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# TEMPERATURE CHARACTERISTICS FOR CERTAIN FRESH WATERS.

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ABSTRACT.

Bodies of water can be readily compared insofar as temperature is concerned, and the suitability of any body of water for various aquicultural purposes can be readily determined by fitting the "sine curve" to temperature observations. Ponderous masses of temperature readings, otherwise difficult to handle, are reduced and made easy of treatment (liquidated). A measure of that very important ecological factor, temperature, is hereby provided.

"The considerable effect of relatively small changes of temperature on the biological processes of marine plants and animals has been discussed, and it is apparent that the temperature of the water is a factor of primary importance in its effect on the population."—H. W. Harvey.[1]

## I.

The importance of knowledge of the temperatures and heat content (calories) of bodies of water is recognized universally, but it is of greatest interest, probably, to meteorologists and biologists. Welch[2] states that "since the heat content of a lake is a matter of vital importance in limnology, methods of determining *heat budgets* are necessary". He discusses the methods developed by Birge[3] and states that such heat budgets "form an important method of comparing lakes with respect to heat content".

Hachey[4] developed a method of comparing water and air temperatures, by means of fitting a sine curve to accumulated temperature data. The purpose of the present paper is to demonstrate the value of this latter method, as a measure or means of comparing habitats insofar as the important ecological factor, temperature, is concerned.

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[1] Harvey, H. W., "*Biological Chemistry and Physics of Sea Water*", Cambridge. 1928, p. 133.

[2] Welch, P. S., "*Limnology*", McGraw-Hill Book Co., New York and London, 1935, p. 58.

[3] Birge, E. H. and Juday, C., "*A Limnological Study of the Finger Lakes of New York*", U. S. Bur. of Fish. Bull. 32, 1912, pp. 525-610.

[4] Hachey, H. B., *Monthly Weather Review*, 61 (9), 264-265 (1933).

## II.

There was available a mass of temperature readings from many fresh-water bodies, chiefly streams, used for fish hatchery water supplies, and as it was important to be able to make comparisons between the temperature conditions of various hatcheries, mainly for pathological studies upon the various hatchery populations, attempts were made to use some method of comparison.

By developing the method devised by Hachey, it is believed that very satisfactory comparison can be made. Because fresh-water is covered during the winter first, by a layer of water of less density as the temperature falls below 4°C., and then by ice, radiation is so interfered with that for many weeks the temperature changes little. The sine curve indicates continuous symmetrical change, so that this departure during cold weather of observed values from a smooth or symmetrical rate of change forms an important objection to the use of such a means of comparison as a sine curve, but since one can fit such a curve quite accurately to the temperature data above 4°C., in practice it proves a very satisfactory means of comparison.

In fitting the sine equation to fresh-water data, Hachey's equation has been slightly modified, since a period (P) of fifty-two weeks, instead of one of twelve months, has been employed, and degrees rather than radians have been used in order to facilitate the use of ordinary mathematical tables by those who are not mathematicians. For the same reason, a table of sines and cosines (natural) has been worked out to half-weeks and added to this paper as an appendix. It will enable non-mathematical workers interested to use the equations.

The theory on which this application of the sine equation rests depends upon the fact that the motion of any body or point in a circular path, when plotted graphically against time, results in a characteristic curve called the sine curve, and which is expressed by the mathematical expression called the sine equation. In this case, the earth in its annual orbit around the sun in fifty-two weeks is the moving body. It is a familiar fact to all that it is this same revolution of the earth around the sun which causes the marked changes of temperature which we call the seasons.

The sine equation used here is of the form:

$$Y = A - B \sin 6.92^\circ (x - K),$$

in which Y is the temperature in degrees for any week x: A, B and K are values (constants) which must be determined for each body of water, and which are the "characteristics" of this paper. In the resulting equations, A is an average temperature (not usually, in fresh-water, the true average), B is the amplitude, and K a factor which may be called the "lag" factor, and which depends upon the point selected as the starting point.

