VALUE OF INFORMATION IN CONDITION BASED MAINTENANCE USING PROPORTIONAL HAZARDS MODELS: A COMPARISON OF MAINTENANCE STRATEGIES

by

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ABSTRACT

In this study, a comparison of equipment maintenance strategies' cost behaviour is performed under different assumptions regarding equipment failure behaviour, availability of condition monitoring data, and its accuracy. Each strategy uses certain level of available information to support decision making in maintenance. Information such as observed state of equipment, actual state of equipment, state transition behaviour, and failure data are used. Long-run average costs of the maintenance strategies are calculated for different sets of system parameters. The value of information is obtained by comparing the long-run average costs of the maintenance strategies while taking into account the time value of money.

The significance of this study is that it shows the value of various sorts of information from equipment, which can be compared to the cost of obtaining those information to magnify the financial profit of choosing the proper strategy.

LIST OF ABBREVIATIONS AND SYMBOLS USED

AFT	Accelerated Failure Time
AI	Artificial Intelligent
ANN	Artificial Neural Network
CBM	Condition Based Maintenance
CCNN	Cascade Correlation Neural Network
HMM	Hidden Markov Model
Im-Im	Imperfect-Imperfect Strategy
Im-P	Imperfect-Perfect Strategy
MCDM	Multi-Criteria Decision Making
MLE	Maximum Likelihood Estimation
MRL	Mean Residual Life
MTTF	Mean Time to Failure
PHM	Proportional Hazards Models
P-P	Perfect-Perfect Strategy
RCM	Reliability Centered Maintenance
RtF	Run to failure Maintenance
RUL	Remaining Useful Life
TBM	Time Based Maintenance
TPM	Total Productive Maintenance

C	Cost of a preventive replacement
c_i	Associated cost of test data i
K	Extra cost of a failure replacement
k	Observation interval number
P	State transition matrix
P_1	1st row of three-state matrix P
p_{ij}	The probability of moving from state i to state j in the next transition
Q	Observation-State matrix
Q_i	ith row of the three-state matrix Q
T	Failure time of equipment
T_d	The planned preventive replacement time
t_i	Replacement time of test data i
Z_k	State of equipment at observation point k
α	Scale parameter of the Weibull distribution
β	Shape parameter of the Weibull distribution
γ	State dependent function parameter
Δ	Length of observation interval
θ_k	Observation value at k
π^k	Conditional probability distribution at k

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Chapter 1: Introduction

According to International Standard IEC50 (191), maintenance is defined as:

"The combinations of all technical and corresponding administrative actions, including supervision actions, intended to return an entity in, or restore it to, a state in which it can perform its required function"

According to Dekker and Scarf [1], rising expenditures of maintenance operations is attributed to two factors: 1) the rapid expansion of industries, and 2) the increased demand for availability and properly functioning systems. Maintenance tasks are performed mainly to return equipment to a state of increased reliability, availability or optimal performance for the foreseeable future. In addition to these objectives, reducing the cost of equipment maintenance actions is also of high importance. There are various methods to classify maintenance types and objectives, however one common classification is given by Figure 1 [2]:

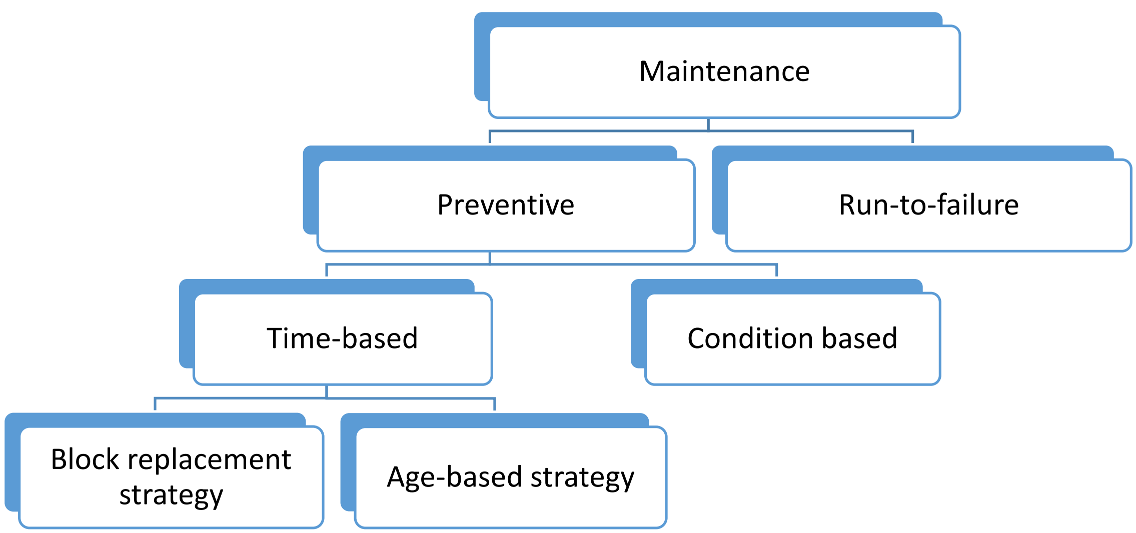


Figure 1: A classification of maintenance strategies. Source: [2]

When implementing a condition based maintenance (CBM) policy a fundamental design issue is whether one has perfect information of a system's condition, or imperfect information. CBM with perfect information assumes that any observations from equipment represent the actual health states. Contrarily, CBM with imperfect information uses observations from equipment that do not represent actual health states, but are stochastically related to the actual health states.

1.1 Run-to failure maintenance

A run–to-failure maintenance strategy simply replaces the equipment after it fails. Run-to-failure is also called corrective maintenance or break down maintenance. It's a common strategy in many small industries where downtimes and maintenance costs have a low monetary impact. However, in industries where downtime and failure costs are significantly higher a preventive maintenance strategy is often preferred.

1.2 Time-based maintenance

Time-based maintenance (TBM) is a traditional maintenance strategy that is widely used in different industries. In TBM the failure rate of equipment is assumed to be solely dependent on its age. Mathematical models in TBM are used to model the failure rate, based on historical failure data. Replacement decisions of TBM are based on the analysis of failure rate and balancing the associated costs of preventative versus corrective maintenance. One common approach is to fit the failure data into statistical distribution models (e.g. Weibull, Normal and Lognormal) [3]. These models are used to estimate the reliability of equipment, or mean time to failure (MTTF) of the equipment which forms the basis for an optimization model that can minimize costs of the maintenance strategy, or risk of a failure.

Weibull is the most common distribution used in reliability modelling which usually has two parameters: scale parameter (corresponding to age of equipment) and shape parameter (corresponding to attributes of equipment lifetime) [2], [4]. Equipment failure time modeling in TBM has been broadly done by using the Weibull distribution: Ebeling [4] discussed the Weibull distribution in great detail. Some of the earliest research in time-based maintenance was done by Barlow and Hunter [5]. They studied two types of preventive maintenance. For the first type, preventive maintenance is performed after a specific time (t_0) of equipment operation without failure (known as Age-Replacement). In this strategy if a failure occurs earlier than t_0, corrective maintenance is performed to return the equipment to a desired state. Normally, it is assumed that preventive maintenance and corrective maintenance will return the equipment to as good as new condition. In the second strategy, known as block-replacement, a preventive maintenance is performed at kt^*, (k=1,2, …) time since the beginning of equipment operation, no matter if any failure has occurred before kt^*. For each failure that occurs before t^* only a minimal repair is performed. It is assumed that a minimal repair puts the equipment in working condition but does not affect the failure rate. Preventive maintenance is only performed at t^* interval which resets the equipment's failure rate. Barlow and Hunter [5] proposed mathematical formulations to model these two strategies and noted that the failure distributions Gamma, Normal or Weibull would perform well used in their model. The significance of their work was to identify which preventive maintenance strategy would lead to lower costs under different conditions, and to compare the effect of different failure distributions on the long-run average costs. Nakagawa [6] further developed Barlow and Hunter's work by presuming that minimal repairs do not return the state of equipment to as good as new. Nakagawa also showed that the Weibull distribution makes optimal calculations easier. Barlow and Proschan [7] also investigated different failure distributions and found that the Weibull distribution works better than other failure distributions to approximate failure rate. The Weibull distribution has also been used by many researchers in reliability and failure analysis, including Weibull [8], Kao [9], Lieblein & Zelen [10], Sheu [11] amongst several others.

The following sections present block replacement and age replacement strategies literature in greater detail.

1.2.1 Block replacement strategy

Block replacement strategy is based on performing maintenance actions at equipment failure or after fixed times kt^*, (k=1, 2 …). However, this strategy can result in unnecessary costs since equipment is replaced at fixed calendar times kt^* regardless of whether any failures have occurred up to time kt^*. Boland [12] extended Barlow's [7] model by implementing minimal repairs to reduce the number of unnecessary replacements. Boland's model assumed the cost of minimal repair was a function of equipment age, then found the optimal replacement period T_0 which minimizes long-run cost. Tahara and Nishida [13] introduced a breakdown cost to Barlow Porschan's model to distinguish unplanned maintenance costs from planned maintenance costs. They used the renewal reward theorem to compute the optimal replacement strategy by minimizing expected cost per unit of time. Sheu et al. [11] recognized the relationship between long-run costs of age replacement and block replacement and developed a cost mechanism that considered both strategies by using two types of failures, a minimal failure and a "catastrophic failure". They investigated the effect of shocks on equipment arriving according to a Poisson process and considered preventive replacements costs, failure replacement costs and minimal repairs costs. Sheu et al also developed a general model that considers both age-based and block-based replacement strategies to improve maintenance decision making. Tango [14] introduced the idea of using old equipment while performing unplanned replacements. He assumed that replacing the equipment by a used equipment will cost lower than replacement by a new equipment. Tango compared this method by general model of block replacement strategy and showed lower costs are obtained using his model. Tango also used Erlang distribution to model the life time of the equipment.

1.2.2 Age-based strategy

Age-based strategy requires replacements to be performed at a certain age T or upon failure of the equipment. Barlow and Hunter's age-based model was extended and generalized by Glasser [15], Block et al. [16], Muth [17], Nakagawa [18], Bai & Yun [19].

Glasser [15] generalized Barlow & Hunter's [5] age replacement model with various failure distributions including Gamma, truncated Normal, and Weibull to obtain the optimal replacement strategy for each distribution. Block et al. [16] created a stochastic age replacement model where two repair types are considered: complete repair, and minimal repair. At each failure, either a minimal or complete repair is performed with a probability function/distribution dependent on operation duration. Their model assumed that only complete repairs set the state of equipment to as good as new.

An alternative to age-based (continuous distribution) failure models is to derive a failure distribution from number of times equipment has been used (discrete distribution). Nakagawa [18] combined both continuous and discrete age distributions to reach a general strategy that considers both the age of equipment and number of times equipment is used. Equipment is replaced at a failure, at age T, or when the number uses reaches a limit N. He considered different replacement costs for replacement at failure, age T, and N. He then defined a function dependent on both age and number uses to predict failure of equipment, and solved an example with a Weibull failure distribution and calculated T^* and N^*. Bai and Yun [19] developed another age-replacement model assuming that equipment would be repaired when failure occurs before age T only if the cost of repair is less than a value L, otherwise it would be replaced. At age T equipment is replaced. This work is a development of the model proposed by Muth [17], while taking into account the repair costs more realistically.

In summary, the concept of TBM replacement strategies were introduced by Barlow and Hunter [5]. Different distributions were considered by several researchers for equipment failure, but by majority of researchers Weibull distribution was preferred. Especially in the work done by Jardine and Buzacott [20], the use of Weibull distribution for TBM was justified and recommended. Berg and Epstein [21] compared age-based, block replacement, and run-to-failure strategies to find which strategy would generally result in lower maintenance costs. The results allow for the prediction of least maintenance cost as a function of failure distribution parameters. It can be observed from their results that for a wider range of the Weibull distribution parameters an age-based strategy would lead to lower maintenance costs.

1.3 Condition Based Maintenance

Condition based maintenance (CBM) strategy triggers maintenance actions based on information collected regarding the state/health condition of equipment. CBM can reduce the number of unneeded preventive maintenance actions and therefore reduce associated costs of maintenance. CBM requires condition monitoring data from equipment. Condition monitoring data can come from various sources. The data can come from vibration of equipment or vibration of components, oil analysis, temperature, moisture, pressure or from the environment [22]. The information is collected through different types of sensors, tests and inspections. One remarkable work to review CBM was conducted by Jardine, Lin, and Banjevic [22]. In their review three steps were identified for CBM modeling; a) Information collection, b) processing of collected data, and c) Decision making.

a. Information collection

Information collected from equipment can be event data or the data from condition monitoring. Event data would capture the types of problems, their causes, and the actions which were subsequently performed. Data from condition monitoring is the observations from equipment which identifies the health/state condition of equipment whether perfectly or imperfectly. Some research concerning condition monitoring data are represented by Austerlitz [23], Kirianaki et al. [24], and Davies & Greenough [25].

b. Processing collected data/information

The first step of data processing is data cleansing, which removes unacceptable data (Xu & Kwan [26]). The next step is to analyze the cleansed data. There are various methods for data analysis including: waveform analysis through vibration signals (Russ [27], Nixon & Aguado [28], Wang & McFadden [29], Heger & Pandit [30], Bialie & Mathew [31], and Ho & Randall [32]), value type analysis (Stellman et al. [33], and Allgood & Upadhyaya [34]), and analysis of data when event and condition monitoring data are combined. Some methods which combine the event data and condition monitoring data include proportional hazards modelling (PHM) (Jardine et al. [35], Newby [36], Kalbfleisch & Prentice [37], Ghasemi et al. [38]), proportional intensity modelling (PIM) Vloak et al. [39], and Markov modelling (Dong & He [40], Elliot & Moore [41], and Bunks et al. [42]).

c. Decision making

The last step is decision making through CBM modeling to achieve effective strategies. Decision making in CBM can be divided into two categories; diagnostics, and prognostics. Diagnostics is the method of detecting faults and mapping it to a fault space. Prognostics is the prediction of a fault occurrence in future. Below we can see references to the works done in diagnostics and prognostics, categorized by the type of method.

1. Diagnostics
    a. Statistical methods
        i. Hypothesis test (Ma & Li [43])
        ii. SPC method (Skormin et al. [44], and Arts et al. [45])
        iii. Pattern recognition (Schurmann [46])

        iv. Hidden Markov Model, HMM (Bunks et a. [42], Ying et al. [47])

    b. Artificial Intelligence (AI)methods

        i. Artificial neural networks, ANN (Roemer et al. [48])

        ii. Cascade correlation neural network, CCNN (Spoerre [49])

        iii. Fuzzy logic theory (Collins et al. [50], and Du & Yeung [51])

2. Prognostics

    a. Remaining useful life, RUL (Li et al. [52] [53], Wang et al. [54], and Phelps et al. [55])

    b. Prognosis by optimizing maintenance strategies based on certain criteria, such as cost, risk, and availability. (Wang [56] [57], Grall et al. [58], Ohinishi et al. [59], Jardine et al. [35], Ghasemi et al. [60])

    c. The interval of condition monitoring adjustment, continuous or periodic monitoring (Wang [56] [61], Okumura [62])

The technologies of data collection sensors are developing rapidly and it is becoming more affordable and less expensive to monitor equipment state/health conditions. Therefore, implementation of CBM has become more feasible as a strategy for performing maintenance actions. Jardine et al. [22] concluded that instead of implementing common methods such as run-to failure method or performing as many as possible maintenance actions, in many cases CBM is a better choice particularly when cost of failure or maintenance is high.

An important aspect of CBM is the interpretation of the monitoring data to derive the current state of equipment. We can divide CBM condition monitoring data into two general categories as described below:

1- Perfect information: Collected data from equipment represents actual state/health condition of equipment.
2- Imperfect information: The actual state/health condition of equipment cannot be known. Instead, indirect measures of state/health conditions are collected from the equipment which is then related to the actual state/health condition of equipment stochastically.

1.3.1 CBM with Perfect Information

The majority of previously referenced CBM research considered perfect information as an assumption in their CBM modeling. Other studies which also make this assumption include Bergman [63], Taylor [64], Gotlieb [65], Posner & Zuckerman [66], and Makis and Jardine [67].

Makis and Jardine [67] proposed a CBM strategy with perfect information using a proportional hazards modeling (PHM) in which the rate of failure is dependent on the age and state/health condition of the equipment. Before introducing the CBM model proposed by Makis and Jardine, it is important to describe the proportional hazards models.

Proportional hazards model (PHM) is a statistical regression model that incorporates descriptive factors of interest (indicators) and relates them to the equipment failure rate [68]. Using PHM the failure rate of equipment can be derived by multiplying the baseline failure rate (depending only on age of equipment) and a function that is dependent only on the covariates' factors. In a CBM strategy with perfect information the descriptive factors (covariates) are assumed to identify the actual state/health condition of equipment. In the work of Makis and Jardine, covariates are represented by values of a stochastic process and are observed at equal time intervals. The covariates are assumed to represent the state/health condition of equipment.

