Dynamics.

of the distinguishing properties of matter

Philosophers have been at great pains in endeavouring to ascertain in what consists the essence of matter, or of that substance of w̃ all body is composed. But their want of success has arisen from their inaccurate Idea of ẏ object of their search— If by ẏ nature or essence of a thing we mean ẏ which constitutes with the thing & which renders it that thing & no other or is the principle from w̃ all its proporties flow & rendering it susceptible of all ẏ modifications to w̃ it is subject it follows ẏ the essence of every thing is peculiar to itself & common to it w̃ no other being in ẏ universe— every body has its own essence— If we take it in this sense, it is evident ẏ we are ignorant of the essence of matter. we do not know the principle w̃ constitutes the essence of one body as distinguished from another. a stone from water much less do we know the circumstance w̃ is the original of all the common properties of mater, distinguishing it from what ever is not material— The essence of a thing must be totally distinct from any proporty of ẏ thing however general may the

universal. what we call y^e property of a thing ha[s refe]
rence to some other thing, & the name which w[e give]
it is only an expression of an event. When we say
that gravity is a property of matter it is only a
short way of expressing the event that every body
will fall when not supported.

But y^e essence of matter must be competent to it
independent of every other thing & must refer only to itself
Not only so but it must be peculiar to matter, belonging
to nothing else; farther, it must be the cause of all y^e
properties of M^r. But our knowledge does not yet
reach so far - we cannot discover y^e manner in which
the different properties of m^r exist in one substance, &
what is the bond w^h unites them - our knowledge is con-
-fined because in the examination of m^r we proceed on
the information of our senses; these give us our ultimate
principle, & yet by their means we can never penetrate
beyond y^e surface of things As to y^e constitution & nature
of an atom we are entirely ignorant. Nor are we
conceive a notion of an atom by what we observe in
the mass - an atom of Gold cannot be ductile nor can-
not dissolve in aquaregia - the atom of an animal mus-
cular fibre cannot be irritable we can know only
those properties which effect our senses, & it is not at all
necessary y^t these properties should do so -

[...] This what we formerly delivered w^h respect to
[caus]e & effect all our meaning by y^e term power is
only an expression of a connection w^h subsists in the
mind between two events. But did we know y^e essence
of matter we would be perfectly acquainted with every
cause. Upon the whole, all y^t we can do is to discover
the properties of matter & to arrange them according to
their various generalities, taking it for granted that those
w^h are more general & nearest allied to y^e essential &
first properties of matter accordingly some philoso-
phers proceeding in this matter found y^t after all pos-
sible abstractions have been made that extension is y^e ge-
neral property of m^r being competent to all matter, &
distinguishing it from every other kind of being, & they
have even attempted to shew y^t from it flowed every
other property, & therefore that extension constituted y^e
essence of matter.

But this doctrine is ill founded, hasty and such an
abuse of common language a[s] entirely destroys its mea-
ning. all can and do conceive extensions without any m^r
and space or vacuity w^h is nothing but extension w^tout M^r
extension then is not peculiar to m^r and is common to it w^h
space, & all that is commonly said about the infinite
divisibility of M^r relate only to space y^e object of geo-
metry but by no means of nat. philosophy.

Others have placed ye essence of matter in m[...]
But it is evident from Geometry yt this may be [...]municated without matter; and such motions are every day perceived by us. no one can say yt the shadow of a sun-dial is Mr. Tho' by its motion we compute ye time—no one can deny ye mobility of the thinking principle of animals, when he sees it always accompanying the body. nor let any man say that these motions are ye consequences of the motions of Mr. they are distinguishable from the motions of Mr. & therefore seperable from them—

The fact is that not only we shall & must remain ignorant of the nature of Mr. so long as all our information comes from experience, but these philosophers have taken the improper method for attaining ye notions of Mr. wh we may acquire.—

It is not by abstractions that we have acquired the distinctive notions of of any substance. It is by observation, where we see ye substance in its individual & complex state vested with all its qualities. I therefore pay little regard to all that ye refinements of philosophy has discovered, and in no case allow them to have concurred in giving us any of ye usefull knowledge of different subjects wh is acquired by all men. I am inclined to do the same thing here, & assign yt as the original of our distinction that is to say I consider these qualities

[...] matter hinders any other Mr. from occupying same place wt itself as constituting the distincture property of matter; but ye same time acknowledging yt I do not know any thing of its essence—

Solidity is an universal property of Mr and completely distinguishes it from spirit, and whatever is not material whatever has this property is considered by us as matter.

Mobility belongs to it in common wt other things, & is not essentially necessary; for there is no absurdity or contradictions in supposing that Mr must be absolutely immoveable; the powr of communicating motion only belongs to Mr when in motion and is therefore not competent to it at all times. Even extension belongs to it only accidentally, for if some, we admit ye divisibility of Mr without end, and thro' infinite of infinitesimal orders, we must conceive ye ultimate object of our contemplation as unextended. In short, without any farther enumeration, every other quality which has been ascribed to Mr may be taken from it; but strip it of solidity & we destroy ye very Idea, & confound it wt space or nonentity.—

But let us form just notions of ye quality wh I thus assert to be the distinctive property of matter—

It is quite distinct from the solidity of ...
for he talks of prisms standing on the same ...
Solidity in our view means yͭ quality by wͪ no two
parts of matter can exist in yͤ same place, or yͭ
power by wͪ Mʳ excludes all other matter from
its place. We acquire yͤ notion by our sense of touch,
and had we never touched a body, our sense of seeing
never could have given us any Idea of it —
An artful painting, the picture in a camera
obscura, & the figures in yͤ air formed by a concave
mirror have every other appearance; & might
have suggested to us all its other properties. —
But had a person passed his life only among such
appearances he could have acquired no Idea of solidity.
And yet some philosophers have attempted to
deduce this property from extension because no
two points of space can occupy yͤ same place this is
very true, but arrises from yͤ immobility of the
parts of space, & no one can doubt of the spectrums
formed by mirrors existing in yͤ same place
It is yͤ particular view of solidity wͪ is called the im-
penetrability of mʳ, & wͪ is asserted to be an universal
property of mʳ. If a body were not impenetrable
any pressure would be sufficient to destroy it —

... yͤ matter wͪ composes the globe, & wͪ is all
... by a tendency to yͤ centre, presses on yͤ inferior
parts wͭ very great force, & yet an annihilation of them
does not follow we justly conclude yͭ all yͤ mʳ of the Earth
has this property. As yͤ accurate examination of yͤ
celestial phaenomena shews us yͭ there is the same
tendency of yͤ mʳ wͪ composes ye body of the sun & the
planets to their respective centre we justly conclude that
all mʳ has yͤ property — Altho yͤ resistance wͪ
we feel opposed to our attemps to compress mʳ is yͤ
orginal of our notions of yͤ property it does not inform
us of its nature; it is true yͭ if we only suppose yͭ yͤ
parts cannot change their respective situation it
will give us a distinct notion of impenetrability.
Now yͤ only way in wͪ we can suppose yͭ is to conceive
yͭ yͤ parts of Mʳ are in contact; for sensible measures
of mʳ. from wͪ by analogy we from our conceptions of its
atoms, are not possessed of such a power, but may be
made to change their situation by very moderate
forces, except when in contact. This is therefore yͤ most
natural way of accounting for yͤ impenetrability of
body, & by wͪ we explain it —
Let us examine it —
There is hardly a body wͪ does not admit a passage

thro' it to some other body. & they seem [illegible]
together — thus water penetrats into marb[le]
Gold is penetrated by mercury tho' both are extremely compact — one fluid penetrates another — light and fire penetrate all bodies. what is more to the present purpose — In many mixtures of metals in many mixtures of fluids, in many cases of solution, y^e mixture occupies less room than than the sume of y^e ingredients — can the particles of any body come nearer than contact? this is impossable.

All this is allowed; but it is answered by saying that y^e parts of m^r. touch only in a few points, & leave great interstices between them. the other body intersects these, and is lodged there — thus a box filled w^t musquet balls will yet receive a considerable quantity of small shot, to which we may afterwards add a considerable quantity of sand, and after all a quantity of water —

But this does not explain some of the difficulties. There occur in Chemestry some facts where a fluid will occupy a smaller space after another body has been added. this happens in the case of water and some salts. & in the case of air & y^e principle of inflamability. If a diamond y^e hardest of all bodies be struck on a piece of marble it will rebound — this it cannot do without compression. nor does it avail to say y^t y^e parts are not brought nearer by y^e compression,

[necessar]ily bent; for it remains to shew how they can be [be]nt without being compressed in some parts. Take y^e same diamond, measure accurately its bulk, expose it to a great degree of cold. & it will be found to have contracted all its dimensions. this it could never do had the parts been in contact — I mus here observe y^t y^e resistance we feel in our attempts to compress body does not allow us to cunclude y^t y^e parts of m^r touch each other; for ther are many instances of this resistance where we know y^t the bodies do not touch. If y^e impossibility of reducing water to smaller dimension leads us to think that y^e particles do not touch the impossibility of doing y^e beyond a certain degree in vapour & air should lead us to form y^e same conclusion. where we hang up a chain. we think its links touch each other. for they suport each other but electrical experiments shew y^t they do not touch till forced together by a great weight, & it is not demonstrated that they do touch. Optical experiments shew y^t two object glasses of long telescopes do not touch even when pressed w^t a very considerable weight. & even then we can shew y^t even tho' y^e appearances of y^e want of contact cease it does not follow y^t they touch. —

From all these we may conclude y^t sol[...]
arise from contact of parts nor authorises us to [...]
y^t y^e parts of body touch.—

I thought it y^e best & shortest way of saving you a great deal of trouble of reading y^e attempts of philosophers to account for the external form, & many of y^e sensible qualities of bodies from y^e form of their parts. Very hard bodies are by them supposed to consist of parts w^c adhere by many touching parts. Brittle bodies have few points of cohesion. Water is fluid & almost without cohesion, because its parts are globular. But plaster of Paris when very hot exhibits every appearance of fluidity; tho' its parts are rough and kept asunder by the acid vapour w^c comes thro' them in stream. It is evident y^t all forms are indifferent if y^e parts do not touch, nay if it can be demonstrated y^t they cannot touch.—

These considerations will also save you the trouble of reading y^e attempts of philosophers to explain y^e constancy of natural productions from the perfect hardness of elementary particles. Now there is no doubt but perfect hardness is a necessary quility of an elementary because they consist of Calx & phlogiston.