For any body of water which does not freeze, as sea-water and certain springs, a sine curve fits the temperature data throughout the year, and A will be then the true average temperature. In fitting the sine curve to temperatures of the non-freezing period, for waters which do freeze, the theoretical values below 4°C. will depart from the actual values observed, and hence A will be lower than the true average by an amount corresponding to the duration of freezing and depending upon the amplitude.

It will be clear that, since the equation contains three unknown quantities, three separate determinations of Y (at different weeks) must be available to determine A, B and K. The most simple determinations will result by taking temperatures (Y) for the points where x corresponds to 52, 39, 26, or 13. This is because at these values of x the sine reduces to 0 or ±1.

If we place $Y_0$ as the temperature for $x = 52$, $Y_1$ for $x = 39$, $Y_2$ for $x = 26$, the following relationships hold:

$$A = \frac{Y_0 + Y_2}{2}$$

$$B = 0.5\sqrt{(Y_0 + Y_2 - 2Y_1)^2 + (Y_0 - Y_2)^2}$$

$$\sin \epsilon = \frac{Y_0 - A}{B} \qquad [K = 0.14\epsilon].$$

## TABLE 1.

### Temperature Equations—°C.

| Hatchery | Latitude | Temperature Equation (maximum) | Temperature Equation (minimum) |
|---|---|---|---|
| Newtown, Ohio | 39.1°N. | $Y = 18.7 - 13.8 \sin 6.92(x - 0.5)$ | $Y = 11.3 - 14.2 \sin 6.92(x - 1.0)$ |
| Piqua, Ohio | 40.2 | $Y = 17.4 - 14.4 \sin 6.92(x + 0.5)$ | $Y = 10.9 - 15.5 \sin 6.92(x - 0.5)$ |
| Xenia, Ohio | 39.7 | $Y = 16.7 - 12.6 \sin 6.92(x + 0.5)$ | $Y = 10.8 - 13.1 \sin 6.92(x - 0.5)$ |
| Normandale, Ontario | 42.7 | $Y = 13.6 - 11.6 \sin 6.92(x)$ | $Y = 6.61 - 11.3 \sin 6.92(x - 1.0)$ |
| Erwin, Tenn. | 36.1 | $Y = 13.1 - 0.28 \sin 6.92(x)$ | |
| Springville, Utah | 40.2 | $Y = 12.3 - 0.56 \sin 6.92(x - 1.0)$ | |
| Yarmouth, N. S. | 44.0 | $Y = 11.6 - 12.6 \sin 6.92(x - 1.0)$ | $Y = 7.33 - 11.6 \sin 6.92(x - 1.5)$ |
| Middleton, N. S. | 44.8 | $Y = 11.4 - 10.2 \sin 6.92(x)$ | $Y = 6.95 - 10.2 \sin 6.92(x - 1.5)$ |
| East Orland, Me. | 44.5 | $Y = 11.2 - 9.6 \sin 6.92(x - 2.5)$ | $Y = 6.89 - 10.5 \sin 6.92(x - 3.0)$ |
