Poverty Durations and Poverty Measurement

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**Abstract**

Currently available measures of poverty typically use an annual accounting period, which implicitly ignores both the duration of deprivation experience within a particular year and after or before the year in question. This article argues that since the incomes of the poor often fluctuate within a year, and since long duration poverty is of great social concern, measures of aggregate poverty should reflect "duration aversion", in addition to the other axioms of poverty measurement now urged in the literature. Two composite, multiplicatively decomposable measures of poverty intensity which incorporate duration aversion are proposed and discussed.

1.      Introduction

How should the extent of poverty be measured? Assuming that agreement can be reached on the identification of poor individuals,[1] the issue remains of how to aggregate poverty across individuals, and how best to make comparisons of aggregate poverty over time and across jurisdictions. In the popular press, the poverty rate (the percentage of the population whose incomes lie below the poverty line) is the most commonly encountered poverty measure, supplemented occasionally by reference to the average poverty gap ratio (the average percentage shortfall of poor individuals' incomes below the poverty line). However, since Sen (1976) noted that the former measure is insensitive to the depth of poverty and the latter ignores the number of poor individuals (and neither statistic is transfer sensitive) the literature has gone in two directions. One strand has urged the use of dominance criteria and examination of the distribution of deprivation as a whole,[2] while another strand of the literature has attempted to establish a single index which is an ethically defensible aggregate measure of poverty.[3]

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[1]Hagenaars (1986, 1991) and Atkinson (1998) provide representative summaries of the debates on how to identify poor individuals -- i.e., whether the "poverty line" should be defined in terms of income, consumption or wealth, the appropriateness of a relative or an absolute poverty norm, how equivalence scales for different family sizes should be established, etc. More recently, Short et al (1999) have a sophisticated discussion of the appropriate measurement of income flows and consumption needs in which, however, the only statistic used to compare the implications of alternative measurement choices is the annual poverty rate.

[2]See, for example, Atkinson (1987), Anderson (1996),  Davidson and Duclos (1998), Jenkins and Lambert (1997),  Xu and Osberg (1998)  and Zheng, Cushing and Chow (1995)

[3]Blackorby and  Donaldson (1980), Chakravarty (1983),  Foster, Greer and Thorbecke (1984), Shorrocks (1995) and Thon (1979, 1983) are examples.

However, none of this literature treats duration of deprivation in a very satisfactory way. As indicated recently by Sen (1999), the theory of measurement needs to focus on some interrelated implications of the significance of non-income factors. Overwhelmingly,[4] the duration of low income spells is ignored and the literature usually uses an annual accounting period to identify poor individuals.[5] A person is considered to be poor if their total annual income falls below the annual income poverty line. In general, poverty indices are unaffected by when income is received during the accounting period or by what happens to individuals before or after [see Zheng (1997)].

The timing of income receipt and the duration of deprivation experienced should, however, matter -- for at least three reasons:

(1) The long-term poor are clearly worse off than the short-term poor, other things equal. Analysts who share the concern of Rawls (1971) for the "most disadvantaged" will therefore be most concerned with the long term poor and with shifts in the distribution of poverty spell durations which increase the prevalence of long durations.

(2) Income-based measures of poverty are known to be imperfect indicators of economic deprivation, since individuals with low income can sometimes get access to credit, run down their stock of consumer durables, obtain transfers from friends and family, etc. However, the longer and the more frequent spells of low income are, the higher is the probability that such buffers to current consumption will be exhausted, and the greater will be the actual material deprivation that corresponds to poverty in current measured income.

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[4]Ruggles (1990), Bane and Ellwood (1986) and Tienda (1990) are exceptions.

[5]Jenkins (1999, p. 7) , for example, notes that the British data which he uses actually measures income during the month prior to the interview, but is converted to an annual flow and an annual incidence of poverty.

(3) Much social concern with poverty is based on a presumption that poverty has adverse psychological and sociological implications - especially for children, but also for adults. For any given set of social norms, these impacts may be relatively small for short-term deprivation but are highly likely to increase with the duration of an individual's poverty experience. Moreover, social norms and the social stereotypes associated with low income status may also depend heavily on the distribution of poverty durations. A given rate (or depth) of poverty may be produced by either a high turnover/short duration process or a low turnover/long duration process. The degree of social exclusion that being poor implies may depend heavily on whether a poor person lives in a society where quite a few of today's non-poor have in the past experienced a short duration poverty spell, or whether the poor live in a community in which a preponderance of long duration spells, among a small minority of the population, has created a pervasive set of social stereotypes.

The purpose of this article is, therefore, to argue that the duration of poverty spells should be an explicit part of more general poverty measurement. Section 2 provides a motivating example of how the dynamics of short term flows into and out of poverty may affect annual income-based measures of poverty. Section 3.1 proposes that "duration aversion" should be an additional criterion for the evaluation of poverty measures and section 3.2 proposes two new compound, multiplicatively decomposable indices of poverty intensity. Sections 3.3 and 3.4 discuss communicability and calculation issues. Section 3.5 is an auto-critique. Section 4 concludes.