[...] does not change either its external form or [che]mical properties. There is no more reason for supposing it necessary y^t y^e calx and phlogiston w^c compose an atom should touch, them that y^e atoms should touch in order to form a sensible mass.—

Accordingly it has been y^e opinion of some eminent philosophers y^t y^e parts of m^r are not in contact.—

Sir Isaac Newton expresses his opinion in this manner— Suppose y^t God had endowed mathematical point w^c insuperable powers of repulsion extending to minute distance only, what Idea cou'd we acquire from y^e but y^t of solidity— Sir I. N. and M^r Lock conversing on this subject shewed y^t from y^e we could form a notion of creation. It was only necessary y^t God should render impenetrable what was before penetrable. For y^e purpose it was only necessary to ordain y^t y^e other portions of space should not penetrate a particular portion of space & then y^e portion becomes impenetrable. many look on creation as an absurdity, but in this view it does not appear so. The defficulty of any operation proceeding from certain obsticles, and mere nothing opposing no obstacle, it appears y^t creation brings neither absurdity nor defficulty. This consequence is y^e principle advantage w^c we can reap from the opinion of these philosophers—

The celebrated Leibnitz also supposed m[illegible] trable. He supposed y universal composed of elem[illegible] w he called monads, kept asunder by spheres of activity, w were not unsurmountable, and they might therefore be forced to unite or pentrate. The person who has taken y most accurate view of y subject is father Boscovich. who has built a system of Nat. philosophy on y principle, w he endeavours to support by many arguments drawn from abstract considerations, and he shews several ways in w y most perfect impenetrability may be produced without y smallest continuous extension of y elementary parts of m^r. But at y same time he observes y this absolute impenetrability does not probably obtaing in nature but that every mass of m^r is compenetrable, y is will allow y freest passage to every other mass of M^r without disturbance of y parts of either; precisely in y same manner as we see y rays of light pass freely thro' transparent bodies without y smallest disturbance of their parts or of its own motion. He supposes matter to consist of nothing but mathe-matical points, kept at their proper distance by certain powers of attraction & repulsion, so y each point is in y limit between an attracting & repulsive power. It will not approach y other on account of y repulsive pow^r. Now says he let another particle of such matter approach slowly to a body. as soon as it comes into the sphere of repulsion of the part of that body it velocity will be dimenished while

[illegible] it pushes y particles a little from each other [illegible]s motion is stoped & it does not penetrate. If it moved with a greater velocity it will be able to approach nearer & seperate y parts farther but yet stops. If it moves still swifter, it may be just able to pass thro', but at last extremely slow. During all the time of its passage it is employed in seperating y particles of the body. Suppose it moves still swifter, it passes thro' more quickly, & as its repulsive powers act on y particles of y body for a shorter time, it disturbs them less from their place. This disturbance will be lessened by every augmentation of its velocity. & when y is enormous, it will pass thro' without any sensible effect on y body. this he illustrates by y well known fact of a musket ball discharged on an open door, w it pene-trates without making it move on its hinges; whereas were its velocity much less it would shatter it exceedingly. & give it a sensible motion. This is a fact well known to y sailors who observe y a great shot almost spent does much more danger to y ship than when it comes to its greatest force. It is in this manner y light & heat penetrate bodies with-out any sensible disturbance of their parts, on account of their almost immense velocity. From such considerations it must follow y by sufficiently augmenting y velocity of any body we shall not only enable it to overcome the impenetrability of bodies but even to pass thro' them with-out disturbing their parts or itself suffering any disturbance.

You must remark yt all yt may happen wh[...]
penetration of ye material particles; nay if ye particle[s]
are only points, it is impossible yt ye point of different bodi[es]
can touch, and therefore ye compenetration is consistent both wt
ye aparent but false impenetrability wch we usually ascribe to bod[ies]
& also wt its real impenetrability.—

If ye particles of mr are not mathematical points
but physical points of their minuteness, endowed wt such
powers of attraction & repulsion as I have been mentioning,
& wch I will afterwards explain at great length, all the
sensible properties of mr will follow, without any contact
wch I apprehend You will soon be convinced is a thing wch
does not exist in nature. You see then yt by all that I have
said on this quality considered as a distinguishing property of mr
I have adhered to ye vulgar notion of it meaning by it
nothing more than this fact yt mr opposes any attempt
to penetration; & yt what manifests ye truth of ys opinion
is matter. I may only observe that it is a name for
a primary quality of mr, and not a name for a sen-
-sation of ye mind. When I speak of ye smell of a
rose, I commonly do no more then name my sen-
sation but when I speak of the solidity of a diamond
I speak of a real quantity of ye body, wch existed tho'
it had never been handled, altho' the smell of a rose
has no existence but in ye mind of him who smells.
The sensation by wch we perceive ye solidity of mr

[...]ately has got no name because we seldom
[...]nd to it, but pass immediately from it to the thing
perceived, the solidity of ye body we can however fix our
attention on it if we please, & find it totally unlike ye
thing of wch it conveys ye notion to our minds. we probably
did attend to it at first, while we were infants, & emplo-
yed in making ourselves acquainted to our means of
information, tho' we have afterwards in most cases totally
overlooked it; just as a person in learning a language
first attends to ye sounds only, & afterwards neglecting
them entirely, turning his whole attention to the thought
the thing signified by the sound.

I shall only farther observe that this
quality of mr is one of wch we have ye distinctive Idea.
It is not like colour, sound, smell, of wch we know nothing &
about wch philosophers have formed many hypothesis.—
If ye parts of mr exclude other mr from their place, it is
solid When I pretend to go a little farther than ye vulgar, &
shew that we conclude very hastily when we infer from ye
solidity of mr yt its parts are in contact, or when we deduce
solidity from ye contact, and when I ascribe ye quality to
certain powers of attraction & repulsion, I do not assign a
cause, because in saying yt a repulsive power keeps
asunder the particles of mr I only say yt they keep asunder.

In calling ye discontinuity of the parts of mr [...]
of power, it is only from the analogy of language. [...]
every thing a power by which animal power is opposed,
& we may therefore call ye quality a power meaning by ye
term only yt mr is solid. I formerly observed yt ye
expression of a coincidence in matter of fact was called
a natural law. I therefore assume as ye first law of
nature.

That all matter occupies its own place to
the exclusion of all other Mr. and yt no body can pe-
netrate another, so as to exist along with it in ye same place.
Since we find yt in a known space we can put very
different quantities of mr. & yet sensibly fill it, as appears
by ye box filled wh musquet Balls, and afterwards admit-
-ting ye addition of small shot; since we find yt a cubic foot
contains a quantity of water when cold wh will do mor than fill
it when hot, and since we find yt it is highly probable that
ye particles of mr are not in contact, & yt we are uncertain
at what distance they are placed from each other, it follows
yt in a given bulk there may be more or fewer particles of mr.
The mass, or number of material particles of wh a
body consists depends on ye space wh it occupies, or its bulk, &
what is called its density.

[...] any given bulk there is ye greater number of
particles, or ye greater quantity of mr. ye nearer they are
to each other. It is ye vicinity wh we express by ye term
density. we say yt a body is denser than another body if
its particles ~~are~~ nearer to each other, & rarer, if they are
farther removed. Now we do not know their distance,
we cannot say that one is denser than another except
by finding yt a given bulk has more of some other
sensible quality wh is competent to all matter. thus
for instance, supposing yt all particles are equally
heavy, we say yt mr has twice the density of wh a
given bulk has twice the weight - and thus density
wh really means ye closeness of ye material par
-ticles, becomes like velocity a term of relation, expressing
the proportion wh ye number of particles of mr in ye
same bulk of another body.

In thus representing ye density as expres
-sing ye number of particles contained in a deter-
mined bulk wh we call unity, it is plain yt in order to
have ye mass or total number of particles of mr in
any body we must multiply the density by the bulk.

We shall for ye future use M to express ye
mr. and Δ to express the density, & B to express
bulk from wc arrises ye following formula

$$M = B\Delta, \quad \Delta = \frac{M}{B}, \quad B = \frac{M}{\Delta}$$

Always remembering yt by density we do not mean the real number of particles, but only proportion of ye number in a given bulk of one body to the number of parts contained in an equal bulk of the other body.

II. Second of ye perserverance of mr. vid. book from page 13 to page 15.

Corollary 1. A body wc is perceived at rest, has always been at rest, unless affected by some external cause. For if not, it must have come into its present place either from one side or from another; but there is no cause to determine from wc side it should come - it must therefore have come from neither, or must have been at rest.

2. If a body has acquired motion from any external cause, it cannot accellerate or retard this motion - for it cannot determine itself either to motion or reast.

3 Whenever a body at rest is brought into motion we ascribe this to some external cause.

Proposition 2.

In order to preserve a body in motion, ye instantanious action

...ving cause is sufficient. This is very defferent from ...pinion entertained by some philosophers, who imagine yt ye continual action of ye cause is necessary, & thence find themselves obliged to ascribe all motions to ye perpetual agency of intelligent beings. Other philosophers, who see no reason for having recourse to the immediate action of Deity or intelligent beings, ascribe ye continuation of motion to ye continual agency of some power inherent in mr, called into exertion ye moment that ye body is put in motion, & keeping exact proportion to whatever is the original cause of ye motion, so that when ye original cause is double, & ye motion is double, the continual exertion of this inherent power shall also be double. But these opinions are in themselves extremely obscure & do not tend to remove what is here thought to be a deficiency, for it will appear that if ye instantanious action of ye moving cause is not sufficient to make the body continue in motion neither ye continued action of the ... Motion is conceived by all as ye effect of a cause, and their would be no room to infer a cause if there was no effect, that is if a body were not moving. Now, motion is a change of place, or is ye successive existence of a body in different parts of space. Apply this to ye present case. The action of ye moving cause produces motion in ye body, that is causes ye body to be in a different place now from what it was in before now ye motion is successive & continual. If therefore ye action of ye cause in any one instant does not produce a continual change of place, it does not produce motion, & is not a moving cause. Continuation of motion therefore is implied in the very notion of a moving cause, & is ye consequence of its instantanious action otherwise ye instantanious exertion of ye moving cause would be intirely without effect. And if its instantanious exertion is without effect its continual exertion must also be without effect. for since in no instant of its continuation it produces any effect, it is impossible ye it can produce any by its continuance. If therefore motion exists ye instantanious exertion of ye moving cause is sufficient for producing a continuation of motion. There is not therefore any necessity of supposing in a moving body any power by ie it continues in motion, or for having recourse to the continual action of external causes; & ye exertions of such power or such external causes would be ineffectual.

Proposition 3

See book from page 17 to 19 The demonstration of these fundamental propositions proceeds on the truth of ye axiom that

Let any force Q act in the direction AQ and let the free motion which it would produce be AQ. Drawing the necessary lines as before, AB will represent the rotatory effort. And AN the pressure on the Centre of Motion.

AB is to AB in the compounded ratio of AB to AF, AF to AQ. and AQ to AB. that is

$$\begin{aligned} &\text{of } PD : PA \\ &\text{of } FA : QA \\ \text{and } &\text{of } PA : P\alpha \end{aligned}$$

that is of $PD \times FA \times PA : PA \times QA \times P\alpha$ or $PD \times FA : QA \times P\alpha$.