PHM was introduced by Cox [69] to analyze the effect of multiple covariates on failure rates. It originally was developed for biomedical purposes but later its application was investigated for reliability engineering and especially condition-based maintenance [68]. Bendell et al. [70] underlined the possibility and usefulness of applying PHM to reliability assessment in engineering. They suggested that factors such as temperature, vibration, pressure and so forth can be considered as effective covariates on life of a system. Jardine and Mann [35] applied a Weibull based PHM to analyze marine and aircraft engine failure data. Using simulation it was shown that the failure data can be fitted properly by the model. Aside from CBM, PHM has also been used in age-based maintenance (e.g. Kumar and Westberg [71]). Newby [36] studied Weibull PHM to illuminate some of its advantages and disadvantages. He compared PHM to accelerated failure time (AFT) method, and concluded that most significant advantage of PHM over AFT is that the relative degree of effect of system variables can be recognized. AFT is a method that considers the effect of system variables to accelerate or decelerate the failure rate by a constant value, however PHM considers the effect of system variables to multiply the failure rate by a constant value.

It is shown that by utilizing information obtained from condition of equipment and modeling equipment failure using PHM, maintenance decision making can be improved by reducing long-run average costs of maintenance [67]. The following assumptions were made in the CBM model proposed by Makis and Jardine [67]:

1. Equipment is subjected to random sudden failures
2. Baseline age-dependent failure rate distribution is a Weibull distribution
3. Indicators can be observed at discrete points in time
4. At each observation point a decision is made whether to replace the equipment or to leave the equipment to operate
5. Observed indicators from equipment perfectly represent actual state of equipment
6. State process (represented by indicators) is a Markov chain with triangular transition probabilities matrix (which means state of equipment does not improve by itself)
7. Maintenance actions include failure replacement, preventive replacement and no replacement
8. Replacing the equipment brings its condition to as good as new
9. Cost of failure replacement is greater than a preventive replacement

It is important to note that assumption 5 is not always correct in reality. For example, although observations collected on the vibration of a system, are stochastically related to the state of the system but they are not a perfect indicator of the state of the system. Also the value of vibration indicator is not monotone with the state of the system which contradicts item 6 above (from now on state of equipment (system) will be used instead of health/state condition of equipment).

1.3.2 CBM with Imperfect Information

The CBM approach of Makis and Jardine [67] assumes that the information obtained from equipment perfectly represents the actual state of the equipment. In reality the actual state of equipment can rarely be observed. Therefore it is reasonable to consider that what we observe from equipment does not represent the actual state of equipment but it somehow relates to the actual state of equipment.

White [72], Ohnishi [59], Sinuany-Stern [73], Makis et al. [74], and Daming et al. [75] all have studied CBM considering unknown state and a stochastic relationship between observed value(s) from equipment and actual state of equipment. The terms “partially observable state” or “partial information” are also used to describe models with unknown state where an inspection cannot reveal the actual state [60] [76] [77] [78] [69].

Ghasemi et al. [60] assumed the actual state of equipment is unknown while using PHM and introduced a CBM strategy to minimize the long-run average maintenance costs. In constructing the maintenance strategy other assumptions are similar to the work of Makis and Jardine [67]. The maintenance strategy proposed by Ghasemi et al. [60] assumed that at each observation point the obtained values can be an indirect indicator of the actual state. The indicator value is stochastically related to the actual state of equipment which in turn are used as PHM covariates. If the actual state of equipment is x, then an indicator value y is observed with probability q_{xy}. These probabilities and state transition probabilities as well as parameters of PHM can be estimated from historical data [38].

1.4 A Comparison of TBM and CBM strategies

Ignoring the importance of selecting a proper maintenance strategy for equipment that is subject to random failures can severely limit profits due to higher long-run average costs of maintenance. Associated maintenance costs and potential lost profit due to lack of appropriate maintenance strategy are dependent on the characteristics of equipment and type of maintenance strategy (TBM, CBM, or run-to failure) chosen for that equipment. The characteristics of equipment are as follows:

1- Degradation process: internal physical and external environmental factors that cause state transition and finally failure of the equipment. Identify these factors and relating their effects on failure of equipment is a non-trivial task which can significantly impact strategy effectiveness.
2- Cost of maintenance: depending on the type of equipment, the maintenance action (repair or replacement) might require different amounts of time, costs and resources.

The cost of maintenance in a manufacturing industry can be responsible for a significant percentage of the total production cost. Once the characteristics of equipment have been identified a proper maintenance strategy can be chosen in order to reduce the cost of maintenance. Several studies have been done in this regard which described the importance of a cost comparison. In 1978, Berg & Epstein [79] investigated age-based, block replacement and run-to failure strategies, and provided a rule by which the practitioner can choose the least costly maintenance strategy. Their proposed rule was based on the parameters of failure distribution of equipment. Duc & Ming [80] compared cost of an age-based strategy with a CBM strategy. In those strategies they also considered minimal repair for both continuous and sequential inspection schemes. The age-based strategy was similar to the model proposed by Barlow & Hunter [5] and the CBM strategy was modeled using a Markov-Multi state method, where equipment degradation was represented by a continuous-time Markov process. They showed that for a wider range of failure distribution parameters, generally CBM is more effective than the age-based strategy, in terms of failure-reduction and cost. Alnajjar & Alsyouf [81] used a fuzzy multi-criteria decision making (MCDM) method to select the proper maintenance strategy. However, they mainly compared Total Productive Maintenance (TPM) with Reliability Centered Maintenance (RCM) and did not consider any specific model. Carlo & Arleo (2013) [82] studied the economic feasibility of CBM for a heating, ventilation and air conditioning system. For that purpose, they compared the cost of run-to-failure maintenance, age-based maintenance and CBM. They compared these three strategies for different reliability parameters.

System elements	C(RtF)	C(TBM)	C(CBM)
Element1	1.74	0.84	0.33
Element2	2.63	1.53	1.05
Element3	1.65	0.8	0.2

Table 1: cost (in pounds) comparison of Run-to-Failure, TBM and CBM by Carlo & Arleo

The obtained results suggested that a CBM strategy would always provide lower maintenance costs. Table 1 shows that the relative cost of CBM is lower than RtF (run-to-failure) and TBM (in this example the costs are in Pounds). However adopting a CBM strategy requires proper fault detection and monitoring technology which can be relatively very costly.

Ahmad & Kamaruddin [3], also argued that CBM is more powerful than time-based maintenance strategy but more effective results from CBM relies on development of costly equipment monitoring technologies as well as better computerized CBM methods by developing user friendly applications.

To the author's knowledge there is currently no research comparing the performance of different maintenance strategies with CBM using PHM considering the level of information perfection. This thesis develops a simulation based model to compare maintenance strategies by incorporating parameters of failure behaviour, level of knowledge of actual state, and calculated the cost of each maintenance strategy for different sets of parameters. The following sections will describe the structure of the each maintenance strategy investigated in this research.

1.5 An Introduction to Mathematics of Reliability

Reliability is the probability of performing a desired function by an item in a specific period of time and under specific conditions [2]. The term "item" may refer to a system, component, subsystem or equipment. As noted by Hoyland and Rausand [2], the objective of reliability is to provide an information basis for decision making; which can be applied to a wide range of areas such as safety analysis, environmental protection, quality management, engineering design, and optimization of maintenance programs. Its application ranges from insurance companies, biology, the aircraft industry, to industrial machinery. Since we will address the cost analysis of maintenance strategies, and because all maintenance strategies take into account the reliability of the equipment, it is essential to have a good understanding of the mathematics of reliability in the area of optimization of maintenance, and its application in industrial machinery.

1.5.1 Reliability Function

Failures can be classified as sudden or gradual, and hidden or evident. A sudden failure may be easily recognized while a gradual failure is harder to detect. A hidden failure can only be detected by inspection of equipment, while an evident failure can be observed instantly [2]. The maintenance strategies in this research will consider sudden and evident failures.

A sudden failure can occur at any time. If the failure time is identified by a random variable T, then the reliability function of equipment provides the probability that the equipment will function at least up to time t [4].

$$R(t) = Pr\{T \geq t\} \quad (1)$$

It follows that the probability of a failure before time t can be calculated as:

$$F(t) = 1 - R(t) = Pr\{T < t\} \quad (2)$$

Where R(t) is the *reliability function* and F(t) is the *cumulative distribution function* (CDF) . the probability density function (PDF) f(t), can be calculated by:

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt} \quad (3)$$

1.5.2 Hazard Rate Function

Hazard rate or failure rate is the immediate rate of failure at time t [4]. In other words, hazard rate is the conditional probability that a failure will occur in the period [t, t+Δt[; Δt → 0, knowing that it has not occurred until t.

$$h(t) = \frac{f(t)}{R(t)} \qquad (4)$$

The failure rate is either increasing failure rate (IFR), decreasing failure rate (DFR), or constant failure rate (CFR), depending on h(t) being an increasing, decreasing, or constant function of t.

1.5.3 Mean Time to Failure

Mean time to failure (MTTF) is the expected time to the occurrence of the first failure:

$$MTTF=E(T)= \int_0^{\infty} tf(t)dt = \int_0^{\infty} R(t)dt \qquad (5)$$

MTTF also represents the time between failures, if average replacement time is fairly smaller than MTTF [2].

1.5.4 Conditional Reliability

Conditional reliability of equipment can be defined as the equipment's reliability during period t, given that the equipment has already survived up to time T_0:

$$R(t|T_0) = Pr\{T > T_0 + t|T > T_0\} = exp\left[-\int_{T_0}^{T_0+t} h(s)ds\right] \qquad (6)$$

1.5.5 Weibull distribution

Analysis of failure times and understanding failure trends of equipment is a critical task in reliability engineering. As discussed earlier Weibull is one of the most useful probability distributions in reliability studies. Research by Nakagawa [6] and Scarf [83] are closely related to the work of this thesis.

The Weibull hazard rate function is of the form:

$$h(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} \qquad (7)$$

Where α is the scale parameter and β is the shape parameter. And its failure time probability distribution function is:

$$f(t) = -\frac{dR(t)}{dt} = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-(\frac{t}{\alpha})^{\beta}} \qquad (8)$$

1.5.6 Weibull PHM

PHM takes into account the effect of working condition ("concomitant variables" [67]) and the age into the hazard function. It provides a hazard rate function which is the product of an age-dependent baseline hazard function and a state dependent function.

$$h(t, Z_t) = h_0(t)\psi(Z_t) \qquad (9)$$

Where $h_0(t)$ is the age-dependent function, which in this work is assumed to follow a Weibull hazard rate function (equation 7), and $\psi(Z_t)$ is the state dependent function which for this research, is expressed as:

$$\psi(Z_t) = exp\{\sum_{i=1}^{m} \Upsilon_i Z_{it}\} \qquad (10)$$

In equation 10, Z_t is the state of equipment at time t, and Υ is the covariate parameter which indicates the degree of effect that the state variable has on the failure rate.

Hoyland & Rausand [2], and Ebeling [4] provide a complete reference of reliability, its statistical methods, and mathematics of reliability models.

1.5.7 Stationary Markov Process

State of equipment represents the health condition of equipment. A state space $\mathbb{Z}$={0,1,2,…,N} can be defined to include all possible values of the equipment state. State of equipment may change due to usage conditions, aging or environmental factors. Markov process can model this random system according to probability p_{ijt} which is the probability of non-stationary transitioning from state $i \in \mathbb{Z}$ at the current moment to state j at time t The probability of going from state i to state j is independent on what states the chain was before state i.

The p_{ij} is the stationary transition probability. A stationary Markov Process is a Markov Process with the following properties: (1) there are finite states, (2) the future states is only dependent on the current state and independent of the past states, (3) transition probabilities do not change with time.

All possible transitions from one state to another, are expressed by a transition matrix which contains the transition probabilities from row i to column j, rows being the current state and columns the future state [84]:

$$P = \begin{bmatrix} p_{00} \; p_{01} \; p_{02} \; p_{03} \cdots \; p_{0N} \\ p_{10} \; p_{11} \; p_{12} \; p_{13} \cdots \; p_{1N} \\ . \\ . \\ . \\ p_{N0} \; p_{N1} \; p_{N2} \; p_{N3} \cdots \; p_{NN} \end{bmatrix}$$

1.5.8 Renewal Process

In general, if the time between random events (replacements in this case) are positive independent and identically distributed random variables, the process is a renewal process. In a renewal process, the first event occurs at a time with some distribution, and the time between the first and second event follows the same distribution but is independent of the first event, and this goes on for the upcoming events. The renewal process refers to a failed equipment that is replaced with a new equipment or restored to as good as new. Let T_1,T_2,

..., T_k be independent and identically distributed random variables representing replacement times of equipment, and the counting process $\{N(t), t \geq 0\}$ represents number of events (replacements) occurring until time t. Since the sequence of $\{T_1, T_2, \ldots, T_k\}$ are positive independent and identically distributed random variables, the counting process $\{N(t), t \geq 0\}$ is renewal process [84]. Renewal process is the basis for optimizing the long-run average costs of maintenance strategies used in this thesis.

1.6. Mathematical description of the maintenance strategies

The detailed mathematical description of each maintenance strategy is provided in the following.

1.6.1. Run-to failure strategy

A run-to-failure strategy is equivalent to no preventive maintenance strategy or to only performing corrective maintenance. There is only one type of replacement which is a failure replacement with cost K+C. C is the cost of preventive replacement and K is the extra cost due to failure replacement which can be obtained from historical data regarding associated costs of unplanned replacement, such as production loss. The long-run average cost of performing this strategy is:

$$\bar{C} = \frac{K+C}{MTTF} \tag{11}$$

Consider equipment which follows a Weibull failure distribution with parameters β and α:

$$f(t) = \frac{\beta t^{\beta-1}}{\alpha^{\beta}}(exp(\frac{-t}{\alpha})^{\beta}) \quad ; \quad t \geq 0 \tag{12}$$

And the MTTF is calculated as [6]:

$$MTTF = \alpha\Gamma(\frac{1}{\beta}+1) \tag{13}$$

Where:

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt \quad (14)$$

No information is needed to apply this strategy and therefore no failure prediction and decision making is considered in this strategy. This strategy is investigated in this research as a benchmark to compare other maintenance strategies, and to compare the value of having different levels and accuracies of information available for decision making.

1.6.2 TBM Strategy

The TBM model used in this work is an age-replacement strategy using Weibull as the failure distribution. The age-replacement strategy was first presented by Barlow & Proschan [7], Barlow & Hunter [5], and later by Glasser [15].

The main assumptions of this strategy include:

- Replacements occur either at failure (failure replacement) or before failure at a replacement time (preventive replacement)
- Cost of a preventive replacement is less than failure replacement
- The time to perform failure or preventive replacements is negligible
- Equipment replacement will reset the equipment to as good as new (i.e. no minimal repair is considered in this strategy)

Note that this model is an age-replacement strategy and not a block-replacement strategy. These two TBM models were discussed in details earlier in this chapter. The difference between the two mentioned models is that while in latter the replacements occur at failures and times (T*, 2T*, 3T*,…) in the former the replacements occur at failures and when age of equipment reaches T, or on failure, whichever occurs first.

Figure 2, and Figure 3 help to better understand how the described age-replacement maintenance strategy works. The pointers represent age-replacement actions, and the block-replacement actions are only performed at (T*, 2T*, 3T*, 4T*).

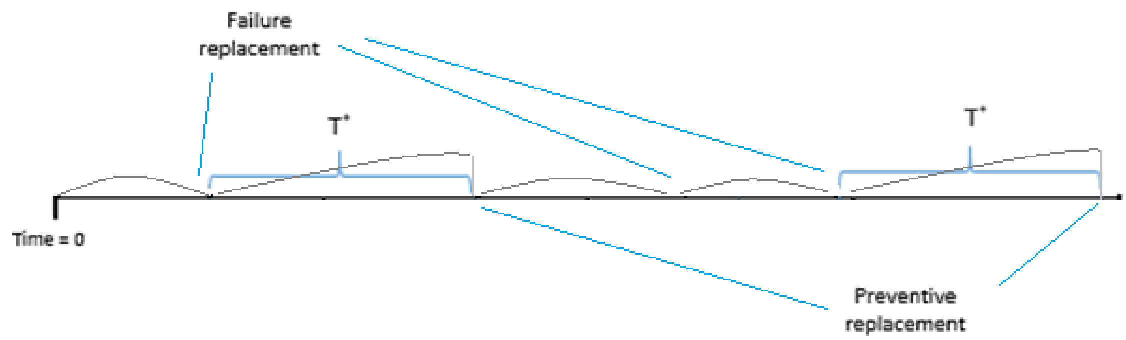


Figure 2: Age-replacement

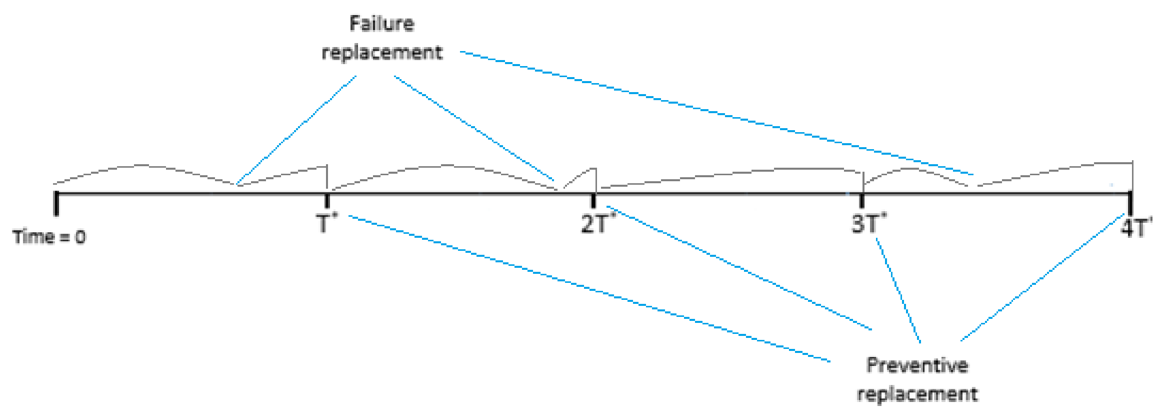


Figure 3: Block replacement

It can be observed that the block replacement strategy performs greater number of replacements compared to an age-replacement strategy. For that reason the age-replacement strategy was chosen as the proper candidate to represent the TBM strategy in our study. The following section describes how the average replacement cost of the optimum replacement time T^* is found for an age-replacement strategy.