2.      Poverty Dynamics

Suppose that the poverty line is defined in terms of annual income at some level $z$, but individuals actually receive income on a more frequent basis - for example, monthly. The monthly equivalent of the annual poverty line is $z_M = z/12$. For expository purposes, imagine that there is a population of homogeneous individuals, who are either deprived or not deprived and that income dynamics are of the simplest possible sort. Specifically, suppose that: (1) individuals receive a monthly income of either $y_1$ in the non-deprived state or $y_2$ in the deprived state with $y_1 \geq z_M > y_2$ and (2) the individual's transition from one state to the other follows a Markov process.[6] The transition probability of entering the deprived state from the non-deprived state in any month is $\alpha$ while the transition probability of moving from the deprived state to the non-deprived state is $\beta$. The individual's monthly deprivation is $z_M - y_2$.

On a monthly basis, the deprivation dynamics of such a simple system are fully described by $[\alpha, \beta, y_1, y_2, z_M]$. This dynamics is a sequence of *dependent* Bernoulli trials with two states - the deprived and non-deprived states. We can denote the initial likelihood in the non-deprived state as $\pi_{01}$ and in the deprived state as $\pi_{02}$. Regardless of the values of these initial probabilities, they will change over time following the transition probabilities given in the transition probability matrix:

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[6] As Jenkins (1999) emphasizes, transitions into and out of poverty are driven by both income events and demographic events. For specificity, one might think of $y_1$ as a person's equivalent income while employed or married and $y_2$ as income while unemployed or separated. An individual's probability of entering the deprived state $y_2$ (i.e., becoming unemployed or separated) is $\alpha$ while the probability of escaping from deprivation in any given month (by getting a job or by remarrying) is $\beta$.

$$(1) \qquad P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}.$$

As $t$ increases, this system converges to a limit in which the population proportions are $[p_1 p_2]$ as

shown in equation (2):

$$(2) \qquad \lim_{t \to \infty} [p_{01} \quad p_{02}] \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}^t = [p_1 \quad p_2].$$

In the limit, population proportions in any given month simply converge to $p_1 = \dfrac{b}{a+b}$ and

$p_2 = \dfrac{a}{a+b}$ ,which depend only on the transition probabilities $\alpha$ and $\beta$.

Note that the duration of deprivation experienced by an individual $i$ within a longer period of

time is the product of the length of the longer period ($n$) and the fraction of time spent in deprivation

spells within the period ($d_i$) - that is $nd_i$. Total deprivation experience is thus a function of $n$ - which

is set to 12 months in annualized analysis. The total duration of deprivation $nd_i$ can be naturally

viewed as an additive process defined on a finite Markov chain,[7] which has an asymptotic normal

distribution with mean

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[7]See Cox and Miller (1965, Pp. 135 - 139).

$$(1) \qquad E(nd_i) = n\frac{a}{a+b}$$

and variance

$$(2) \qquad V(nd_i) = n\frac{ab(2-a-b)}{(a+b)^2}.$$

At this point we must emphasize the distinction between "deprivation" and "poverty". We use the term "deprivation" to refer to those months in which an individual's current income falls short of the monthly equivalent of the annual poverty line and we label as $d_i$ the fraction of a year that a person spends in deprivation. However, in most current discussion, an individual is only identified as "poor" if their annual income $y_{i,A}$ falls below the annual-income-based poverty line $z$. If we think of income as being actually received on a monthly basis, the annual income of individual $i$, $y_{i,A}$, can be readily computed from his or her average monthly incomes in two states ($y_1$ and $y_2$) and the fractions of a year actually spent in each state [$(1-d_i)$ and $d_i$].

$$(5) \qquad y_{i,A} = 12[(y_1(1-d_i) + y_2 d_i]$$

Individual $i$ is labelled as "poor" only if his or her annual income is less than the annual-income-based poverty line, $y_{i,A} < z$. This implies :

(6)
$$y_1(1\text{-}d_i) + y_2 d_i < z_M = z/12$$

or

(7)
$$d_i > \frac{y_1 - z_M}{y_1 - y_2}.$$

In other words, only those people with "long enough" experience of deprivation are classed as poor in terms of annual income - the annual income poor have a lower bound to their deprivation duration, $12d_i$, namely $12\dfrac{y_1 - z_M}{y_1 - y_2}$.

More generally, if an individual is to be classed as "poor", the deprivation duration in a period of the length $n$ (i.e. $nd_i$), has a lower bound $n\dfrac{y_1 - z_M}{y_1 - y_2}$. The probability that deprivation duration,

$nd_i$, is greater than its lower bound $n\dfrac{y_1 - z_M}{y_1 - y_2}$ over that longer period can be readily computed by:

(8)
$$\Pr\left(w > n\frac{y_1 - z_M}{y_1 - y_2}\right) = \int_{n\frac{y_1 - z_M}{y_1 - y_2}}^{n} \left(2pn\frac{ab(2 - a - b)}{(a + b)^2}\right)^{-1/2} \exp\left(-\frac{w - n\frac{a}{a + b}}{2n\frac{ab(2 - a - b)}{(a + b)^2}}\right) dw,$$

where $n = 12$ is used in annualized analysis, with a monthly income flow.

We note that the probability (measured on a longer period basis) depends on the transition probabilities ($\alpha$ and $\beta$), the length of the period considered ($n$), the poverty line ($z_M$), and the income flows in each state ($y_1$ and $y_2$). In an economy with homogenous individuals, this probability can be interpreted as the poverty rate which, as does the annual income poverty gap, depends on all the variables in the system.[8] The underlying economic intuition is that because an annual accounting period averages an individual's experiences over periods of deprivation and non-deprivation, the poverty rate and poverty gap[9] (measured in annual income) depend on both the probabilities of transition and *how much* incomes exceed (fall short) of the monthly poverty line in good (bad) periods.

