How Should We Measure Global Poverty in a Changing World?

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Abstract

Before effective anti-poverty policy can be designed and implemented, the extent, trend and distribution of poverty must be identified – in this sense, poverty measurement is a crucial intermediate step in public policy making and development planning. This paper asks whether the estimated proportion of the world's population with income below US $1 (adjusted according to purchasing power parity) per day is a good measure of trends in global poverty. We argue that the answer depends on three important issues in the measurement of poverty – the definition of the poverty line, how best to summarize the level of poverty and how to infer statistical estimates of poverty from sample data. In this paper, we survey the literature on poverty measurement, demonstrate the importance of considering poverty incidence, depth and inequality jointly, present a simple but powerful graphical representation of the Sen and SST indices of poverty intensity (the "poverty box") which is the FGT index of order 1 and extend our empirical work to China using the commonly accepted international poverty line definition of one half median equivalent income.

1. Introduction

According to the United Nations (UN) International Development Report (2004), the proportion of the world population with income below US $1 (adjusted according to purchasing power parity[3]) per day[4] has dropped from 40% in 1981 to 21% in 2001. However, this paper asks the question: is this alone a good indicator of global anti-poverty progress?

One of the primary targets of the UN Millennium Development Goals is the poverty rate or head-count ratio - i.e. the proportion of population with income below the US $ 1 poverty line. As a measure of poverty, this has the enormous advantage of simplicity. The poverty line – one US dollar per day – seems immediately understandable as an indicator of absolute deprivation. The calculation of the percentage of people who are poor is similarly direct. This measure can therefore easily be used in public debates – even though it implicitly embodies the assumption that the degree, and inequality, of deprivation of the poor is not important.

A secondary indicator of Millennium Development Goals is the poverty gap ratio (also called the average poverty gap of the population[5] or the poverty gap index[6]), which is the mean distance for the entire population of income shortfalls below the poverty line as a proportion of the poverty line.[7] The poverty rate and the average poverty gap ratio[8] are the two most used

[3] Aten and Heston (2004) note that since the consumption of the poor is more heavily weighted to food than the consumption of the population as a whole, and since food is relatively highly priced in developing countries, the PPP adjustment appropriate for comparisons of GDP per capita is inappropriate for comparisons of absolute poverty – they argue that a more appropriate poverty line PPP would increase substantially the global poverty rate. However, in this paper we cannot directly address this issue.
[4] Chen and Ravallion (2001, p.285) note that initially the $1per day standard was set in 1985 prices, but they use $1.08 in 1993 prices.
[5] See Xu and Osberg (2002, p.140).
[6] See Lipton and Ravallion (1995, p. 2579).
[7] See Chen and Ravallion (2001, note for Table 3).
[8] Two closely related ideas are the average poverty gap ratio of the population (where the deprivation of the nonpoor is taken to be zero – see equation 2.3 below) and the average poverty gap ratio of the poor (or the income gap ratio), which is defined as the average income shortfall below the poverty line as the proportion of the poverty line for the poor – see equation 2.2 below [see Lipton and Ravallion (1995, p. 2579), Raj (1998, p. 255 and Xu and Osberg (2002, p.140)]. Clearly, the average poverty gap ratio of the population equals the product of the average poverty gap ratio of the poor and poverty rate.

poverty measures in many countries and international organizations - largely because they can be easily understood and, as a consequence, *actually used* in the broader public debate.

A third dimension of poverty measurement (although not calculated widely[9]) is the inequality of poverty. Although this is not part of the Millennium Development Goals, economists such as Amartya Sen have argued for the use of poverty measures which jointly incorporate the incidence, depth and inequality of poverty. In this paper, we explore such possibilities, and possible simplifications.

Measures of the incidence, depth and inequality of poverty presuppose specification of the poverty line. In common language usage, poverty is about deprivation of necessities - the primary dictionary definition of "poverty" is "want of the necessities of life" [see Oxford (1998, p.1135)]. However, any operational criterion of poverty for statistical measurement necessarily involves some approximation in the measurement of individuals' command over resources – and some balancing of the risks of misclassification. For any given poverty line, there is some probability that a person who is actually deprived may not be identified as a poor person (Type I error) and there is also some chance that a non-deprived person may be identified as poor (Type II error) – minimizing the first source of error seems particularly important. As well, analysts have often debated whether to measure poverty in terms of a generalized command over resources (i.e. income) or in terms of command over specific commodities – i.e. a minimum food and non-food basket.

A poverty line threshold can be based on either an absolute or relative criterion. Typically, an absolute poverty line has been used in developing countries, often based on the minimum food consumption basket for a specific level of calories (say 2200) and a minimum non-food consumption basket. However, economic growth means that even absolute poverty lines tend to change over time, as consumption items which were considered non-essential in the past, become seen as essentials now. The rapidity of economic growth in recent years in some

[9] The Philippines is one of the few developing countries which report inequality regularly as a part of official poverty statistics.

countries provides some evidence that the absolute poverty line methodology may be becoming less appropriate in some countries in this changing world.[10]

In affluent countries, extreme deprivation may be rare, and in practice poverty research in most developed countries uses an explicitly relative definition of the poverty line[11] (often defined as a fraction – usually 50% - of median income). An absolute poverty line (such as $1 US per day) has been more common in research on developing countries. However, some developing countries are very rapidly becoming more affluent – at least in average incomes. Hence, although many researchers would agree that absolute deprivation remains the important issue in countries with very low per capita incomes, this division of focus has become harder to justify in recent years. Rapid economic growth in countries such as China, Maldives, Thailand, and some others (comprising a large fraction of the world's population) raises the question: how should we measure poverty when average income is growing rapidly?

Amartya Sen (1985) has noted that there is the broader question of whether a poverty line income threshold can be representative of other dimensions of capacities. As well, at the operational level, researchers need to decide which measurement units to use. The recipient unit – individuals or households – must be defined and identified, as is culturally appropriate. Researchers must also decide whether *income* or *consumption* or *expenditure* is the most appropriate concept to use in assessing *command over resources* and how exactly each concept can be best approximated, in the real world of statistical practice. But given these research decisions, poverty still has to be summarized by some index - and one example of current practice is Chen and Ravallion (2001), who use the head-count ratio and poverty gap ratio based on the international absolute consumption poverty lines (the 1993 PPP adjusted $1.08 and $2.15 respectively).

[10] For example, in Maldives, Thailand, some regions in China, no absolute poverty exists if an absolute poverty line were used in 2003-2004.

[11] See, for example, the recent OECD study by Forster (2005). Even when the rhetoric of an "absolute" poverty line is used, the redefinition over time of a "subsistence" consumption bundle in developed economies often means that the poverty line is implicitly, if periodically, drawn relative to prevailing norms of consumption [see Fisher (1995) and Osberg (2000)]. The USA is an exception, since the Social Security poverty line was initially set in 1963 at three times the level of a "subsistence" food budget, and has been adjusted only for price increases since then.

In the economic literature, many poverty measures have been proposed in the literature primarily based on the axiomatic approach advocated by Sen (1976) [see Zheng (1997) for a review]. However, most are not actually used in practice. The more communicable and often-used poverty measures are the head-count ratio, poverty gap ratio, income gap ratio, and Foster-Greer-Thorbecke (FGT) indices of different orders [see Lipton and Ravallion (1995), Ray (1998), and Todaro and Smith (2003)]. Although perhaps desirable from a theoretical perspective, more complex poverty measures such as Sen (1976) that measure poverty incidence, depth and inequality jointly appear more difficult to calculate and harder to communicate. Hence, it is the simpler poverty measures that tend to be actually used despite their insensitivity towards distribution among the poor, which is considered important by Sen (1976), Foster, Greer, and Thorbecke (1984), Shorrocks (1995), Lipton and Ravallion (1995), among others.

In this paper, we analyze the benchmark poverty measures such as the Sen and SST indices of poverty intensity in order to (1) find their simplified representations, (2) relate them to an illustrative tool called the "poverty box," which combines the incidence and depth of poverty in a two-dimension space, and (3) apply these measures to a developing country (China). Osberg and Xu (2001) find that in the developed countries, where the poverty rate is relatively low (typically considerably less than 20%), inequality among the poor is small and fairly constant over time and across jurisdictions. Hence Osberg (2000) and Xu and Osberg (2001) advocate the "poverty box" approach as a way of simplifying communication and facilitating comparative studies.[12] This approach offers a solution to how to measure poverty incidence and depth jointly and graphically and gives the poverty gap ratio a geometric interpretation. This paper addresses the issue of whether the same should be done in developing countries such as China, where the poverty rate is much higher and the variations in inequality of poverty are greater.

Finally, since all poverty statistics are estimates from samples surveyed from the population, sampling variability of these estimates is a perennial concern. Particularly in many

[12] Fields (1977, p. 576 or 1980, p. 26 and p. 212) study of Brazil's poverty, includes a figure in which the poverty rate and average poverty gap in local currency are shown in a coordinate system – but for international comparison one needs to use the poverty gap ratio. Based on international data in 1987 and 1998, Chen and Ravallion (2001) note that the poverty rate based on the 1993 PPP US$ 1.08 (or 1993 PPP US$2.15) poverty line, poverty rate is higher than 40% (70%) in South Asian and Sub-Saharan Africa.

developing countries (such as China) where estimates of poverty outcomes for absolutely large numbers of people may depend on quite small numbers of sample observations, it may be important to ask – what are the statistical properties of poverty estimators?

Section 2 of the paper reviews what we have learned from the literature on a set of useful poverty measures. Section 3 provides some empirical evidence from China and Section 4 examines related statistical issues. Section 5 concludes.

2. What Have We Learned about Poverty Measurement?

The most common measure of poverty is to calculate the proportion of the population whose incomes are below a designated poverty line. If we use N for the size of a population and Q for the number of the poor, then the *poverty rate* is given by

$$H = \frac{Q}{N}. \tag{2.1}$$

This "head count" measure presupposes the definition of recipient unit (individual or family or household) and income concept, and the specification of a poverty line (z), below which the income of individual i (y_i) is unacceptably low. However, the poverty rate cannot show the depth of poverty - in two countries (or the same country at two different points in time). With identical poverty rates, the two poor subpopulations may have very different average income levels. More disturbingly, if the poverty rate is used as the main measure of the effectiveness of anti-poverty policy, policy makers may be tempted by "cream-skimming", because the most cost effective way to reduce poverty is to give a small transfer to the richest of the poor so that his or her income is lifted just above the poverty line.