If we consider the cost of a failure replacement to be K+C and the cost of a preventive replacement C, then the expected cost function per unit of time is written as [19]:

$$C(T) = C\int_{T}^{\infty} f(t)dt + (K + C)\int_{0}^{T} f(t)dt \qquad (15)$$

The first term of (equation15) is the product of equipment reliability up to time T and the cost of a preventive replacement, and the second term is the product of the cost of a failure replacement multiplied by the probability that a failure would happen before the scheduled preventive maintenance time T. Therefore, the expected time of replacement can be calculated by:

$$S(T) = T\int_{T}^{\infty} f(t)dt + \int_{0}^{T} tf(t)dt \qquad (16)$$

The first term of this expression is the product of the planned replacement time (T) and the probability that the equipment survives until T, and the second term is the expected value of failure probability distribution up to time T.

The optimum time to perform preventive replacement would be reached when the expected cost per unit of time is minimized. The expected cost per unit of time can be calculated by dividing the expected cost of replacement over the expected time of a replacement:

$$E(T) = C(T)/S(T) \qquad (17)$$

E(T) identifies the average cost per unit of time. The lowest long-run average costs can be achieved if the equipment is replaced after working for an optimal unit of time, T^*. T* is the value of T that minimizes equation 17.

In equations 15 & 16, *f(t)* is the equipment failure time pdf which is considered to be a Weibull distribution with parameters α and β (equation 12). The Weibull distribution function represents increasing failure rate for $\beta > 1$, which makes it an appropriate model for aging process of equipment. Any value of $\beta \leq 1$ means that the failure rate is not increasing and a preventive maintenance will not improve the cost of the maintenance strategy. The reason is that when the failure rate is not increasing, replacing the equipment and therefore brining the state to as good as new, will increase the failure rate. However, for an increasing failure rate, bringing the state of equipment to as good as new decreases the failure rate and therefore improves the cost of the maintenance strategy.

1.6.3. CBM with perfect information strategy

In this study a CBM strategy is used that considers a stochastically deteriorating system (equipment) whose failure is dependent on both age and state of equipment. The state of equipment follows a stochastic process Z revealing the exact state of the equipment, which is observed at equidistant discrete points of time. The failure rate function is modeled by PHM, which considers both age and state of the equipment as factors affecting failure time of equipment.

Failure rate is the product of two function (refer to equation 9): a baseline failure rate function $h_0(.)$, and a state dependent function $\psi(z) = e^{\Upsilon Z}$. The product of these two functions is the hazard rate function of equipment [67]:

$$h(t, Z_t) = h_0(t)\psi(Z_t) \qquad (18)$$

In this strategy the equipment is only replaced and no minimal repair is performed on it. The equipment should be replaced at failures or at discrete points of times when observed state and age of equipment recommend a preventive maintenance. The cost of a replacement is C while a failure replacement would cost C + K. The replacement strategy objective is to minimize the long-run average cost of equipment.

Assumptions below are considered in this CBM strategy:

- Observations from equipment perfectly reveal state of equipment
- Observations are made at equidistant discrete points of time (Δ, 2Δ, 3Δ, …, kΔ)
- A preventive replacement is less costly than a failure replacement
- A replacement would turn the equipment to as good as new state, $Z_0=0$
- The damage process Z is stochastically increasing, meaning that state of equipment is degrading and therefore $\psi(z)$ is non-decreasing
- $h_0(t)$ is a non-decreasing function, meaning that system is deteriorating with age

The following describes the mathematical model of the CBM strategy introduced by Makis and Jardine (1992) [67].

The stochastic process Z:

$$Z = \{Z_t;\ t = 0, \varDelta, \dots, k\varDelta\} \tag{19}$$

With a state space:

$$R^{+} = [0, \infty) \tag{20}$$

Because Z_t represents the exact deterioration state of the equipment and there is no repair between the replacements, it is none decreasing. It is assumed that state information Z_t can only be observed at discrete times Δ, 2Δ, 3Δ, …, kΔ and $Z_0 = 0$. State transitions follow a Markov process with the following transition matrix:

$$P = [P_{ij}]\ , \tag{21}$$

Where P_{ij} is the transition probability from state i to state j during one observation period Δ.

The conditional reliability of equipment is calculated as below [67]:

$$R(k, Z_k, t) = P(T > k\Delta + t | T > k\Delta, Z_1, \ldots, Z_k) = exp(-\psi(Z_k) \int_{k\Delta}^{k\Delta+t} h_0(s)ds) \quad (22)$$

This function is the probability that no failure will occur before t+kΔ, knowing that no failure has occurred until kΔ, and the observed states of equipment were Z_1, …., Z_k at each observation point (Δ, 2Δ, …, kΔ).

The mean sojourn time, the average remaining life of equipment at observation moment k with state Z_k, for the equipment is calculated as below [69]:

$$\tau(k, z, +\infty) = \int_0^{+\infty} R(k, z, t)dt \quad (23)$$

From equation 23 and in the absence of replacement action at the observation point k while the state of equipment is Z_k, the expected time that the equipment would remain in working condition is: $\int_0^{+\infty} R(k, z, t)dt$.

The expected average cost of equipment, considering failure replacement costs and actions taken at each observation to preventively replace the equipment or to leave it operating, is derived from renewal theory [67]:

$$\Phi_{T_d} = \frac{C_{T_d}}{P_{T_d}} = \frac{[C + KP(T \leq T_d)]}{E[min[T, Td]]} \quad (24)$$

Expected cost C_{T_d} over the expected length of replacement P_{T_d} is the expected average cost per unit of time Φ_{T_d}. T is the failure time of equipment and T_d is the planned preventive time to replace the equipment. $P(T \leq T_d)$ represents the expected probability that the failure occurs before replacement time and therefore an additional cost K would occur. $Emin[T, T_d]$ is the expected replacement time which is either T (failure time) or T_d (preventive replacement time). T_d is calculated using the following function:

$$T_d = \Delta . inf\{r \in R^+ : K[1 - R(r, Z_r, \Delta)] \geq d . \tau(r, Z_r, \infty)\}, \quad (25)$$

Where Δ, is the equidistant observation intervals. In the function above, the left hand side of inequality shows the additional cost of failure replacement multiplied by the probability of a failure in the next period at observation point k $(k = \lfloor r \rfloor)$ while the state of the equipment is Z_k. The right hand side is the product of d, an average long-run cost per unit of time and the mean sojourn time of the equipment. This means that if the expected cost of performing a failure replacement is equal or greater than the cost of preventively replacing the equipment, then the equipment must be replaced at that point of time. For any value of d, a value of T_d can be calculated. The objective of this model is to find the optimum value of T_d that minimizes the long-run average cost *d*. In order to find the optimal replacement strategy, equation 24 is solved iteratively and optimal long-run average cost is calculated. A dynamic programming approach is used to solve each iterative equation 24.

Iterative function $W(j,i)$ is defined to calculate $Emin[T,Td]$. It is shown that:

$$E[min[T,Td]] = W(0,0), \tag{26}$$

Where $W(j,i)$ is defined as below [67]:

$$W(j,i) = \begin{cases} 0 \quad if\ j \geq k_i \\ \int_0^{t_i - j\Delta} R(j,i,s)ds \quad if\ j = k_i - 1 \\ \int_0^{\Delta} R(j,i,s)ds + \sum_{r=i}^{m} W(j+1,r)P_{i,r}(j)R(j,i,\Delta) \quad if\ j < k_i - 1 \end{cases} \tag{27}$$

In which i is the state of equipment at the observation point j. And k_i is the interval in which the replacement occurs $\{\Delta.T_d(i) < k_i < \Delta.(T_d(i)+1)\}$. $T_d(i)$ is calculated from equation 25. The forward recursion of $W(0,0)$ starts from time j=0 with state i=0 and continues forward calculating each possible state value at each time step until the equipment is eventually replaced.

$P_{i,r}(j)$ is calculated from equation below:

$$P_{i,r}(j) = P(Z_{j+1} = r | T > (j+1)\Delta, Z_j = i), \qquad (28)$$

As the probability of going from state i to state r at time j+1 knowing that no failure will occur until (j+1)th observation point.

Similarly in order to calculate expected probability that a failure will occur before replacement time $P(T \leq T_d)$, an iterative auxiliary function is defined as below [67]:

$$Q(j,i) = \begin{cases} 0 \quad if \;\; j \geq k_i \\ 1 - R(j,i,t - j\Delta) \quad if \;\; j = k_i - 1 \\ 1 - R(j,i,\Delta) + \sum_{r=i}^{m} Q(j+1,r) P_{i,r}(j) R(j,i,\Delta) \end{cases} \qquad (29)$$

Where i is the state of equipment at j_{th} point of time. And k_i is the interval in which the replacement occurs. It has been shown that the probability $P(T \leq T_d)$, is equal to the value of $Q(0,0)$ [67]. Starting from time j=0 with state i=0 and continuing forward, before reaching the observation interval at which the replacement occurs, the probability of a failure occurring before replacement is calculated from third line of Equation 29. Second like of equation 29 is the probability of a failure at the observation point j, knowing that the state is i. The recursive calculations continue until the condition of the first line of equation 29 is met. In order to calculate the optimal long-run average cost, equation 24 should be solved iteratively. By considering a starting value for d (a cost value) using equations (25) to (29), the optimal value for d or the optimal long-run average cost can be found. An example is given below to clarify how this model works:

Consider a piece of equipment with a Weibull baseline distribution having parameters β =4 and α=3 (equation 7):

And consider $\psi(z) = e^{0.8z}$, with K=4 and C=5 which leads to failure replacement cost of K+C and preventive replacement cost of C=5. The observation interval is considered to be equal to 1 (Δ=1) and the state of equipment can be either 0 or 1 (Z is represented by a homogenous Markov chain) with transition probabilities:

$$P = \begin{bmatrix} 0.74 & 0.26 \\ 0 & 1 \end{bmatrix}$$

We would like to find the minimal long-run average cost of the replacement and the supporting optimal decision strategy at each observation interval. To find the minimal long-run average cost, we begin with an optional value d=15 and continue the calculations until d=Φ_{T_d}. The results from calculations are summarized in Table 2.

Table 2: Step by step calculations to obtain optimal d

i	d	T_d	K_i	W(0,0)	Q(0,0)	$d_{new}=\Phi_{Td}$
1	15	4.02	5	2.493	0.880	3.416
2	3.416	2.308	3	1.966	0.23	3.03
3	3.03	2.212	3	1.911	0.210	3.056
4	3.056	2.219	3	1.915	0.212	3.053
5	3.053	2.218	3	1.914	0.212	3.053

Firstly T_d is calculated via equation 25. For instance at observation k=1, T_d is calculated as below:

$$T_d = 1.\inf\{r \in R^+ \geq 0: 4[1 - R(r, 0, \Delta)] \geq 15.\tau(r, 0, \infty)\} = 4.02$$

k_i is the interval in which the failure occurs, and is equal to ceil (4.02) =5. Then W(0,0) and Q(0,0) are calculated using equation 27 and 29. Finally d_{new} is calculated using equation 24, by replacing $P(T \leq T_d)]$ with Q(0,0) and $Emin\{T, T_d\}$ with W(0,0) as follows:

$$d_{new} = \frac{[C+KP(T \leq T_d)]}{E[min[T,Td]]} = \frac{[C+KQ(0,0)]}{W(0,0)} = 3.416$$

Now d= d_{new} and the above process is repeated until the optimal long-run average cost is found to be d^*= 3.053. Therefore for each observation, *n* with Z_n as the state of the equipment, a decision a can be made based on the result of the cost function as follows:

$$a = \begin{cases} Replace & if \quad 4[1 - R(n, Z_n, \Delta)] \geq 3.053 * \tau(n, Z_n, \infty) \\ Do\ nothing & if \quad 4[1 - R(n, Z_n, \Delta)] < 3.053 * \tau(n, Z_n, \infty) \end{cases}$$

1.6.4. CBM with imperfect information strategy

The mathematical structure of this strategy is very similar to CBM with perfect information strategy described in section 3.3. However, this approach assumes that the observations are imperfect and do not necessarily reveal the exact state of equipment, but the indicator value θ_k, which is observed at kth discrete points of time is stochastically related to the exact state of the equipment Z_k.

$$Z_k \in \{0,1,2,\dots,N-1\} \quad ; \quad k \in \{0, \Delta, 2\Delta, \dots, k\Delta\} \qquad (30)$$

$$\theta_k \in \{1,2,\dots.,M\} \quad ; \quad k \in \{0, \Delta, 2\Delta, \dots, k\Delta \qquad (31)$$

The stochastic relation between state and observation is known via the probability of observation value $q_{j\theta}$, where j is the state of the system and θ is the observation value. Q is the matrix including all probabilities $q_{j\theta}$ for j $\in \{0,1,2,\dots,N-1\}$ and $\in \{1,2,\dots.,M\}$.

The mathematical structure of this model was developed by Ghasemi et al. [60]. The failure rate is modeled using PHM using equation 18. The conditional reliability and mean sojourn time functions can be derived using equations 22 and 23.

In order to take into effect the assumption of imperfect information and stochastic relation of indicator value θ and state Z, a conditional probability distribution of the equipment's state is defined as [41]:

$$\pi^k = \{\pi_i^k:\ 0 \le \pi_i^k \le 1 \ \ for\ \ i = 0, \dots, N-1, \sum_{i=0}^{N-1} \pi_i^k = 1\}, k = 0,1,2, \dots \quad (32)$$

Where π_i^k is the probability of being at state i at the k^th^ observation, given all the observations and actions until the k^th^ observation point. It is also assured that $\pi_0^0 = 1$, meaning that equipment at time zero is new.

Since the exact state is unknown, the state transitions are calculated differently from the previous strategy. While knowing the conditional distribution probability π^k at kth observation, the probability of observing θ at observation point k+1 would be calculated as below [60]:

$$Pr(\theta|k, \pi^k) = \sum_{i=0}^{N-1} \sum_{j=1}^{N} \pi_i^k P_{ij} q_{j\theta} \quad (33)$$

At each observation point, when a value θ is observed, the conditional probability distribution is updated to $\pi_j^{k+1}(\theta)$ according to Bayes' formula [60]:

$$\pi_j^{k+1}(\theta) = \frac{\sum_{i=0}^{N-1} \pi_i^k P_{ij} q_{j\theta}}{\sum_{i=0}^{N-1} \sum_{l=1}^{N} \pi_i^k P_{il} q_{l\theta}} \quad (34)$$

As it can be seen in the formula above, the conditional probability carries all the history of equipment up to the current observation point. At each point the conditional probability is calculated based on the current observed value and the conditional probability of the previous observation point. This produces a set of probabilities, indicating the chance of being in either of states when θ is observed.

To obtain the minimum long-run average cost, a dynamic programming approach is used. The long-run expected average cost can be calculated as below [60]:

$$\Phi_{T_g} = \frac{C_{T_g}}{P_{T_g}} = \frac{[C+KP(T\leq T_g)]}{E[min[T,Tg]]} \tag{35}$$

In the dynamic programming formulation an iterative auxiliary function $W(j,\pi^j)$ is defined as:

$$W(j,\pi^j) = \begin{cases} 0 & if\ \ j \geq k \\ \int_0^{t_g(\pi^j)-j\Delta} \bar{R}(j,\pi^j,s)ds & if\ \ j = k-1 \\ \int_0^{\Delta} \bar{R}(j,\pi^j,s)ds + \sum_{\theta=1}^{M} W\left(j+1,\pi^{j+1}(\theta)\right)\bar{R}(j,\pi^j,\Delta)\,Pr(\theta|j,\pi^j) & if\ \ j < k-1 \end{cases}$$

(36)

It was shown that:

$E[min[T,Tg]]= W(0,\pi^0)$

In addition $Q(j,\pi^j)$ is defined as:

$$Q(j,\pi^j) = \begin{cases} 0 & if\ \ j \geq k \\ 1-\bar{R}(j,\pi^j,t_g(\pi^j)-j\Delta) & if\ \ j = k-1 \\ 1-\bar{R}(j,\pi^j,\Delta) + \sum_{\theta=1}^{M} Q\left(j+1,\pi^{j+1}(\theta)\right)\bar{R}(j,\pi^j,\Delta)\,\mathrm{Pr}(\theta|j,\pi^j) & if\ \ j < k-1 \end{cases}$$

(37)

It can be shown that $Q(0, \pi^0)$ is equal to the expected probability of having a failure before replacement time, i.e. $P(Tg \geq T)$, where T_g is the time to perform replacement and is calculated as below:

$$Tg = \Delta . inf\{k \geq 0: K[1 - \bar{R}(k, \pi^k, \Delta)] \geq g. \bar{\tau}(k, \pi^k, \Delta)\} \tag{38}$$

Where $\bar{R}(k, \pi^k, \Delta)$ and $\bar{\tau}(k, \pi^k, \Delta)$ are respectively the conditional probability that the equipment is working at observation point k+1 and the mean sojourn time of equipment at k+1 observation point, calculated as follows:

$$\bar{R}(k, \pi^k, t) = Pr\left(T > k\Delta + t \middle| T > k\Delta, (k, \pi^k)\right) = \sum_{i=1}^{N} R(k, i, t)\pi_i^k \tag{39}$$

$$\bar{\tau}(k, \pi^k, a) = \int_0^a \bar{R}(k, \pi^k, t)dt \tag{40}$$

Assuming a starting value for g (a cost value) the optimal value for optimal long-run average cost g^* can be found via the iterative solution of equation 35.