| Antigonish, N. S. | 45.6 | $Y = 10.8 - 10.8 \sin 6.92(x - 2.0)$ | $Y = 4.66 - 12.7 \sin 6.92(x - 3.0)$ |
| Ithaca, N. Y. | 42.4 | $Y = 10.7 - 12.7 \sin 6.92(x - 0.5)$ | $Y = 6.06 - 13.8 \sin 6.92(x - 0.5)$ |
| Bedford, N. S. | 44.7 | $Y = 10.2 - 13.6 \sin 6.92(x - 0.5)$ | $Y = 5.89 - 12.5 \sin 6.92(x - 1.5)$ |
| *St. Andrews, N. B. | 45.1 | $Y = 9.83 - 10.8 \sin 6.92(x - 1.0)$ | $Y = 7.33 - 10.6 \sin 6.92(x - 1.0)$ |
| *Birch Cove, N. B. | 45.2 | $Y = 9.44 - 9.6 \sin 6.92(x - 3.5)$ | $Y = 4.44 - 11.2 \sin 6.92(x - 3.5)$ |
| Kellys Pond, P. E. I. | 46.3 | $Y = 9.33 - 13.2 \sin 6.92(x - 1.0)$ | $Y = 4.72 - 12.2 \sin 6.92(x - 1.5)$ |
| *Magog, Quebec | 45.6 | $Y = 8.22 - 16.0 \sin 6.92(x - 1.5)$ | |
| Dorion, Ontario | 48.9 | $Y = 8.06 - 2.4 \sin 6.92(x + 1.5)$ | $Y = 5.56 - 2.1 \sin 6.92(x)$ |
| *Chamcook, N. B. | 45.2 | $Y = 8.06 - 2.0 \sin 6.92(x - 13.0)$ | |
| Miramichi, N. B. | 46.9 | $Y = 7.95 - 12.6 \sin 6.92(x - 0.5)$ | $Y = 2.00 - 10.3 \sin 6.92(x - 2.0)$ |
| Saint John, N. B. | 45.3 | $Y = 7.84 - 12.3 \sin 6.92(x - 1.0)$ | $Y = 6.17 - 12.4 \sin 6.92(x - 2.0)$ |
| Margaree, N. S. | 46.2 | $Y = 7.61 - 9.3 \sin 6.92(x - 2.0)$ | $Y = 4.17 - 10.1 \sin 6.92(x - 3.0)$ |
| Sault Ste. Marie, Ont. | 46.6 | $Y = 7.27 - 2.3 \sin 6.92(x - 1.0)$ | $Y = 6.17 - 2.7 \sin 6.92(x - 2.0)$ |
| Lac Manitou, Que. | 45.9 | $Y = 7.00 - 9.0 \sin 6.92(x - 2.0)$ | $Y = 1.39 - 9.9 \sin 6.92(x - 2.0)$ |
| Grand Falls, N. B. | 47.2 | $Y = 6.73 - 8.3 \sin 6.92(x - 1.0)$ | $Y = 3.28 - 7.4 \sin 6.92(x - 2.5)$ |
| Florenceville, N. B. | 46.4 | $Y = 6.67 - 10.3 \sin 6.92(x - 1.5)$ | $Y = 2.61 - 8.7 \sin 6.92(x - 2.0)$ |
| Lac Tremblant, Que. | 46.3 | $Y = 5.44 - 12.4 \sin 6.92(x - 1.5)$ | $Y = 3.77 - 12.0 \sin 6.92(x - 1.5)$ |
| Tadoussac, Que. | 48.2 | $Y = 4.66 - 14.8 \sin 6.92(x - 2.0)$ | $Y = 2.84 - 14.5 \sin 6.92(x - 2.0)$ |
| St. Félicien, Que. | 48.7 | $Y = 4.40 - 18.7 \sin 6.92(x - 1.5)$ | $Y = 1.39 - 15.4 \sin 6.92(x - 1.5)$ |
| Restigouche, N. B. | 48.0 | $Y = 4.06 - 5.8 \sin 6.92(x - 3.0)$ | $Y = 2.44 - 6.2 \sin 6.92(x - 3.0)$ |

*In these cases, temperature readings have been available for only a single year.

## TABLE 2.

Temperature Equations—°F.