Concern with the depth of poverty motivates two closely related measures – the average poverty gap ratio of the poor and that of the total population. The former is denoted by

$$I = \frac{1}{Q}\sum_{y_i<z}\left(\frac{z-y_i}{z}\right) \tag{2.2}$$

and the latter

$$HI = \frac{Q}{N}\frac{1}{Q}\sum_{y_i<z}\left(\frac{z-y_i}{z}\right) = \frac{1}{N}\sum_{y_i<z}\left(\frac{z-y_i}{z}\right), \tag{2.3}$$

where the poverty gap ratio is set to zero for the nonpoor population because they have zero deprivation of income.

Although these measures illustrate incidence and average depth of poverty, they cannot reveal whether deprivation differs substantially among poor people. Further, the average poverty gap ratios are not sensitive to whether poverty alleviation targets the poorest of the poor and those who are only marginally poor. In 1976 Amartya Sen proposed a set of fundamental axioms as the basis for poverty measurement. Similar to the debate in establishing inequality measurement, where the Pigou-Dalton transfer principle became a guidepost or an axiom [see Dalton (1920) for the original work and Xu (2003) for an intuitive explanation], Sen's (1976) axioms, refined further later [see Shorrocks (1995) and Chakravarty (1997)]], formed the foundation for subsequent poverty measures. One of the key points made by Sen is that all the existing poverty measures at that time were insensitive to the distribution aspect of poverty.

Seven well-known axioms or principles for evaluating poverty measures are:[13]

(1) Focus axiom (F): the poverty measure should be independent of the nonpoor population.
(2) Weak monotonicity axiom (WM): a reduction in a poor person's income, holding other incomes constant, must increase the value of the poverty measure.
(3) Impartiality axiom (I): A poverty measure should be insensitive to the order of incomes.
(4) Weak transfer axiom (WT): An increase in a poverty measure should occur if the poorer of the two individuals involved in an upward transfer of income is poor and if the set of poor people does not change.
(5) Strong upward transfer axiom (SUT): An increase in a poverty measure should occur if the poorer of the two individuals involved in an upward transfer of income is poor.
(6) Continuity axiom (C): The poverty measure must vary continuously with incomes.
(7) Replication invariance axiom (RI): The value of a poverty measure does not change if it is computed based on an income distribution that is generated by the *k*-fold replication of an original income distribution.

[13] See Hagenaars (1986, 1991) or Xu and Osberg (2001) in Chinese; the English version is available at http://is.dal.ca/~econhome/RePEc/dal/wparch/sensw.pdf.

For some observers, these axioms or principles are pre-conditions to judge the reasonableness of a poverty measure. Of course, as shown later, some axioms impose stronger conditions than other axioms do (WT versus SUT or with or without C).

The poverty rate *H* satisfies the Focus, Impartiality, and Replication axioms but it violates the Weak Monotonicity, and Weak Transfer axioms. Hence, many economists find the poverty rate unacceptable, since it captures the incidence of poverty but is insensitive to the depth of poverty. The average poverty gap ratio of the poor *I* satisfies the Focus, Weak Monotonicity, and Impartiality axioms but not the Weak Transfer axiom - which means that *I* captures the depth of poverty but is insensitive to the distribution aspect of poverty. Because of these deficiencies in the poverty rate and average poverty gap ratio, Sen (1976) proposed two version of the same poverty measure. The first is

$$S_0 = H\left[1-(1-I)\left(1-G(y_p)\right)\left(\frac{Q}{1+Q}\right)\right], \tag{2.4}$$

where $G(y_p)$ is the Gini index of the distribution of the poor. As the population size gets larger, $\frac{Q}{1+Q} \to 1$. Thus another version is given by

$$S = H\left[I+(1-I)G(y_p)\right]. \tag{2.5}$$

These two versions of the Sen indices will satisfy the other axioms but not the Strong Upward Transfers and Continuity axioms. S_0 does not satisfy the Replication Invariance axiom while S does. Clark et al. (1981) applied equation (2.5) in their empirical study.

Immediately after Sen's work many economists proposed a wide range of poverty measures [see Zheng (1997) and Xu and Osberg (2002) and references therein]. Among those, Shorrocks (1995) proposed a modified Sen index which is identical to the limiting case of the Thon index (1979, 1983), and can be called the Sen-Shorrocks-Thon (SST) index of poverty, which is defined as

$$S_{SST} = \frac{1}{N^2}\sum_{y_i<z}\left(2N-2i+1\right)\left(\frac{z-y_i}{z}\right). \tag{2.6}$$

Note that the poverty gap ratio for the nonpoor $\left(\frac{z-y_i}{z}\right)$ is set to zero. The application of this poverty index can be found in Xu (1998).

Foster, Greer and Thorbecke (1984) proposed a class of decomposable poverty indices (the FGT indices) of the form:

$$FGT_\alpha(y,z) = \frac{1}{N}\sum_{y_i<z}\left(\frac{z-y_i}{z}\right)^\alpha, \qquad (2.7)$$

where y represents the income distribution and y_i represents the income of individual i. Within this family of indices, the FGT index with some values of α ($\alpha = 0,1$) does not satisfy all of the above axioms. However, higher order FGT indices (i.e. α > 1) do satisfy axioms WM, WT, and SUT. As can be seen below, the FGT indices include those that are criticized by Sen (1976). When $\alpha = 0$,

$$FGT_0(y,z) = \frac{1}{N}\sum_{y_i<z}\left(\frac{z-y_i}{z}\right)^0 = \frac{Q}{N} = H. \qquad (2.8)$$

The FGT index of order 0 is the poverty rate. When $\alpha = 1$,

$$FGT_1(y,z) = \frac{1}{N}\sum_{y_i<z}\left(\frac{z-y_i}{z}\right) = HI. \qquad (2.9)$$

The FGT index of order 1 is the average poverty gap ratio of the population, which equals the product of the poverty rate and the average poverty gap ratio of the poor. FGT indices of an order higher than 1 are distribution-sensitive. For example, when $\alpha = 2$,

$$FGT_2(y,z) = \frac{1}{N}\sum_{y_i<z}\left(\frac{z-y_i}{z}\right)^2. \qquad (2.10)$$

In this formulation, when α > 1, a larger poverty gap ratio $\left(\frac{z-y_i}{z}\right) > 0$ receives more than proportionately higher weight in the FGT index. Schady (2002) is an example where the FGT index of order 2 is used. Researchers often face the question as to what value should be assigned to α. It is unclear how to weight each of the FGT indices in terms of their importance. This family of indices themselves do not provide any guidance on this issue. However, as Osberg (2004) has noted, in the FGT family of indices, when Luxembourg Income Study data on affluent countries are used, it appears that over the range α = 2, 3, …, 6 index values tend to be clustered and there is not much additional gain of information. We consider below whether a similar conclusion is warranted in the very different circumstances of rural China.

It is somewhat surprising to see that the FGT indices are applied more widely in empirical poverty studies than the Sen indices, S_0, S, and S_{SST}, although the latter were proposed earlier and have been improved upon over time [see Osberg (2004)]. Fields (1980, p.170) noted that it is sometimes impossible to calculate *S* because of the unavailability of data on income inequality. Although the FGT index of order α < 2 does not satisfy many of the important axioms, the FGT indices are considered attractive by many analysts, in particular for their additive subgroup decomposability. Osberg and Xu (1999, 2001), Osberg (2000), Xu and Osberg (2001, 2002) argue that the Sen indices are not as simple to the policy analysts, but should and can be substantially simplified. Indeed, as soon as these simplifications become known, the Sen indices, in particular the SST index, become a powerful tool in policy analysis as shown by Myles and Picot (2000).

In particular, we have argued[14] that the Sen index *S* and the SST index S_{SST} [given in equations (2.5) and (2.6) respectively] should, and can, be simplified into their multiplicative components - the poverty rate, average poverty gap ratio of the poor, and a measure that is related to the Gini index of poverty gap ratios of the poor (for the Sen index) or of the population (for the SST index).

Formally, let x_p represents the poverty gap ratios $\left(\frac{z-y_i}{z}\right)$ for the poor and *x* those of the population. The Sen index given in equation (2.5) can be written as

$$S = HI\left[1+G(x_p)\right]. \tag{2.11}$$

[14] See Osberg and Xu (1999, 2001), Osberg (2000), Xu and Osberg (2001, 2002).

Note that in order to calculate $G(x_p)$, one can use the regular Gini index formula15 with poverty gap ratios sorted in non-decreasing order [see Xu and Osberg (2002, p. 143)]. The higher is the value of $1+G(x_p)$, the greater is inequality among the poor. A verbal expression of equation (2.11) above is:

The Sen Index = [the poverty rate] $\times$ [the average poverty gap ratio of the poor]
$\times$ [the inequality of poverty gap ratios of the poor].

The interpretation of the above is that the Sen index measures poverty incidence, depth and inequality *jointly* while permitting decomposition into commonly used poverty measures. Poverty is high when the incidence of poverty is high (a higher poverty rate), or when the depth of poverty is increasing (a higher average poverty gap ratio), and or when the poverty gap ratios of the poor are more unequal [a higher $1+G(x_p)$]. When poverty gap ratios of the poor are identical, then the Sen index becomes:

The Sen Index = [the poverty rate] $\times$ [the average poverty gap ratio of the poor]

because the poor are approximately equally deprived, $G(x_p)=0$, so $1+G(x_p)=1$. The Sen Index thus collapses to the FGT index, with α = 1. As well, when the inequality of poverty gap ratios is a constant at the level *K*, the major sources of changes in poverty can be expressed as the sum of changes in the poverty rate and the average poverty gap ratio of the poor alone. Hence, when either when the poor are equally deprived or the inequality of poverty is constant, the combination of two simple concepts – the *rate* and *average depth* of poverty - leads to a powerful illustrative tool: the "poverty box". Under the condition that inequality of poverty is nil

[15] For a data set $\{y_1, y_2, \ldots, y_N\}$, the Gini index or coefficient is given by
$G(y)=1-\frac{1}{N^2\overline{y}}\sum_{i=1}^{N}\left(2N-2i+1\right)y_i$, when $\{y_1, y_2, \ldots, y_N\}$ are sorted in non-decreasing order. Alternatively,
$G(y)=\frac{\sum_{i=1}^{N}\sum_{j=1}^{N}|y_i-y_j|}{2N^2\overline{y}}$, where $\{y_1, y_2, \ldots, y_N\}$ do not have to be sorted. Note that $\overline{y}$ is the mean of $\{y_1, y_2, \ldots, y_N\}$. See Xu (2003) for more details.

or that there is little change in this inequality, the size of poverty box can represent the welfare loss caused by poverty and the change in the box size can be interpreted as the change in welfare loss.