That is, in general, the rotatory efforts of powers applied in this manner are as the products of the forces by the perpendicular distances of their directions from their centre of rotation.

On the other hand supposing that their perpendicular distances are the same, but that the distance of the body is changed. As to $P\alpha$ (Fig 28) then calling the effort in the former case E, and the effort in the present case ε, and the perpendicular distance P,

$$F : E = PA : P$$
$$\varepsilon : F = P : P\alpha$$
$$\text{therefore } \varepsilon : E = PA : P\alpha$$

That is when the perpendicular distances are the same the Efforts are inversely proportionals to the distance of the points of application. Call this distance D; then the effort in all cases, may be expressed by this formula $\frac{P \times P}{D}$, which will represent the Quantity of Motion communicated to A in the

direction

direction of the tangent by any power com-
ing force is F

Farther as this quantity of Motion communica[ted] to A is proportional to the product of its Matter M by its velocity V and as this velocity will consequently be reciprocally proportional to the quantity of Matter, when the motion F, or $\frac{FP}{D}$ is given it follows that the velocity which any force can ~~give~~ communicate in this Matter will be directly proportional to the Quantity of Motion F, and inversely as the quantity of Matter; therefore we shall have this proportion $V \div \frac{F}{M}$ or $\frac{F\times P}{M\times D}$.

Lastly, as this Motion of Rotation is an Angular Motion; it is ~~not~~ measured by its Angular velocity. but the Angular velocity of any body describing a Circle is as the Arch directly and the distance from the Centre inversely, or as its real velocity directly and the distance inversely, that is in the present case as $\frac{F\times P}{MD}$ directly and D inversely, or as $\frac{F\times P}{MD^2}$. we shall find this formula of the most extensive use in practical mechanics.

Let us now consider the resistance which the body A makes to this rotatory Motion of F. It must be considered as precisely equal and contrary to the Motion, that is it Must be proportional to $M\times D^2$. And since $F\times P$ is constant it must be proportional to MD^2 —

[The]re is another way of conceiving which resistance [m]ay in every case whatever be considered as a power opposed, contrary and precisely equal to the motion impressed. For a double body opposes a double resistance to any given velocity. And any body opposes a double resistance to a double velocity. A body A at rest when hit by an equal body B moving with a velocity 1 takes away from its velocity a velocity ½. If ~~If~~ B moves with a velocity 2 it takes away a velocity 1 double the former. Now the dimensions of Bs velocity are the measures of the resistances of As inertia. Therefore the effect of As resistance in this case of rotatory motion is proportional to $M\times D$ (for $F\times P$ is constant and does not change the proportion) And its effect is the same as if A ~~did~~ not resist but were but were held back by a power proportional to $M\times D$. Now the effect of this power $M\times D$ is proportional to the perpendicular distance of its direction from the Centre of rotation But since the motion of A is along the Arch of the Circle always perpendicular to PA its perpendicular distance from P is D. Therefore the effect of this power in opposing the Motion of rotation is $\overline{M\times D\times D}$ or MD^2.

While therefore the effort of the power F in producing a rotation is expressed by $F\times P$, the effort of As inertia in opposing it will be expressed by $M\times D^2$. This truth is also of the most extensive use in practical Mechanics.

suppose

Suppose now that the force F instead of acting im[me]diately on the body, acts on it by the intervention of [an] immaterial and inflexible line AF, the effect would be the same. Suppose that PD also becomes an inflexible line, connected with PA and AF. The effect must remain the same. Suppose that the part AD of the inflexible line AF is annihilated, but that the line DPA is inflexible. The effect must still remain the same. Lastly suppose that the force F, instead of acting on D by the intervention of the line DF, acts immediately upon it. The effect must still remain the same. It follows therefore that the Motion produced in the body A is the same whether the power F acts immediately on, or acts immediately on D

The resistance opposed by the inertia of A may be considered as a power equal and contrary to the motion produced in A. Let there be applied to any other part of the line PA for instance O a body whose resistance in opposing the motion of rotation shall be precisely equal to that of A. we do not yet know what proportion this body must bear to A, in respect of its distance from P. it is enough that it opposes at A a resistance equal to that of the body A applied at A. Let now the power act again at A It must have the same effect as before in producing a rotation round P. Since the resistance is the same

×D But it will have the same effect if applied at B.×

From this arises a most useful truth, that the effort of a force in producing a rotation round a fixed point, in opposition to a resistance exerted to prevent this Motion, is always pro-

portional to the perpendicular distance of its direction from the Centre of rotation, and that it is indifferent to what part of the assemblage of inflexible lines connecting the power, resistance & Centre it is applied. Or in other words, the effort which any force exerts in turning an assemblage of inflexible and immaterial lines round a fixed point, is proportional to the perpendicular distance of its direction from that point.

Lemma

If from any point P (fig 29) be drawn PD, PE, PF perpendicular to the sides and diagonal of any Parallelogram ABRC, then the rectangle of the diagonal and its perpendicular shall be equal to the sum or to the difference of the Rectangles of each side and its respective perpendicular according as the point P lies without or within the angle A thro' which the diagonal passes. That is $AR : PE = AC \times PF \pm AB \times PD$.

Case 1st Let P be without the angle BAC. Draw PA, PB, PC, PR, to the angles of the parallelogram, and let RB cut PF in G. Then the triangle APR, or $\frac{AR \times PE}{2}$ = to the triangle $ABR + PBR + PBA = \frac{AC \times GF}{2} + BR$ or $\frac{AC \times EG}{2} + \frac{AB \times PD}{2} = \frac{AC \times PF}{2} + \frac{AB \times PD}{2}$ double the whole, and then $AR \times PE = AC \times PF + AB + PD$.

Case 2d Is demonstrated in the very same Manner.

Let there be now (fig 30) two forces HK, IL, having the proportion and directions of these lines, act on the assemblage of lines HPI lying in the same plane

plane and produce a rotation round P in opposi-
tion to some resistance opposed, for instance at
point O of the inflexible line PO connected with them.
Produce KH, LI till they meet in A, make AB, AC
= KH, IL, and compleating the parallelogram
draw the diagonal AR. Draw PD, PE, PF ⊥ to AB.
AR, AC. Then AR × PE = AC × PF + AB × PO. or AR
× PE = IL × PF × HK × PD. Let now PD, PE, PF be-
come inflexible lines connected with PH, PI. Then
the efforts of HK, IL acting at D, F are equal to the
effort of AR acting at E. But the efforts are the
same should they act at H and I and AR at C.
Hence we deduce the general Theorem that the
effort of any two forces acting in any direc-
tions lying in the plane of rotation, is equal
to the effort of their resulting force which is always
equal to the sum or difference of their particular
efforts according as the centre of rotation lies
without or within the angle formed by their
directions, or according as they tend to turn the
lines the same or contrary ways.
The same may be demonstrated of the force resul-
ting from the combination of any number of
forces acting in the plane of rotation.
And this effort is equal to the sum of all
the efforts tending one way — the sum of
all the efforts tending the contrary way. The point
O thro which the direction of the resulting force
passes is called the centre of effort of these
forces.

...re is one case to which the foregoing reasoning
does not immediately seem to apply namely when
the forces are parallel to each other, in which case
they do not meet in any point such as A. But
we can easily find the centre of effort of such forces.
Since these directions are parallel then, the line
which cuts any one of their directions HK, Fig 31
at right angles, cuts also the others IL & TS at
right angles. In order to take in all the varieties,
we shall suppose that the force TS acts in a
contrary direction to the forces HK, IL.

Let us suppose that O is the point thro' which
their resulting force OR passes: this must be
parallel to the others, since there is no reason
that it should incline more to the one than to the
other. This may be rigidly demonstrated. Let
there be two forces AB, AC their directions always
passing thro the points F, H, and AB keeping its
direction and their resultent AR (Fig 32) with the
Centre A, and distance AF describe the Circle FDE,
cutting AR in D, and AH in E. Draw DG, DH perpen-
dicular to AF, AK, and EI perpendicular to AF
then the lines DG, DH, EI are proportional to
AB, AC, AC, AR, because the forces represented by
the sides and the diagonal of a Parallelogram are
always proportional each to the sine of the angle
contained by the other two. Then as the point A
moves off, and the angle A diminishes, the line EI
continually approaches to an equallity.
and

and coincides with the arch EDF and is ultimately equal and coincident with it when AE is parallel. In like manner DG, DH ultimately coincide with the arches DE, DE and are equal to them, and these arches are ultimately coincident and equal to the straight line FL perpendicular to AF; and since each of the lines AF, AD, AE is perpendicular to the arch on which it stands, the three lines AE, AD, AF are ultimately perpendicular to FL. AD is therefore ultimately parallel to AE and AF, and EI is ultimately equal to DG ± DH and AB, AC, AR are ultimately proportional to MF, ML, LE. Hence it follows

1. The Resultant of two parallel forces is parallel to them.

2. It is equal to their sum or difference according as they act one or different ways.

3. If a line be drawn across their direction, each is proportional to the part of it intercepted by the direction of the other two. In the line HT take any point P. Then from the property of the resulting force but now demonstrated.

$$HK \times HP + IL \times IP - TS \times TP = OR \times OP.$$

Take any other point p then

$$HK \times Hp + IL \times Ip - TS \times Tp = OR \times Op.$$

therefore $HK \times Pp + IL \times Pp - TS \times Pp = OR \times Pp.$

and consequently $HK \times IL - TS = OR$

and $\dfrac{HK \times HP + IL \times IP - TS \times TP = OP}{HK \times IL - TS.}$

Hence we see 1st. That the resultant of any number of parallel forces is equal to the prevalent sum of the particular forces, and acts in that direction

2 Its distance from any point is had by dividing the prevailing efforts by the prevailing sum.

These things may be demonstrated without the lemma thus. Let their directions meet in A as before. And suppose a body in A (fig 33) connected with the inflexible line PH, PI by means of the invariable angle IPA, and such as to oppose by its inertia a resistance equal to the resistance hitherto conceived to have been opposed; take AB, AC = HK, IL and compleating the parallelogram AR will be the resisting force. Draw BD, CE, RF perpendicular to the tangent AF, & BG, Ch, Ri perpendicular to AP. Then AF will be the effort of the resulting force in producing a rotation round P and Ri will be the pressure which it occasions on P. Now AF evidently is equal to AE ± AD, & Ai = AG ± Ah.