Why might this matter?

The benefit of constructing a very simple model of poverty dynamics is that it can show how different combinations of $[\pi_1, \pi_2, y_1, y_2, z_M]$, which produce different distributions of poverty durations, can be observationally equivalent in terms of annual-income-based poverty measures. The poverty rate, the poverty gap or a more complex poverty intensity index may be the same, for a variety of underlying inflow and outflow probabilities.

For example, based on annual income, a poverty rate of 15% and a poverty gap ratio of 18.5% could be produced by two very different processes. If poverty were really an annual phenomenon, and

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[8]More exactly, the annual income poverty rate and poverty gap depend on all the variables in the system in those countries that actually measure annual income. As footnote 5 notes, the UK actually measures monthly income and scales it up to an annual total, implying that the poverty rate there depends only on $\alpha$ and $\beta$, while the poverty gap depends only on $z_M$ and $y_2$.

[9]And the same is true of any other measure of poverty based on annual income (such as the Sen-Shorrocks-Thon or Foster-Greer-Thorbecke indices or generalized calculations of deprivation dominance).

the poor were poor for exactly a year, then a process in which every year a different fifteen per cent of the population fell 18.5% below the poverty line would produce these aggregate poverty statistics. In this world, poverty experience would rotate among the population, and the living standards of the poor would be fairly close to the poverty line.

However, if the Markov model described above were true, and the vector $[\pi_1, \pi_2, y_1, y_2, z_M]$ were [0.1, 0.2, 1000, 300, 500], one would get the same annual poverty rate and average annual poverty gap ratio. Every month, the non-poor would face a 10% probability of entering poverty and the poor would have a 20% chance of escaping poverty. At any given time, one third of the population would have an income ($300) which was 40% below the monthly poverty line ($500). If capital markets for the smoothing of consumption were unavailable to low income groups, the level of deprivation at any point in time would be much larger, and experienced by more people, than annual income poverty numbers would indicate. Most people (71.8%) would have at least one month of low income in the year, but since the annual income poor are the upper tail of the distribution of low income durations, (i.e. those people who have nine or more months of low income), the annual income poor would have, in the past ten year period, an average duration of 46.8 months of low income. Overall, this is a very different picture of poverty than if 15% of the population were poor for exactly a year, but one could not tell from looking just at the poverty rate or the poverty gap.

From the point of view of the concerns which motivate much analysis of poverty this is unfortunate, because different distributions of low income duration may imply different levels of deprivation and different ethical evaluations. Many people might, for example, be less concerned with poverty statistics if a given rate and depth of annual poverty was produced by an underlying process

where poverty was broadly experienced but of short duration, compared to a process that produced narrowly focussed, long duration poverty.[10]

From a pragmatic policy design perspective, many analysts would also like to know whether changes in poverty stem from shifts in the inflow ($\alpha$), or changes in the outflow ($\beta$), probability since the types of policies that reduce the chances that people will become low income in the first place are likely to be quite different from the types of policies that increase their odds of leaving low income status.[11]

3.      Duration Aversion as a Principle

Although poverty duration does not enter current aggregate measures of poverty, there has long been a concern in the literature about the prevalence of long duration poverty. Recently, Atkinson (1999) has argued that reducing the percentage of the population that is "consistently" poor should become an explicit, rather than a "flexible", policy target. Implicitly, this builds in a duration dimension to the poverty issue, but in a discrete way, since occasional poverty is not to be counted. As well, because this measure focusses solely on the *rate* of consistent poverty, the poverty *gap* of the poor is implicitly ignored.

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[10]The term "social exclusion" hardly appeared at all in the discussion of poverty until the 1990s, but is now commonly distinguished from "poverty" - partly on the basis that broadly experienced short spells of poverty do not cut the poor off from society in the same way as longer spells, which fewer of the non-poor have ever experienced.

[11]Divorce and unemployment are two examples of the demographic and economic events which can precipitate a poverty transition. In these examples, the probability of inflow ($\alpha$) might be lessened by marital counselling services or legislation which constrains layoffs, while the probability of outflow ($\beta$) may be assisted by dating services or job finding clubs  - since $\alpha$ or $\beta$ positive policies are likely to be quite different, policy makers may want to know which type of policy to emphasize.

Since Atkinson has, for 30 years, been at the forefront of the academic literature on the measurement of poverty and inequality, it is clear that this suggestion is made in full knowledge of all its theoretical defects.[12] However, the point of his article is that poverty measurement should affect policy and if the objective is to have a measure of poverty that will actually be used in public policy debates, comprehensibility is a crucial consideration. Atkinson argues that if poverty alleviation is to receive the same level of attention as other social objectives [such as the prevention of inflation], then it is probably necessary that there be a "single headline number" to summarize trends in poverty, and that number has to be one that can be generally understood.

One can add that a "single headline number" is more useful if it: (1) is consistent with a defensible set of ethical axioms, and (2) can provide a framework for informed debate on the determinants of poverty -- i.e. be decomposed into the major dimensions of poverty. This article therefore agrees with Atkinson's emphasis on public communicability, but attempts to construct two somewhat more defensible indices of poverty which also build in consideration of the duration of poverty spells.