The same example considered for CBM with perfect information is considered and solved by the CBM with imperfect information strategy. The only extra input to the problem is the indicator θ which is assumed to obtain any value from the set {1,2,3}. Z_k (Unobservable state at k) and θ (Observed indicator value at k) are related by the probability matrix Q:

$$Q = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} 0.67 & 0.23 & 0.1 \\ 0.1 & 0.47 & 0.43 \end{bmatrix} \end{matrix}$$

The problem is to find the minimum long-run average cost which leads to optimum decisions at each observation interval. To find the minimum long-run average cost we begin with a starting value g=15 (a good starting value of g is the long run average cost of the run-to failure strategy), and the algorithm continues until g = Φ_{T_g}. The results from calculations can be found in Table 3:

Table 3: Step by step calculations to obtain optimal g

i	g	T_g	K_i	$W(0,\pi^0)$	$Q(0,\pi^0)$	$g_{new}=\Phi_{Tg}$
1	15	3.998	4	2.486	0.903	3.463
2	3.463	2.141	3	1.858	0.194	3.109
3	3.109	2.041	3	1.799	0.169	3.154
4	3.154	2.054	3	1.807	0.172	3.147
5	3.147	2.052	3	1.806	0.171	3.148
6	3.148	2.052	3	1.806	0.171	3.148

The optimal long-run average cost is g^*= 3.148. Therefore at each observation point k, a decision can be made based on the value of the following cost function:

$$a=\begin{cases} Replace & if \quad 4[1-R(k,\pi^k,\Delta)] \geq 3.148 * \bar{\tau}(k,\pi^k,\Delta) \\ Do\ Nothing & if \quad 4[1-R(k,\pi^k,\Delta)] < 3.148 * \bar{\tau}(k,\pi^k,\Delta) \end{cases}$$

1.7 Parameter Estimation of the Maintenance Strategies

A mathematical description of parameter estimation for the maintenance strategies used in this thesis are described in this section.

1.7.1 TBM and Run-to-Failure strategies

Parameters of TBM and run-to-failure strategy in this research are the Weibull parameters α, and β. Weibull parameter estimation is done using Maximum Likelihood Estimation (MLE). When a set of independent and identically distributed failure times are available, the MLE method can be used to find the parameters of the distribution. This is done by calculating the joint density function and maximizing it for distribution parameters as variables. Consider the set of independent and identically distributed variables $\{T_1, T_2, \ldots, T_n\}$, where the joint density function is:

$$f(T_1, T_2, \ldots, T_n|\delta) = f(T_1|\delta) \times f(T_2|\delta) \times \ldots \times f(T_n|\delta) \quad (41)$$

In the above function, the values of δ are the set of parameters to be estimated. The joint density function is referred to as the likelihood function. It is usually more practical to work with the logarithm of the likelihood function.

The simplified equations for each parameter of the Weibull distribution described by Balakrishnan & Kateri [85]are:

$$\alpha = \left\{\frac{1}{n}\sum_{i=1}^{n} T_i^{\beta}\right\}^{\frac{1}{\beta}} \quad (42)$$

$$\frac{1}{\beta} = \frac{\sum_{i=1}^{n} T_i^{\beta} ln(T_i)}{\sum_{i=1}^{n} T_i^{\beta}} - \frac{1}{n}\sum_{i=1}^{n} ln(T_i) \quad (43)$$

Where T_i, the ith failure time is in the historical data and n is the number of failures that are available to estimate the Weibull distribution parameters.

1.7.2 CBM with perfect information strategy

The parameters to be estimated are those of PHM, state transition probabilities; i.e. α, β, ϒ and $P=[P_{ij}]$. Banjevic et al. [86] used MLE technique to estimate PHM parameters. They derived the total logarithm of likelihood and maximized the logarithm. The log likelihood function is:

$$l = nln\left(\frac{\beta}{\alpha}\right) + (\beta - 1)\sum_{i=1}^{n} ln\left(\frac{T_i}{\alpha}\right) + \sum_{i=1}^{n} \Upsilon z^{(i)}(T_i) - \sum_{i=1}^{n}\int_0^{T_i} exp\{\Upsilon z^{(i)}(t)\} d(\frac{t}{\alpha})^{\beta} \quad (44)$$

Where n is the number of analyzed failure times, T_i is the failure time and $z(T_i)$ is the state of equipment at failure. Estimated parameters are the values that maximize the log-likelihood function. The estimation of transition probabilities (P_{ij}) is done by counting the number of transitions from i to j at time kΔ, $n_{ij}(k)$, and dividing it to the number of all transitions from state i at time kΔ, $n_{i0}(k)$:

$$P_{ij}(k) = \frac{n_{ij}(k)}{n_{i0}(k)} \tag{45}$$

1.7.3 CBM with imperfect information

The parameter estimation of this strategy is more complex since it should also estimate parameters of matrix Q and considers the relationship of matrix P and Q (Bayes' formula equation 34). A parameter estimation technique is introduced by Ghasemi et al. [38]. They derived a probability mass function for failure times which states:

$$f\big(T, \theta(T)\big) = \left[\prod_{k=0}^{l-1} Pr(\theta^{k+1}|T > (k+1)\Delta, \pi^k\right]\left[\prod_{k=0}^{l-1} \sum_{i=1}^{N} \pi_i^k exp(-\int_{k\Delta}^{(k+1)\Delta} h(s,i)ds)\right] \sum_{i=1}^{N} \pi_i^k exp(-\int_{k\Delta}^{T} h(s,i)ds) h(T,i) \tag{46}$$

Where l is the number of observation intervals, N is the number of possible states, T is the failure time, and θ^{k+1} is the observed indicator value at observation point k+1. This function calculates the probability density function of time to failure while observed indicator values up to time T are $\theta(T)$. The likelihood function is therefore:

$$L(\delta) = \prod_{i=1}^{n} f(T_i, \pi_i;\ \delta)\ , \tag{47}$$

Where n is the number of available histories and δ is the set of PHM parameters including probability matrices P and Q.

Parameter estimation for censored data in all above categories are also available, but it is not in the scope of this study and will not be discussed in this thesis.

This chapter provides the reader with a brief background required to understand the general purpose of this study. The importance of maintenance and some of the common maintenance strategies are discussed and the maintenance strategies investigated in our study are listed and the rationale behind these strategies are stated. Next the basic mathematics of reliability required for general understanding of the maintenance strategies are explained. Finally, the mathematics of each maintenance strategy and their parameters are described; and how the long-run average costs are calculated for each maintenance strategy.

To summarize, the purpose of this research is to illustrate the value of information in maintenance decision making, and to show how it is dependent on the equipment hazard rate function parameters and state transition behaviour in order to provide a framework by which the practitioners can choose the proper maintenance strategy. The run-to-failure strategy was introduced as the strategy that does not incorporate any information. The TBM strategy uses historic failure times of equipment to estimate parameters of Weibull distribution, and the decision making is done based on that information (only failure data). The CBM with perfect information uses historic failure data and observations from equipment at failure to estimate the parameters of PHM. In this strategy observations are assumed to be performed at equal times to collect observations from equipment and to decide whether to replace the equipment or to perform no action. This strategy assumes the observation information to perfectly represent actual state of equipment. The CBM with imperfect information acts similar to CBM with perfect information, except that it does not consider observations from equipment to represent the actual state of equipment. Instead observations from equipment are stochastically related to the actual state of equipment. Value of perfect information from equipment and the value of applying the CBM model assuming imperfect information will be derived by comparing the long-run average costs of CBM assuming perfect information and CBM assuming imperfect information.

Chapter 2: Methodology

In this study we compare the long-run average cost of CBM, TBM and run-to-failure strategies for various levels of condition monitoring accuracy. The accuracy ranges from "no condition monitoring data available" to "perfect condition monitoring information available" regarding the state of equipment. Furthermore, we investigate the effect of system parameters on the value of information. Survival data is generated through simulation and long-run average costs of maintenance strategies are calculated for a range of the model parameters. The goal is to provide a guideline for practitioners to identify the value of each maintenance strategy and to select a suitable strategy.

Prior to an analysis of maintenance strategies, survival data must be either gathered from a real system or generated through simulation. For this research, simulation is the preferred method since it would be infeasible to obtain real survival data with so many different sets of parameters. Simulation enables us to consider a wider range of parameters and therefore to examine a greater number of parameter sets. After generating survival data from simulation, parameters are then estimated for each type of maintenance strategy. Finally, the long-run average costs are calculated using test simulated data for each maintenance strategy.

The following figure illustrates the steps required for calculating long-run average costs:

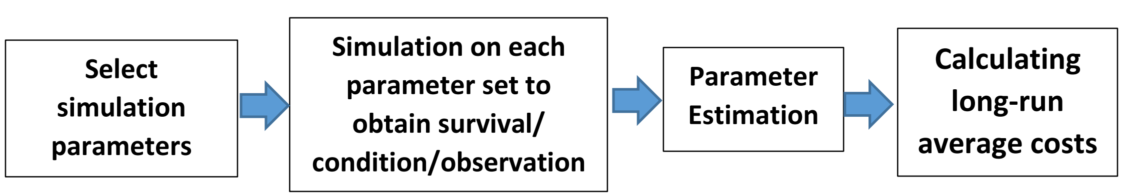


Figure 4: Steps towards calculating long-run average costs

The following sections describe in more detail the methodologies of this research. Firstly the system parameters are described, secondly the design of experiment is explained, and thirdly the maintenance strategies will be elaborated. Finally it will be shown how the long-run average costs are calculated and employed towards the goal of our study.

2.1. The System parameters

The system parameters fall into two categories: those that represent age of equipment, and those that represent state transition of equipment. PHM is used to model the hazard function while the state transition probabilities are modeled using a Markov Process.

PHM parameters are α and β for the baseline hazard function, and ϒ is the state-dependent function parameter. The state transition probabilities are probabilities of matrix P, and matrix Q. This study considers two possible state sets; first with two possible states (new, degraded), and second with three possible states (new, degraded, and worn-out) of equipment. For both cases, three possible values for observation indicator is considered. In terms of the state and observation sets defined, this means observations can have any value of $\theta \in \{1,2,3\}$ while the equipment state can have a value of $z \in \{1,2\}$ for two state equipment and $z \in \{1,2,3\}$ for three state equipment. Banjevic et al. [86] argue that considering more than three states requires significantly greater number of steps of recursive calculations and therefore more computation time. We do not consider Equipment with more than three states due to the high amount of complexity and CPU time that would be required. Makis et al. [67] and Ghasemi et al. [60] consider cases with two-state equipment. Kim et al [87] also argue that considering only two states is usually sufficient. Kim et al found through working with diagnostic data, that in many cases the equipment moves only through two distinct states. The system in the first state behaves normally and the observations are stationary, however in the second state the behaviour of observations change significantly meaning that the equipment is passed a certain level of degradation. Kim et al also point out that defining more states may not be advantageous because the practitioner may want to perform a maintenance action only when the equipment is under sever degradation.

The reason for considering a three-state equipment was to investigate if dividing the observations into more intervals will have any effect on the savings. The results will show that considering three states does not provide any improvements compared to two states in CBM decision making. The following shows the P and Q matrices for equipment with two and three states.

Two-state: $$P = \begin{bmatrix} P_{11} & 1 - P_{11} \\ 0 & 1 \end{bmatrix} \quad \& \quad Q = \begin{bmatrix} q & q_1 & q_2 \\ q_1 & q & q_2 \end{bmatrix}$$

Three-state: $$P = \begin{bmatrix} P_{11} & P_{12} & 1 - P_{11} - P_{12} \\ 0 & P_{22} & 1 - P_{22} \end{bmatrix} \quad \& \quad Q = \begin{bmatrix} q & q_1 & q_2 \\ q_1 & q & q_2 \\ q_1 & q_2 & q \end{bmatrix}$$

The matrix P represents the state transition probabilities. For example P_{12} is the probability that the equipment goes to state 2 from state 1 after a time step t. The Q matrix represents the stochastic relation between the state of equipment and monitoring observations from the equipment. The observation either represents the exact state of equipment with probability q, or a state which is not the exact state of equipment with probability $q_1 + q_2 = (1 - q)$. For simplicity, q_1 & q_2 are assumed roughly equal to $(1 - q)/2$.

As the probability of q increases, so does the probability of observing the actual state of equipment. In other words, perfect information is collected from the equipment if q = 1.

Another parameter which has been taken into account is the cost ratio of K/C. A failure replacement costs K+C and a preventive replacement costs C. If K≈0 the cost of failure replacement would be equal to a preventive replacement. The effect of parameters and the value of information is explored for different ratios of K/C in the next chapter.

For two-state equipment, each experiment includes the following parameters:

$$\{\alpha, \beta, \Upsilon, \frac{K}{C}, p_{11}, q\}$$

And for a three-state equipment:

$$\{\alpha, \beta, \Upsilon, \frac{K}{C}, P_1 = \{p_{11}, p_{12}, 1 - p_{11} - p_{12}\}, p_{22}, Q_1 = \{q, q_1, q_2\}, Q_2 = \{q_1, q, q_2\}, Q_3 = \{q_1, q_2, q\}\}$$

Where Q_1, Q_2, Q_3 are the first to third rows of the three-state matrix Q. And P_1 is the first row of the three-state matrix P. There are 6 parameters for two-state equipment and 9 parameters for three-state equipment. In order to analyze the effect of parameters on long-run average costs, we specify desired levels for each parameter. Investigating all the parameter combinations would require significant effort and computation. For example if each parameter of the two-state equipment is considered to have 5 levels, there would be 5^7 =78125 different possible combinations. Therefore, it is essential to design the experiments in a way that the number of experiments is reduced.

2.2. Design of Experiments

Despite the restrictive number of parameter combinations it is possible to consider fewer combinations and still derive robust conclusions on the effect of each parameter on long-run average costs and value of information. Taguchi [88] introduced a method for designing experiments which investigates how the mean and variance of the outputs from a system are affected by the inputs of the system. Taguchi method uses orthogonal arrays to pair different levels of parameters in a way that the effect of each parameter and their levels can be compared sensibly. Taguchi orthogonal arrays are predefined Tables selected for different number of parameters and different number of parameter levels. To implement this method, the levels for each parameter must first be defined. Selecting the number of levels depends on the sensibility of the results to the parameter, while selecting the values for each level is based on knowledge of the process (long-run average costs of the equipment). The minimum and maximum and normal value of the parameter are helpful in determining the levels.

For the 6 parameters of the two-state equipment, the orthogonal arrays indicate that there should be 2 levels for one parameter and 5 levels for the remaining parameters. For this study this recommendation is adhered to by considering two levels for P_{11} parameter, while the rest of parameters have 5 levels. The levels are defined based on our understanding of the process (long-run average costs obtained via preliminary experiments). For instance when parameters α and β have values greater than 6 the long-run average costs of the CBM strategies do not vary significantly. Table 4 and Table 5 show how the long-run average costs change for different values of α and β. (In the Tables 4, 5, and 6, Im-Im refers to the CBM strategy assuming imperfect information and P-P is the CBM strategy assuming perfect information. They will be explained in detail in section 3 of this chapter)

Table 4: The variations of long-run average costs of Im-Im and P-P strategies for different values of α. β=6, γ=3, K/C=2.5

α	Im-Im	P-P
2	12.73	11.93
3	8.72	8.05
4	7.44	6.79
5	6.12	5.65
6	5.07	4.82
7	4.38	4.12
8	3.89	3.65
9	3.44	3.28
10	3.1	3.01

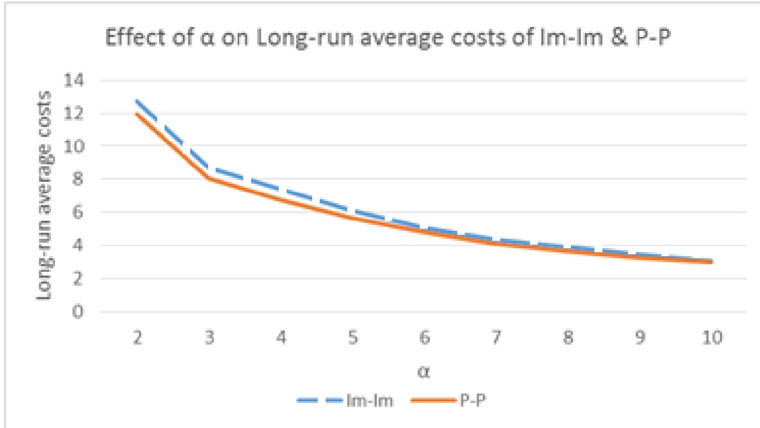
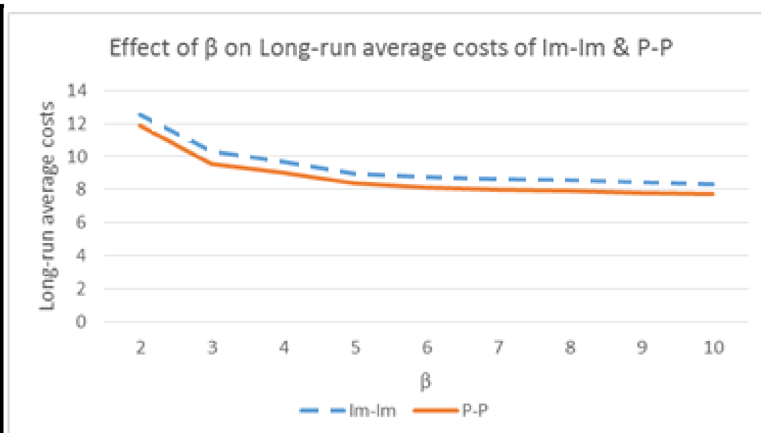


Table 5: The variations of long-run average costs of Im-Im and P-P strategies for different values of β. α =3, γ=3, K/C=2.5

β	Im-Im	P-P
2	12.52	11.85
3	10.34	9.55
4	9.67	9.03
5	8.98	8.38
6	8.77	8.15
7	8.65	8.02
8	8.6	7.96
9	8.45	7.82
10	8.34	7.76



As the value of parameters α and β increases, the steepness of long-run average costs decreases. It was possible to consider wider range of parameters α and β, but that would require greater amount of computation. Therefore, since the change in long-run average costs are not significant for higher values of α and β, the simulation was designed for the range [2-6].