The "poverty box" is, in fact, related to the poverty profile [originally due to Jenkins and Lambert (1997)], which we show in Figure 1. In this coordinate system, the poverty profile draws the curve of cumulative percentage of poverty gap ratios, for the total population, from the highest to the lowest (zero ratio for the nonpoor) corresponding to the percentage of the population. The poverty profile curve rises from the origin (at point 0) at a faster rate, increases at a decreasing rate, reaches the plateau (at point a) when the last and least poor individual's poverty gap ratio is added, and then becomes flat to the end (at point HI) when zero poverty gap ratios of the nonpoor are add to the cumulative percentage. When the inequality of poverty gap ratios is nil, the curved segment of the poverty profile becomes a straight line.

As shown in Figure 2, the geometric interpretation of the Sen index with reference to the poverty profile curve according to Xu and Osberg (2002), is as shown in the upper-right panel. Let the triangle area of OHH' be Area E. The Sen index is given by the sum of Areas C and D divided by Area E. Hence the Sen index will take the curvature (in relation to Area C) into account. In the lower-left panel of Figure 2, the poverty box is drawn in relation to the poverty profile. In the event that there is no curvature in the poverty profile curve or when the curved segment varies little in a relative sense, the poverty box can convey all the information needed for poverty comparisons.

Figure 1 Geometry of the Poverty Profile

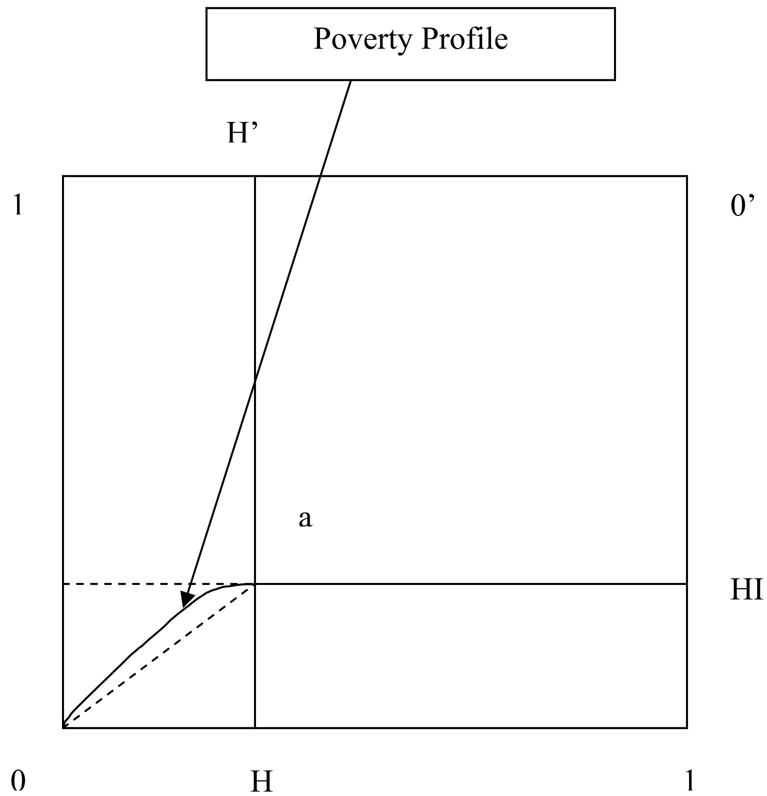
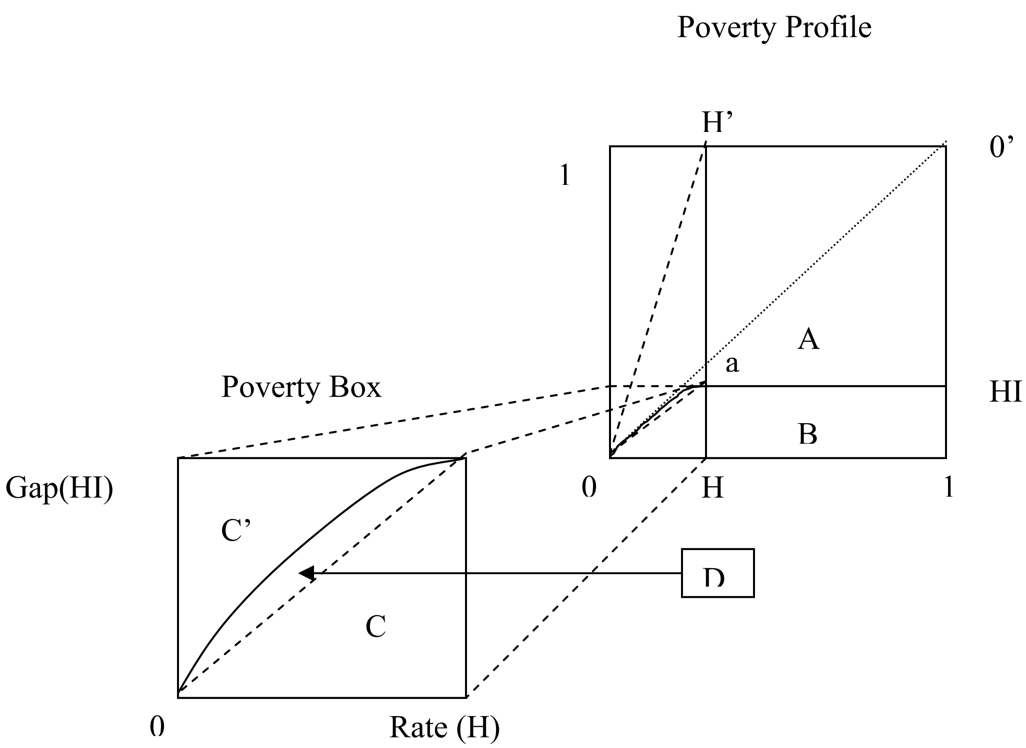


Figure 2 Geometry of the Sen index and Poverty Box



As shown in Osberg and Xu (1999, 2000), the SST index [equation (2.6)] proposed by Shorrocks (1995) following Sen (1976), can be simplified into

$$S_{SST} = HI(1 + G(x)), \qquad (2.12)$$

where x represents the poverty gap ratios of the total population. A less mathematical expression of the above is

The SST Index = [the poverty rate] $\times$ [the average poverty gap ratio of the poor] $\times$ [the inequality of poverty gap ratios of the population].

As shown previous for the Sen index, the SST index also measures the welfare loss caused by poverty and it can measure poverty incidence, depth and inequality *jointly* while permitting the SST index to be decomposed into commonly used poverty measures. The difference between the Sen and SST indices is the Gini index of poverty gap ratios. Unlike $G(x_p)$ which is in the Sen index and can take value zero when all the poor are equally poor – i.e. have the same poverty gap ratio, $G(x)$, which is in the SST index, cannot be zero simply because even if the poor are equally poor the nonpoor have zero poverty gap ratios. As shown in Xu and Osberg (2002, p. 145, equation 24), $G(x) = 1 - H$ when the poor have an identical poverty gap ratio. For example, if the poverty rate is 15% and the poor are equally poor, the Gini index of poverty gap ratios of the population will be 1-0.15 = 0.85. The inequality component in the SST index will then be $1 + G(x) = 1 + 0.85 = 1.85$. Any inequality in poverty gap ratios among the poor will add to [1 + $G(x)$] but with an upper bound value 2 – so there is a fairly narrow possible range, particularly if the poverty rate is relatively low.

The "common sense" explanation for the small role that inequality among the poor plays in an aggregate measure of poverty intensity is that the differences in income among the poor are relatively small when compared to income differences among the nonpoor. The upper bound on the incomes of poor people is the poverty line. The lower bound, leaving aside measurement error aside, is subsistence. The money value of the difference is not large, particularly when compared to the differences among the nonpoor population.

When the inequality of poverty gap ratios of the population changes little over time and across countries/regions/social groups, the value of the SST index is in proportion to ($\propto$) the product of the poverty rate and the average poverty gap ratio of the poor; that is

$$\text{The SST Index} \propto [\text{the poverty rate}] \times [\text{the average poverty gap ratio of the poor}].$$

To a logarithmic approximation, the percentage change in the SST Index is then equal to the sum of the percentage changes in the poverty rate and the average poverty gap ratio of the poor.

The geometric interpretation of the SST index, according to Xu and Osberg (2002) is as follows. Let the lower triangle of the poverty profile box in the upper-right panel of Figure 2 surrounded by O, O', HI, 1, and H be Area A, which is the half of the unit box. The Sen index is the ratio of the sum of Areas B, C, and D to Area A. Hence, the poverty box is directly connected to the poverty profile.

For both Sen and SST indices[16], it appears that the inequality of the poor in developed countries is fairly constant, and thus plays little role in comparisons – either internationally or over time [Osberg and Xu (2000)].). Hence a two-dimensional poverty box can present poverty reasonably accurately and can be used for across country/region/social group comparisons. The "poverty box" is formed by the poverty rate *H* and the average poverty gap ratio of the poor *I* [see Xu and Osberg (2001) and Osberg (2004)].