3.1    Duration Aversion as a Principle

The use of a particular accounting period for income (such as a year) implicitly averages income flows and household needs for income within that period. When researchers assess economic deprivation and poverty status on an annual basis, this may be problematic.  In fact, income is received,

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[12] Indeed, the article goes on to discuss the advantages and disadvantages of presenting "a richer array of statistics".

and spent, with much greater than annual frequency -- receipts are usually weekly, bi-weekly or monthly, while food shopping may be nearly continuous, rent and utility payments are typically bi-weekly or monthly[13] and clothing purchases may be seasonal.  If credit markets are easily available, or if individuals can acquire sufficient transactions balances, then the non-synchronicity of consumption and income may be a minor issue. This is likely largely true for the affluent, but is far from typical for the poor.

For most of the population, cash flow problems are nothing more than an occasional annoyance [two-thirds of US households have at least one credit card to finance consumption expenditures; see Black and Morgan (1999)].  However, for the third of US households who do *not* have a credit card, or for the 28.7% of households with zero or negative financial wealth,[14] cash flow problems can be a more significant issue.  Indeed, a substantial fraction of US households are quite cut off from the formal financial system -- Carney and Gale (1998, p.14) note that "45% of black families and 49% of those on public assistance do not have basic transactions accounts."

It is not particularly surprising that lenders hesitate to advance credit to individuals with low (and often uncertain) income and little if any assets to offer as security.  Nor is it surprising that poor households usually cannot accumulate much by way of precautionary or transaction balances. Wolff (1998, p.45) notes that the accumulated financial reserves of families in the bottom quintile of the US

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[13]As a practical matter, rent and utilities (gas, electricity, water, telephone, etc.) may have a monthly payments schedule *only for those households which can provide a sufficient deposit.* In many countries, households which are unable to post the required payments bond are driven into a more informal (and higher unit cost) spot market of nightly rentals, payment in advance, metered delivery, etc.

[14]Based on 1995 Survey of Consumer Finance data for US households; see Wolff (1998, p.36, Table 1).

income distribution in 1995 would finance 0.0 months of consumption, while the second quintile of families could support their consumption (at 125% of the poverty standard) for only 0.6 months (i.e. about 18 days). As Ruggles (1990) and others have noted, the annual accounting framework of poverty statistics can miss a good deal of real deprivation. If there is no food in the pantry, no cash coming in, no credit available and no friends or family to beg from, even a week can seem like a very long time. Since it is in practice very difficult for many poor individuals to smooth their consumption over the course of a year, social welfare program administrators have long recognized the existence of *immediate* needs for financial assistance.

By contrast, the measurers of poverty have assumed that it makes no difference to a person's poverty status when income is received during a given year -- which can be seen as equivalent to implicitly assuming a perfect "within year" capital market. In current measures of poverty, income flows before or after the year in question are also not considered at all -- implicitly, this is equivalent to assuming a total inability to borrow or lend between years. The contrast in assuming perfect "within-year" credit markets and non-existent "between-year" credit markets is striking. If a year were really an "instant" of time, these assumptions might be reasonable, but it is more likely that they have been accepted as a data driven constraint, since much income distribution data comes in an annual format. Historically, the lack of reliable data on the shorter term income flows of households undoubtedly explains much of the research focus on annual income measures of poverty, but some data sets *are* becoming available with a finer grained observation of income flows.[15]

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[15]For example, the Survey of Income and Program Participation (SIPP) in the USA and the Survey of Labour and Income Dynamics (SLID) in Canada.

For the purposes of poverty measurement the optimal "granularity", or frequency of observation, of household income data may be quite short. In principle, poverty researchers would like income data on the poverty population which has a  frequency which reflects the actual ability of low income households to smooth their consumption over time.  In practice, poverty statistics are often derived from data bases, such as Surveys of Consumer Finance, whose main focus is the income distribution of the population as a whole - and since most of the population is not as limited in their access to credit as poor people are, the accounting period appropriate for such surveys is not necessarily optimal for poverty measurement.  However, even if poverty researchers have no choice but to work with the data that does exist, there are strong reasons for paying attention, wherever possible, to the duration of poverty spells.


3.2     Two Duration Averse Poverty Measures

If it can be agreed that: (1) a longer poverty duration, ceteris paribus, signifies increasingly severe deprivation compared to a shorter poverty spell and (2) the purpose of a measure of aggregate poverty is to indicate the aggregate level of deprivation, then "duration aversion" should be a property of an ethically acceptable poverty measure.  A general specification of duration aversion would be:

"an acceptable index of poverty should increase if, other things being equal:

(a) a poor person is deprived for a longer period of time or

(b) a poor person with a shorter period of deprivation experience escapes deprivation at the expense of a similar or greater increase in the duration of deprivation experience of another similar individual with longer initial experience of deprivation."

As already noted, duration aversion can be justified on the basis of a Rawlsian concern with the least well-off, *or* because the exhaustion of access to formal and informal sources of credit means that longer spells of low income correspond to increasingly severe material deprivation *or* on the grounds that the social, psychological, and health consequences of poverty are increasing functions of the duration of deprivation.