Similarly, ϒ values considered here are in the range [0.5, 2.5]. However, there is no limit in the range of ϒ.

Table 6: The variations of long-run average costs of Im-Im and P-P strategies for different values of ϒ. α=3, β=5, K/C=2.5

ϒ	Im-Im	P-P
0.2	7.84	7.41
0.5	8.39	7.87
1	8.92	8.33
1.5	8.88	8.24
2	9.09	8.53
2.5	9.31	8.82
3	9.37	8.94
5	9.44	9.03

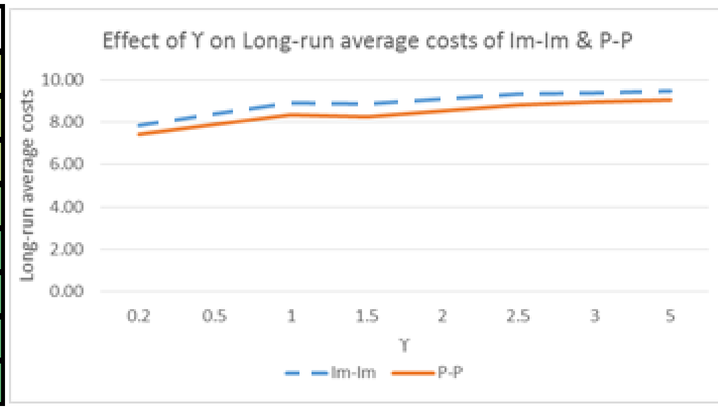


Table 7 reveals the parameters and their levels designed for two-state equipment:

Table 7: Designed parameters and their levels for two-state equipment

	p_{11}	β	α	K/C	ϒ	q
Level 1	0.2	2	2	0	0.5	0.1
Level 2	0.8	3	3	0.5	1.	0.3
Level 3	-	4	4	1	1.5	0.5
Level 4	-	5	5	2	2.	0.7
Level 5	-	6	6	4	2.5	0.9

From the orthogonal array and the designed parameters from Table 7, the Taguchi design of experiments for two-state equipment is constructed as follows:

Table 8: Taguchi experiments design for two-state equipment

	P_{11}	Beta	alpha	K/C	Gama	q		P_{11}	Beta	alpha	K/C	Gama	q
Experiment1	0.2	2	2	0	0.5	0.1	Experiment26	0.8	2	2	0	2	0.9
Experiment2	0.2	2	3	0.5	1	0.3	Experiment27	0.8	2	3	0.5	2.5	0.1
Experiment3	0.2	2	4	1	1.5	0.5	Experiment28	0.8	2	4	1	0.5	0.3
Experiment4	0.2	2	5	2	2	0.7	Experiment29	0.8	2	5	2	1	0.5
Experiment5	0.2	2	6	4	2.5	0.9	Experiment30	0.8	2	6	4	1.5	0.7
Experiment6	0.2	3	2	0.5	1.5	0.7	Experiment31	0.8	3	2	0.5	0.5	0.5
Experiment7	0.2	3	3	1	2	0.9	Experiment32	0.8	3	3	1	1	0.7
Experiment8	0.2	3	4	2	2.5	0.1	Experiment33	0.8	3	4	2	1.5	0.9
Experiment9	0.2	3	5	4	0.5	0.3	Experiment34	0.8	3	5	4	2	0.1
Experiment10	0.2	3	6	0	1	0.5	Experiment35	0.8	3	6	0	2.5	0.3
Experiment11	0.2	4	2	1	2.5	0.3	Experiment36	0.8	4	2	1	1.5	0.1
Experiment12	0.2	4	3	2	0.5	0.5	Experiment37	0.8	4	3	1	1.5	0.1
Experiment13	0.2	4	4	4	1	0.7	Experiment38	0.8	4	4	4	2.5	0.5
Experiment14	0.2	4	5	0	1.5	0.9	Experiment39	0.8	4	5	0	0.5	0.7
Experiment15	0.2	4	6	0.5	2	0.1	Experiment40	0.8	4	6	0.5	1	0.9
Experiment16	0.2	5	2	2	1	0.9	Experiment41	0.8	5	2	2	2.5	0.7
Experiment17	0.2	5	3	4	1.5	0.1	Experiment42	0.8	5	3	4	0.5	0.9
Experiment18	0.2	5	4	0	2	0.3	Experiment43	0.8	5	4	0	1	0.1
Experiment19	0.2	5	5	0.5	2.5	0.5	Experiment44	0.8	5	5	0.5	1.5	0.3
Experiment20	0.2	5	6	1	0.5	0.7	Experiment45	0.8	5	6	1	2	0.5
Experiment21	0.2	6	2	4	2	0.5	Experiment46	0.8	6	2	4	1	0.3
Experiment22	0.2	6	3	0	2.5	0.7	Experiment47	0.8	6	3	0	1.5	0.5
Experiment23	0.2	6	4	0.5	0.5	0.9	Experiment48	0.8	6	4	0.5	2	0.7
Experiment24	0.2	6	5	1	1	0.1	Experiment49	0.8	6	5	1	2.5	0.9
Experiment25	0.2	6	6	2	1.5	0.3	Experiment50	0.8	6	6	2	0.5	0.1

Similar to the two-state equipment, parameter levels and the Taguchi experiments are designed for three-state equipment as follows:

Table 9: Designed parameters and their levels for three-state equipment

	Υ	β	α	P_1	p_{22}	Q1	Q2	Q3	K/C
Level 1	0.5	2	2	(0.1,0.1,0.8)	0.2	(0.1,0.4,0.5)	(0.4,0.1,0.5)	(0.4,0.5,0.1)	0
Level 2	1.	3	3	(0.3,0.3,0.4)	0.8	(0.3,0.3,0.4)	(0.3,0.3,0.4)	(0.3,0.4,0.3)	0.5
Level 3	1.5	4	4	(0.2,0.7,0.1)		(0.5,0.3,0.2)	(0.3,0.5,0.2)	(0.3,0.2,0.5)	1
Level 4	2.	5	5	(0.6,0.2,0.2)		(0.7,0.1,0.2)	(0.1,0.7,0.2)	(0.1,0.2,0.7)	2
Level 5	2.5	6	6	(0.8,0.1,0.1)		(0.9,.05,.05)	(.05,0.9,.05)	(.05,.05,0.9)	4

Table 10: Taguchi experiments design for three-state equipment

	P2	Beta	alpha	K/C	Gama	P1	q		P2	Beta	alpha	K/C	Gama	P1	q
Experiment1	0.2	2	2	0	0.5	(0.1,0.1,0.8)	0.1	Experiment26	0.8	2	2	0	2	(0.6,0.2,0.2)	0.7
Experiment2	0.2	2	3	0.5	1	(0.3,0.3,0.4)	0.3	Experiment27	0.8	2	3	0.5	2.5	(0.8,0.1,0.1)	0.9
Experiment3	0.2	2	4	1	1.5	(0.2,0.7,0.1)	0.5	Experiment28	0.8	2	4	1	0.5	(0.1,0.1,0.8)	0.1
Experiment4	0.2	2	5	2	2	(0.6,0.2,0.2)	0.7	Experiment29	0.8	2	5	2	1	(0.3,0.3,0.4)	0.3
Experiment5	0.2	2	6	4	2.5	(0.8,0.1,0.1)	0.9	Experiment30	0.8	2	6	4	1.5	(0.2,0.7,0.1)	0.5
Experiment6	0.2	3	2	0.5	1.5	(0.2,0.7,0.1)	0.9	Experiment31	0.8	3	2	0.5	0.5	(0.1,0.1,0.8)	0.5
Experiment7	0.2	3	3	1	2	(0.6,0.2,0.2)	0.1	Experiment32	0.8	3	3	1	1	(0.3,0.3,0.4)	0.7
Experiment8	0.2	3	4	2	2.5	(0.8,0.1,0.1)	0.3	Experiment33	0.8	3	4	2	1.5	(0.2,0.7,0.1)	0.9
Experiment9	0.2	3	5	4	0.5	(0.1,0.1,0.8)	0.5	Experiment34	0.8	3	5	4	2	(0.6,0.2,0.2)	0.1
Experiment10	0.2	3	6	0	1	(0.3,0.3,0.4)	0.7	Experiment35	0.8	3	6	0	2.5	(0.8,0.1,0.1)	0.3
Experiment11	0.2	4	2	1	2.5	(0.8,0.1,0.1)	0.7	Experiment36	0.8	4	2	1	1.5	(0.2,0.7,0.1)	0.3
Experiment12	0.2	4	3	2	0.5	(0.1,0.1,0.8)	0.9	Experiment37	0.8	4	3	1	1.5	(0.2,0.7,0.1)	0.3
Experiment13	0.2	4	4	4	1	(0.3,0.3,0.4)	0.1	Experiment38	0.8	4	4	4	2.5	(0.8,0.1,0.1)	0.7
Experiment14	0.2	4	5	0	1.5	(0.2,0.7,0.1)	0.3	Experiment39	0.8	4	5	0	0.5	(0.1,0.1,0.8)	0.9
Experiment15	0.2	4	6	0.5	2	(0.6,0.2,0.2)	0.5	Experiment40	0.8	4	6	0.5	1	(0.3,0.3,0.4)	0.1
Experiment16	0.2	5	2	2	1	(0.3,0.3,0.4)	0.5	Experiment41	0.8	5	2	2	2.5	(0.8,0.1,0.1)	0.1
Experiment17	0.2	5	3	4	1.5	(0.2,0.7,0.1)	0.7	Experiment42	0.8	5	3	4	0.5	(0.1,0.1,0.8)	0.3
Experiment18	0.2	5	4	0	2	(0.6,0.2,0.2)	0.9	Experiment43	0.8	5	4	0	1	(0.3,0.3,0.4)	0.5
Experiment19	0.2	5	5	0.5	2.5	(0.8,0.1,0.1)	0.1	Experiment44	0.8	5	5	0.5	1.5	(0.2,0.7,0.1)	0.7
Experiment20	0.2	5	6	1	0.5	(0.1,0.1,0.8)	0.3	Experiment45	0.8	5	6	1	2	(0.6,0.2,0.2)	0.9
Experiment21	0.2	6	2	4	2	(0.6,0.2,0.2)	0.3	Experiment46	0.8	6	2	4	1	(0.3,0.3,0.4)	0.9
Experiment22	0.2	6	3	0	2.5	(0.8,0.1,0.1)	0.5	Experiment47	0.8	6	3	0	1.5	(0.2,0.7,0.1)	0.1
Experiment23	0.2	6	4	0.5	0.5	(0.1,0.1,0.8)	0.7	Experiment48	0.8	6	4	0.5	2	(0.6,0.2,0.2)	0.3
Experiment24	0.2	6	5	1	1	(0.3,0.3,0.4)	0.9	Experiment49	0.8	6	5	1	2.5	(0.8,0.1,0.1)	0.5
Experiment25	0.2	6	6	2	1.5	(0.2,0.7,0.1)	0.1	Experiment50	0.8	6	6	2	0.5	(0.1,0.1,0.8)	0.7

Once the experiments are properly designed, the simulation to generate survival data can now be performed. For each experiment, 100 sets of survival data are simulated using PHM and the state transition probabilities. The simulation provides the failure time of equipment, actual state of equipment at each observation interval, and the observed value at each interval. Consider the 35th experiment of Taguchi Table for two-state equipment, a sample simulation set is shown in Table 11:

Table 11: 35th experiment of Taguchi Table for two-state equipment

k	State	Observed θ	S/F
0	-	-	S
1	1	2	S
2	2	2	S
3	2	3	F
Failure time of equipment: 3.4			

k is the time interval, *State* is actual state of equipment at each interval, *Observed θ* is the observed value at each point, and *S/F* indicates whether the equipment survives (S) at or fails (F) during the period. The equipment survival or failure is decided by comparing a randomly generated number from [0, 1] to the value of reliability function (eq. 22) at each observation point. The failure time is decided by adding the observation interval value k to the result of the following function:

$$Pt = \frac{1 - R(k, Z_k, t)}{R(k, Z_k, 0) - R(k, Z_k, 1)} \qquad \textit{Equation 48}$$

Where Z_k, is the state of equipment at the interval in which the equipment fails. This function returns the probability of failure up to time t. The time between two consecutive observations is defined discretely by the set t={.05,.15,.25,.35,.45,.55,.65,.75,.85,.95} with probability P_t. For example, if the equipment fails at observation interval k=4, and for each value of t the calculated probabilities from the above function are equal to P_t={0.51,0.33,0.14,0.01,0.01,0,0,0,0,0}. With a randomly generated number 0.45; since 0.45<0.51 the failure time would be 4+0.05=4.05. The flow chart algorithm for generating one set of failure data is presented in Figure 5:

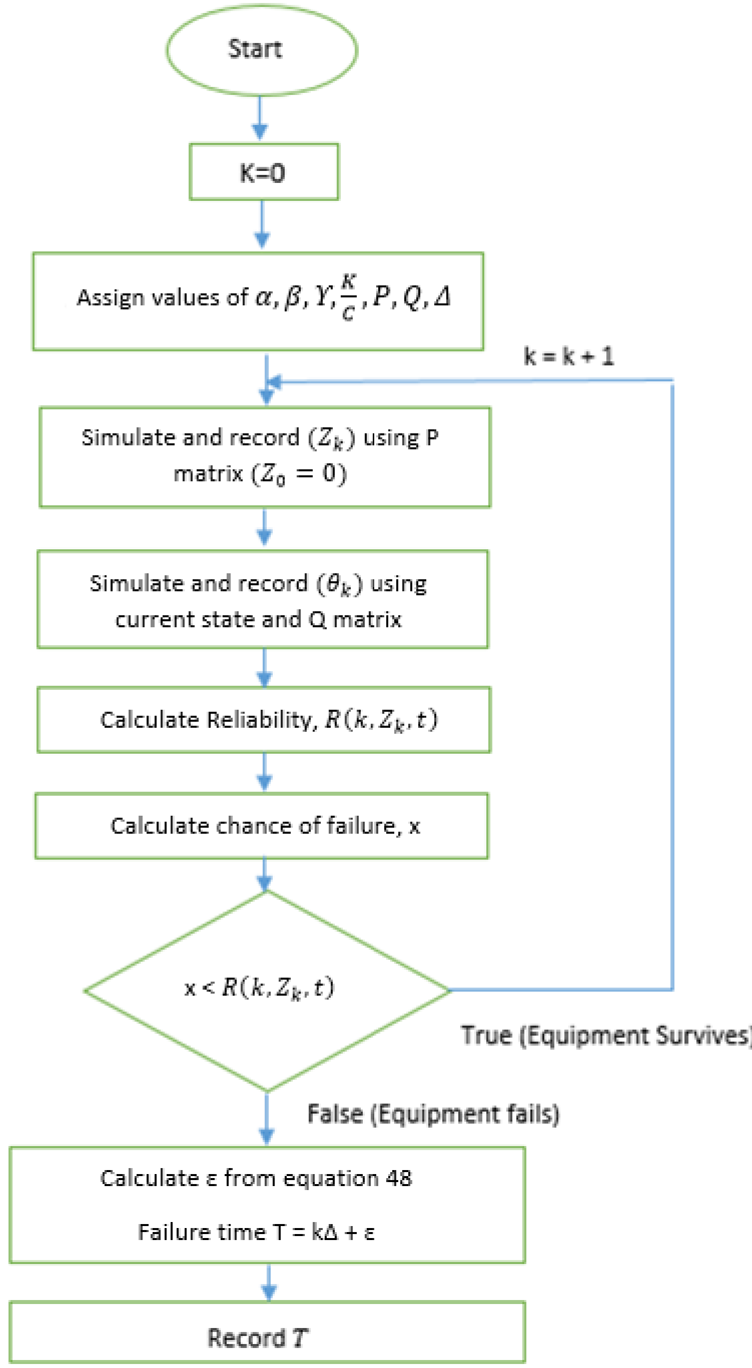


Figure 5 Flow chart algorithm for generating survival data

For each experiment, 100 sets of survival data is simulated to achieve acceptable accuracy from parameter estimation. In order to show that 100 sets of survival data provides good estimation of system parameters, simulation with parameters $\alpha = 4, \beta = 2, and\ \Upsilon = 0.5$ is done to obtain 10, 20, 50, and 100 sets of survival data and the results of parameter estimation of CBM with perfect information is performed on each set are provided in Table 12 and Table 13:

Table 12: Parameter estimation results for CBM assuming perfect information strategy

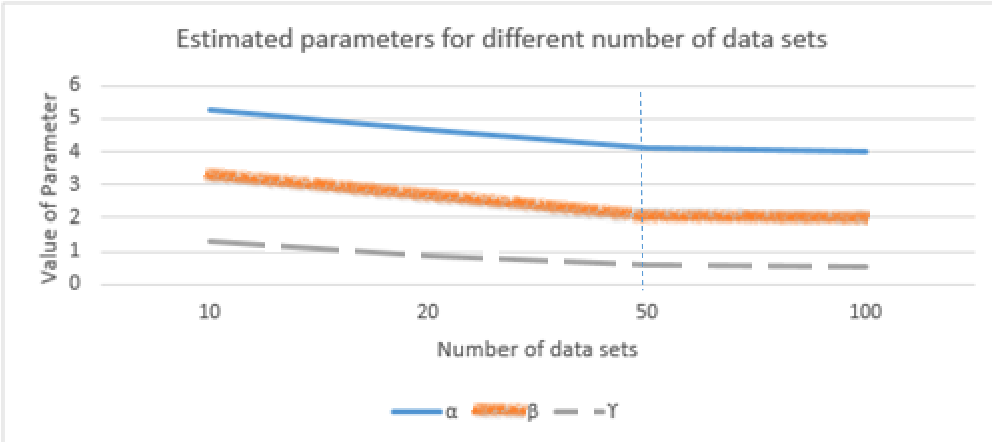
Number of survival data sets	α	β	ϒ
10	5.25	3.26	1.32
20	4.69	2.62	0.85
50	4.12	2.05	0.58
100	4	1.99	0.52

Table 13: Ratio of estimated parameters for different number of data sets

Ratio of estimated parameters for:	α	β	ϒ
20 sets of data /10 sets of data	0.89	0.80	0.64
50 sets of data /20 sets of data	0.88	0.78	0.68
100 sets of data/50 sets of data	0.97	0.97	0.90

As the number of data sets increases, the estimated parameters converge to initial parameters of the simulation. Accuracy of parameter estimation based on the number of sets of survival data is presented in Figure 6:

Figure 6: Estimated parameters for different number of data sets for CBM assuming perfect information strategy



It appears that roughly for more than 50 sets of survival data, the estimated parameters converge to a constant value. We considered 100 sets of survival data to be sufficient and estimated the parameters of the maintenance strategies accordingly.

After specifying the design of experiments and simulating the survival data, the next step is to estimate the parameters for each maintenance strategy and calculate long-run average costs based on the estimated parameters.

2.3. Maintenance strategies under study

Four strategies were introduced in chapter 1; run-to-failure, time-based maintenance and condition-based maintenance assuming perfect information, and condition-based maintenance assuming imperfect information. The run-to-failure strategy (RtF) does not take into account any information while time-based maintenance strategy goes one step further and considers one type of information, normally the equipment history of age at failure. Condition-based maintenance strategy considers both the age of equipment at failure and the state of the equipment. Hence, by comparing the RtF strategy with the TBM strategy we can infer the value of age information, and similarly, comparing CBM and TBM strategies give us the value of state information.

The Perfect-Perfect (P-P) strategy uses the ***perfect state information*** (column 2 of Table 11), which is the actual state of equipment, and models the maintenance decision making based on the CBM strategy assuming ***perfect*** information. The Imperfect-Perfect (Im-P) strategy uses the ***imperfect state information,*** i.e. the observations made from the equipment (column 3 of Table 11), and simulates the maintenance decision making based on the CBM strategy assuming ***perfect*** information. The Imperfect-Imperfect (Im-Im) strategy uses the ***imperfect state information*** (same as the Im-P strategy), which are the observations made from the equipment, and models the decision making based on the CBM strategy assuming ***imperfect*** information.

To better understand the strategies used in this study, consider that the health state of an engine is defined to have three states; good as new, medium degraded health state, and highly degraded health state. To find out the health state of the engine we collect a sample of the oil and analyze it to measure the amount of particles in the oil. If the amount of particles is high we can conclude that the engine is degraded. Assume that 0% to 3% of additives identifies a like new engine, 3% to 6% of particles show a medium degraded engine, and between 6% and 9% of particles show a highly degraded engine, and higher than 9% of particles is characterized as a failed engine. An oil analysis aims at identifying the actual particles in an engine oil, however if the test is subject to error and inaccuracy in measurement; analysis results showing 7% of particles may be due to a different particles level. In this example the 7% of particles indicates the imperfect state information obtained from the engine. Therefore it is possible that the actual amount of particles is in fact less than 6%, or in other words the actual state of engine is medium degraded. In that case (<6%) "medium degraded" is the perfect information that we want to obtain from the engine and 7% is the observation from the equipment or the imperfect state of the equipment. In reality, achieving high accuracy in measurement is usually not feasible. For that reason we can use historical data of the measurements (observations) from equipment and failure time data of the equipment to find a relationship between the actual health state of the equipment and the observed health state of the equipment.

The Perfect-Perfect strategy assumes that we know the actual health state of the engine and the decision making is done by CBM model assuming perfect information. The Imperfect-Perfect strategy which is the real world application of the CBM model assuming perfect information, uses the imperfect information obtained from the engine but ignores its imperfection and uses the CBM model assuming perfect information for maintenance decision making. The Imperfect-Imperfect strategy considers the reality that the actual state of equipment can hardly be measured and the CBM model assuming imperfect information is used.

The P-P strategy requires knowledge of the exact state of equipment, which is obtained from the simulation of survival data for this research. The P-P strategy and the Im-P strategy both use the same decision making model (CBM assuming perfect information), the only difference between these two strategies is that the P-P strategy is fed with exact state of equipment but the Im-P strategy uses the observations from equipment assuming that the observation is a direct (perfect) indicator of the state of the system. However, the Im-P strategy and the Im-Im strategy both use observations from equipment (imperfect state information) but they use different decision making models; Im-P uses CBM assuming perfect information model and Im-Im uses CBM assuming imperfect information model. Therefore the comparison of Im-P strategy and Im-Im strategy provides us the value of the CBM strategy assuming imperfect information. Furthermore, we show that the long-run average costs of Im-Im strategy converges to P-P strategy if the practitioner is provided with more accurate state information. Hence, it can be inferred that comparing Im-Im strategy and P-P strategy gives us the value of having perfect state information from equipment in CBM using PHM. Figure 7 illustrates the CBM strategies.

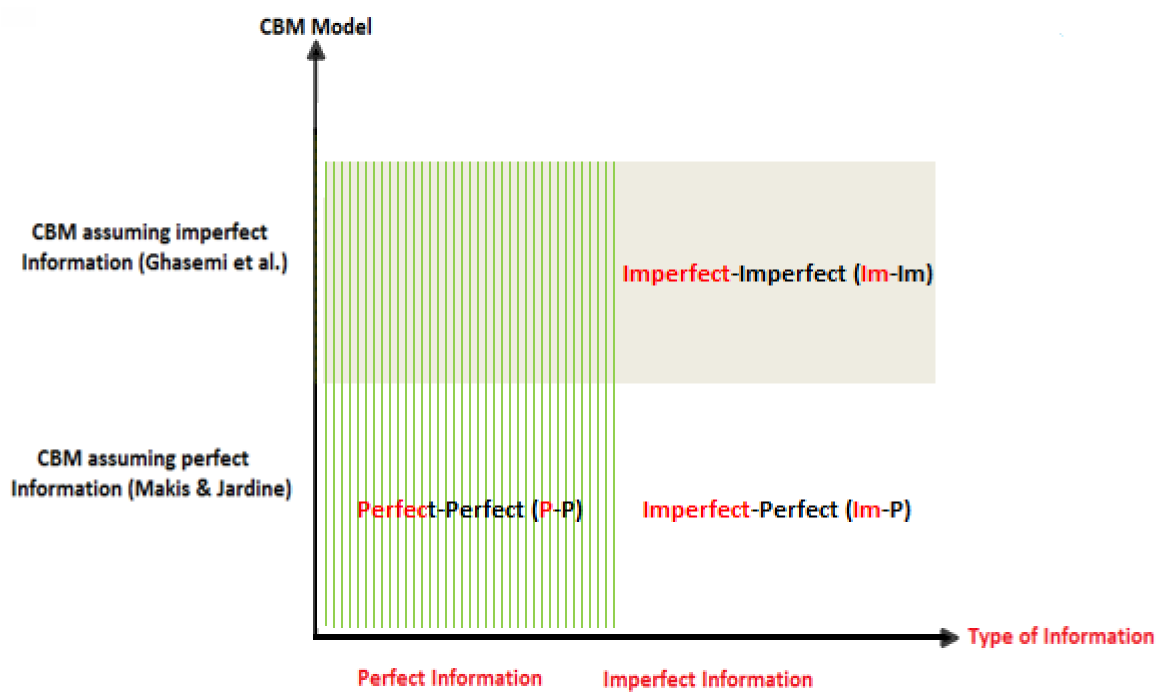


Figure 7: Description of CBM strategies by the model they incorporate and the assumption regarding observations from equipment

A significant part of this research is to understand the value of applying the CBM strategy with perfect information versus imperfect information. The former is achieved by comparing Perfect-Perfect (P-P) CBM strategy with Imperfect-Imperfect (Im-Im) CBM strategy, and the latter is achieved by comparing the Imperfect-Imperfect (Im-Im) CBM strategy with Imperfect-Perfect (Im-P) CBM strategy.

2.4. Long-run average costs of strategies using simulated survival data

The long-run average costs of the maintenance strategies can be calculated after the survival data is generated and the parameters are estimated accordingly. In the first chapter of this thesis we introduced the detailed mathematics and information required to estimate parameters for each of the maintenance strategies. In summary, for RtF strategy the equipment is only replaced at failure, and for TBM strategy equipment is replaced at failure or at the optimal replacement time T^*. However, for CBM strategies, at each observation moment, a decision should be made to whether replace the equipment or not, based on the threshold of the observed values, for each maintenance strategy we need to simulate the preventive replacements and failure replacements using the estimated parameters and replacement policies of each strategy. For that reason a set of test data should be generated. Test data is generated similar to the survival data except that, the survival data is used to estimate parameters and the test data is used to simulate replacement policies and calculate long-run average costs.

The decision making policy is simulated on 100 sets of the test data. The reason behind selecting 100 sets of simulated test data is the statistical behaviour of calculated long-run average costs. The long-run average costs for different number of test sets are provided below:

Table 14: Long-run average costs for different number of test data sets

Number of data	Im-P	Im-Im	P-P	TBM
10	7.15	7.14	6.88	7.64
20	7.45	7.35	7.13	8.17
50	7.38	7.26	6.93	8.06
100	7.31	7.2	7.04	7.91
200	7.31	7.2	7.04	7.91

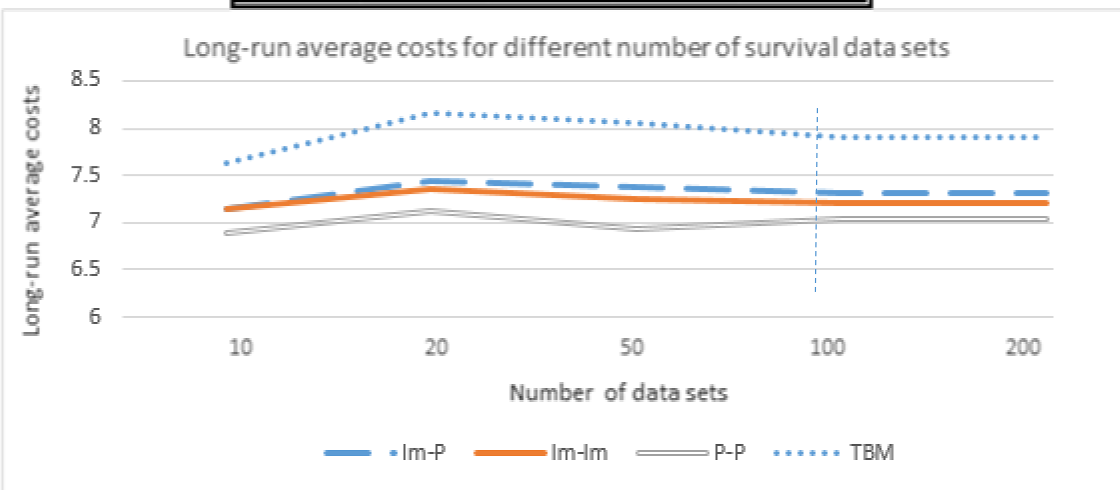


Table 14 graphs the mean values of survival data for ϒ=2.5, α=2, 4, 6, and β=2, 4, 6. The results show that for slightly less than 100 sets of data the long-run average costs converges to a constant value. Therefore, in this work we will use 100 sets of test data.

Figure below summarizes the simulation procedure of calculating long-run average costs (note that for conformity we refer to survival data as training data):

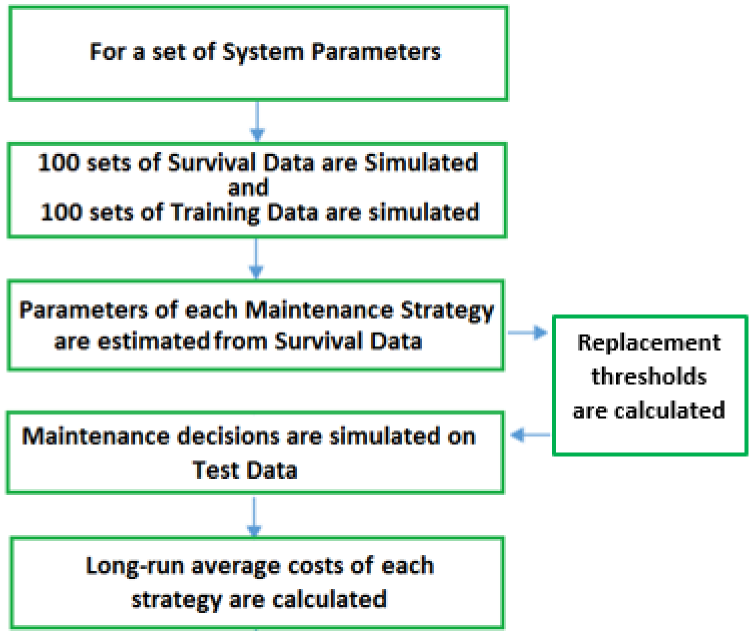


Figure 8: The simulation procedure of calculating long-run average costs

Each experiment with predefined sets of parameters is simulated to produce 100 sets of training data (including observation time, observation value, state value, and failure time). Each 100 sets of training data is used to calculate the parameters and decision criteria for each of the 5 maintenance strategies. Another set of 100 data (test data) is simulated using each set of predefined parameters. Using the above process, long-run average costs of the Rtf, TBM, Im-P, Im-Im, and P-P strategies are calculated for each of the 100 sets of test data.

Detailed description of calculating long-run average costs for each strategy are provided below:

2.4.1. RtF strategy

Given that all replacements in this strategy occur at the point of failure, the expected cost of applying this strategy is always K+C (cost of a failure replacement). The expected time to failure is the average of failure times from the survival data. The long-run average cost of this strategy is calculated by simply dividing the expected cost of the failure to the expected time of failure (in test data).

2.4.2. TBM strategy

The long-run average costs of this strategy are calculated by first deriving the optimal time of replacement using equation 16. If the failure occurs before T^*, the replacement time is equal to the time of failure and the replacement cost is equal to K+C. However if the failure occurs after T^* the replacement time is T^* and the replacement cost is equal to C. The expected cost of replacement and the expected replacement time are then calculated by taking averaging of the test data sets and dividing the expected cost of replacement with the expected time of replacement.

2.4.3. CBM strategies

Calculating the long-run average costs of the CBM strategies (Im-P, Im-Im, P-P) requires simulation of replacement decisions at each observation point of the test data. The expected cost of the CBM strategies and the expected replacement time of these strategies are based on the decisions that are made at each observation.

2.4.3.1. Im-P and P-P strategy

Both of these models assume that the data fed to the model is perfect state information, however for Im-P the observation values obtained from monitoring are assumed to be imperfect indicator of the state of the system. Following is valid for both Im-P and P-P strategies, however in Im-P θ_k (observation value obtained from the system) is assumed to be the indirect indicator of equipment health at time k. P-P uses the actual state of the system.