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[16] The Sen and SST indices are closely related. According to Xu and Osberg (2002),

$$S_{SST} = HS + 2H(1-H)I.$$

That is, given H and I, it is always possible to compute S_{SST} from S and vice versa. For example, if we know S_{SST}, H, and I based on the data, we can compute S using

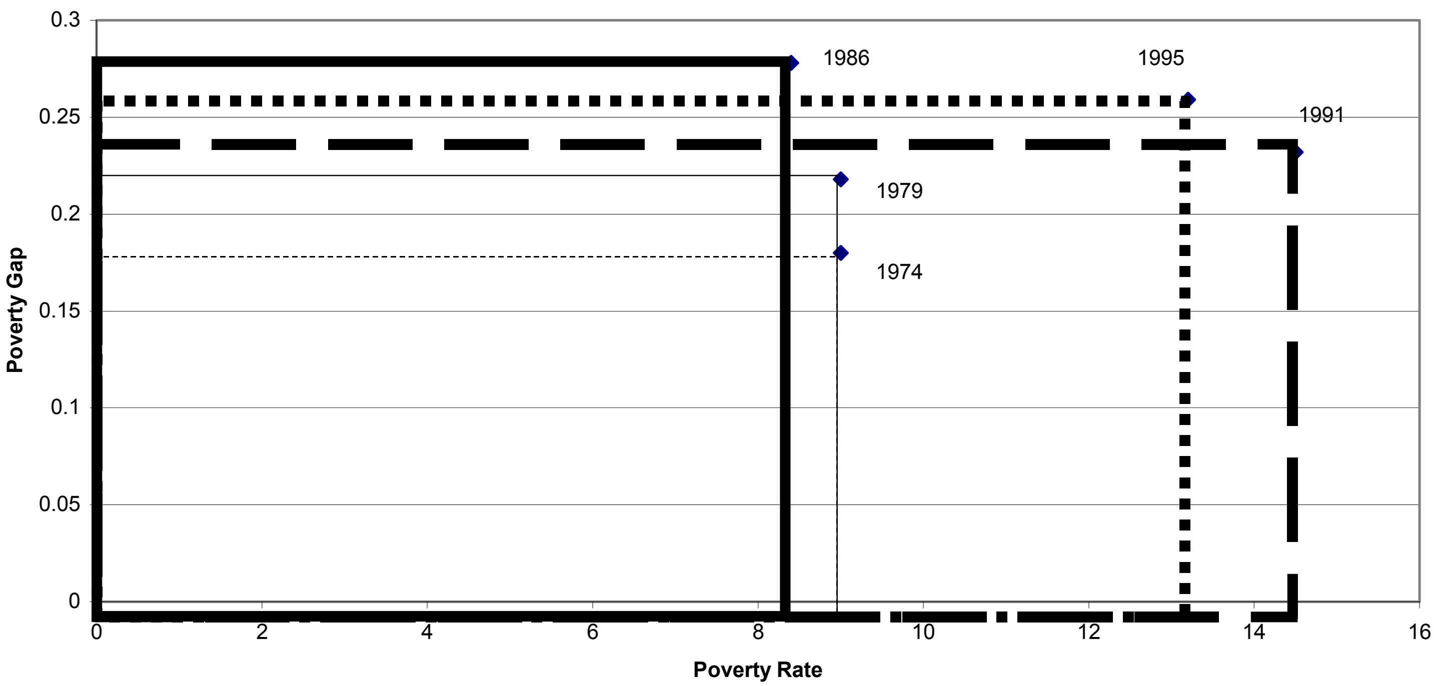
$$S = \frac{S_{SST} - 2H(1-H)I}{H} = \frac{S_{SST}}{H} - 2(1-H)I.$$

What does the "poverty box" add to the debate? Figure 3, which is taken from Osberg (2004) illustrates its potential usefulness for comparisons of poverty in the context of the United Kingdom, where the average poverty gap ratio and the poverty rate moved in different directions over time. An assessment of poverty policy in the UK which looked only at the poverty *rate* would score the 1979 to 1986 period as a success, since the poverty rate fell (from 9 % to 8.4 %), but would miss completely the significant increase in the average poverty gap of the poor (which rose from 21.8 % of the poverty line to 27.8 %). This divergence between trends in the poverty rate and average poverty gap ratio is not uncommon in developed nations [see Osberg (2002, p.18)], but is crucial for the assessment of poverty policy "success". If there is little change in inequality among the poor, the "poverty box" represents the welfare loss clearly and is precisely the poverty gap ratio index ($H \times I$) advocated by the United Nations as the secondary indicator of poverty. Having established the link between the poverty box and the Sen and SST indices, the remaining question for this paper is whether or not the poverty box approach can be effectively used in analyzing poverty in developing countries – and to assess this issue we turn to evidence from China.

Figure 3 The Poverty Box for the United Kingdom in 1974, 79, 86, 91, and 95

Poverty Line = ½ Median Equivalent Disposable Income

Source: Osberg (2004).



3. How Should We Measure Poverty Trends in China?

In assessing the level and trend of global poverty, a crucial variable stands out in importance – the rate of growth of the Chinese economy. With 1.29 billion citizens, roughly 20% of the world's population, China dominates global poverty trends in Asia and the world – and in recent years, the Chinese economy has been growing strongly.[17] In 1980, GDP per capita in China was $708, (PPP, constant 1995 international $[18]), but by 2003 that had risen six-fold to $4,344.[19] Over the 1995-2003 period, the average annual growth rate of per capita GDP was 7.55 %. Extrapolation of these recent trends would imply that in twenty years, in 2023, per capita GDP in China would be about $20,000 in PPP terms – a level of income that is well above the income levels in Europe at the time when a "relative income" conception of poverty became recognized as appropriate.[20]

At current exchange rates, the US dollar value of China's per capita GDP is far lower – at $1,024 in 2003. Clearly, when the ratio between PPP and the exchange rate is of the order of 4:1, adjustment for PPP has an enormous impact on the estimated level of average real income of 1.29 billion people. In fact, the calculation of PPP values can be done in a number of ways – each with its own advantages and disadvantages. Hill (2000, p.294) has compared the range of estimates of PPP adjusted average income levels that thirteen available methodologies imply, noting that calculated average income ratios can nearly double, depending on PPP methodology chosen.[21] Hence, one has to worry that estimates of the extent of global poverty are extremely sensitive to very technical choices about PPP methodology – particularly since the income distribution is typically very dense in the region of the poverty line, implying that small changes in the calculation of the poverty line can affect the measured poverty status of fairly large fractions of the population. The technical uncertainties involved in PPP calculations, and their enormous impact on poverty measurements, are a strong argument for the use of a relative income criterion of the

[17] India's 1.06 billion inhabitants (and faster rate of population growth), and lower level of average income (in 2003, GDP per capita of $2529 – PPP, constant 1995 international $) mean that India's growth rate is also crucial to global poverty trends – between 1995 and 2003, growth in GDP per capita in India averaged 4.3 %, with strong acceleration in most recent years (8% GDP growth in 2003).

[18] Unless otherwise noted, all aggregate data in this section are PPP constant 1995 $, drawn from the World Bank web site - http://devdata.worldbank.org/dataonline/

[19] To put this in context, the comparable per capita PPP GDP of Portugal was at $7,499 in 1975.

[20] The GDP per capita (PPP, constant 1995 international $) of Canada was $23,842 in 2003.

[21] For example, although (when evaluated at observed exchange rates) the ratio of per capita income in the USA in 1990 to that of Turkey was 8.1 to 1, the range of PPP income ratios was between 3.3 to 1 and 6.4 to 1 (with the Geary-Khamis price index method favoured by the ICP project generating a ratio of 3.7 to 1).

poverty line, measured in own currency units – on the grounds of transparency and robustness. As well, developing countries such as China are moving rapidly from the group of nations in which absolute poverty is the key concern to the group of countries in which relative poverty will be in the spotlight. While it is still possible to continue to calculate the absolute $1 per day poverty line, the concept of relative poverty is becoming steadily more socially relevant.

The usual methodology for international comparisons of poverty among developed countries is to use micro-data on the incomes of individual households (from a data set such as the Luxembourg Income Study) in order to calculate the equivalent income of individuals and to draw the poverty line relative to median equivalent income –most commonly at 50% of median individual equivalent income. Typically, all individuals within households are assumed to share equally in household resources, and have no claim on the resources of other households.[22] The LIS definition of total family money income after tax (disposable income)[23] is often used as the basis for calculation of the after tax money "equivalent income" of all individuals within families. The concept of equivalent income is used to reflect the fact that members of larger households can benefit from economies of scale in their consumption expenditure. In the literature, a number of equivalence scales have been used to account for the economies of scale of household consumption [see Burkhauser et al. (1996), and Phipps and Garner (1994), among others] but recent literature[24] has predominantly used the LIS equivalence scale, which calculates the equivalent income of each household member as:

$$y_i = \frac{y_f}{n_f^{\,0.5}} \qquad (3.1)$$

where y_f is total household income after tax,[25] and n_f is the number of persons in the household.[26]

[22]Admittedly, these are strong assumptions about the social context of income flows since the effective resources available to each person depend on the degree of inequality in the intra-household distribution of consumption. See Phipps and Burton (1995, p. 194).

[23]Disposable income consists of the sum of gross wages and salaries, farm self-employment income, non-farm self-employment income, cash property income, sick pay, disability pay, social retirement benefits, child or family allowances, unemployment compensation, maternity pay, military/veteran/war benefits, other social insurance, means-tested cash benefits, near cash benefits, private pensions, public sector pensions, alimony or child support, other regular private income, and other cash benefits; minus mandatory contributions for self employed, mandatory employee contribution, and income tax.

[24]See, for example, Buhmann et al. (1988), Coulter et al. (1992), Burkhauser et al. (1996), and Figini (1998) for comparison of the LIS, OECD and other equivalence scales. Figini (1998, p. 2) notes that "OECD and other two-parameter equivalence scales empirically used show a similarity of results [in measurement of inequality] to one parameter equivalence scales with elasticity around 0.5."

[25]"Disposable Personal Income" in the LIS data sets.

This methodology lies behind the poverty estimates for the UK discussed in Section 2 (and much of the broader literature on poverty in affluent nations), but this paper started with a discussion of global poverty trends using an absolute poverty line concept (specified as the local currency equivalent, in purchasing power parity terms, of US $1 per day). How does the relative poverty line methodology compare with the absolute US $1 standard for China in 1995?