In the literature on poverty measurement, there has been general agreement on a set of axiomatic principles that an acceptable index of poverty should satisfy. Chief among these are focus, monotonicity, symmetry, replication invariance, and transfer sensitivity.

Axiom 1 (Focus Axiom) The poverty index should be independent of non-poor population.

Axiom 2A (Weak monotonicity axiom for income) A reduction in a poor person's income, holding other incomes constant, must increase the poverty index.

Axiom 3A (Impartiality axiom for income) The poverty index may be defined over ordered income profiles without loss of generality.

Axiom 4A (Weak transfer axiom for income) An increase in the poverty index occurs if the poorer of two individuals involved in an upward transfer of income is poor and if the set of the poor people does not change.

Axiom 5A (Strong upward transfer axiom for income) An increase in the poverty index occurs if the poorer of two individuals involved in an upward transfer of income is poor.[16]

Axiom 6A (Continuity axiom for income) The poverty index varies continuously with incomes.

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[16]Note that the strong upward transfer axiom implies the weak transfer axiom since the former allows the number of poor either to be the same or to change while the latter does not. allow the number of poor to change.

Axiom 7A (Replication invariance axiom for income) The poverty index does not change if it is computed based on an income distribution that is generated by the *k*-fold replication of an original income distribution.

When the weak definition of the poor is adopted (i.e., the poor all have income less than the poverty line), Axioms 1, 2A, 3A, and 4A are equivalent to requiring that for a fixed number of poor people, the poverty index should be a decreasing, strictly S-convex function of income of the poor or should correspond to an increasing strictly S-concave social evaluation function in a negative monotonic way [see Donaldson and Weymark (1986) and Chakravarty (1990)].

The commonly used poverty rate $H$ satisfies Axiom 1 but is inconsistent with Axioms 2A and 5A. The average poverty gap ratio (or gap) of the poor $\overline{X}_p$ is inconsistent with Axiom 4A. These dissatisfactions led Sen (1976) to work on the Sen index (or the S index) based on Axioms 1, 2A, 3A, and 4A. According to Clark, Hemming and Ulph (1981) and Xu and Osberg (1997, 2000), the S index can be expressed as

(8)
$$I_S = H \cdot \overline{X}_p \cdot [1 + G(X_p)]$$

where $G(X_p)$ is the Gini index of poverty gap ratios of the poor $(X_p)$. In other words, the S index of poverty intensity has three multiplicative components - the poverty rate, the poverty gap and one plus the Gini index of poverty gap ratios of the poor. As shown in Xu and Osberg (1999), this index is a special case of the index proposed by Blackorby and Donaldson (1980).

Nevertheless, because the S index is inconsistent with Axioms 5A, 6A, and 7A, Shorrocks (1995) worked out a modified Sen index, which is identical to the limit of the Thon index (1979,

1983). Hence, we call it the Sen-Shorrocks-Thon index of poverty intensity (or the SST index).[17] The

SST index satisfies Axioms 5A, 6A and 7A. That is, it  is consistent with the transfer axiom, is

continuous in individual incomes, and is replication invariant. It is also homogeneous of degree zero

in individual incomes and the corresponding poverty line. It takes values in the range from zero to one

and admits a geometric interpretation.[18]  As shown in Xu and Osberg (1999) and Chakravarty (1997),

the SST index is a special case of the more general specifications advocated by Chakravarty (1983).

It possesses the same appealing property of multiplicative decomposability as the S index does and can

be written as:

(9) $$I_{SST} = H \cdot \overline{X}_p \cdot [1 + G(X)]$$

where $G(X)$ is the Gini index of poverty gap ratios of all individuals in the population ($X$).

The SST index satisfies all the axiomatic requirements.  However, all these axioms (and the

measures of poverty that satisfy them) are defined *given* a particular accounting period for income.

Income variability before, during or after that accounting period has been ignored.  When we consider

both income shortfall and poverty duration, individuals' welfare must be ranked according to two

criteria jointly. To evaluate another space of poverty -- poverty duration,  we extend Axioms 2A-7A

to Axioms 2B-7B as follows:

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[17]This name is first used in Osberg and Xu (1997, 2000) and Xu (1998).

[18]See Shorrocks (1995), Osberg and Xu (1997, 2000), and Xu and Osberg (1999)  for
details regarding the properties of the index.  Note that the S index and SST index differ in the
argument of the  $G$  term: The former has the poverty gap ratios of the *poor* while the latter has
those of the *population*.

Axiom 2B (Weak monotonicity axiom for poverty duration) A reduction in a poor person's deprivation duration, holding all else constant, must decrease the poverty index.

Axiom 4B (Weak transfer axiom for poverty duration)  An index of poverty should increase if a poor person with shorter duration of deprivation experience escapes poverty at the expense of a similar or greater increase in the duration of deprivation experience of another similar poor individual,  with longer initial deprivation experience and if the set of the poor people does not change.

Axiom 5B (Strong upward transfer axiom for poverty duration) An index of poverty should increase if a poor person with shorter duration of deprivation experience escapes poverty at the expense of a similar or greater increase in the duration of deprivation experience of another similar poor individual with longer initial deprivation experience.

Axiom 6B (Continuity axiom for poverty duration) The poverty index varies continuously with deprivation durations.

Axiom 7B (Replication invariance axiom for poverty duration) The poverty index does not change if it is computed based on a deprivation duration distribution that is generated by the $k$-fold replication of the original population of deprivation durations.