| Hatchery | Latitude | Temperature Equation (maximum) | Temperature Equation (minimum) |
|---|---|---|---|
| Newtown, Ohio | 39.1°N. | $Y = 65.7 - 24.9 \sin 6.92(x - 0.5)$ | $Y = 52.4 - 25.5 \sin 6.92(x - 1.0)$ |
| Piqua, Ohio | 40.2 | $Y = 63.3 - 25.9 \sin 6.92(x + 0.5)$ | $Y = 51.6 - 27.9 \sin 6.92(x - 0.5)$ |
| Xenia, Ohio | 39.7 | $Y = 62.0 - 22.6 \sin 6.92(x + 0.5)$ | $Y = 51.4 - 23.6 \sin 6.92(x - 0.5)$ |
| Normandale, Ontario | 42.7 | $Y = 56.5 - 20.9 \sin 6.92(x)$ | $Y = 43.9 - 20.4 \sin 6.92(x - 1.0)$ |
| Erwin, Tenn. | 36.1 | $Y = 54.5 - 0.5 \sin 6.92(x)$ | |
| Springville, Utah | 40.2 | $Y = 54.2 - 1.0 \sin 6.92(x - 1.0)$ | |
| Yarmouth, N. S. | 44.0 | $Y = 52.9 - 23.1 \sin 6.92(x - 1.0)$ | $Y = 45.2 - 21.2 \sin 6.92(x - 1.5)$ |
| Middleton, N. S. | 44.8 | $Y = 52.6 - 18.4 \sin 6.92(x)$ | $Y = 44.5 - 18.4 \sin 6.92(x - 1.5)$ |
| East Orland, Me. | 44.5 | $Y = 52.1 - 17.3 \sin 6.92(x - 2.5)$ | $Y = 44.4 - 18.9 \sin 6.92(x - 3.0)$ |
| Antigonish, N. S. | 45.6 | $Y = 51.5 - 19.5 \sin 6.92(x - 2.0)$ | $Y = 40.4 - 22.8 \sin 6.92(x - 3.0)$ |
| Ithaca, N. Y. | 42.4 | $Y = 51.3 - 22.9 \sin 6.92(x - 0.5)$ | $Y = 42.9 - 24.9 \sin 6.92(x - 0.5)$ |
| Bedford, N. S. | 44.7 | $Y = 50.3 - 24.4 \sin 6.92(x - 0.5)$ | $Y = 42.6 - 22.4 \sin 6.92(x - 1.5)$ |
| St. Andrews, N. B. | 45.1 | $Y = 49.7 - 19.5 \sin 6.92(x - 1.0)$ | $Y = 45.2 - 20.9 \sin 6.92(x - 1.0)$ |
| Birch Cove, N. B. | 45.2 | $Y = 49.0 - 17.3 \sin 6.92(x - 3.5)$ | $Y = 40.0 - 20.1 \sin 6.92(x - 3.5)$ |
| Kellys Pond, P. E. I. | 46.3 | $Y = 48.8 - 23.7 \sin 6.92(x - 1.0)$ | $Y = 40.5 - 21.9 \sin 6.92(x - 1.5)$ |
| Magog, Que. | 45.6 | $Y = 46.8 - 28.9 \sin 6.92(x - 1.5)$ | |
| Dorion, Ontario | 48.9 | $Y = 46.5 - 4.3 \sin 6.92(x + 1.5)$ | $Y = 42.0 - 3.6 \sin 6.92(x)$ |
| Chamcook, N. B. | 45.2 | $Y = 46.5 - 3.5 \sin 6.92(x - 13.0)$ | |
| Miramichi, N. B. | 46.9 | $Y = 46.3 - 22.7 \sin 6.92(x - 0.5)$ | $Y = 35.6 - 18.5 \sin 6.92(x - 2.0)$ |
| Saint John, N. B. | 45.3 | $Y = 46.1 - 22.2 \sin 6.92(x - 1.0)$ | $Y = 43.1 - 22.4 \sin 6.92(x - 2.0)$ |
| Margaree, N. S. | 46.2 | $Y = 45.7 - 16.7 \sin 6.92(x - 2.0)$ | $Y = 39.5 - 18.1 \sin 6.92(x - 2.0)$ |
| Sault Ste. Marie, Ont. | 46.6 | $Y = 44.9 - 4.1 \sin 6.92(x - 1.0)$ | $Y = 43.1 - 4.9 \sin 6.92(x - 2.0)$ |
| Lac Manitou, Que. | 45.9 | $Y = 44.6 - 16.2 \sin 6.92(x - 2.0)$ | $Y = 34.5 - 17.8 \sin 6.92(x - 2.0)$ |
| Grand Falls, N. B. | 47.2 | $Y = 44.1 - 15.0 \sin 6.92(x - 1.0)$ | $Y = 37.9 - 13.3 \sin 6.92(x - 2.5)$ |
| Florenceville, N. B. | 46.4 | $Y = 44.0 - 18.6 \sin 6.92(x - 1.5)$ | $Y = 36.7 - 15.7 \sin 6.92(x - 2.0)$ |
| Lac Tremblant, Que. | 46.3 | $Y = 41.8 - 22.4 \sin 6.92(x - 1.5)$ | $Y = 38.8 - 21.7 \sin 6.92(x - 1.5)$ |
| Tadoussac, Que. | 48.2 | $Y = 40.4 - 26.7 \sin 6.92(x - 2.0)$ | $Y = 37.1 - 26.0 \sin 6.92(x - 2.0)$ |
| St. Félicien, Que. | 48.7 | $Y = 40.1 - 33.7 \sin 6.92(x - 1.5)$ | $Y = 34.5 - 27.8 \sin 6.92(x - 1.5)$ |
| Restigouche, N. B. | 48.0 | $Y = 39.3 - 10.5 \sin 6.92(x - 3.0)$ | $Y = 36.4 - 11.1 \sin 6.92(x - 3.0)$ |