To assess this we use data from the 1995 CHINESE HOUSEHOLD INCOME PROJECT (1995 CHIP)[27] whose purpose was to measure and estimate the distribution of personal income in both rural and urban areas of the People's Republic of China. The concept of "income" used was considerably broader than that used in most studies of OECD nations - it included both cash payments and a broad range of additional components: payments in kind valued at market prices, agricultural output produced for self-consumption valued at market prices, the value of food and other direct subsidies, and the imputed value of housing services.[28]/[29] Although calculation of the value of in kind or own account self-production is arguably an appropriate adjustment to the context of rural China, none of the nations whose data is included in the Luxembourg Income Study make an imputation of

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[26] Note two important special cases of the "equivalent income" calculation $y_i = \frac{y_f}{n^\alpha}$ .Household income is assigned to each individual if α = 0 and per capita income is assigned if α = 1.

[27] Riskin, Carl, Zhao Renwei, and Li Shi. CHINESE HOUSEHOLD INCOME PROJECT, 1995 [computer file]. ICPSR version. Amherst, MA: University of Massachusetts, Political Economy Research Institute [producer], 2000. Ann Arbor, MI: Inter-university Consortium for Political and Social Research [distributor], 2000. The Chinese Household Income Project is a joint research effort sponsored by the Institute of Economics, Chinese Academy of Social Sciences, the Asian Development Bank and the Ford Foundation. Additional support was provided by the East Asian Institute, Columbia University.

[28] Disposable rural household income = Income from wages pensions and other compensations received by individual members of the household + Household income from township, village, collective and other types of enterprise (other than compensation for labor) + Cash income from farming and industrial and subsidiary activities + Gross value of self-consumption of farm products + Income from property + Rental value of housing equity + Net transfer from/to collective and state entities + Miscellaneous income (including private transfer) + Net cash income from the sale of farm products + Net income from non-farm subsidiary activities.

[29] Disposable urban household income = Cash income of the working members + Income of the retired members + Income of the non-working members + Income from private/individual enterprises + Income from property + Miscellaneous income (including private transfer and special income) + Subsidies less taxes (except housing subsidy and ration coupon subsidy) and income in kind + Ration coupon subsidy + Housing subsidy + Rental value of owner occupied housing equity.

the rental value of owner occupied housing.[30] Thus, maintaining a comparable estimate of poverty implies similarly disregarding the imputed value of housing services.

The 1995 CHIP dataset is based on a survey of 7,998 rural households (together representing 34,739 individual household members) in 19 provinces plus 6,931 urban households (with 21,698 members) in 11 provinces. Eliminating observations with negative incomes produces 7,988 rural and 6,929 urban households. Table 1 presents estimates, based on one half the median equivalent income (in local currency) as the poverty line, of the SST index, poverty rate, average poverty gap ratio, and inequality of poverty gap ratios. The top panel uses the comprehensive definition of income, while the bottom panel excludes the imputed value of owner occupied housing.

If the comprehensive definition of income is adopted, then half the median equivalent income is 2,555 Yuan (Renminbi). At the official exchange rate of 8.28 Yuan per US $1, this is equivalent to a poverty line of US $308.57, or US $0.85 per day. However, excluding the imputed value of owner occupied housing implies that half the median income is 2289 Yuan, which is equivalent to $276.44 per year ($ 0.76 per day) at official exchange rates. Clearly, however, the official exchange rate is a poor guide to relative purchasing power. If the PPP exchange rate is 1.9 Yuan per US $1,[31] this implies that calculating a relative poverty line of half the median equivalent income produces a poverty line equivalent to $1,344 per year ($3.68 per day) using the comprehensive income concept, or $1,204 per year ($3.30 per day) excluding the imputed value of home ownership. In 1995, therefore, a relative poverty line would be set substantially above the $1 or $2 absolute standard.

Of course, *if* incomes at the bottom end of the income distribution in China were to have grown over the period 1995 to 2003 at the same 7.55 % rate as per capita GDP, a person earning $1 in 1995 would be making $1.83 in 2003, and someone making $2 per day in 1995 would make $3.66 in 2003. Hence, a relative poverty line of one half median equivalent income in 1995 is, in absolute terms, about what somebody who was just at the $2 per day income level in 1995 would now be making, *if* their incomes had grown at the national average rate – *which implies that in China in 2003 a relative poverty line may not actually have been so different from an absolute ($2 per day) poverty line, in*

[30] The method used in the 1995 CHIP is to assume an 8% return on the respondent-estimated value of home equity.

[31] See World Bank, *World Development Indicators 2003*, Pages 282-285.

practice. [Of course, one clear concern about the path of China's development is precisely this assumption – that people at the bottom of the income distribution are sharing in the benefits of economic growth.]

Implicitly, the use of a common national poverty line criterion for poverty measurement in developed countries is based on the idea that the nation as a whole is the relevant comparison group for the assessment of interpersonal equity. The motivation for this idea is not really a sociological presumption that individuals in all parts of the nation actually compare themselves with each other – China is a vast country, and survey evidence indicates that interpersonal comparisons tend to be highly local, in all countries.[32] However, the nation state is the political entity within which redistribution of income, or other forms of anti-poverty policy might conceivably occur – and it is the political unit within which any expressions of political discontent with poverty outcomes will primarily be managed.

If a common national poverty line is used, one clear implication of Table 1 is the concentration of poverty in China in rural areas. Focussing on the lower panel of Table 1, we see that by this definition of the poverty line, the SST index of poverty in urban areas is approximately 18 times larger in rural areas than in urban China (0.1180 compared to 0.0065) – not primarily because the depth of poverty in rural areas is so much greater (the average rural poverty gap is 0.309, compared to an average urban poverty gap of 0.255) but because the rate of poverty is so very much higher (32.3 % in rural areas, compared to just 1.3% in urban areas). The poverty box for the information in Table 1 is given in Figure 4. As can be seen in Figure 4, the divide between the rural and urban China is huge.

Table 2 shows that if rural and urban China are analyzed as if they were separate countries, and if the urban poverty line were drawn at half the median equivalent income *of urban areas*, while the rural poverty line is similarly drawn at half the median equivalent income *of rural areas*, the poverty line would be set over twice as high in urban areas (3862 Yuan) as in rural China (1527 Yuan). Interestingly, the level of poverty in rural China would still be twice as high as in urban areas (a rural SST index of 0.072, compared to an urban index of 0.036). This is again illustrated clearly in Figure 5, which shows the differences between the poverty box between rural and urban China when each is evaluated by its own poverty standards.

[32] See Kluegel et al (1995:20) or Evans and Kelley (2003)

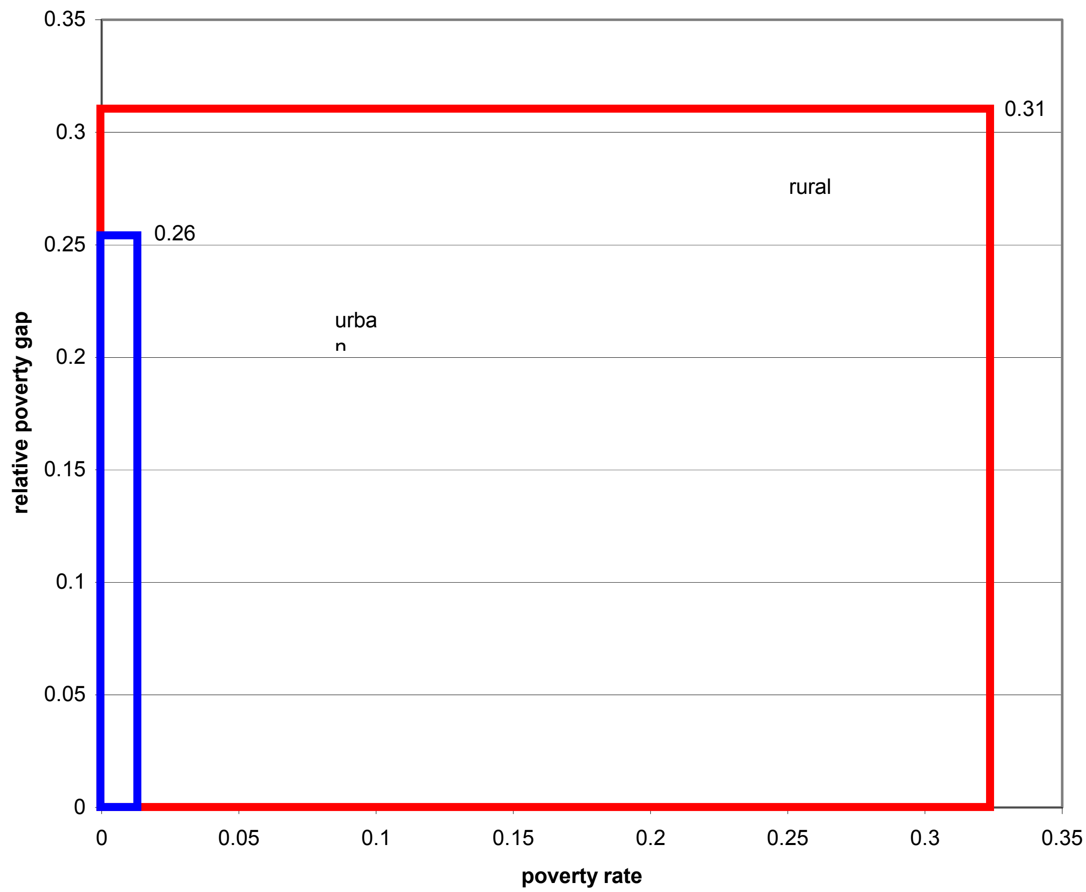
However, since the CHIP data go to some lengths to account for possible sources of in-kind income that might reduce the money cost of living in rural areas, there seems to be little technical reason why rural and urban incomes cannot be compared. If Chinese society comprises a common polity, the application of a common national poverty line to both urban and rural China therefore seems defensible. Table 3 therefore compares the SST index of poverty across the rural areas of the sampled provinces of China. Even leaving aside the capital region Beijing, because of its absolutely low fraction of rural dwellers, there is a huge range of variation in the SST index of poverty – with large differences across provinces in all three components of the SST index. As Table 3 indicates, the rural poverty rate (excluding Beijing) is as high as 61.9 % and as low as 9.7%. The average rural poverty gap ranges from about 38.9% to about 7% of the poverty line. These differences – of the order of a 5:1 ratio – are huge, so large as to swamp the observed differences in inequality of the poverty gap in the population – which varies between 1.567 and 1.962. The variation in $(1+G(x))$ across the rural areas of Chinese provinces are relatively large compared to the variation observed across other data sets in developed countries, but small compared to the variation in poverty rate of poverty gap – which may indicate the usefulness of the poverty box emphasis on poverty incidence and depth of poverty.