The S index constructed from the deprivation durations of the poor  rather than from the poverty gap ratios of the poor will be naturally consistent with Axiom 1 and Axioms 2A-4A. The SST index constructed from the deprivation durations of all people rather than from the poverty gap ratios of all people  will be naturally consistent with Axiom 1 and Axioms 2B-7B, which reflect the fact that longer deprivation durations and more dispersed distribution of deprivation duration will increase the modified SST index. In general, the depth of deprivation ($X_p$ and $X$) and the duration of deprivation ($D_p$ and $D)$  are two different but inseparable spaces of poverty experience and should be evaluated jointly.

Given that the SST index  is an index  which satisfies all of  the axioms listed above  (Axiom 1 and Axioms 2A-7A or Axiom 1 and Axioms 2B-7B) and that the  SST index is multiplicatively

decomposable into several commonly used poverty measure - the poverty rate, poverty gap, and Gini index of inequality, we propose a new poverty index that is an extension of the SST index in a multiplicative way:

(10)
$$I_{OX} = H \cdot \overline{X}_p \cdot \overline{D}_p \cdot [1 + G(X)] \cdot [1 + G(D)]$$

where $\overline{D}_p$ is the average deprivation duration of the poor and $G(D)$ is the Gini index of deprivation durations among all people of the population ($D$).

This new index remains consistent with Axiom 1 and Axioms 2A-7A and is also consistent with Axioms 2B-7B. Essentially, the index incorporates two dimensions of poverty experience into the measure -- (1) the depth of deprivation and (2) the duration of deprivation.

Currently, the implicit assumption of poverty measurement is that poverty duration is 100 percent (12 months in a year) and identical for all poor people. If this is actually the case, then:,

(11)
$$\overline{D}_p = 1, \ G(D) = 1 - H.$$

The index given in equation (10) then changes to

(12)
$$I_{OX} = H \cdot \overline{X}_p [1 + G(X)] \cdot (2 - H)$$

or

$$(13) \qquad\qquad I_{OX} = I_{SST} \cdot (2 - H).$$

If one is satisfied with Axiom 1, Axioms 2A-4A, and Axioms 2B-4B, the S index can also be extended using the above multiplicative principle:

$$(14) \qquad\qquad I_{OX'} = H \cdot \overline{X}_p \cdot \overline{D}_p \cdot [1 + G(X_p)] \cdot [1 + G(D_p)].$$

Note that the Gini index of poverty gap ratios of the poor $G(X_p)$ and that of deprivation durations of the poor $G(D_p)$ are used in the index.

This alternative new index remains consistent with Axiom 1 and Axioms 2A-4A and is also consistent with Axioms 2B-4B. When the poverty duration is 100 percent (12 months in a year) and identical for all poor people,

$$(15) \qquad\qquad \overline{D}_p = 1, \qquad\qquad G(D_p) = 0.$$

In this case, $I_{OX'}$ becomes $I_S$ for income shortfalls. Thus, $I_S$ is a special case of $I_{OX'}$ in the sense that the latter is more general than the former.

The OX index can be easily decomposed into the changes of its sources as follows:

(16) $$\Delta I_{OX} = \Delta H + \Delta \overline{X}_p + \Delta \overline{D}_p + \Delta[1 + G(X)] + \Delta[1 + G(D)].$$

where $\Delta w = \ln w_t - \ln w_{t-1}$ represents the percentage change in $w$ from time $t$ - 1 to time $t$. The OX' index can be decomposed similarly into the changes of its sources as follows:

(17) $$\Delta I_{OX'} = \Delta H + \Delta \overline{X}_p + \Delta \overline{D}_p + \Delta[1 + G(X_p)] + \Delta[1 + G(D_p)].$$

Equations (16) and (17) show that a percentage change in each newly proposed index can be traced to percentage changes in six intuitive dimensions of poverty - the poverty rate, the average poverty gap, the average duration of deprivation, the inequality in income gaps, and the inequality in duration of deprivation.

### 3.3    Communicability

When poverty is discussed, a seemingly simple question is often heard: "Has poverty increased or decreased?". In addition to comparisons over time, the debate on poverty also often centres on comparisons across jurisdictions -- i.e. on whether poverty is greater, or less, in Jurisdiction A than in Jurisdiction B. The motivation for either sort of comparison is rarely just measurement for its own sake alone. Rather, people measure poverty, and make comparisons over time and space, because they hope

that such comparisons can help guide economic and social policy. In deciding whether poverty is in general getting worse or better, or in deciding whether or not a specific policy has been successful in reducing poverty, it is extremely useful to have a "single headline number" with which to answer seemingly simple questions.

The problem is that some headline numbers -- e.g. the poverty rate -- may not always be an appropriate guide to policy. The poverty rate is not affected by the size of the poverty gap and is not transfer sensitive. Although it is useful to have a single headline number with which to answer seemingly simple questions, that index should satisfy the axioms of focus, symmetry, replication invariance, continuity, transfer sensitivity, replication invariance  - and (we would argue) duration aversion.