By means of these relationships, equations were determined for the various waters for which adequate temperature readings were available. These equations will be referred to as temperature equations to distinguish them from other equations discussed. In most cases there were maximum and minimum readings for every day, and a period of five years was used. These temperature equations appear in Table 1 in degrees Centigrade, and in Table 2 in degrees Fahrenheit. The readings were in most cases taken in the Fahrenheit scale, by means of the Six's type of maximum and minimum thermometer.

The temperature data were tabulated by weeks, and the maximum and minimum temperatures for corresponding or equivalent weeks of the five-year period were averaged. These weekly averages were then plotted graphically, and the best-fitting sine curve ascertained.

## III.

The order of arrangement of the various bodies of water in the tables has been in the descending order of the constant A of the maximum temperature equations. Consideration of this order will show that there is a definite relation between the magnitude of this constant and the latitude of the bodies of water. By plotting the values of the constant A (maximum) against the latitude, a curve is obtained which can be expressed by the parabolic equation, $A = -0.0192\, z^2 + 48.5$, in which A is the constant of the temperature equation, and z is degrees of north latitude. Apparently the cosine equation, $A = 1368.\cos z - 87.2$, will fit the available data equally well, so that more data are needed to determine which of these equations best expresses the true relationship. In similar manner, the constants A of the minimum equations are related to latitude. The corresponding parabolic equation for the constant A (minimum) is $A = -0.0143\, z^2 + 34.3$, and the cosine equation is $A = 101.9 \cos z - 66.6$. While this relationship holds for the region under investigation, namely, the north and eastern part of North America, between latitudes 35°N. and 50°N., and, with one exception, between 60°W and 85°W longitude, it will not necessarily hold without modification for other regions, such, for example, as the corresponding western portion of Europe, or for regions higher in altitude, or further from the ocean, and so forth. It is of interest that a similar relationship may be derived for the average annual atmospheric isotherms and the latitude where these isotherms intersect the eastern coast of North America from Florida northwards, and that the constants are of almost the same magnitude as those for the minimum temperature-latitude relationship.