Table 1
SST and Components
China 1995
Poverty Line = 1/2 the Median for the Country

Region	Poverty Line (½ Median Equivalent Income)	SST	Poverty Rate	Relative Poverty Gap	1 + Gini of Gap	Number of Poor Observations
Income Includes Imputed Return Owner Occupied Housing						
All	2555	0.100	0.189	0.282	1.886	2474
Urban	2555	0.0063	0.014	0.225	1.993	94
Rural	2555	0.154	0.298	0.283	1.818	2380
Income Excludes Home Wealth						
All	2289	0.118	0.204	0.309	1.875	2677
Urban	2289	0.0065	0.013	0.255	1.993	86
Rural	2289	0.180	0.323	0.310	1.801	2591

Figure 4 The Poverty Box China 1995: Urban and Rural

Poverty Line 1/2 Median for Country

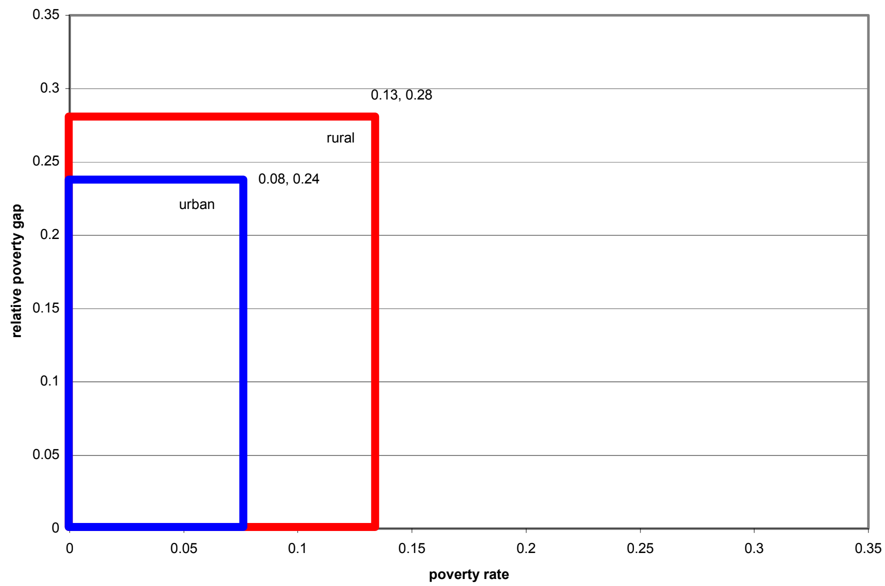


note: Income excludes home wealth.

Table 2
SST and Components
China 1995
Poverty Lines: 1/2 the Urban Median for the Urban; 1/2 Rural Median for the Rural

Region	Poverty Line (½ Median Equivalent Income)	SST	Poverty Rate	Relative Poverty Gap	1 + Gini of Gap	Number of Poor Observations
Income Includes Imputed Return Owner Occupied Housing						
Urban	4159	0.033	0.073	0.230	1.958	494
Rural	1753	0.057	0.120	0.245	1.931	974
Income Excludes Home Wealth						
Urban	3862	0.036	0.076	0.238	1.956	515
Rural	1527	0.072	0.133	0.281	1.924	1084

Figure 5 The Poverty Box China 1995: Urban and Rural Comparison
Poverty Lines: 1/2 Median for Urban; 1/2 Median for Rural



Note: Income excludes home wealth.

Table 3 SST and Components Rural China 1995 by Province Poverty Line = 1/2 the Median for the Country (including urban) Income Excludes Home Wealth						
Region	Poverty Line (½ Median Equivalent)	SST	Poverty Rate	Relative Poverty `Gap	1 + Gini of Gap	Number of Poor Observations
11 - Beijing	2289	0.023	0.021	0.558	1.985	2
13 - Hebei	2289	0.184	0.312	0.328	1.801	159
14 - Shanxi	2289	0.342	0.559	0.373	1.643	166
21- Liaoning	2289	0.166	0.288	0.316	1.820	92
22 - Jilin	2289	0.146	0.253	0.312	1.848	75
32 - Jiangsu	2289	0.303	0.220	0.070	1.962	36
33 - Zhejiang	2289	0.052	0.129	0.210	1.918	53
34 - Anhui	2289	0.117	0.247	0.256	1.853	112
36 - Jiangxi	2289	0.108	0.252	0.231	1.852	88
37 - Shandong	2289	0.142	0.249	0.307	1.850	178
41 - Henan	2289	0.129	0.271	0.258	1.847	203
42 - Hubei	2289	0.194	0.279	0.381	1.828	111
43 - Hunan	2289	0.229	0.412	0.319	1.741	204
44-Guangdong	2289	0.059	0.097	0.310	1.946	46
51 - Sichuan	2289	0.248	0.485	0.301	1.697	388
52 - Guizhou	2289	0.272	0.547	0.301	1.657	165
53 - Yunnan	2289	0.215	0.472	0.268	1.701	146
61 - Shanxi	2289	0.308	0.578	0.328	1.625	177
62 – Gansu	2289	0.378	0.619	0.389	1.567	190

Since one of the purposes of poverty measurement is to rank the severity of the problem of poverty in different jurisdictions, one can ask to what extent using the "normalized poverty gap" (FGT1) or poverty box concept[33] will alter the ranking of rural poverty among Chinese provinces based on the SST index and to what extent using higher order FGT indices would alter the ranking based on the SST index. As noted earlier, in equation (2.7), the Sen and SST indices are well justified poverty measure which contain, as their components, both the FGT index of order 0 - the poverty rate (H)- and -the FGT index of order 1 - the average poverty gap of poor people (I) times the poverty rate (H).

Table 4 reports, for each province of China whose data is available in the CHIP, the computed level of rural poverty, using as measure the SST index and FGT $\alpha = 0 \ldots 6$. One way of evaluating the loss of information entailed by using the poverty box is to see how much the ranking of the provinces based on the SST index is altered by using the poverty box (the poverty rate times poverty gap or FGT1). Similarly, one way of thinking about how much the FGT index of a higher order (i.e. FGTα when $\alpha > 1$) matters relative to the benchmark SST index is to see how much the poverty ranking of the provinces based on the SST index can be altered by the FGT index of a higher order ($\alpha > 1$). As explained previously, the FGT index of a higher order can be interpreted as giving larger poverty gaps higher weights in the weighted sum in the FGT index so as to incorporate more aversion to poverty inequality.

For many purposes it is not so much the absolute, but the comparative, level of a poverty index that matters - for example, if one wanted to know which province of China has the greatest problem of rural poverty. Since the various poverty indices discussed thus far have different ranges, it is not very informative to compare their numeric value – so this paper adopts the "Linear Scaling Technique (LST)" to standardize the range of all poverty measures. To do this, the high and low observed values are taken to represent the possible range of a poverty measure for all countries, and denoted Min and Max, respectively. The data is then scaled according to

[33] Although the FGT1 does not satisfy the transfer axiom, it does possess the socially desirable property of easy comprehensibility and subgroup decomposability and is equal to the area of the poverty box.

these values. Figure 6 then reports for each province[34] the value for each poverty index scaled according to the formula $(value - \min)/(\max - \min)$.

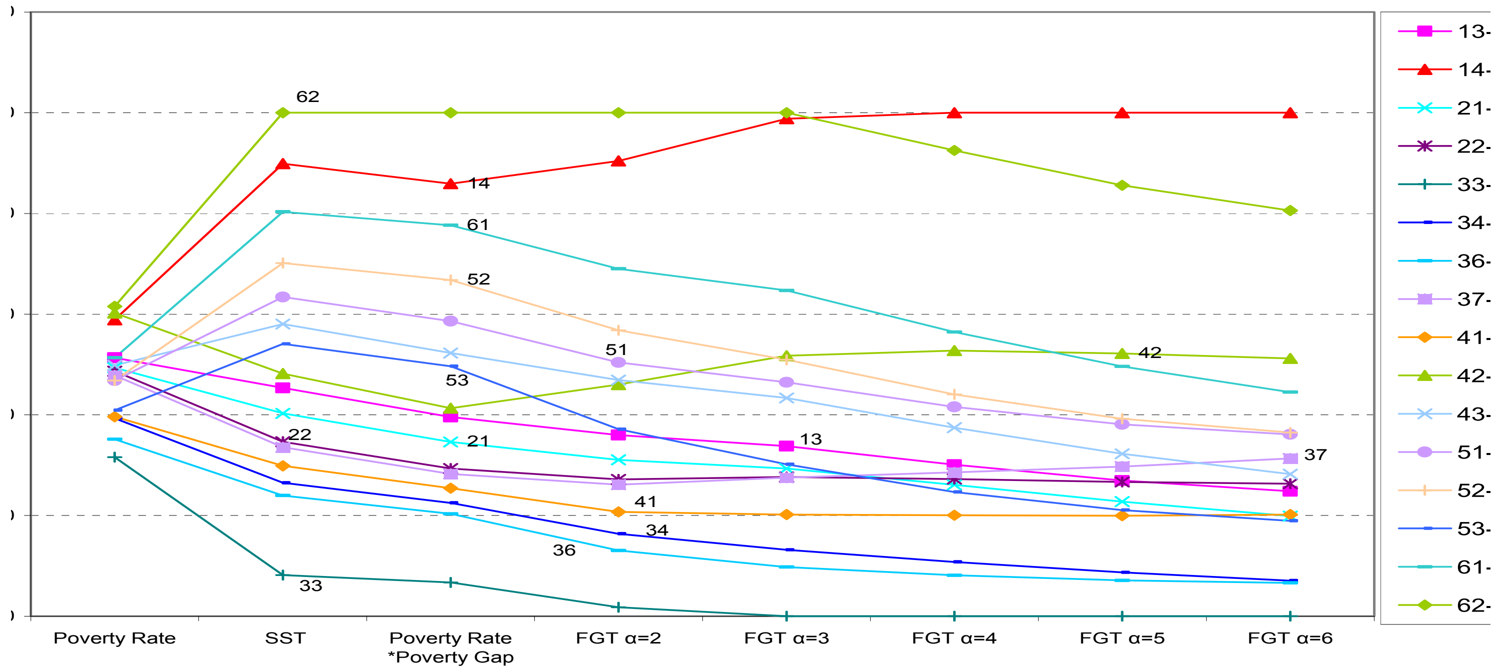
Figure 6 indicates that the ranking of the provinces based on the poverty rate is very different from the ranking based on the benchmark SST index. As an example, Yuanan Province (code 53) was ranked the 9th in poverty intensity based on the poverty rate but is 7th according to the SST index. However, the ranking of the provinces based on the poverty box is consistent with the ranking based on the benchmark SST index – which indicates that the poverty box is a good approximation of the benchmark SST index.35 The FGT indices of order higher than 2 give increasing weights to inequality in poverty and hence may change the ranking of provinces based on the benchmark SST index substantially, in particular among the middle ranked provinces. A case in point again is Yuanan Province (code 53), which has the most prominent decline in ranking as one increases the order of the FGT index. Yuanan province is ranked the 7th in poverty intensity based on both the SST index and poverty box. But as one increases the order of the FGT index from 2 to 6, Yuanan province experiences a rapid decline in the poverty ranking to the 8th, 9th, 11th, 11th, and 13th, respectively. However, for the most poverty-stricken provinces such as Gansu (code 62) and Shanxi (code 14) and the least poverty-stricken provinces such as Zhejiang (code 33) and Jiangxi (code 36), the higher order FGT indices do not provide any additional information in terms of relative rankings to those based on the benchmark SST index. Hence, Figure 6 can be read as indicating that there is relatively little gained in inter-provincial poverty comparisons if one uses “higher order” [FGT $_{\alpha = 2 \ldots 6}$] poverty indices.