Cor. 1 If the diagonal, or the direction of the resulting force passes thro' the centre of rotation, there will be no motion; for then the perpendicular from the Centre of rotation vanishes, which makes one of the factors of the general effort. Or in the other way of conceiving it, AF vanishes which represents the motion of the resisting body

body applied at A. and is equivalent to the a[c]
against the resisting power.
In this case as no rotation ensues, the powers
are in equilibrio with each other which they
would not be without this connection established
between them by the inflexible lines. There is
therefore an equilibrium whenever ~~and~~ the
direction of the resulting force passes thro the
Centre of rotation, and in no other. In this case
too the Centre sustains the whole pressure of the
resulting force, and it is easy to see that this
pressure is the same as if the particular force
had been immediately applied to it in their own
directions. This appears by making the parallelo-
gram Pbcr equal, similar, and similarly
situated as ABRC. I may observe in general,
even when a rotatory motion happens, this is
the case if we consider (which we ought) the
resistance opposed to the rotation as one of
the powers applied. For make (Fig 34) PM=AI,
and PN equal and parallel to AF, and PQ equal &
parallel to AR and Join QM, MN, it is evident
that it constitutes a parallelogram having
lines equal and parallel to AF & AR for its sides,
and AI for its resultant, the pressure on P.
This observation is of the greatest use in the construc-
tion of Machines. What has been hitherto said is
true only of forces which act in the plane of rotation.
but

But suppose they act in parallel planes, and
that the rotation is not performed round a
point P. but round an axis passing thro' P perpen
dicular to those planes. The result and the de
monstration will be precisely the same, and the
only variety which this Circumstance introduces
is the equilibrium of the ends of this axis. since we
see that by considering the resistance as one of
the principle powers applied, we may consider
all the forces as immediately applied to this axis, and
then making any plane pass thro' it, resolving
each force into two, one of which shall be parallel
and one perpendicular to this plane, and all of
them perpendicular to the axis, we can easily
find the two points of the axis thro' which the
resultants of each set shall pass, and thus determine
the equilibrium of the Axis.
Should the powers not act even in parallel planes,
we can resolve each into three, one of which shall
be in the parallel planes of rotation, and
then we shall find the state of motion, or of equi
librium as before. but I must leave this to your
private consideration, contenting myself
with having pointed out to the method of
investigation.

with

With this I conclude what I had to say conc[...]
ning the motions of single bodies considered as
all collected in their Centre, and now proceed to
consider the laws which regulate the general
motions of Systems

Of the motions of a System.

In order to discourse on the Motion of a system of bodies, it is necessary to attend both to its general progressive Motion, and to the relative motion of its parts. In order to investigate the general Motion, it is evident we must select some point of the System by whose motion we may esteem the Motion of the whole. Now in every System of bodies a point may be found such that supposing the whole Matter of the System were collected there, and moving with its Motion, the whole motion counted in any direction will be the same with the real whole Motion counted in the same direction.

Such a point may therefore properly be called the centre of the System, and it is worth while to examine into the method of ascertaining in what part of the System it may be found.

It is easy to see from what was formerly said of the centre of a single body, that the centre of a System is to be found in the very same manner. It will be a point such that if a plane be made to pass thro' it, and from every particle of Matter in the System be drawn a perpendicular to this plane, then the sum of all the perpendiculars on one side will be equal to the sum of all those on the other side.

But a circumstance occurs which renders the Investigation of the Centre of a System much easier. It was shewn that if from every particle of Matter in any body were drawn a perpendicular to any plane whatever, the sum of all these perpendiculars was equal to the distance of the centre of that body from the plane multiplied by the quantity of matter. In order therefore to find the Centre of a System, we may consider each body as condensed to its own Centre, and then the centre of the System will be such a point, that if any plane pass thro' it, and the Matter of each body be multiplied by its distance from that plane, the sum of the products on one side will be equal to the [sum] of the products on the other.

Let A,B,D (Fig 35) be any three bodies any how placed and let C be the Centre of this System. It has this property similar to the Centre of a single body. that if any plane EF be drawn, the sum of the products of each body Multiplied by its distance from this plane is equal to the sum of the bodies Multiplied by the distance of the Centre from the plane.

For draw thro C a plane parallel to EF viz IL

$$\text{then } A \times AI = B \times BK \times D \times DL$$

$$\text{now } A \times AE = A \times CH \times A \times AI$$

$$\text{and } B \times BF = B \times CH - B \times BK$$

$$\text{and } D \times DG = D \times CH - D \times DL$$

$$\text{therefore } A \times AE \times B \times BF \times D \times D \times DG = \overline{A \times B \times D} \times CH \times A \times AI - B \times BK \times D \times DL.$$

$$\text{but } A \times AI - B \times BK - D \times DL = 0$$

$$\text{therefore } A \times AE \times B \times BF \times D \times DG = \overline{A \times B \times D} \times CH$$

hence CH may easily be found it being = to

$$\frac{A \times AE \times B \times B \times BF \times D \times DG}{A \times B \times D}$$

Therefore to find the distance of the centre of the System from any plane, multiply each body by its distance from that plane, and divide the sum of the products by the sum of the bodies — the Quotient is the distance sought. Hence we have an easy Method of finding the Centre. Find its distance TI from any given plane Fig 36 ABDE the Centre must be in some point of a plane OPQR parallel to the given plane, and having this distance from it; then find its distance TO from another given plane ABFG (it is generally convenient to make this plane perpendicular to the former) then the Centre must be somewhere in the plane IKML parallel to this second given plane and having the distance now now found from it. It must therefore lay in the Common intersection TW of the two planes having the distance now found from given planes Find now its distance ZD from any third plane BDHF (it is generally convenient to make this plane perpendicular to the two former) It will therefore be somewhere in a plane YZab parallel to this third plane, and having the third found distance

from

from it. It will therefore be in the point of section C of all the three planes, and thus its place is determined. We must now attend to some geometrical properties of this point.

1. The Centre of a System of two bodies lies in the line joining their Centres and their distances from it are reciprocally proportional to their Quantity of Matter, or to the bodies. (For Fig 37) Join the Centres of two bodies A B and divide the line in C so that A : B = BC : AC. Then draw thro C any line; then BE : AD = BC : AC = A : B and therefore A × AD = B × BE which is is the property of the Centre.

2 Add now a third body D (Fig 38) then the Centre of the System of three bodies is in the line joining the third body with the centre of the other two. Let C be the centre, and joining it with D produce it towards E. Then since D is a plane passing thro' the Centre if we draw AF, BE perpendicular to it, we must have A × AF = B × BE & A : B = BE : AF therefore A : B = BC : AC, &c. is the Centre of A & B.

In the next place CD is so divided by the Centre of the System C that A × B : D = DC : cC. (For Fig 39) draw thro C the plane EF perpendicular to DC, and draw the perpendiculars AE, BF. Then since C is the Centre of A, B it follows that A × AE + B × BF = $\overline{A \times B} \times cC$.

But since C is the Centre of A, B, D, A × AE + B × BF = D × DC that is $\overline{A \times B} \times cC = D \times DC$ and $\overline{A \times B} : D = DC : cC$.

3 In like Manner it may be shewn that if a fourth body E is added the common Centre C of this System is in the line joining E with the Centre of the other three and so situated that $\overline{A \times B \times D} : E = EC : cC$. and so on of any Number of bodies. (Fig 40)

II, of the perseverance of matter –

or ye property by wch mr always continues in ye state in wch it is whether of rest or of motion –
Every body at rest will continue in yt state unless it is disturbed from it some external cause –

For simplicity's sake, and in order to rid ourselves from every foreign circumstance, I shall suppose yt there exists nothing else in nature but ye body of wch I am speaking; & of consequence yt it is without weight, wch is a quality bearing an invariable reference to ye Earth, an external body. I farther suppose ye body at perfect liberty, exposed to no friction or resistance all wch flow from external things –

Now if it is asserted yt a body in circumstances can determine itself to motion, it must move either to one side or another – But there is no cause wch will determine it to one side rather than another; & as no determined event can be conceived to happen wout a cause of determination it follows yt it will neither move to one side nor to ~~another~~ i, e it will remain at rest –

Prop. III

A body put in motion by any cause whatever — will continue to move uniformly in a straight line, unless acted on by some other external Cause —

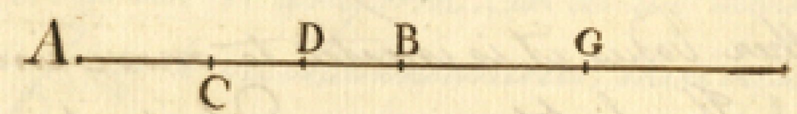
For after ye first instant of ye action of ye moving cause exists no longer & yet ye motion subsists ~~but~~ a body in motion cannot accelerate or retard its own motion, for it must either accelerate or retard it, and as there is not supposed an intervention of any cause to determine it to accelleration, more than to retardation or ye contrary, its motion during this time must have been uniform

In like manner as no Cause is supposed to intervene to determine it to deviation to one side more than to another, it must have moved in a straight line.

Since then it can move for any time, however small, independent of ye moving cause, it must move during ye time uniformly in a right line —

But a body wh has moved for one moment uniformly in a straight line, must continue to do so, if nothing disturb it.

For supposed a body setting out from A, & capable of moving uniformly thro' AB in any time however small — Take any points C, D between A & B the body when at D is, in every, in ye same condition as when at C — But at C it was in a condition to move uniformly to B in a certain time — at D therefore it is in a condition to move uniformly thro' an equal space in an equal time; that is it is in a condition of itself of moving uniformly thro DG = CB, & so on & thus a body in motion tends to move uniformly in a straight line till disturbed by some external cause —



Hitherto we have been considering a body as brought into motion by the instantanious action of one single power — But by much ye greatest number of ye phaenomena of nature shew us bodies moving in consequence of ye combined action of several powers — a ball wh I project in ye air is at once acted upon by ye impulse wh it acquired from ~~my~~ my hand, & by its own weight — the one would carry it forward in ye direction in wh it was projected, & ye other would make it fall to ye ground. When a body receives at once a stroke from two other bodies it is incited to move in ye direction of both stroks — If a lightor is dragged by two horses on different sides of a river it tends to both —

But in all these cases, ye body cannot follow ye action of either power, but must move in some third direction — whatever this direction is, & to whatever velocity the body moves, its direction & velocity, in short its motion must be considered as resulting from & equal to ye motions wh are impressed on it at ye same instant.

The motion wh ye body takes is considered as compounded of ye two or more motions wh it would take by ye action of each power singly, & ye joint action is called the composition of motion —

Fig 2 The general question is this. Suppose ye a body A is acted on at once by two forces P, Q, one of wh P would carry it uniformly from A to B in a certain time, & ye other Q would carry it from A uniformly to C in ye same time, what will be ye path & velocity of ye body — This question is of ye utmost consequence in Nat. philosophy, & ye greatest part of what will follow will be only making applications of it —

It is evident ye ye path must be a straight line, & ye motion must be uniform. For as we suppose ye joint action of ye two forces to be instantanious, there occurs nothing afterwards to induce any change ——

Fig. 2 Let AG be ye unknown path & at any point G of this path let two other forces act on it one that would carry it along GE equal & parallel to AC, but in ye opposite direction, in ye same time ye ye body would have moved along AC by ye force Q alone & another that would carry it along GF equal parallel & contrary to AB. It is plain that ye body must rest in ye point G For had those new forces acted on it at A, at ye same time wh Q & P, it must then have rested, as ye equal and opposite forces must have destroyed each other. But ye body at G is in precisely ye same situation as at A, for it has the same velocity & direction as at A.