This information has a peculiar value in that by means of it one can determine readily what ought to be suitable waters, with regard to temperature, anywhere in this region for various purposes. For successful rearing of the eastern brook or speckled trout (*Salvelinus fontinalis*), the maximum temperature ought not to exceed 75°F. (24°C.), or be less than 57°F. (14°C.), an intermediate value being probably better, namely 64°F. or 18°C. (Embody[5], M'Gonigle[6]). By means of the latitude equation above, the average temperature (constant A) for a given latitude can be ascertained very nearly. Then having the average temperature, waters with a definite amplitude (constant B) must be sought for the particular purpose desired. For example, what ought to be the characteristics for the maximum temperatures for suitable brook trout waters in latitude 44°N.? We can say that the average (constant A) will be close to 52°F. (11°C.). Therefore the amplitude ought not to be greater than 23 Fahrenheit degrees or 13 Centigrade degrees, and the better values would be 12 Fahrenheit degrees or 7 Centigrade degrees. Warmer waters seem to be associated with more numerous, and more severe outbreaks of various bacterial and parasitic diseases, as well as with better growth rates, hence, for the control of disease, more temperate conditions are better, though not so good from the point of view of rapid growth. This gives for our maximum temperature equation for latitude 44°N., $Y = 11 - 7 \sin 6.92 (x - 2)$, in degrees Centigrade. The

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[5] Embody, G. C., *Fish Culture*, **2** (1), 1-5 (1936).
[6] M'Gonigle, R. H., *Trans. Amer. Fish. Soc.*, **62**, 119-125 (1932).

value of K will not affect the result; it merely indicates the time of year that result will occur. In the case of all the temperature equations and results in this article, the starting point selected is the last week of October, which is week 1.

Decided advantages result from having temperature readings reduced to the form of a simple well-known mathematical equation. It will be apparent that no two bodies of water, of all those analysed and tabulated in tables 1 and 2, are exactly alike. Thus the constants in these equations serve as a means of making easy comparisons between bodies of water. Since the first of these constants (A) depends on the latitude of the body of water, it must be apparent that the second (B) depends upon local characteristics, such as the shape of the basin, depth of the water stratum being measured, either in a lake, or in the earth (springs), and so on. As previously indicated, K depends upon the point of reference, the starting point, and larger or smaller K's depend on the position of the maximum (or minimum) temperature, with relation to this starting point.

By means of these equations, one can ascertain accurately the average amount of heat gained or lost, in calories per unit volume of water, between any particular times one may desire, subject only to the condition that the period is that when the surface of the water is unblanketed. This may be done by integration. It will provide the heat budget for the stratum being studied (Welch[2]) which is used by limnologists. By simple algebra, one may determine for how long the temperature will be above any selected value, on the average. One can readily find out how fast the water is heating or cooling at any given time by means of differentiation.

Another interesting possibility of this method is to use it for the prediction of temperatures which have not and perhaps cannot be measured. In the case of a spring at Chamcook, N. B., (see Tables 1 or 2), there was available only a series of summer temperatures. These had been taken by Dr. M. W. Smith, of the Atlantic Biological Station, in the summer of 1930. He obtained the following.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| June | 13 | 7.2°C. | July | 7 | 7.7°C. | Aug. | 18 | 8.9°C. | Sept. | 29 | 9.9°C. |
| | 16 | 7.2 | | 14 | 8.0 | | 25 | 9.2 | Oct. | 6 | 9.9 |
| | 18 | 7.2 | | 21 | 8.2 | Sept. | 1 | 9.5 | | 13 | 10.1 |
| | 20 | 7.4 | Aug. | 1 | 8.5 | | 8 | 9.5 | Nov. | 1 | 9.7 |
| | 23 | 7.2 | | 4 | 8.8 | | 15 | 9.6 | | 12 | 9.3 |
| | 30 | 7.8 | | 11 | 8.7 | | | | | | |

From the above values, a temperature equation was calculated, which, when checked against the observed values of Dr. Smith's by graphical means, proved to fit very well indeed. Since there were no temperature readings for this spring between November 12 and June 13, the equation could be further checked by taking readings in this interval. and comparing them with those predicted by the equation. Accordingly, the following readings were made:

| Readings. | Observed. | Theoretical, of Calculated. |
|---|---|---|
| December 26, 1935 | 8.8°C. | 8.8°C. |
| February 11, 1936 | 7.6 | 7.1 |
| March 12, 1936 | 6.9 | 6.6 |
| April 1, 1936 | 6.3 | 6.2 |
| April 15, 1936 | 6.1 | 6.1 |
| May 1, 1936 | 5.9 | 6.1 |
| May 18, 1936 | 6.1 | 6.3 |
| March 15, 1937 | 6.4 | 6.4 |

It can be easily seen that the agreement between the theoretical temperature calculated from the equation determined from Dr. Smith's data and that actually observed later is quite good.