At each observation the maintenance decision is made based on the equations below.

$$a = \begin{cases} Replace & if \quad K[1 - R(k, Z_k, \Delta)] \geq d^* * \tau(k, Z_k, \infty) \\ No\ Action & if \quad K[1 - R(k, Z_k, \Delta)] < d^* * \tau(k, Z_k, \infty) \end{cases}$$

As explained chapter 1 d^*, $R(n, Z_n, \Delta)$, and $\tau(n, Z_n, \infty)$ are calculated from equations (25) to (29) using the estimated parameters from the simulated data for CBM assuming perfect information. Im-P strategy uses observations from equipment and the CBM with perfect information to model the decision making process. Therefore, for this strategy the values of Z_k come from the Observation column of the test data. Table 15 provides an example of the test data and the decisions made at each observation for Im-P strategy:

Table 15 Test data data and simulation of replacement policy for Im-P strategy

Test Data				Simulation of replacement Policy		
k	**State**	**Observation**	**S/F**	**Decision**	**Cost**	
0			S	No Action	0	
1	1	2	S	No Action	0	
2	2	2	S	Replace	C	
3	2	3	F			
Failure time of equipment					Replacement time = 2	
3.45						
k	**State**	**Observation**	**S/F**	**Decision**	**Cost**	
0			S	No Action	0	
1	1	2	S	No Action	0	
2	2	2	F	No Action	K+C	
Failure time of equipment					Replacement time = 2.15	
2.15						

In the first test data set of the Table 15, the equipment is replaced before the failure occurs; therefore, the replacement cost is C and the replacement time is k=2. For the second test data set, the failure occurs before a preventive replacement decision is made; therefore, the replacement cost is K+C and the replacement time is the failure time which is equal to 2.15 according to the simulation. The above was an example for a 2 sets of test data. The simulation is done for 100 test data sets and the expected replacement cost and the expected replacement time are calculated by averaging the results from the 100 sets of data. Consider the cost associated with test data i is c_i and the replacement time of test data i is t_i, and the number of sets of data is n (n = 100 in our simulation), the expected average cost per unit of time is calculated from the equation below:

$$\text{Average cost per unit of time} = \frac{\text{Total cost } of\ replacement}{\text{Total time } of\ replacement} = \frac{\sum_{i=1}^{n} c_i}{\sum_{i=1}^{n} t_i} \qquad (49)$$

The long-run average costs for P-P strategy are calculated similar to Im-P strategy. The difference is the P-P strategy uses the exact state of equipment (from State column of the test data set) for the parameter estimation and decision making. For P-P strategy values of Z_k are from the state column and for Im-P strategy the values of Z_k come from Observation column.

2.4.3.2. Im-Im strategy

This strategy uses the CBM with imperfect information to model the decision making process. At each observation the maintenance decision is made based on the equations below:

$$a = \begin{cases} Replace & if \quad K[1-\bar{R}(k,\pi^k,\Delta)] \geq d^* * \bar{\tau}(k,\pi^k,\Delta) \\ No\ Action & if \quad K[1-\bar{R}(k,\pi^k,\Delta)] < d^* * \bar{\tau}(k,\pi^k,\Delta) \end{cases}$$

Equations (36) to (48) from chapter 1 are used to calculate the values of d^*,$\bar{R}(k, \pi^k, \Delta)$, and $\bar{\tau}(k, \pi^k, \Delta)$.

In this chapter the design of experiments as well as the maintenance strategies were described, and it was pointed out that from the long-run average costs of the maintenance strategies the value of information can be extracted. In the next chapter, the long-run average costs are presented and the value of information is extracted and discussed. Furthermore, the effect of system parameters on long-run average costs and value of information is examined. Value of age information is derived by comparing long-run average costs of RtF and TBM strategies, value of imperfect state information, from TBM and Im-P strategies, value of CBM model assuming imperfect information is obtained by comparing Im-P and Im-Im strategies, and finally the value of perfect information is acquired by comparing Im-Im and P-P strategies.

Chapter 3: Experiments and Results

In this chapter, we compare the long-run average costs of the discussed maintenance strategies for different parameter combinations in order to illustrate the value of information in maintenance decision making. The detailed mathematical description of the maintenance strategies are provided in the first two chapters. Experiments are designed using the Taguchi method to investigate the effect of parameters on long-run average costs. The long-run average costs of the maintenance strategies are compared to long-run average cost of the RtF strategy to illustrate the average savings that can be obtained from each strategy. Paired comparison of the long-run average costs of the maintenance strategies demonstrate the value of information and the value of CBM strategy assuming imperfect information from equipment. We present the **value of equipment's age information** by comparing the *RtF strategy* with the *TBM strategy*, **the value of observation** by comparing *Im-P strategy* and *TBM strategy*, **the value of CBM model assuming imperfect information** by comparing *Im-P* and *Im-Im strategy*, and **the value of perfect information** by comparing the *Im-Im strategy* and the *P-P strategy*.

Table 16 Comparisons of long-run average costs of the maintenance strategies to obtain the value of information/model

Type of Information/Model	**How the value of Information/Model is obtained**
Age Information	Long-run-average cost of RtF – Long-run-average cost of TBM
Observation (Imperfect State Information)	Long-run-average cost of TBM – Long-run-average cost of Im-P
CBM model assuming Imperfect information	Long-run-average cost of Im-P – Long-run-average cost of Im-Im
Perfect State Information	Long-run-average cost of Im-Im – Long-run-average cost of P-P

Table 16 summarizes the definitions of value of information/model. To provide better insight, the present net value of information is also shown by considering different interest rates over different periods of time. Having the present net value of information, the practitioner can decide whether it is profitable to implement each type of information or CBM model while taking into account the time value of money and comparing with present value of required investment and operation cost of the implemented type.

This study also focuses on analyzing the effect of system parameters on long-run average costs of the maintenance strategies. We investigate the effect of PHM hazard rate function parameters (α, β, and ϒ), as well as the effect of state transition matrix P and state-observation probabilities matrix Q. The effects are studied for two-state equipment and three-state equipment.

In the last section of this chapter, an application developed to compare long-run average costs of the maintenance strategies is introduced.

3.1. Results from Taguchi Experiments for two state equipment

The parameters and design of experiments were explained in details in chapter 2. The system parameters are PHM parameters (α and β for the baseline hazard function, and ϒ is the state-dependent function parameter), and the state transition matrix P and state-observation probabilities matrix Q. PHM is used to model the hazard function while the state transition probabilities are modeled using a Markov Process. The parameters and design of experiments were explained in chapter 2. For every experiment, simulation is performed to obtain long-run average costs per unit of time (day, week, month …) of the maintenance strategies. The simulation is performed on 100 sets of test data and the results are provided in

Table 17 (for two-state equipment). The darker cells show the worse strategies and the brighter cells show the better strategies. For instance, the results for experiment 17, as expected, show that the lowest long-run average cost is achieved by using a Perfect-Perfect strategy (P-P) and the highest long-run average cost is a result of using a Run-to Failure maintenance strategy.

Table 17: Taguchi results for two-state equipment

	P-P	Im-Im	Im-P	TBM	RtF		P-P	Im-Im	Im-P	TBM	RtF
Experiment1	6.49	6.49	6.49	6.49	6.49	**Experiment26**	6.27	6.27	6.27	6.27	6.27
Experiment2	7.84	7.85	7.86	7.95	7.97	**Experiment27**	6.27	6.34	6.55	6.74	6.87
Experiment3	8.82	8.89	9.17	9.38	9.93	**Experiment28**	5.96	5.98	6.01	6.03	6.15
Experiment4	9.03	9.47	10.71	11.19	14.74	**Experiment29**	6.68	6.85	6.9	6.95	8.22
Experiment5	9.84	9.99	10.05	11.98	25.31	**Experiment30**	8.06	8.07	9.14	9.09	12.8
Experiment6	10.17	10.38	10.63	10.68	10.93	**Experiment31**	8.39	8.41	8.47	8.39	8.65
Experiment7	9.27	9.38	9.68	10.14	12.02	**Experiment32**	7.06	7.13	7.16	7.24	8.17
Experiment8	8.73	9.47	9.49	10.95	16.43	**Experiment33**	6.63	6.77	7.01	7.21	9.84
Experiment9	7.14	7.19	7.34	7.3	13.03	**Experiment34**	6.82	6.87	7.34	7.46	14.34
Experiment10	2.4	2.7	2.77	2.78	3.14	**Experiment35**	2.6	2.6	2.6	2.6	2.6
Experiment11	10.44	10.58	10.6	13	16.42	**Experiment36**	9.78	9.85	10.09	10.18	11.7
Experiment12	7.97	7.99	8.07	8.05	12.38	**Experiment37**	6.17	6.62	6.76	6.82	8.21
Experiment13	7.59	7.72	7.8	7.95	17.96	**Experiment38**	7.33	7.45	7.82	8.19	16.72
Experiment14	3.15	3.15	3.15	3.15	3.15	**Experiment39**	2.36	2.36	2.36	2.36	2.36
Experiment15	4.17	4.2	4.26	4.18	4.46	**Experiment40**	2.9	2.95	2.99	2.96	3.2
Experiment16	11.45	11.98	12.37	11.61	18.81	**Experiment41**	11.35	11.71	11.72	11.38	17.63
Experiment17	9.3	10.03	10.05	9.78	24.97	**Experiment42**	7.81	7.91	7.9	7.84	18.73
Experiment18	3.97	3.97	3.97	3.97	3.97	**Experiment43**	2.96	2.96	2.96	2.96	2.96
Experiment19	4.88	4.89	4.91	4.88	5.31	**Experiment44**	3.34	3.46	3.68	3.47	3.74
Experiment20	3.07	3.09	3.24	3.12	4.05	**Experiment45**	3.26	3.38	3.48	3.64	4.47
Experiment21	13.3	13.76	14.23	14.31	35.64	**Experiment46**	11.45	12.11	12.52	12.45	27.2
Experiment22	5.18	5.18	5.18	5.18	5.18	**Experiment47**	3.9	3.9	3.9	3.9	3.9
Experiment23	3.93	3.94	3.95	3.93	4.39	**Experiment48**	4.05	4.11	4.32	4.21	4.7
Experiment24	3.78	3.81	3.9	3.81	5.08	**Experiment49**	3.79	3.83	4.15	4.17	5.31
Experiment25	3.8	3.87	3.91	3.83	6.91	**Experiment50**	3.1	3.21	3.27	3.13	5.68

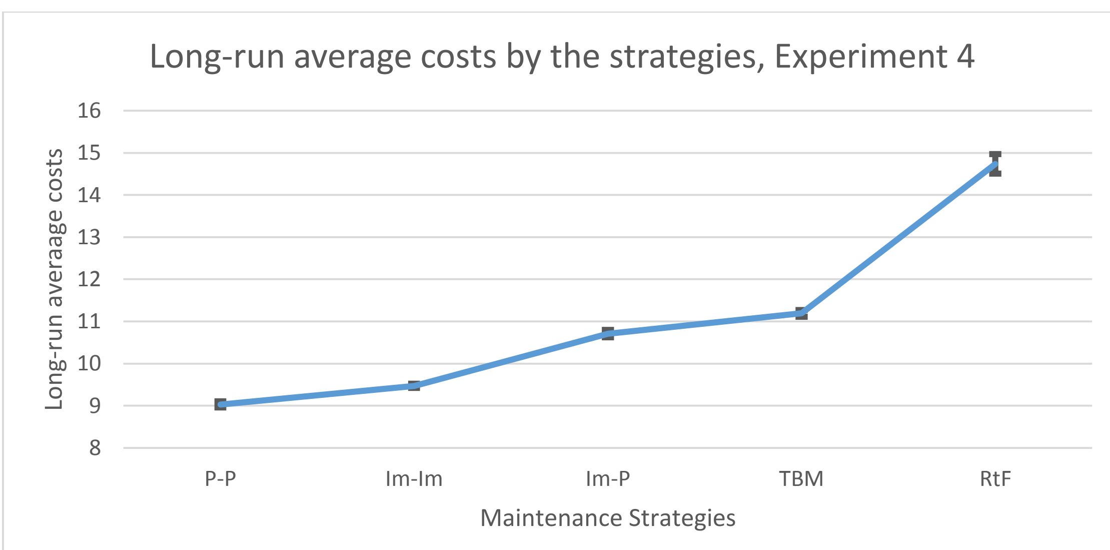
The long-run average costs provided in Table 17 are the average of 5 runs of the simulation of the long-run average costs. In Table 18 some of the variances are provided (experiment1-

10). In the first part of Table 18 the variances are provided and in the second part the percentage of the variances over the corresponding value of long-run average cost is illustrated. It can be observed that except the RtF strategy, the variances for other strategies are less than or equal to %1.

Table 18: Variances of the long-run average costs of the maintenance strategies for the first 10 Taguchi experiments

	P-P	Im-Im	Im-P	TBM	RtF
Experiment1	0.02	0.02	0.02	0.02	0.02
Experiment2	0.09	0.05	0.09	0.06	0.09
Experiment3	0.05	0.04	0.11	0.06	0.14
Experiment4	0.07	0.05	0.09	0.08	0.23
Experiment5	0.03	0.02	0.07	0.03	0.44
Experiment6	0.01	0.04	0.06	0.06	0.16
Experiment7	0.04	0.02	0.05	0.1	0.19
Experiment8	0.02	0.04	0.06	0.12	0.3
Experiment9	0.09	0.05	0.12	0.1	0.17
Experiment10	0.01	0.01	0.02	0.01	0.02
	P-P	**Im-Im**	**Im-P**	**TBM**	**RtF**
Experiment1	0%	0%	0%	0%	0%
Experiment2	1%	1%	1%	1%	1%
Experiment3	1%	0%	1%	1%	1%
Experiment4	1%	1%	1%	1%	2%
Experiment5	0%	0%	1%	0%	2%
Experiment6	0%	0%	1%	1%	1%
Experiment7	0%	0%	1%	1%	2%
Experiment8	0%	0%	1%	1%	2%
Experiment9	1%	1%	2%	1%	1%
Experiment10	0%	0%	1%	0%	1%

Figure 9: Long-run average costs of the maintenance strategies for experiment 4 of the Taguchi experiments



Experiments designed based on Taguchi method are composed of a uniform combination of different levels of parameters which provides the possibility of deriving meaningful conclusions. We perform an ANOVA [88] analysis on the results to show that the means of the long-run average costs of strategies are in fact different, and to rank the maintenance strategies based on the long-run average costs.

ANOVA is used to compare more than two means [89]. This method can be used to test the hypothesis of all the means of several groups are equal and they are from the same population. ANOVA may reject or accept this hypothesis. Hypothesis is rejected if the calculated F value is greater than the $F(\nu_1, \nu_2)$ distribution otherwise it cannot be rejected, meaning that the groups are in fact from the same population [89] (ν_1 & ν_2 are the degrees of freedom, ν_1 = {number of groups – 1} & ν_2 = {Number of observations – number of groups}). In this study it is of interest to know whether the differences in the means of long-run average costs of each maintenance strategy are result of randomness or due to different populations. The results for two-state equipment are as below:

Table 19 Average of Taguchi results (long-run average costs) for two-state equipment

Maintenance Strategy	P-P	Im-Im	Im-P	TBM	RtF
Long-run average cost per unit of time	6.48	6.62	6.78	6.90	10.18

The values in Table 19 show the long-run average cost per unit of time of each strategy, the unit of time can be hour, day, week, month, and so forth. These values are the average costs over a wide range of system parameters, therefore in the absence of knowledge of the system parameters the practitioner may use these values to compare the strategies.

The following is the calculation of F value for the ANOVA test.

$\bar{Y} = \frac{\sum_{i=1}^{n} y_i}{n} = 7.39$

$SS_B = n\sum_{i=1}^{k}(\bar{Y}_i - \bar{Y})^2 = 490.84$ $\qquad$ $MS_B = \frac{SS_B}{k-1} = 122.71$

$SS_W = \sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij} - \bar{Y}_i)^2 = 4467.52$ $\qquad$ $MS_W = \frac{SS_W}{N-k} = 18.23$

$$F = \frac{MS_B}{MS_W} = 6.72$$

$F(k-1, N-k) = F(4,245) = 3.81$ Null hypothesis is rejected $P < 0.00003$

Where $\bar{Y}$ is the mean of long-run average costs of all groups, SS_B is the sum of squares between groups and SS_W is the sum of squares within groups. N is the size of all groups combined, n is the size of each group, and k is the number of groups (ν_1=k-1=4, ν_2=N-k=245). The Null hypothesis is that the means of all the five groups are equal and they are from the same population. Since the critical value for F distribution is lower than the calculated F, it can be concluded that the differences in means of each group suggests that the groups are in fact from different populations. Thus, we can conclude that the P-P strategy provides lower long-run average costs than all other strategies. The results show that the RtF strategy has the highest long-run average costs, and the CBM strategies (P-P, Im-P, and Im-Im) have lower long-run average costs than TBM strategy. Consequently, we can observe the profitability of each strategy compared to another. In the absence of historical failure data, therefore no knowledge of system parameters, the practitioner can use the results from

Table 20 to obtain expected cost savings of implementing each maintenance strategy compared to RtF strategy.