& is as if actuated by ye two forces w acted on it at A; that is by two forces equal & contrary to GE & GF. Whatever therefore is ye joint effect of the two forces GE & GF ye joint effect of AC & AB must be equal & contrary— Let us therefore substitute for GE & GF two forces whose joint effect we perfectly know.

For this purpose let us suppose yt A adheres to a plane void of weight & inertia, & therefore incapable of inducing any change on ye motion of A. when A has got to G, suppose that this plane is carried along ye grooves MI, LK parallel to AC, in a contrary direction & w a velocity equal to ye velocity along AC, in this case it is evident yt ye point G of ye plane will describe GE equal & parallel to AC, & therefore ye body will have impressed on it a force equal & contrary to yt along AC— Suppose farther yt ye grooves MI, LK are carried along ML parallel to AB w a velocity & direction equal & contrary to yt along AB— in consequence of this second motion ye body has impressed on it a motion = & contrary to yt along AB.— Now it is unnecessary to demonstrate yt ye joint effect of these two motions must be a motion along GH, ye deagonal of ye parallelogream FGEH.

Since therefore ye joint effect of ye forces P Q must be equal & contrary to this. it follows yt ye body actuated by them at A must describe a line AD equal & parallel to HG, but in a contrary direction— This must evidently be ye diagonal of ye paralellogram ABDC.——

I therefore assume it as a 3d law of motion yt "If a body is acted on at once by two forces each of which would make it describe ye sides of a paralellogram w a uniform motion in a given time, it will by their joint action describe ye diagonal of ye paralellogram uniformly in ye same time"——

we see then yt it is indifferent whether we conceive ye body A as solicitated by two forces P, Q, or by one in the direction AD & fit for carrying it along AD in ye time yt P, Q would have carried it along AB, AC—

The two or more forces w combine in producing a motion along some indeterminate direction are called ye simple & constituent forces, & ye motion w results from their joint action is called ye compound, ye resulting motion— And as ye motion might have been produced by a force in its own direction, & having it for its measure, it is called ye equivalent, ye resulting force— The conceiving two motions or forces combined is called y^e composition of motion, & y^e conceiving a motion or force as resulting from y^e combination of two or more motions, or forces, is called y^e resolution of motion & forces

As y^e is of so much consequence in y^e science of motion y^t y^e greatest part of what is to follow is a continual application of it, it is necessary to give it a particular attention —— Therefore observe —

1^st That y^e constituent & resulting motion & forces acting on a material point are all in one plane.

For y^e sides & y^e diagonal of any paralellogram are all in one plane.

2^d The constituent & resulting motions are ÷ to y^e sides of a triangle w^ch are parallel or any how = by inclined to y^er directions

For AB, AC, AD (fig. 3) are ÷ to AB, AC, CD, because CD = AD. Now if all y^e sides of y^e triangle a b c are parallel to y^e respective sides of y^e triangle ABC, y^e triangles are similar & y^e sides proportional — The same may be said of y^e triangle α β κ whose sides α β, β κ, κ α are respectively perpendicular to AB, BC, CA — & y^e same may be said if they are any how equally inclined —

3^d The constituent & resulting motions are each equal to the Sine of y^e angle made by y^e directions of y^e other two

For (fig. 3) y^e sides of any triangle are ÷ to y^e sines of their opposite angles, therefore AB : AC :: Sine ∠C : sine ∠B; & AB : BC :: Sine ∠C : sine ∠A —

4^th If two forces conspire in y^e same direction they will produce a motion equal to that produced by a force = to their sum — or if two motions conspire in one direction y^e resulting motion will be = to y^e sum —

For let BAC, y^e inclination of y^e constituent forces AB, AC (fig. 4) continually diminish & at last vanish, in w^ch case these forces or motions have but one direction AC —

It is evident y^t AB becomes AE, BD becomes EF, & y^e resulting force AD becoms AF = AB + BD, = AB + AC

Hence it follows y^t y^e joint action of two forces conspiring in one direction is = to y^e sum of their seperate actions — or more properly y^e velocity communicated by y^e joint actions of two conspiring forces is = to y^e sum of y^e velocities communicated by their seperate actions.

It appears almost needless to have made this observation but it is of the greatest importance to our just conception

of ye actions of forces It has been generally assumed on
ye credit of ye maxim ye effects are proportional to ye
causes, a maxim wh, to say ye best of it, is a mere tauto
-logy, in ye sense in wh it is used in Nat. philosophy: for we
know nothing of ye proportions of ye causes but ye propor
tions of ye effects — It is a maxim too wh has led some
of ye most eminent philosophers into great errors, & has
occasioned great disputes in philosophy wh never can be
can be terminated till the maxim is given up.
S. I. N. has introduced ye into ye science of motion under
ye name of a law, in these terms — The change of motion
is ÷l to ye force impressed
But in all ye use wh he makes of it he never takes any
other view of ye degree of ye force impressed than ye change
wh it produces. Leibnitz & others, reasoning from ye same
maxim, asserted, yt ye forces are not ÷l to ye velocities
wh they communicate, but to ye squares of these velocities;
for there are many cases, in wh we think ourselves certain
of ye proportion of ye acting forces, & we find the velocities
which these communicate in ye subduplicate ratio of ye
forces from wh these philosophers have been led to one

of these two conclusions. either yt ye velocities are not the
proper measures of ye effects. or that there exists in a
moving body a force differing from yt wh is impressed on it.
But ye present determination was founded on no hypo-
thesis concerning ye nature of forces, or any previous
knowledge of their proportions All that we know of a
force is yt it produces motion. & it is conceived as a force
only in so far as it produces motion; and from this
we are led clearly to the conclusion that ye forces are
really ÷l to ye velocities wh they communicate, provided
only if we call yt a double or triple force wh is = to ye
sum of 2 or 3 = forces: Now we have no other notion of
ye proportion of any quantity whatever.

5. If two forces act in opposite directions ye velocity
wh they communicate is equal to that wh would be
communicated by a force = to their difference or if
two motions are in opposite directions ye resulting
motions is equal to ye difference.

For let BAC, (fig. 5) ye inclination of ye constituent
motions AB, AC, continually increase till it becomes 180°,
then AB becomes AE, CD becomes CF. DB becomes

FE, & ye resulting motions AD becomes AF = AC — DC = AC — AB —

Hence it follows yt if AB = AC the diagonal AD vanishes that is rest ensues from ye joint action of ye constituents AB, AC. This is called ye equili-brium or ballance of motions & moving powers, & it is ye foundation of a principal branch of Dynam & of partical mechanics. The construction of machi the construction of arches, ye stability of ships & man others of ye most useful application in Nat. Phi-losophy to ye arts of life depending on ye principle of ye ballance of powers - all may be reduced to one generical proposition —

6, three forces or motions which have ye directions & proportions of ye sides & diagonal of paralellogram will be in Equilibrio, provided yt ye force wch has ye proportion of ye diagonal acts in an opposite direction to ye diagonal — For AE (fig. 6) is ye motion resulting from ye motions AB, AC; but if AD is equal & opposite to AE, it will (by ye last observation) ballance it - It will therefore ballance ye two motions AB, AC.

7. Let it now be req. to find the motion or force resulting from the joint actions of ye motions or forces AB, AC, AD, AE &c (fig. 7)

Compleat ye paralellogram BACF, & it appears yt AF is ye motion or force resulting from ye joint action of AB, AC — Compleat ye paralellogram ADGF, & then AG is ye resulting force of AF, AD, or of AB, AC, AD. Compleat ye paralellogram AE, HG & AH is ye resultant of AE, AG, or of AB, AC, AD, AE —

NB, In these compositions of motions it is by no means necessary yt ye constituent motions AB, AC, AD, AE ly in one plane; for however they ly, still ye parm ABFC will be in one plane & AF will be ye resultant of AB, AC. In like manner ye parm ADGF will ly in one plane, whatever is ye position of AD, & ye parm AEHG will lie in one plane whatever is ye position of AE —

8. On ye other hand ye motion AH may be resolved into any two AE, AG, provided only that they are ye sides of a parm of wch it is ye diagonal; and AG may on the same conditions be resolved into any two AF, AD; & AF may be resolved into any two AB & AC - & thus for AH

may be substituted AB, AC, AD, AE, which will produce the very same effect —

9. The most useful of all ye resolutions of forces is ye following — Let AB (fig. 8) be any force or motion produced by it. Let CD, EF be two lines cutting each other in P at right angles, & let GPH pass thro' P at right angles to ye plane in wh CD, EF are — Let it now be req.d to resolve ye motion AB into three, wh shall be respectively parallel to PG, PC, PF. Thro' A draw AI, AL, AM parallel to PG, PC, PF From B draw BK parallel to IA & meeting ye plane in wh AL, AM ly in K, draw KA & draw BI, KM, KL parallel to KA, AL, AM then it is plain yt for AB may be substituted AI, AK, & for AK may be substituted AL AM — therefore for AB may be substituted AI, AL, AM — To give an instance of ye application: suppose a ship sailing at right angles to ye wind; in this situation her stay sails have a threefold operation — they partly tend to drive her forward, in ye direction PC partly to drive her to leeward, in ye direction PF, a very detrimental effect, & partly to lift her up in ye direction PE, & thus diminish ye resistance of her prow. In order to reap all ye advantages of this noble machine it is necessary that ye sail should be so disposed yt it effect in ye direction PF should be as little as possible & yt ye effect in ye direction PC should be as great as possible —

In like manner ye action of ye sun on ye moon (fig. 2) in ye direction MS tends partly to pull her down to ye Ecliptic in ye direction MP, partly to pull her away from ye earth in ye direction MC, & partly to pull her back in her orbit, in ye direction MD — The motion of ye moon cannot be ascertained till it is determined what proportion of ye whole disturbing force of ye sun is employ'd in producing each of these effects —

10. Any number of motions or forces AB, AC, AD, AE, will be ballanced (fig. 7) by a force AK equal & opposite to their resulting motion or force AH — This needs no demonstration for it follows from observation 6 —

Such then are ye general Laws of Motion, ye principle from wh we are afterwards to reason synthetically in deducing ye subordinate laws of motion, in investigating ye powers & in explaining the phaenomena of Nature —

Of the free motions of a single body

And here at first setting out I must observe yt ye smallest conceivable body may be considered as a system, and during ye general motion of ye whole, ye different parts may have their own motion. It is necessary to abstract entirely from these varieties and to fix on some one point of ye body by whose motion we may esteem ye general motn of ye body,

In every body a point may be foun, such yt if a plane be made to pass thro' it, and if from every particle of matter there be drawn a line perpendicular to this plane, the sum of all ye perpendiculars on one side shall be =l to ye sum of all those on ye other side –

If another such plane be found then the point will be in the intersection of these two planes –

If a third such plane be drawn then the point will be in that point of this third plane where the common intersection of ye other two planes passes thro' it –

This point I call the centre of the body, & it has the followg geometrical properties wc render it convenient for our purpose

1. If from every particle of matter in the body there be drawn a perpendicular to any plane wc in or wc out ye body, ye sum of them all is equal to ye distance of this point multiplied into the quantity of matter — For (fig 10) draw thro' C the plane DE parallel to it B & from P, a particle on ye same side of DE without AB draw ye perpendicular FPE, & from H, lying on the other side, draw the perpendicular HKI Then ye sum of all the lines such as FE = the sum of all ye lines such as PE + ye sum of all ye lines such as FP; and the sum of all the lines such as KI = to ye sum of all such a HI – ye sum of all such as HK – but by the property of ye centre ye sum of all the FP = to ye sum of all ye HK.