[34] More exactly, Figure 6 presents data for provinces included in the CHIP data with 50 or more poor observations.
[35] Although this paper does not report the ranking based on the Sen index, it is shown in Footnote 16 of this paper that when I and H are known, the Sen index and SST index have a one-to-one correspondence relationship. Hence, both Sen and SST indices can be used as benchmarks.

Table 4
Comparison: SST and Components versus FGT indices of order 1 to 6
Rural China 1995 by Province
Poverty Line = 1/2 the Median for the Country (including urban)
Income Excludes Home Wealth

				FGT					
Province	poverty rate	poverty gap	SST	alpha=2	alpha=3	alpha=4	alpha=5	alpha=6	alpha=1 rate*gap
11-Beijing	0.021	0.558	0.023	0.0085	0.0067	0.0053	0.0042	0.0034	0.012
13-Hebei	0.312	0.328	0.184	0.0472	0.0254	0.0152	0.0099	0.0070	0.102
14-Shanxi	0.559	0.373	0.342	0.1094	0.0687	0.0479	0.0355	0.0274	0.209
21-Liaoning	0.288	0.316	0.166	0.0415	0.0224	0.0134	0.0085	0.0056	0.091
22-Jilin	0.253	0.312	0.146	0.0371	0.0213	0.0139	0.0098	0.0074	0.079
32-Jianhsu	0.070	0.220	0.303	0.0060	0.0031	0.0019	0.0012	0.0008	0.015
33-Zhejiang	0.129	0.210	0.052	0.0081	0.0029	0.0012	0.0005	0.0002	0.027
34-Anhui	0.247	0.256	0.117	0.0247	0.0117	0.0062	0.0036	0.0022	0.063
36-Jiangxi	0.252	0.231	0.108	0.0209	0.0094	0.0050	0.0030	0.0020	0.058
37-Shandong	0.249	0.307	0.142	0.0359	0.0212	0.0145	0.0109	0.0087	0.076
41-Henan	0.271	0.258	0.129	0.0297	0.0164	0.0105	0.0075	0.0057	0.070
42-Hubei	0.279	0.381	0.194	0.0586	0.0374	0.0258	0.0188	0.0141	0.106
43-Hunan	0.412	0.319	0.229	0.0597	0.0318	0.0187	0.0118	0.0079	0.131
44-Guangdong	0.097	0.310	0.059	0.0155	0.0099	0.0071	0.0055	0.0045	0.030
51-Sichuan	0.485	0.301	0.248	0.0636	0.0338	0.0206	0.0138	0.0100	0.146
52-Guizhou	0.547	0.301	0.272	0.0710	0.0368	0.0217	0.0142	0.0101	0.165
53-Yunnan	0.472	0.266	0.215	0.0485	0.0230	0.0127	0.0079	0.0054	0.125
61-Shanxi	0.578	0.328	0.308	0.0849	0.0460	0.0275	0.0179	0.0123	0.190
62-Gansu	0.619	0.389	0.378	0.1204	0.0695	0.0444	0.0305	0.0221	0.241
minimum	0.021	0.210	0.023	0.0060	0.0029	0.0012	0.0005	0.0002	0.012
maximum	0.619	0.558	0.378	0.1204	0.0695	0.0479	0.0355	0.0274	0.241

**Figure 6 Comparison: Poverty Rate, SST Index and FGT index
Rural China 1995 by Province
Poverty Line = 1/2 the Median for the Country (including urban)
Income Excludes Home Wealth**



4. Statistical Issues

However, observant readers may have noted the right hand side column of Table 3, which reports the number of poor observations, by province. In many provinces, the sample is absolutely small – and it is being used to portray the outcomes experienced by millions.[36] For a theoretically desirable poverty measure to be applied in the real world where we rely on sample data, it is necessary to examine the statistical properties of the sample in order to implement appropriate statistical procedures.[37] In this section, asymptotic theoretical results for the poverty measures advocated here are reviewed first and then the bootstrap method is discussed.

The poverty measures advocated in this paper include the Sen and SST indices of poverty intensity and their components - the poverty rate (the FGT index of order zero), the average poverty gap ratio of the population (the FGT index of order one), the poverty gap ratio of the poor (the ratio of the FGT index of order one to that of order zero), the Gini index of poverty gap ratios of the poor or of the population (and hence 1 plus the Gini index of poverty gap ratios of the poor or of the population). These poverty measures, whether used as an aggregate (the Sen or SST index) or as a component (the poverty rate, or the average poverty gap, or 1 plus the Gini index), have desirable statistical properties. For income or poverty data, we will primarily rely on the asymptotic theories although some of the measures have good finite sample properties.

The poverty rate H is essentially a proportion of a distribution. If we use the sample to estimate H assuming that q is the number of the poor in the entire sample of size n,[38] then the point estimator,

$$\hat{H} = \frac{q}{n}, \tag{4.1}$$

is a consistent estimator of the true, but unknown, H.[39] The large sample variance of $\hat{H}$ is

[36] In Figure 6, provinces with less than 50 observations of rural poverty are not plotted.
[37] See Osberg and Xu (2001) for the bootstrap illustration. The statistical procedures proposed here will be implemented in another project.
[38] For simplicity, we use $\{y_1, y_2, \ldots, y_n\}$ as the sample and z as the poverty line for the sample data.

$$Var(\hat{H}) = \frac{H(1-H)}{n}. \tag{4.2}$$

Since H can be consistent, $Var(\hat{H})$ can be estimated consistently as well by replacing H with $\widehat{H}$ in equation (4.2). In addition, with some minor modification, equations (4.1) and (4.2) can be used for a sample with sampling weights.

The average poverty gap ratio of the poor is a mean concept, which can be estimated by

$$\hat{I} = \frac{1}{q}\sum_{x_i>0} x_i = \frac{1}{q}\sum_{y_i<z}\left(\frac{z-y_i}{z}\right). \tag{4.3}$$

This estimator $\hat{I}$ is also consistent with a variance

$$Var(\hat{I}) = \frac{1}{(q-1)}\sum_{x_i>0}\left(x_i - I\right)^2. \tag{4.4}$$

Again, $Var(\hat{I})$ can be estimated consistent by replacing I with $\widehat{I}$ given $\hat{I}$ is a consistent estimator. Equations (4.3) and (4.4) can be modified for a sample with sampling weights. Sometimes, researchers are interested in the average poverty gap ratio of the population. It can be estimated by $\widehat{HI} = \frac{1}{n}\sum_{x_i>0} x_i = \frac{1}{n}\sum_{y_i<z}\left(\frac{z-y_i}{z}\right)$ with $\widehat{Var}(\widehat{HI}) = \frac{1}{(n-1)}\sum_{x_i>0}\left(x_i - \widehat{HI}\right)^2$. These results can be found in Kakwani (1993).

For the term of 1 plus the Gini index, we can discuss the Gini index first. The probably the earliest contribution was made by Hoeffding (1948) based on the U-statistics. Following Glasser (1962) developed the variance for the Gini index based on the definition of the Gini index based on the Gini's relative mean difference [Xu (2003)]. Note that this paper contains two versions of the Gini index: one is the Gini index of poverty gap ratios of the poor, $G(x_p)$ and the other is the Gini index of poverty gap ratios of the population, $G(x)$. Since these two indices are closely related by

$$G(x) = (1-H) + HG(x_p), \tag{4.5}$$

[39] Note that under the binomial distribution, the estimator is unbiased and has the same variance.

we can simply focus on $G(x_p)$.