In addition to these axiomatic criteria, we would argue that decomposability is an important attribute of a good poverty index. Poverty is a complex phenomenon. Knowing that poverty has "worsened", or "improved", is not by itself much help in the design of appropriate social and economic policies. The ability to decompose trends in a poverty index into the underlying components which drive those aggregate trends can be a useful guide to the direction of policy development, and the decomposition itself can be a useful framework for the discussion and analysis of poverty.

A major advantage of the modified SST and S indices  we propose in equation (10) and (14) is the fact that they are composite indices and  multiplicatively decomposable. By taking logs, one can therefore decompose the percentage difference in poverty intensity (over time or space) into the percentage change that is due to  changes in the rate or depth or duration of deprivation, or changes

in inequality of depth[19] or duration of deprivation. Changes in the overall rate, average depth or average duration of poverty can then be further decomposed into the proportion of those changes arising from the altered experience or changing relative weights of different sub-groups - for example, different regions or demographic cohorts.[20]

We would also argue for communicability as an important practical criterion for a poverty measure. If a measure of poverty is actually to be used in policy discussions, and thereby have a chance of influencing those discussions, it has to be a measure that can be fairly broadly understood. We think that although equation (10) is more complex than the simple poverty rate calculation which now dominates public discussion, each of its components (the poverty rate, the average depth of poverty, the average duration of deprivation, inequality in depth of poverty, inequality in deprivation duration and the correlation of duration and depth of deprivation) has an intuitive and easily understandable meaning.

Finally, we would argue that since poverty duration data is now only occasionally available, it may be useful to maintain as much comparability as possible with poverty index calculations which cannot, for data reasons, incorporate consideration of poverty duration. The index we propose in

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[19]As a practical matter, Osberg and Xu (1997, 2000) note that over time or across jurisdictions the inequality of annual income poverty gaps among all people of the population $[1+G(X)]$ is very nearly constant - hence differences in the rate or average depth of poverty dominate comparisons of poverty intensity, as measured by the SST index.

[20]It has long been the case, for example, that demographic groups (such as single young adults and lone parent families) differ in the frequency and duration of poverty spells. Historically, single youth have been a high turnover group, hence the observation of a trend to increasing duration of youth poverty may be of great social concern - but it is important to be able to assess its relative importance in the determination of aggregate poverty.

Equation (10) differs from the annual income SST index (which is already in use in the literature[21]) by

the multiplicative factor: $\overline{D}_p \cdot [1+G(D)]$  -- so it easy to see how much of a difference consideration

of poverty durations is making to poverty measurement.

3.4     Calculation

In order to calculate a measure of poverty in a particular year, the first step is to identify those

individuals who were poor during that year, and the second step is to calculate some measure of their

poverty. This article is primarily concerned with introducing duration aversion into the second step, but

Sections 3.4.1 and 3.4.2 discuss two alternative ways of identifying the poor.

3.4.1   A Duration Averse Poverty Measure for the Annual Income Poor

Although economic deprivation often occurs on a much shorter than annual time scale,[22]  if we

want a measure of poverty which includes consideration of poverty duration *and* is comparable with

the existing poverty literature we must start from the same criterion of deprivation in identifying the

poor -- i.e. those individuals whose annual income,[23] in a given year, falls below the relevant annual

income poverty line for that year. Following this approach, the calculation of the poverty rate is

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[21]See, for example, Myles and Picot (2000). In the discussion henceforth we refer only to the modified SST index presented in equation (10).

[22]Ruggles (1990, Pp. 89-119) has noted the prevalence of sub-annual spells of poverty - in the US in 1984, the rate of annual income poverty was 11.0%, but 26.2% of the population were poor for one or more months.

[23]"Income" may be calculated in money income terms, before/after taxes and transfers, "equivalized" for household size or adjusted for effective non-cash consumption flows received.

unchanged, as is the calculation of each person's poverty gap, or aggregate income deficiency for the year.

In equation (10), the head count ratio (*H*), average poverty gap ratio ( $\overline{X}_p$ ) and inequality of poverty gap ratio [*1 + G(X)*] terms are therefore calculated in exactly the same way as in equation (9), which reports the Sen-Shorrocks-Thon index of poverty intensity. To add consideration of deprivation durations, measurement of deprivation experience over some longer period of time than the year immediately under consideration is required.

*Given* that we have identified the set of people who are poor, we want to know what the total duration of deprivation experience (*d_i*) is within some specified period of time (*n*), or window of their life (*W*). To calculate individual deprivation durations requires access to reliable micro data from panel data sets in order to track the changes in monthly equivalent income which people experience as they move between household units and as the aggregate income of those households fluctuates.[24] Given such data and the specification of a monthly poverty line, an individual's deprivation status for each month of observation can be determined.

An important issue is the specification of the window of calculation of poverty status (*W*). Since observed lifetime experience of poverty depends partly on age,[25] it would be undesirable to calculate lifetime poverty experience - a fixed window of observation is preferable. Because a very long window of observation of potential poverty status would bias the observed distribution of poverty durations

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[24]In the Canadian *Survey of Labour and Income Dynamics* (SLID), individual incomes are in fact reported in exactly this way (in order to preserve confidentiality of individual data records).