Therefore $\int FE + \int KI = \int PE + \int HI$ – but $\int FE + \int KI = CE \times \int F + \int K = CE \times \int P + \int H = CE \times M$ expressing the quantity of matter or number of particles P & H —

2. If ye body moves in a straight line the sum of all ye motions of each part equal the motion of $C \times M$. This will be true even tho' ye body should during its motion turn round C; for ye sum of all ye retrogradations of ye

particles which go backward may be shown to be equal to ye sum of all ye accellarations of ye particles w come forward, and it will afterwards be demonstrated that the body moving freely can turn round no other point but C —

Therefore the motion of ye whole body will always be equal to ye motion of C×M —

Thus then the distance of the body from any plane, & its motion may be conveniently and accurately esteemed by ye distance & motion of C, & therefore ye whole body must be supposed to be reduced to ye point C —

^ During ye time represented by LM; but it terminated in P, and KP is ye space really described —

Lemma Fig. 11

If AB, BC, CD, &c are taken to represent any successive portions of time, and BE, CF, DG &c are the spaces run over uniformly from the beginning of ye time; the points E, F, G. will be in a straight line. For the spaces are ÷l to ye measures of ye times. & therefore AB:AC = BE:DF therefore, by a known theorem A, E, F are in one straight line.

The same may be shewed of G and of any other point, BE, HF, KG are the spaces described during ye portions of time represented by AB, BC, CD —

If BC = CD, then HF = KG. The velocity w ie any part HF is described $= \frac{HF}{BC}$ — This then will serve as a very convenient characteristic of uniform motion, that if any line such as AD is taken for ye measure of ye time, & ordinates are drawn from its different points equal to ye spaces described in the portions of time counted from ye beginning, the extremities of all these ordinates will be found in a straight line —

On the other hand if, after having taken such a line to be ye measure of ye time, the extremities of ye ordinates are not found to ly in a straight line the motion is not uniform.

Thus (fig. 11. no. 2) if the points A, E, F, G. O, P, Q are found not to ly in a straight line, the motion during the time represented by AN is not uniform — If between A & F they are found to be in a straight line then the motion has been uniform during the time represented by AC; produce AF till it cuts DG in g. Had the extremity of DG been in g, then the motion during the time represented by AD would have been uniform, & h g would have been the space described in y^e portion of time represented by CD. But since y^e extremity of y^e ordinate is in G, h G is y^e space described during the part of time represented by CD. Now hG exceeds h g by by the space G g; the motion therefore during y^e time represented by CD has been swifter than before, and y^e motion has been accelerated at y^e instant of time represented by C —

If F,G,O, ly in a straight line, then the motion during y^e time represented by CL has been uniform, tho' swifter than before —

Did y^e ordinate MP terminate in p in GO produced, then the motion during the time represented by DM would have been uniform, & Kp would have been the space described during [see p 52] that time and is less then kp by Pp — The motion therefore has been retarded at y^e instant of time represented by L —

such motions come next to be considered —

Of Varied Motions

Motions are susceptible of two variations Velocity & direction —

we shall first consider y^e variations in velocity, or the acceleration and retardation of rectilineal motions, these are susceptible of infinite varieties & may all be investigated in the same way, by the proportions of y^e abscissa and ordinate of such figures as I have now exhibited. In order to shorten language, I shall for the future drop y^e consideration of AB, BC, CD &^c as measures or representations of the times, & I shall simply say the times AB, BC, CD &^c

Had y^e motion during the time BCD been uniform the body would have described the space h g in the time CD, but since it has really described y^e space hG, gG is y^e effect of y^e acceleration — But when I say that g G is the effect of y^e acceleration, it does not of itself convey a precise notion of this acceleration, unless I consider also y^e time in w^c hG has been described. — The velocities takes in both circumstances of space & time, and is therefore the proper measure of y^e acceleration — the velocity w^c which hG is described is $\frac{hG}{CD}$ y^e $\frac{gG}{CD}$ is the proper measure of the accelleration happening at y^e instant C. In like manner $\frac{Pp}{LM}$ is the proper measure of y^e acceleration at the instant L —

In the present example it is evident that y^e acceleration has happened by fits, at ye instants C & L, & ye during each of the intervals AC, CL, LN, ye body has moved uniformly, for ye velocity during ye time AC is constant, since $\frac{BE}{AB} = \frac{CF}{AC}$ in like manner because $\frac{hG}{CD} = \frac{LO}{CL}$, ye velocity during CL has been constant —

But suppose that no part of the line which passes thro ye extremities of the ordinates is straight (fig 12) Suppose for instance that the motion is continually accelerated — and let BC, CD be two equal portions of time and BE, CF, DG be ye spaces described during the times AB, AC, AD — draw EH, FI paralel to AD; then HF, IG are ye spaces described during the equal times BC, CD, and therefore since the motion is accellerated IG must be greater than HF — The points E, F, G therefore will ly in a curve convex towards AD —

In this case it is plain that we have no constant measure for ye acceleration — If we take $\frac{IG}{CD}$ for the velocity at the instant C it will be too great and $\frac{HF}{BC}$ will be too small — Draw MFL touching ye curve in F, and cutting BE, DG in M, L — let now ye points B, D continually & equally approach ye point C & ultimately coincide wt it, then the points M, L will continually approach, and ultimately coincide wt G, E — MN is always equal to FI & NF is always equal to IL, therefore $\frac{NF}{MN}$ is always equal to $\frac{IL}{FI}$; Hence it follows ye either of yse quantities, suppose $\frac{IL}{FI}$, will be ye proper measure of ye velocity at ye instant C; but IL in ye case is as ye fluxion of ye space CF, & FI as ye fluxion of ye time AC. Hence we have this formula for expressing ye velocity in any accelerated motion whatever

$$V = \frac{\dot{s}}{\dot{t}}$$

With respect to ye acceleration itself. it consists in the augmentation made in the velocity — But an acceleration that is continual & perhaps unequal will produce different augmentations of velocities in different times. In order therefore to be free of such irregularities it is necessary to suppose ye time of the acceleration extremely small, and even to consider it as evanascent — In ye case ye acceleration will be directly as ye increment of ye velocity & inversely as ye increment of ye time; or $\div \frac{\dot{v}}{\dot{t}}$ — Did ye body at the instant C go on to move during ye small moment CD wt ye velocity wt it had at ye moment C then IG, or $\dot{S}$, would be the space wt it would describe in ye moment CD, or $\dot{t}$ — But as ye motion is accelerated, IG is ye space described, & LG = $\ddot{S}$ is ye acceleration — but LG alone does not convey a precise notion of ye acceleration at ye instant C — The velocity is ye proper measure wt is $\frac{LG}{CD}$; were FgG a straight line, then $\frac{LG}{CD}$ would be a constant quantity whatever were ye motion CD — But FL is not to Fi as LG to lg. & therefore $\frac{LG}{FL}$ is not a constant quantity & a proper measure of ye acceleration. But it was formerly demonstrated that ye ultimate ÷ of LG to lg was equal to the ÷ of FI^2 to Fi^2; & ye therefore $\frac{LG}{FI^2}=\frac{lg}{Fi^2}$ this quantity therefore $\frac{LG}{FI^2}$ is ye proper measure of ye acceleration at the instant C; but $FI^2 = CD^2 = \dot{t}^2$; & LG is evidently the increment of IL, wh is ye increment of CF_LG therefore is $\ddot{S}$ — Hence it follows that $\frac{\ddot{S}}{\dot{t}^2}$ is ye proper measure of the acceleration —

But we shall get simpler & more convent. measures — For ye purpose recolect yt $v=\frac{\dot{S}}{\dot{t}}$ (C) therefore $\dot{S}=v\dot{t}$ — therefore $\ddot{S}=v\ddot{t}+\dot{t}\,\dot{v}$ —

But let us suppose $\dot{t}$ constant; ye is convenient on many accounts, for in comparing variable quantities it is always necessary to have one constant quantity —

2 We are now prepared for investigating ye nature of accelerating forces. I have frequently taken occasion to observe yt we have no knowledge of ye pro. of nature but by their effects — The powers by wh ye variety of motion wh we observe is produced are known to us in no other way nay we know nothing of them but their effects. or ye change of motion wh they produce, we judge of ye existence of an accelerating pr only by observing the acceleration, & of its degree only by ye quantity of that acceleration — when we say yt ye pr of gravity acts on a stone we only mean yt ye stone falls or tends to fall — when we say ye gravity is greater at ye poles than at the equator we only mean yt a stone falls faster or farther in ye same time.

From ye it happens yt ye formula $\frac{\dot{v}}{\dot{t}}$, wh I have just now given for ye measure of ye acceleration, is assumed by all ye mechanical writers as ye measure of ye force wh is conceived to produce ye acceleration — In ye they have proceeded on ye truth of maxim yt causes are proportioned to their effects — but ye is a begging the question. if we know nothing of the nature of causes or of ye principle wh connects them to ye effects; & till we know this we are in no wise intitled to affirm yt causes are proportional to their effects —

You will soon see cases where ye maxim has led into every great errors & into disputes wh will never terminate till the maxim is given up as a mere tantology — When Sr I. Newton established his second law of motion "that the changes of motion were proportioned to ye causes wh produced them he did not foresee ye abuse wh was to be made of it; & in all ye uses wh he has made of it he never means any thing by his cause but ye effect itself ~

I intend therefore to keep altogether clear of ye danger, & for ye future, when I use ye word force as signifying a physical quantity, & assign it certain proportions according to ye variable circumstance of ye case, I desire to be understood as meaning nothing more than to express by one word not ye cause of a change of motion, but ye change of motion itself. It is in ys sense that I now say yt ye accelerating force is proportional to $\frac{\dot{v}}{\dot{t}}$ or to the increment of velocity communicated in a moment of time.