It seems to be necessary to exercise caution in attempting to determine temperature equations for bodies of water for which adequate data are not available. The effects of normal fluctuations of weather need to be smoothed out by means of averages in order to have as smooth a curve of observational data as possible. In this study, five-year averages have been employed. Perhaps less than five years would provide sufficiently smoothed curves. Considerable variation resulted in temperature equations calculated for a body of water, by using values of one year only at a time, this resulting in five equations, one for each year.

Whether or not such annual differences as observed in such calculations may be significant of actual differences in the energy received in the different years, it is not possible to do more at the moment than speculate. Abbott[7] and Clayton[8] have shown that solar radiation varies in longer and shorter periods, and that there is a change of 1.4°C. in terrestrial temperatures for a change of one per cent in the solar radiation[9].

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[7] Abbott, C. G., *Sci. Monthly*, 43, 108-121 (1936).

[8] Clayton, H. H., *Monthly Weather Review*, 64, (11), 350-376 (1936).

[9] See also Shelford, V. E., "*Laboratory and Field Ecology*", Williams and Wilkins, Baltimore, 1929 and Kincer, J. B., *Monthly Weather Review*, 61 (9), 251-259 (1933).

## APPENDIX.

Table of Sines and Cosines for Each Week and Half-Week, (When P = 52).