[25]A twenty year old has, obviously, had far less exposure to the probability of poverty than a seventy year old. Even if the lifetime duration of poverty experience to date were available, the presentation of duration data on that basis would be hopelessly confounded by cohort and age effects.

towards the experience of older cohorts, but a very short window would obscure the importance of multi-year poverty spells, and recurrent short poverty spells, some compromise is unavoidable - in our view, a window of observation of five years would seem to be an appropriate choice. Hence, as a practical matter we propose that the duration of individual experience of poverty [variable $d_i$ in Equation (10)] be measured as the fractional number of years of deprivation experience within the last five years of any individual's life.[26]

In practice, panel data sets will differ in the period of time over which actual incomes can be observed. The maximum feasible window of observation of past deprivation status ($W^{Max}$) will be determined by the age of the panel data set from which observations are drawn. Since many of the panel data sets available internationally are far younger than the Panel Study of Income Dynamics (PSID), and since some have a rotation panel design,[27] the maximum feasible window will necessarily be appreciably shorter than the PSID and, in some cases, significantly less than five years.

If the window of observation of deprivation status is equal to or greater than the desired window of measurement of deprivation durations (i.e. $W^{Max} \geq W$ ) then the actual duration of past deprivation ($d_i$) experienced by each currently poor person can be directly calculated. If the window of observation available in the data is less than the desired window of measurement ($W^{Max} < W$), then incomplete durations will be observed. The issue then is how to infer, from the truncated distribution of durations observed within the window of observation, an estimated distribution of durations within the desired window of poverty duration measurement. Econometrically, the issue is similar to that of

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[26]Note that this number is truncated for children under 5 years of age.

[27]For example, respondents to SLID will be followed for six years, and then replaced.

the measurement of the aggregate duration of unemployment spells, on which there is quite a literature.[28]

### 3.4.2   A Duration Averse Poverty Measure for the Short Term Poor

If one takes seriously the possibility that a high proportion of the low income population face significant difficulties in accessing credit markets, then periods of low income within a year may produce significant deprivation, even for some individuals who end the year with an annual income above the poverty line. If so, one may see maintaining comparability with currently available annual--income-based poverty measures as being less important than improving on those measures. If data on income flows are available on a monthly basis, this would argue for an identification of the poor, in any given year, as those individuals whose monthly income had slipped below the poverty line in one or more months.

In this case, in interpreting the terms of equation (10), the poverty gap ratio for any individual would be the average percentage shortfall of their income below the monthly poverty line, for those months in which their income fell short, and the term $[1 + G(X)]$ would as usual be calculated over the population as a whole (counting the poverty gap ratios of persons without poverty experience as zero). In order to maintain the decomposition properties of the SST index, the average poverty gap ratio ($\overline{X}$) would, however, be calculated over the population of people experiencing any poverty within the year. To calculate average duration ($\overline{D}$) or the inequality of poverty durations $[1 + G(D)]$,

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[28]Early references are Kiefer and Neumann (1989) or Kiefer (1989).

the same considerations already discussed in 3.4.1 apply, and there is no reason to alter the units of measurement.

4.      Auto-Critique

Capital market imperfections are typically asymmetric. There is rarely a problem finding a bank that will accept one's deposit, and the maximum cost to not finding such a borrower is the foregone interest (and risk of theft) involved in holding cash. However, finding a bank that will extend a loan can be much more difficult and the cost of not finding a lender is the utility lost due to foregone consumption (which is quite high, should one be starving). Hence, even if the aggregate duration of low income is the same, an individual is better off if they are one of those whose low income period follows a higher income spell, compared to being one of those whose low income spell precedes a higher income period. The sequencing of income flows matters to individuals, as well as the aggregate duration of their income deprivation.

However, the issue this article addresses is how best to compare the extent of deprivation in a population, over time or across jurisdictions. For any given year, there will be a distribution of sequences of low income and high income spells -- some people will have their good times at the start of the year, and some at the end. Implicitly, in focussing only on the total duration of low income experience, this article is presuming that the distribution of sequencing of low/high income spells does not change, over time or across jurisdictions.

As well, some readers may have wondered what precisely was meant by "another *similar* poor individual" in Axioms 4B and 5B above. If "similar" is interpreted to mean, "with an equal poverty gap", and if that poverty gap is calculated as an average for the low income months of the year (as

Section 3.4.2 suggests), then we are implicitly linearizing the utility deprivation of an income deprivation.[29] (For example, two months of a poverty gap of 0.2 is counted as equivalent to one month of poverty gap of 0.1 plus another month with a 0.3 gap).

If the suggestion of Section 3.4.1 is followed and the poverty gap is calculated using income averaged for the year as a whole, the implication of the measures we propose in equations (10) and (14) is stronger. Since a given annual poverty gap could be produced by either a short spell of intense poverty or a longer spell of shallow poverty, the implication of Equations (10) and (14) is that the latter is to be regarded as unambiguously worse than the former. We think that this position can be defended, on the grounds that the whole point of identifying "poverty" as a distinct state is to say that there is something qualitatively different about being "poor" rather than just "low income" -- hence that the duration of deprivation matters, over and above its contribution to the depth of poverty experienced within a year.

5.    Conclusion

This article has argued that duration aversion should be an explicit part of an acceptable measure of poverty, and has proposed two multiplicatively decomposable indices of poverty intensity that incorporate duration aversion. We hope that these measures will be used in the discussion of poverty issues, because the real test of the utility of proposing a new measure of poverty is whether or not anyone actually uses it.

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[29]Note that this problem is also present in any measure of poverty intensity, such as the SST index, which ignores deprivation duration.

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