This shall always be ye meaning of the formula $F = \frac{\dot{v}}{\dot{t}}$; Taking ys view of accelerating forces we have an easy method for comparing ym wt each other ~ Let F & Φ be two such forces, $\dot{v}$ & $\dot{u}$ ye velocities wh they communicate in ye infinitely small times $\dot{t}$, $\dot{T}$; then $F : \Phi = \frac{\dot{v}}{\dot{t}} : \frac{\dot{u}}{\dot{T}}$ ~ It is convenient to choose some determinated time, such as a second; therefore $\dot{t}$ being $= \dot{T}$ $F : \Phi = \dot{v} : \dot{u}$ ~

b. That ut
& n

Mechanical properties of the Centre of a System of bodies

Let any two bodies move uniformly in any velocities and directions AD, BE. in any planes, then y^e following will be the motion of the Centre C of this system.

1. It will either rest or move uniformly in a right line CO given in position & magnitude.

Compleat y^e parallelograms ADFC, BDGC, each of w^ch will be plane figures. Join FG, DE & draw COQ thro' their intersection. CO shall be y^e path of the Centre of the system & its motion will be uniform.

Let y^e bodies come to H and K in the same time. Draw HI, KL ∥ to AB. Join IL, cutting CO in S (it will cut it, for the triangle FCO is in one plane) draw SH, SK.

Since y^e bodies move uniformly AH : AD = BK : BE & CI : CF = CL : CG, and therefore IL is ∥ to FG.

Because BF is ∥ to GE, DF : GE = DO : OE but DF : GE = AC : CB. Therefore DO : OE = AC : CB, & O is y^e place of y^e centre when y^e bodies are at D & E; and FO : OG = AC : CB = HI : LK, = IS : SL. But y^e angles HIS, KLS are equal, on account of the parallels HI, KL. Therefore the triangles HIS, KLS are similar, and y^e angles ISH, LSK are equal,

and therefore HSK is a straight line, & since HS : S[...]
KL = AC : CB. S is the Centre of ye system when ye [...]
are at H and K. The same may be shewn when the bodies are at ye same time in any other parts of ye lines AD, BE. Therefore the Centre moves in a straight line. Farther, CS : CO = CI : CF = AH : AB. Therefore the Centre moves uniformly

2. Their quantity of motion esteemed in the direction perpendicula[r] to the path of their Centre is nothing. For A has moved in this direction thro' a space = FN and B thro' a space = GQ. then FN : GQ = FO : OC, = B : A. Therefore A×FN = B×CQ, & their difference is equal to nothing.

3. Their quantity of motion estimated in the direction of the path of ye path of ye Centre, is equal to the quantity of motion which would have been produced had both bodies been collect. at ye centre, & moved w its velocity.

HI

Thro' O the final place of ye centre draw a plane HI ⊥ CO
Draw AF, DH, BG, EI parallel to CO, and draw
Fig 13 - DK, KL perpendicular to AF & BG.
Then A has advanced in ye direction of CO thro a space equal to AK and its quantity of motion, estimated in ye direction of CO is A×AK, that of B is B×BL. Now by ye property of ye Centre, A×AF + B×BG = $\overline{A+B}$ × CO. and A×DH, or A×KF –

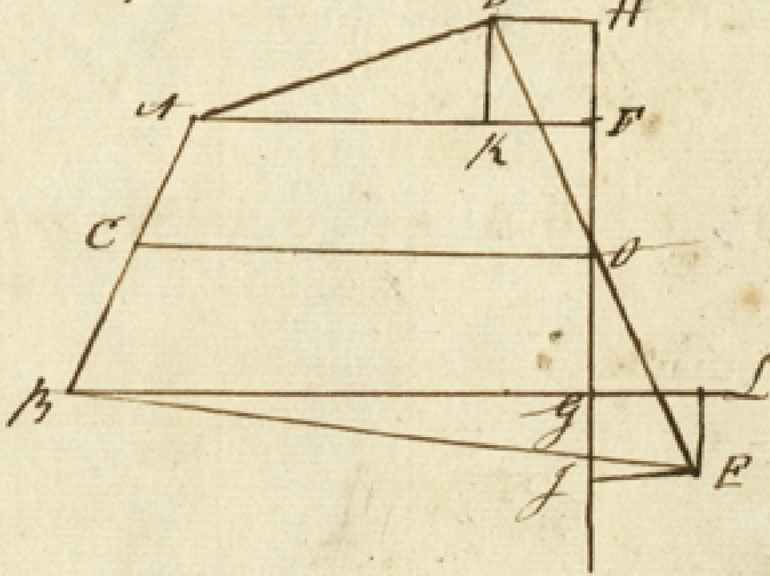
& that w[...]
& [...]I, or GL = O; therefore A×AF – A×FK + B×BG + B×GL = $\overline{A+B}$ × CO or A×AK + B×BL = A+B × CO.

Cor 1. The velocity of ye Centre will be had by dividing ye prevaling quantity of motion of the bodies, counted in the direction of ye Centres motion, by ye sum of ye bodies.

4. If any new force is impressed on one of the bodies, the motion
P× will be +affected in the same manner as if it had been impressed in a parallel direction on all the matter collected at the Centre, & moving w its velocity.

Fig. 42. Let AD, BE be the paths of the bodies, & CO that of ye Centre. And when ye bodies are at A & B, let an external force act on B such that by its action alone B would move thro Bβ uniformly in the time in wch it would move uniformly along BE. Compleat the parallelogram BβεE, then Bε will be the path of B. Draw Dε & make DE : DO = Dε : DO, and draw Cο, Cο will be the path of the Centre of the system. For ο is its place at the end of the motion, and it has been already demonstrated that its path will be a straight line. Compleat the parallelogram COοx, then Cx : Bβ = Oο : Eε = DO : DE = B : $\overline{A+B}$ therefore $\overline{A+B}$ × Cx = B×Bβ. Therefore had the force B×Bβ been applied a C to A+B at rest it would have caused this mass to move along Cx, and if applied to them in motion with the direction and

Fig 13 —



velocity CO. it would carry them thro' CO. Wherefore [...] position is manifest.

Cor 1. If any motions be impressed on both bodies whether at once or seperately, y^e motion of the Centre will be affected in the same way as if the forces had been immediately impressed on the whole matter collected there and moving w^h its velocity & Direction —

All that has now been demonstrated of the Centre of a system of two bodies, may be extended to any number of bodies. For we may consider a system of three bodies as a system consisting of the 3^d body and the other two united in their Centre. In like manner when a 4^th body is added & — Hence we deduce the following general Theorems —

1 If any number of bodies are moving uniformly in straight lines their common Centre either rests or moves uniformly.

2 The quantity of motion of the whole System, estimated in a direction perpendicular to the path of the Centre, is nothing.

3. The quantity of motion of the whole System, estimated in the direction of the path of the Centre, is equal to the sum or difference of the particle motions, estimated in the same direction according as they all tend one way or contrary ways. The velocity of the Centre is to be found by dividing their quantity of motion by the matter of the System.

4. The quantity of motion of the system, counted in the direction of the path of the Centre, is the same w^h the quantity of motion of the whole matter moving w^h the velocity of y^e Centre.

5. If any forces whatever act on the bodies of the system, y^e motion of y^e system, estimated by the motion of the Centre is y^e same as it would have been had y^e same forces been applied to y^e whole matter of y^e system & moving w^h its velocity & direction.

Having shewn the properties of the Centre of a System of bodies detatched from each other, let us attend to the consequences of their mutual actions. It is this w^h properly constitutes a system, and those mutual actions may be very different according to the principle w^h connects the different parts of the system. The bodies w^h compose a System may be connected by tendencies to approach or avoid one another commonly called powers of attraction & repulsion. such as obtain amoung magnetical & electrical bodies. In this case each body is at full liberty to follow the combined motion of all the forces w^h act on it, & its path is not restricted. Or a system may be connected by means of inflexible & inextensible lines —

In this case the bodies are not at liberty to follow [the im]pulses of the impressed forces, but any force impressed [on] any body is immediately diffused over the whole System.

Or lastly the connection of the system may be composed of both of these. We shall consider these in their order.

And first w respect to the motion of a system where each body moves freely.

Suppose that for a moment all the mutual forces should cease; then each body would move on uniformly in the direction in w it was moving the moment before, and the Centre will either rest or move uniformly in some straight line. Restore now the system action between any two of the bodies & suppose first that it acts only on one of them. The Centre will be affected in y same manner as if all y matter of y system was collected there, and this action was exerted one it. But it was formerly shewn that in the mutual actions of bodies there is an equal & contrary reaction. The body w exerts the accelerating force is equally affected in the opposite direction. Thus occasions an equal & opposite motion of the Centre. In consequence therefore of these equal & opposite motions the Centre will rest. It is not affected therefore by the mutual action of any two bodies.

The same may be applied to all the bodies and it therefore follows that the motion of the Centre of any system is not affected by the mutual equal actions of y bodies on each other. It therefore either rests or moves uniformly in a straight line.

We demonstrate in the same manner that the quantity of motion of such a system, estimated in the direction of its path, is the same as if the whole System were collected there, and moving with its velocity & direction.

And we demonstrated that if any external force is impressed on the system, y motion of the Centre is affected in the same way as if the force had been impressed on the whole matter collected there & moving w its velocity and direction.

Cor. 7. The Centre of the whole universe either rests or moves uniformly in a straight line.

By this means we are enabled to transfer what was formerly demonstrated concerning bodies revolving round fixed Centres by means of forces directed to them to bodies revolving round each other by means of mutual attractions & repulsions. It must that such motions, as we then considered hardly obtain in nature. Attractions & repulsions are not exerted by points but by bodies. Farther it must

be observed that in mutual attractions & repulsions
ting body, which we conceive as producing the revolu
the other. cannot remain at rest, and therefore the Centre of
ces will no longer be a fixed point; if no force besides their mutual
attraction and repulsion acts on them they will approach or
recede from each other, with accelerated motion, and velocities re-
ciprocally proportional to their ~~quantity~~ of matter. Should
any oblique act on one or both, they will acquire a motion round
their Centre, while it will either rest or move uniformly in a
straight line.

How these motions may be adjusted comes next to
be considered.

I. Two bodies mutually attracting each other describe round
each other figures similar to those described round their
common Centre.

1. They will describe round y^e Centre similar figures for
the line. joining the bodies always passes thro y^e Centre, and it
is not moved by the mutual action of the bodies. Also their
distances from this Centre are always in a given ratio, namely
y^e inverse ratio of the bodies.

