 (1998).
- ²⁰ A. Punnoose and A. M. Finkel'stein, Phys. Rev. Lett. **88**, 016802 (2002).
- ²¹ A. M. Finkelstein, Sov. Sci. Rev., Sect. A **14**, 3 (1990).
- ²²D. J. W. Geldart and D. Neilson, Phys. Rev. B **67**, 205309 (2003).
- ²³ B. L. Altshuler, A. G. Aronov, and P. A. Lee, Phys. Rev. Lett. 44, 1288 (1980).
- ²⁴H. Fukuyama, J. Phys. Soc. Jpn. **48**, 2169 (1980).
- ²⁵C. Attaccalite, S. Moroni, P. Gori-Giorgi, and G. B. Bachelet, Phys. Rev. Lett. 88, 256601 (2002).
- ²⁶F. F. Fang and P. J. Stiles, Phys. Rev. B **174**, 823 (1968); T. Okamoto, K. Hosoya, S. Kawaji, and A. Yagi, Phys. Rev. Lett. **82**, 3875 (1999).
- ²⁷ V. M. Pudalov, M. E. Gershenson, H. Kojima, N. Butch, E. M. Dizhur, G. Brunthaler, A. Prinz, and G. Bauer, Phys. Rev. Lett. 88, 196404 (2002).
- ²⁸ K. Vakili, Y. P. Shkolnikov, E. Tutuc, E. P. De Poortere, and M. Shayegan, Phys. Rev. Lett. **92**, 226401 (2004).
- ²⁹ M. J. Uren, R. A. Davies, M. Kaveh, and M. Pepper, J. Phys. C 14, L395 (1981).
- ³⁰C. Castellani, C. DiCastro, G. Kotliar, and P. A. Lee, Phys. Rev. Lett. **56**, 1179 (1986).