Among many other approaches, Glasser (1962) proposed the Gini index estimator and its variance estimator. Let the poverty gap ratios of the poor be $\{x_1, x_2, \ldots, x_q\}$,

$$d_i = \frac{1}{q-1}\sum_{i=1}^{q} | x_i - x_j |, \;\; for \;\; i \neq j. \qquad (4.6)$$

and

$$d = \frac{1}{q}\sum_{i=1}^{q} d_i. \qquad (4.7)$$

Let $\bar{x}_p$ be the sample mean of the poverty gap ratios of the poor $\{x_1, x_2, \ldots, x_q\}$. Based on these definitions, the Gini index of poverty gap ratios of the poor can be estimated by the following estimator

$$\widehat{G}(x_p) = \frac{d}{2\bar{x}_p}. \qquad (4.8)$$

The large sample variance of the above estimator is given by

$$\widehat{Var}\left(\widehat{G}(x_p)\right) = \frac{1}{4\bar{x}_p}\left[\widehat{Var}\left(d\right) - 4\widehat{G}(x_p)\widehat{Cov}(d, \bar{x}_p) + 4[\widehat{G}(x_p)]^2 \widehat{Var}(\bar{x}_p)\right]. \qquad (4.9)$$

The term $1 + G(x_p)$ can be estimated by $1 + \widehat{G}(x_p)$. The variance of $1 + \widehat{G}(x_p)$ is the same as given in equation (4.9). The estimator of the Gini index of poverty gap ratios of the population, $G(x)$, and its variance can be developed similarly. Based on equation (4.5) and the assumption that H is fixed, $\widehat{Var}\left(\widehat{G}(x)\right) = \widehat{H}^2 \widehat{Var}\left(\widehat{G}(x_p)\right)$ can be employed.[40]

However, as may be noted, the above discussion is limited to the components of the Sen and SST indices (that is, the FGT indices of order 0 and 1 and the Gini indices of different kinds). In order to explain the statistical inference issues of the Sen and SST index one needs to look beyond the above basic statistical results. Following Hoeffding (1948), Zheng (1993), Bishop et al. (1997, 1998, and 2001), and Zheng et al. (2000) employed the U-statistics to develop the statistical inference for the Sen index and some other extensions. Xu (2004) also

[40] Note that for simplified methods to estimate the standard error of the Gini index, please see Ogwang (2000), Giles (2004), and Ogwang (2004).

worked the statistical inference for the SST index and a uniform framework for all statistics discussed above. The following is a brief presentation of the relevant results in Xu (2004).

In order to apply the U-statistics to the Sen and SST indices, first introduce some U-statistics. Let $I(A)$ be an indicator function: $I(A)=1$ if A is true, $I(A)=0$ otherwise. Let

$$U_1 = \frac{1}{n}\sum_{i=1}^{n} I(y_i < z) = \widehat{H}, \tag{4.10}$$

and

$$U_2 = \frac{1}{n}\sum_{i=1}^{n} y_i I(y_i < z). \tag{4.11}$$

Then the mean of x and that of x_p can be defined using U_1 and U_2:

$$\bar{x} = U_1\left(1 - \frac{U_2}{zU_1}\right) \tag{4.12}$$

and

$$\bar{x}_p = \left(1 - \frac{U_2}{zU_1}\right). \tag{4.13}$$

Now define the U-statistics for the Gini's mean difference:

$$U_3 = \frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{n} | y_i - y_j | I(y_i < z) I(y_j < z). \tag{4.14}$$

According to Xu (2004), the estimator of the Sen index is given by

$$\widehat{S} = \frac{2z^3 U_1^{\,2} - 2z^2 U_1 U_2 - U_3}{2z^3 U_1}. \tag{4.15}$$

The estimator of the SST index is defined as

$$\widehat{S}_{SST} = \frac{2z^3 U_1 - 2z^2 U_2 - U_3}{2z^3}. \tag{4.16}$$

According to Hoeffding (1948, Theorem 7.5), the estimators U_1, U_2, and U_3 are consistent for their population counterparts θ_1, θ_2 and θ_3, respectively. If F_y is the continuous distribution function with finite variance, then the joint distribution function of

$$n^{\frac{1}{2}}\left(U-\Theta\right)=\left[n^{\frac{1}{2}}\left(U_1-\theta_1\right),n^{\frac{1}{2}}\left(U_2-\theta_2\right),n^{\frac{1}{2}}\left(U_3-\theta_3\right)\right]^T \tag{4.17}$$

is asymptotically normal with mean **0** and variance

$$\Sigma=\begin{bmatrix}\theta_1(1-\theta_1) & \theta_2(1-\theta_1) & 2\theta_3(1-\theta_1)\\ \theta_2(1-\theta_1) & \varsigma(\theta_2) & 2\varsigma(\theta_1,\theta_2)\\ 2\theta_3(1-\theta_1) & 2\varsigma(\theta_1,\theta_2) & \varsigma(\theta_3)\end{bmatrix}, \tag{4.18}$$

where

$$\varsigma(\theta_2)=\int_0^z y^2 dF_y(y)-\theta_2^{\ 2}, \tag{4.19}$$

$$\varsigma(\theta_3)=\int_0^z\left(\int_0^z |\, y_1-y_2\,|\, dF_y(y_2)\right)^2 dF_y(y_1)-\theta_3^{\ 2}, \tag{4.20}$$

and

$$\varsigma(\theta_2,\theta_3)=\int_0^z\int_0^z y_1\,|\, y_1-y_2\,|\, dF_y(y_1)dF_y(y_2)-\theta_2\theta_3, \tag{4.21}$$

respectively. Since $\widehat{S}$ and $\widehat{S}_{SST}$ are functions of U_1,U_2,U_3, their variances can be derived from suitable functions of Σ. Due to the space limitation, we refer the interested readers to Xu (2004).

While the asymptotic results discussed above are desirable, the computation of the standard error of the Sen or SST index estimate can be complex. Osberg and Xu (2001) have proposed the bootstrap method for computing the standard error for the SST index estimate. This method is also applicable to the Sen index estimates and to estimates of other poverty measures. In a recent study, Davidson and Flachaire (2004) find that the bootstrap method can generate accurate variance estimates for poverty measures but not for inequality measures because the latter are extremely sensitive to the data in the upper-tail of a distribution. This finding assures that the bootstrap method is suitable for poverty measures.

The bootstrap method can be described as follows: Resample randomly from the sample $y^* = \{y_1^*, y_2^*, \ldots, y_n^*\}$ with replacement, compute the poverty measure of interest using the random sample $P(y^*)$, and repeat this process *B* times (typically *B* is a large number say 200). Then we can compute the bootstrap standard error using

$$se(P) = \sqrt{\frac{\sum_{b=1}^{B}\left(P(y^{*b}) - P(\bullet)\right)^2}{B-1}}\ , \qquad (4.22)$$

where $P(\bullet) = \frac{1}{B}\sum_{b=1}^{B} P(y^{*b})$. In this case, *P* can be $\widehat{H}$, $\widehat{I}$, $\widehat{G}(x_p)$, $\widehat{G}(x)$, $\widehat{S}$, and $\widehat{S}_{SST}$.

A practical example of this methodology was provided by Osberg and Xu (1999) who compared Canadian provinces – and since the provinces of China are an order of magnitude larger in population than Canadian provinces, while the sample size from which inference is made is considerably smaller, the importance of computing bootstrap standard errors is likely to be considerably greater in the context of China.

5. Summary and Conclusion

This paper started by asking whether the estimated proportion of the world's population with income below US $1 (adjusted according to purchasing power parity) per day is a good measure of trends in global poverty. We have argued in this paper that the answer depends on three important issues in the measurement of poverty – the definition of the poverty line, how best to summarize the level of poverty and how to statistically infer estimates of poverty from sample data.

5.1 What Poverty Line?

In common language usage, poverty is about deprivation of necessities - the primary dictionary definition of "poverty" is the deprivation of "the necessities of life"[see Oxford (1998, p. 1135)]. Adam Smith's views on this were drafted at a time – more than 200 years ago – when all nations had much lower incomes than presently, but their relevance endures:

> "Under necessaries, therefore, I comprehend not only those things which nature, but those things which the established rules of decency have rendered necessary to the lowest rank of people." (Vol. 2, Bk. V, Ch. II, Pt II, Art IV – 1961, p. 400)

In thinking about what "the established rules of decency" might be, on a global scale, the criterion of $1 per day – US$, PPP – has the enormous virtue of seeming simplicity, and hence communicability to a global public. However, a good deal of technical complexity sits behind the calculation of $1 per day in Purchasing Power Parity terms – and the issue is crucial to the evaluation of the level of global poverty.

As well, the rapidity of economic growth in China, and in India and South East Asia, means that, for a very substantial fraction of the world's population, the problem of absolute deprivation of commodities is being replaced by a more subtle type of poverty. As Sen has put it:

> "*Relative* deprivation in the space of *incomes* can yield *absolute* deprivation in the space of *capabilities.* In a country that is generally rich, more income may be needed to buy enough commodities to achieve the *same social functioning*, such as 'appearing in public without shame'. The same applies to the capability of 'taking part the life of the community'." (1992, p. 115)

In international poverty comparisons among developed countries, the norm is to calculate the poverty line as a fraction of median income, and to use local currency units throughout – which avoids entirely the problem of the uncertain value of PPP conversions. For the above reasons, this paper argues that more attention should be given to *relative deprivation* (i.e. equivalent incomes less than half the median) as well as *absolute deprivation* (i.e. incomes below $1 or $2 US per day).

5.2 The Summarization of Poverty Outcomes

The Sen and SST indices of poverty intensity measure the welfare loss caused by the incidence, depth and inequality of poverty, have desirable axiomatic properties and can be calculated and decomposed easily. Furthermore, they have simple geometric interpretations that are related directly to an easily communicated illustrative tool - the "poverty box". As demonstrated in the empirical example in this paper, the poverty ranking of regions based on the poverty box is remarkably consistent with that based on the benchmark SST index, which has a one-to-one correspondence relationship with the Sen index. The "higher order" poverty FGT indices (FGT $\alpha = 2\ldots.6$) do not change the rankings of most and least poverty-stricken provinces but will shift the middle range regions primarily due to the overweighting of inequality in poverty. Hence, in addition to being subject to arbitrariness in selecting the order, $\alpha = 2\ldots.6$, the higher order FGT indices add relatively little to comparisons among jurisdictions – in comparisons of rural poverty in China, or of affluent nations. Hence, the poverty box is indeed appealing – particularly since it is a major component of the Sen and SST indices and a special case of the FGT index. Although it is listed as one of many UN Millennium

Development targets, we argue that it should be the primary target and that it should receive more attention than the useful – but sometimes misleading – poverty rate.

5.3 **Statistical Inference Issues**

To implement the desirable poverty measures discussed in this paper, sample data must be used. Hence a full understanding of the statistical properties of the corresponding poverty estimators becomes important. In addition to the common knowledge about the poverty rate and average poverty gap ratios, this paper also shows the desirable statistical properties of the Sen and SST index estimators and their components and explains how to use the bootstrap method to make statistical inference for these poverty measures.

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