6 that when there are many ways in wch an event may happen.
& no cause determining to any, the event itself is impossible.
But this must not be looked as ye cause of ye perseverance
of a body in a state of motion and rest till disturbed by some
external cause the cause must be ye nature of mr; & yt is
only ye means of demonstration.

It is not however allowed by all yt such demonstrations are
irrefragable,

which are of ye kind & to have had no determining cause to
some modes of their existance. Thus in ye immensity of space
and time there appears no cause wch should determine Deity
this place or this time in wch ye world should exist, & therefore ye
existence of the universe is an impossibility.

Altho' ye must be answered by attending more accurately to the
nature of intelligent beings, I will not trouble you wth ye discussion
wch would be long & tedious. I shall rather proceed to lay before you
ye experimental proof of ye proposition. But here I must be
allowed to mean only experimental proofs yt these properties
of mr wth respect to motion, and wch flow from ye nature of
motion, do really obtain in the world. Because there is no
absurdity in supposing yt whatever are ye natural cons-
quences of mr left to itself, yet the author of nature must
have imprinted & perhaps has implanted very different
laws on it. The whole universe, from ye moment of its
creation, may be in a continual progressive motion

we must forever remain ignorant, & there may be implan-
-ted in m^r a power of beginning, regulating, & terminating
its own motions as well as there is a method a power
of beginning regulating, & terminating it own exertion
And accordingly y^e doctrine has been advanced by one of y^e
must penetrating Metaphysicians y^t has ever appeared
& has so little absurdity in it as to have obtained y^e suffrage
of some of the greatest geniuses of Europe.
Such of You as have been much engaged in philosophical
enquiries must have heard of the system of M^r Leibnitz,
who made y^e universe to consist of certain elementary
beings w^h he called monads, & w^h, among other extraordi-
-nary properties, were endowed w^t powers of canging their
situation, & were continually execting these powers. I am
apt to think that his disputes w^h Sir I. N about the
invention of fluxions led him not only to cavel at his phi-
-losophy in general, but in particular at this principle,
w^h makes y^e basis of it. The system is now intirely forgot-
-ten, but y^e supports w^h were given to it were ingenious,
& all concurred in shewing y^t there was no absurdity in
supposing m^r endowed w^t powers of changing its situation.
If a vessel w^t water is suddenly drawn along
y^e floor it leaves part of y^e water behind. If it is suddenly
stoped, part of the water flies ~~outwards~~ out forward.

If persons are in a coach or boat & y^e coach or boat begins sud-
-denly to move, they fall backwards. If it suddenly stops
they fall forward. If a peice of mony is laid on a smooth
card, & y^e card struck smartly horizontally, y^e card flies away
& y^e mony remains, shewing at y^e same time the tendency
of m^r to remain in motion, & to remain at rest. If I
throw any thing straight out of a coach window while y^e
coach is going briskly but smoothly along, it falls oppo-
-site to the window, altho the coach has advanced consi-
-derable whill y^e body was falling. You have seen a man
dismount & remount while his horse was at gallop you
have seen him spring from one horse to another in y^e same
situation. You have seen a man on the slack wire, or on
horseback playing w^t y^e orranges thrown up in y^e air, w^h
fell always into his hand as if he had been at rest. None of
these things could happen if y^e man in mounting or dis-
-mounting, & y^e orranges while in y^e air did not preservere in y^t
motion w^h they had while connected w^t y^e hand or y^e horse.
A stone from the hand or from a sling, an arrow from a bow
all move on by perservering in y^e state of motion in w^h they were
left by y^e hand, y^e sling or the bow. On the other hand, were
it not for y^e tendency to remain at rest, a bullet would not make
a hole in a wall, but carry the wall along with it.

houses would not fall by earthquake but would move along w^t y^e ground.

Thus the fact seems supported by a large induction of experiments.

But the bulk of mankind think y^t rest is y^e natural state of body — that nothing is necessary for a bodies continuing at rest but its continuing to exist. Whereas when a body is put in motion an acting cause is necessary for y^e continuance of y^e effect, w^h is always new, always beginning. And this common opinion is also founded on a very large induction of experiments. We see, say they hardly any motion in nature w^h does not greatly tend to rest a billeard ball rests at last however strong y^e strok — a pendulum ceases to move if not continually acted on by y^e machenary of y^e clock — In short, no machine no body does continue its motion w^tout a renewal of y^e moving cause.

But I must note y^t all y^e observations to y^e purpose are particular, whereas y^e observations for ascertaining general laws should be general in all these particular observations we can see causes for y^e deviation from the assertions.

The retardations w^h we observe are in all probability owing to y^e action of known obsticles roughness of rubbing paths, resistance of bodies & y^e like. For in such cases as we can make experiments, & remove these obstacles the motion continues so much longer The smoother & more slyppery y^e surface on w^h a body moves, y^e longer does y^e motion continue — the rarer the medeum in w^h y^e body moves, y^e more lasting y^e motion as in mercury, water, air, void. There is seen a top w^h will continue to turn in vacuo for a whole day. Since then we see y^t a diminution of the obstacles is invariably attended w^h a diminution of y^e retardation, there is reason to ascribe y^e retardation to these obstacles, and to conclude y^t were they intirely removed, y^e motion would continue forever. And y^s is proved by y^e plantary motions, w^h being performed in a space void of resistance, have continued unchanged ever since Astronomers could accurately observe their motions.

In all y^e cases too of deviation, from uniformity, either of velocity or direction, we find a constant concomitance between y^e deviation & something external. & y^t y^e different degrees of these deviations where they can be examined with accurdcy, bear an invariable relation to y^e canges of these external relations, & when these external relations are removed, y^e deviations are removed at y^e same time. Thus y^e curvilineal path of all bodies projected in y^e air in a direction not perpendicular to y^e horizon bear an invarible relation to y^e position of y^e path w^h respect to y^e lines in w^h gravity acts, & by varying these positions we vary at pleasure y^e curvilineal path of y^e projected body.

The same may be said of every other case.

So far then common experience seems perfectly consistent with the abstract truth. But the experiment alluded to are inaccurate, & only concur in leading us to conclude from observation that m^r has a tendency to contine in its present state; but by no means give us any defined notion of the tendency.

Nay if we come to consider the motion with accuracy we shall find that such observations are totally unfit for deciding the present question, & that we are not authorised to make the deduction from them. You will find it afterwards demonstrated that if once a particle of m^r in the universe has begun to move it is impossible that any one particle should be at rest, unless it should happen to be in one only point of the universe. Rest therefore, so far from being a familiar situation to m^r is one in which we have never found it, & therefore were we to judge from these observations only, we would affirm it a a general law of motion that rest was incompatible with m^r.

We shall find too that every case which we look on as an instance of rest is really an instance of most rapid motion: that many cases which we take to be instance of passing from motion to rest, are in fact instance of a very great increment of motion, & that we are totally insensible of canges of motion inconceivably greater than any which we can observe.

But the reasoning would have been quite the same had the experiment been performed at any other time, & with any other direction only not so simple, on account of the obliquity of the relative & absolute motions.

Now when the experiment is tried, with attention to every circumstance which can introduce any variety in it the result is invariably found to be that that the quantity of motion lost by the overtaking body is precisely equal to the quantity of motion gained by the overtaken body, & the conclusion therefore drawn from it is that the tendency of m^r to continue at rest is precisely equal to its tendency to continue in motion.

We may therefore assume it as the first law of motion that every body perseveres in its state of rest or of uniform rectelineal motion, unless disturbed by the action of some external cause.

And we may hence see that the laws of motion established by the author of nature are the same with what result from the nature of motion abstractly considered. Arguments to the same purpose have been drawn from the exact agreement between the phænomena of nature, & what would be the phænomina ~~of nature, & what would be the phæn~~ supposing that the law took place.

bouses would not fall by earthquake but would move along

But ye argument is not conclusive. for whatever have been ye laws of nature wh respect to motion ye agreement of ye phaenomena to ye calculation would have been ye same. Because assuming any law at pleasure, & meaning every disturbing cause by ye deviation wh it produces from ye law ye result of our conclusions would have been ye same, tho ye quantities introduced into our calculation would have been different. And this leads me to consider ye causes of the change of motion.

III. of the powers and activity of matter.

From ye first law of motion it follows ye motion. and every change of motion is to be ascribed to a cause. To ye cause we give ye name of power. The term is analogical, borrowed from animal life. We call ye quality of ye mind by which it is fitted for producing motion in ye body, or at least we call that faculty of an animal by wh it is fitted for putting mr in motion by the name of power. By analogy we call ye quality of any cause by wh we conceive it as fitted for producing its effect by ye same name & we make a similar application of ye term to ye quality in mr by wh it is fitted for producing motion in ye mr. Animal power exerted is termed force & by analogy when ye powers of mr are exerted ~~exerted~~ in producing motion, they are termed force, in relation to ye body ~~wh is ye cause~~ moved. but powers in relation to ye body which is the cause of ye motion. We find ye same exertions of animal powers necessary for stoping motion or ye tendencies to motion. By analogy we call ye quality of mr by wh it diminishes or stops motion, or ye tendencies to motion Force. When speaking in reference to ye body stoped or having its motion diminished, but powers when speaking in reference to the body employed to stop or diminish them.

As ye tendency wh ye mr manifests to continue in motion is seen in ye change wh it produces in other mr, we consider it as a power, & as employed in producing this change, we consider it as exerting force. as ye is conceived as residing in ye moving body, it is called ye Vis insita, ye inherent force of ye body. But a want of accuracy in our view of ye matter has led some eminent writers into mistakes. Matter is carlessly said to have a <u>power</u> by wh it preserves itself in motion But since all agree in giving ye name of power to what produces a change of motion, it is inaccurate & illogical to give ye same name to what is not employed in producing a change. The continuation of motion therefore indicats no power, but is ye very effect of ye moving cause, by wh I mean what originally produced ye motion. In strict language therefore Vis insita means ye power by wh mr in motion puts another body in motion.

As ye tendency of mr to remain at rest is manifested in producing change in ÿ moving body, it is also carlessly termed a power of continuing at rest, & is termed vis inertiae. In strict language it means only ÿ power by ŵ a body at rest diminishes ÿ motion of another body. It is also called vis resistentiae, be cause its effect & operation are analogous to what we call resistance in animal life. Some use ÿ term inertia (first introduced by Kepler) to express both ÿ power manifested by ÿ moving & ÿ manifested by ÿ resisting body to mentain their present condition and Sir I. N. applied it indiscriminatkly to ÿ tendency ŵ m^r shews to remain in its present condition.

we conceive one man to have twice ÿ power, force, or strength of another. We therefore consider this quality as capable of being doubled, tripled &c In like manner we conceive ÿ inherent force & inertia of m^r as quantities, capable of being doubled tripled &c and thus we make them a subject of mathematical investigation. But phlosophers have frequently too hasty in thus calling in ÿ aid of mathematics in their disquisitions, & have been led into errors. ÿ