| Week No. | Sine | Cosine | Week No. | Sine | Cosine |
|---|---|---|---|---|---|
| 0.0 (52.0) | 0.00000 | 1.00000 | 26.0 | 0.00000 | -1.00000 |
| 0.5 (52.5) | 0.06047 | 0.99817 | 26.5 | -0.06047 | -0.99817 |
| 1.0 (53.0) | 0.12043 | 0.99272 | 27.0 | -0.12043 | -0.99272 |
| 1.5 (53.5) | 0.18023 | 0.98362 | 27.5 | -0.18023 | -0.98362 |
| 2.0 (54.0) | 0.23938 | 0.97093 | 28.0 | -0.23938 | -0.97093 |
| 2.5 (54.5) | 0.29737 | 0.95476 | 28.5 | -0.29737 | -0.95476 |
| 3.0 (55.0) | 0.35456 | 0.93503 | 29.0 | -0.35456 | -0.93503 |
| 3.5 (55.5) | 0.41045 | 0.91188 | 29.5 | -0.41045 | -0.91188 |
| 4.0 (56.0) | 0.46484 | 0.88539 | 30.0 | -0.46484 | -0.88539 |
| 4.5 (56.5) | 0.51728 | 0.85582 | 30.5 | -0.51728 | -0.85582 |
| 5.0 (57.0) | 0.56808 | 0.82297 | 31.0 | -0.56808 | -0.82297 |
| 5.5 (57.5) | 0.61681 | 0.78711 | 31.5 | -0.61681 | -0.78711 |
| 6.0 (58.0) | 0.66306 | 0.74857 | 32.0 | -0.66306 | -0.74857 |
| 6.5 (58.5) | 0.70711 | 0.70711 | 32.5 | -0.70711 | -0.70711 |
| 7.0 (59.0) | 0.74857 | 0.66306 | 33.0 | -0.74857 | -0.66306 |
| 7.5 (59.5) | 0.78711 | 0.61681 | 33.5 | -0.78711 | -0.61681 |
| 8.0 (60.0) | 0.82297 | 0.56808 | 34.0 | -0.82297 | -0.56808 |
| 8.5 (60.5) | 0.85582 | 0.51728 | 34.5 | -0.85582 | -0.51728 |
| 9.0 (61.0) | 0.88539 | 0.46484 | 35.0 | -0.88539 | -0.46484 |
| 9.5 (61.5) | 0.91188 | 0.41045 | 35.5 | -0.91188 | -0.41045 |
| 10.0 (62.0) | 0.93503 | 0.35456 | 36.0 | -0.93503 | -0.35456 |
| 10.5 (62.5) | 0.95476 | 0.29737 | 36.5 | -0.95476 | -0.29737 |
| 11.0 (63.0) | 0.97093 | 0.23938 | 37.0 | -0.97093 | -0.23938 |
| 11.5 | 0.98362 | 0.18023 | 37.5 | -0.98362 | -0.18023 |
| 12.0 | 0.99272 | 0.12043 | 38.0 | -0.99272 | -0.12043 |
| 12.5 | 0.99817 | 0.06047 | 38.5 | -0.99817 | -0.06047 |
| 13.0 | 1.00000 | 0.00000 | 39.0 | -1.00000 | 0.00000 |
| 13.5 | 0.99817 | -0.06047 | 39.5 | -0.99817 | 0.06047 |
| 14.0 | 0.99272 | -0.12043 | 40.0 | -0.99272 | 0.12043 |
| 14.5 | 0.98362 | -0.18023 | 40.5 | -0.98362 | 0.18023 |
| 15.0 | 0.97093 | -0.23938 | 41.0 | -0.97093 | 0.23938 |
| 15.5 | 0.95476 | -0.29737 | 41.5 | -0.95476 | 0.29737 |
| 16.0 | 0.93503 | -0.35456 | 42.0 | -0.93503 | 0.35456 |
| 16.5 | 0.91188 | -0.41045 | 42.5 | -0.91188 | 0.41045 |
| 17.0 | 0.88539 | -0.46484 | 43.0 | -0.88539 | 0.46484 |
| 17.5 | 0.85582 | -0.51728 | 43.5 | -0.85582 | 0.51728 |
| 18.0 | 0.82297 | -0.56808 | 44.0 | -0.82297 | 0.56808 |
| 18.5 | 0.78711 | -0.61681 | 44.5 | -0.78711 | 0.61681 |
| 19.0 | 0.74857 | -0.66306 | 45.0 | -0.74857 | 0.66306 |
| 19.5 | 0.70711 | -0.70711 | 45.5 | -0.70711 | 0.70711 |
| 20.0 | 0.66306 | -0.74857 | 46.0 | -0.66306 | 0.74857 |
| 20.5 | 0.61681 | -0.78711 | 46.5 | -0.61681 | 0.78711 |
| 21.0 | 0.56808 | -0.82297 | 47.0 (-5.0) | -0.56808 | 0.82297 |
| 21.5 | 0.51728 | -0.85582 | 47.5 (-4.5) | -0.51728 | 0.85582 |
| 22.0 | 0.46484 | -0.88539 | 48.0 (-4.0) | -0.46484 | 0.88539 |
| 22.5 | 0.41045 | -0.91188 | 48.5 (-3.5) | -0.41045 | 0.91188 |
| 23.0 | 0.35456 | -0.93503 | 49.0 (-3.0) | -0.35456 | 0.93503 |
| 23.5 | 0.29737 | -0.95476 | 49.5 (-2.5) | -0.29737 | 0.95476 |
| 24.0 | 0.23938 | -0.97093 | 50.0 (-2.0) | -0.23938 | 0.97093 |
| 24.5 | 0.18023 | -0.98362 | 50.5 (-1.5) | -0.18023 | 0.98362 |
| 25.0 | 0.12043 | -0.99272 | 51.0 (-1.0) | -0.12043 | 0.99272 |
| 25.5 | 0.06047 | -0.99817 | 51.5 (-0.5) | -0.06047 | 0.99817 |
| 26.0 | 0.00000 | -1.00000 | 52.0 ( 0.0) | -0.00000 | 1.00000 |