Table 20: Average cost savings of maintenance strategies compared to RtF strategy for two-state equipment

Maintenance Strategy	P-P	Im-Im	Im-P	TBM
Long-run average savings per unit of time	3.7	3.56	3.40	3.28
Variances	25.45	24.23	23.62	22.65

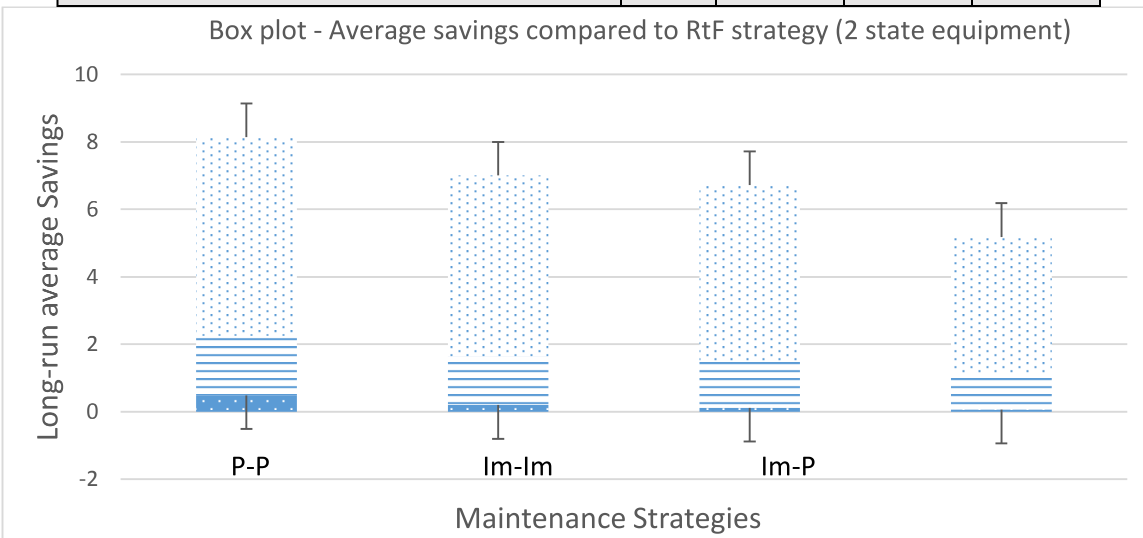


Table 21 presents the cost savings as the percentage of the cost of the RtF strategy. The practitioner may use the results from this Table if he/she only has the knowledge of the cost of a run-to failure strategy.

Table 21: Cost savings presented as the percentage of the cost of RtF strategy for two-state equipment

Maintenance strategy	P-P	Im-Im	Im-P	TBM
Long-run average cost savings per unit of time	%36	%35	%33	%32

3.2. Results from Taguchi Experiments for three state equipment

The long run average costs and the cost savings are also calculated for three-state equipment. Table 22 includes the long-run average costs calculated for Taguchi experiments designed for three-state equipment.

Table 22 Taguchi results for three-state equipment

	P-P	Im-Im	Im-P	TBM	RtF		P-P	Im-Im	Im-P	TBM	RtF
Experiment1	7.36	7.36	7.36	7.36	7.36	Experiment26	7.22	7.22	7.22	7.22	7.22
Experiment2	8.21	8.69	8.73	8.78	8.83	Experiment27	6.31	6.68	6.72	7	7.05
Experiment3	8.88	9.62	9.68	9.69	10.92	Experiment28	7.88	7.94	8.01	7.97	8.36
Experiment4	7.23	8.97	9.53	10.93	12.79	Experiment29	8.65	9.95	10.58	9.8	12.33
Experiment5	8.07	8.08	8.7	12.11	15	Experiment30	10.37	11.67	12.28	12	19.87
Experiment6	10.1	10.27	10.34	10.89	11.16	Experiment31	10.09	10.18	10.27	11.32	11.79
Experiment7	7.55	8.91	9.23	9.44	10.26	Experiment32	8.74	9.21	9.31	9.14	10.51
Experiment8	6.58	8.83	9.07	8.58	10.85	Experiment33	9.05	9.28	9.31	9.47	13.6
Experiment9	7.84	8.28	9.1	8.15	15.34	Experiment34	7.29	9.53	12.44	11.62	18.98
Experiment10	3.22	3.22	3.22	3.22	3.22	Experiment35	2.75	2.75	2.75	2.75	2.75
Experiment11	9.6	10.22	10.27	10.33	12	Experiment36	10.38	10.76	11.02	11.59	14.18
Experiment12	8.95	9.24	9.4	9.46	14.94	Experiment37	8.29	8.69	8.81	8.47	10.38
Experiment13	8.1	9.29	9.46	8.51	20.02	Experiment38	7.23	8.06	9.14	9.97	17.5
Experiment14	3.84	3.84	3.84	3.84	3.84	Experiment39	2.8	2.8	2.8	2.8	2.8
Experiment15	4.1	4.85	4.88	4.9	4.9	Experiment40	3.8	3.89	4.08	3.86	4.09
Experiment16	11.43	11.95	12.82	11.56	19.69	Experiment41	11.13	11.5	12.11	12.26	17.79
Experiment17	9.27	10.12	10.33	9.62	25.13	Experiment42	9.58	10.15	10.73	9.84	21.65
Experiment18	3.91	3.91	3.91	3.91	3.91	Experiment43	3.65	3.65	3.65	3.65	3.65
Experiment19	3.63	4.12	4.13	4.21	4.32	Experiment44	4.23	4.32	4.37	4.33	4.75
Experiment20	3.44	3.46	3.48	3.47	4.42	Experiment45	4.01	4.14	4.25	4.91	5.77
Experiment21	13.35	14.16	15.13	14.42	31.14	Experiment46	13.33	14.18	14.32	13.39	31.65
Experiment22	4.16	4.16	4.16	4.16	4.16	Experiment47	4.67	4.67	4.67	4.67	4.67
Experiment23	4.32	4.39	4.44	4.35	4.76	Experiment48	4.75	5.2	5.33	5.21	5.51
Experiment24	4.26	4.28	4.32	4.41	5.82	Experiment49	3.86	4.6	4.78	4.84	5.53
Experiment25	4.47	4.75	4.78	4.83	8.53	Experiment50	3.48	3.63	3.78	3.5	6.31

For both two-state equipment and three-state equipment, it can be observed that for K/C≈0 all the maintenance strategies have equal long-run average costs (experiments1, 10, 14, 18, 22, 26, 35, 39, 43, 47). It is not surprising since all the replacements happen at failure and have the same cost (K+C=C). In that case the cost of any strategy is equivalent to that of the RtF strategy.

Table 23 Mean long-run average costs for three-state equipment compared between different maintenance strategies

Maintenance Strategy	P-P	Im-Im	Im-P	TBM	RtF
Long-run average cost per unit of time	6.91	7.39	7.65	7.65	10.84

Results from ANOVA analysis on long-run average costs of Taguchi experiments for three-state equipment are as below (refer to appendix for detailed calculations):

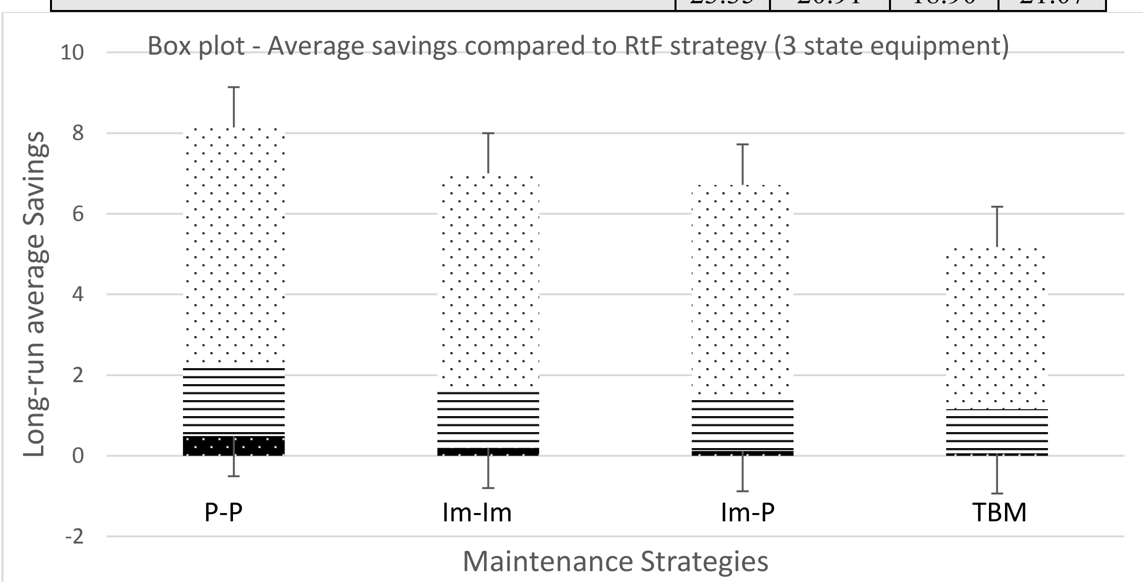
$$F = \frac{MS_B}{MS_W} = 6.76$$

$F(k-1, N-k) = F(4{,}245) = 3.81$ Null hypothesis is rejected $P < 0.00003$

Similar to two-state equipment, the null hypothesis is rejected for three-state equipment. We can see that on average the P-P strategy provides the lowest long-run average costs than all other strategies, and RtF strategy has the highest long-run average costs. From Table 24 we can see the average savings of each maintenance strategy compared to RtF strategy.

Table 24 Average cost savings of maintenance strategies compared to RtF strategy for three-state equipment

Maintenance Strategy	P-P	Im-Im	Im-P	TBM
Long-run average cost savings per unit of time	3.93	3.45	3.19	3.19
Variances	23.35	20.91	18.90	21.07



The cost savings presented as percentage of the cost of RtF strategy is provided in Table 25.

Table 25 Cost savings presented as the percentage of the cost of RtF strategy for three-state equipment

Maintenance Strategy	P-P	Im-Im	Im-P	TBM
Long-run average cost savings per unit of time	%36	%32	%29	%29

It is observed that amongst the maintenance strategies, a Run-to-failure strategy will always lead to higher long-run average costs, the Im-P strategy provides slightly lower long-run average costs than the TBM strategy, the long-run average costs of CBM strategy assuming imperfect information (Im-Im) is lower than the long-run average costs of CBM strategy assuming perfect information (Im-P), and the P-P strategy provides the lowest long-run average costs compared to other maintenance strategies.

The illustrated savings that are obtained from long-run average costs of the experiments designed by Taguchi method, can be used as a general framework to choose the best maintenance strategy. This framework is especially useful when knowledge of system parameters is limited to the practitioner. However, the practitioner is likely to have the knowledge of the cost of a failure replacement and preventive replacement, for that reason the cost savings obtained from the experiments designed by Taguchi method are also presented for each value of K/C separately. Table 26 and Table 27 show the cost savings for each value of K/C = (0, 0.5, 1, 2, and 4) presented as the percentage of the cost of RtF strategy for two-state and three-state equipment:

Table 26 Cost savings presented as the percentage of the cost of RtF strategy for each value of K/C for two-state equipment

	Percentage of savings compared to RtF			
K/C	**P-P**	**Im-Im**	**Im-P**	**TBM**
0	0.00%	0.00%	0.00%	0.00%
0.5	7.92%	6.51%	4.69%	5.03%
1	21.97%	20.72%	18.87%	15.28%
2	37.87%	35.54%	33.61%	32.84%
4	57.12%	55.93%	54.43%	53.39%

Table 27 Cost savings presented as the percentage of the cost of RtF strategy for each value of K/C for three-state equipment

	Percentage of savings compared to RtF			
K/C	**P-P**	**Im-Im**	**Im-P**	**TBM**
0	0.00%	0.00%	0.00%	0.00%
0.5	10.89%	6.53%	5.52%	3.30%
1	21.66%	16.63%	15.28%	14.15%
2	39.25%	33.15%	30.34%	31.19%
4	56.34%	52.14%	48.39%	49.31%

The savings are also presented as the percentage of the cost of a preventive replacement per unit of time. The long-run average costs obtained from experiments designed using Taguchi method are calculated for C=10; however, since the long-run average costs are proportional to the value of C, the effect of C on the percentage of savings is eliminated and the percentage of savings will only be dependent on the ratio K/C and not the value of K or the value of C. The reason is that the ratio K/C is constant in the designed experiments, multiplying C by an x factor forces the value of K to be multiplied by x as well.

$$\Phi_{T_d} = \frac{C_{T_d}}{P_{T_d}} = \frac{x[C + KP(T \leq T_d)]}{E[min[T, Td]]} = \frac{xC + xKP(T \leq T_d)}{E[min[T, Td]]}$$

From equation above (refer to equation 24) it can be seen that if the ratio K/C is multiplied by x, the long-run average costs will also be multiplied by x.

Table 28 and Table 29 show the cost savings for each value of K/C presented as the percentage of the cost of a preventive replacement, C=10, for two-state and three-state equipment:

Table 28 Cost savings presented as the percentage of the cost of a preventive replacement, C=10, for each value of K/C for two-state equipment

	Percentage of savings compared to RtF			
K/C	P-P	Im-Im	Im-P	TBM
0	0.00%	0.00%	0.00%	0.00%
0.5	4.56%	3.75%	2.70%	2.90%
1	18.28%	17.24%	15.70%	12.71%
2	46.55%	43.69%	41.32%	40.37%
4	118.06%	115.60%	112.51%	110.35%

Table 29 Cost savings presented as the percentage of the cost of a preventive replacement, C=10, for each value of K/C for three-state equipment

	Percentage of savings compared to RtF			
K/C	P-P	Im-Im	Im-P	TBM
0	0.00%	0.00%	0.00%	0.00%
0.5	6.92%	4.15%	3.51%	2.10%
1	19.33%	14.84%	13.63%	12.63%
2	50.95%	43.03%	39.39%	40.49%
4	121.85%	112.76%	104.65%	106.65%

If the system parameters are unknown the practitioner may use the average cost savings provided in Table 21 to Table 25. If the system parameters are unknown but the cost of a failure replacement (K+C) and the cost of a preventive replacement (C) are available, the practitioner may use the average cost savings provided in Table 26 to Table 29. However, if the system parameters are known, the practitioner may directly use the long-run average costs provided in Table 22 to calculate cost savings for each maintenance strategy.

3.3. Results from additional experiments

The previous sections provided average cost savings of the maintenance strategies compared to RtF strategy by considering a wide range of parameter levels based on the Taguchi design of experiment. The practitioner may use the illustrated cost savings to have a general view of the average cost savings of each maintenance strategy, and to select the suitable maintenance strategy. In this section we introduce additional experiments designed to validate the results obtained from the Taguchi experiments, and to provide results that can be used to better analyze the effect of system parameters on the value of information. The results from new experiments are the average of cost savings on smaller range of parameters. The new experiments as it will be explained later in this sections, remove the effect of less important parameters (parameters with lowest effect on long-run average costs). We use the long-run average costs obtained from the Taguchi experiments to find the parameters with the lowest effect on long-run average costs. A Level Average Analysis is performed [87] on the results obtained from Taguchi design of experiments for two-state and three-state equipment to observe the effect of parameters on long-run average costs, and the results are provided in Table 30, and 31. The Level Average analysis is used to identify the levels of parameters at which the lowest and the highest long-run average costs are obtained. The effect of each parameter is decided by comparing the level with the lowest value and the level with the highest value. The higher the difference, the greater the effect of that parameter. From the results of Table 30 and 31 we can observe that the effect of P matrix and Q matrix on long-run average costs is less significant, therefore in developing new experiments we do not consider P matrix and Q matrix parameters in order to reduce the number of experiments and computation time and to better analyze the effect of remaining parameters by removing the noise effect of P and Q on long-run average costs. The most effective parameter on long-run average costs of the maintenance strategies, is the K/C parameter. We design and present the new experiments for each value of K/C separately. Presenting the results for each value of K/C is useful because the practitioner is more likely to know the cost of a failure replacement and the cost of a preventive replacement. Furthermore, in depth information of other parameters of the system are not readily available unless required data is collected and some statistical analysis is performed on historical data. We will use parameters α, β, and ϒ (which have the highest effect on long-run average costs after K/C) in designing the new experiments, and for each parameter we consider three levels. Considering three levels necessitate 3^3 experiments for each value of K/C, and that would require fair amount of computation time. However, for more than three levels and three parameters the number of experiments and the computation time increase significantly. The level average results for the maintenance strategies of two-state equipment are summarized in Table 30:

Table 30: Level Average analysis on long-run average costs of the experiments designed using Taguchi method for two-state equipment

Parameter	Long-run average costs of the maintenance strategies				
K/C	P-P	Im-Im	Im-P	TBM	RtF
0	4.10	4.10	4.10	4.10	4.10
0.5	5.30	5.39	5.49	5.47	5.76
1	6.49	6.60	6.75	7.05	8.32
2	7.64	7.92	8.16	8.26	12.29
4	8.86	9.11	9.42	9.64	20.67
Max-Min	***4.77***	***5.01***	***5.32***	***5.54***	***16.57***
α	P-P	Im-Im	Im-P	TBM	RtF
2	9.91	10.16	10.34	10.48	15.98
3	7.08	7.23	7.31	7.36	10.84
4	6.00	6.13	6.25	6.48	9.31
5	5.10	5.19	5.44	5.47	7.53
6	4.32	4.41	4.57	4.73	7.26
Max-Min	***5.59***	***5.75***	***5.77***	***5.75***	***8.71***
β	P-P	Im-Im	Im-P	TBM	RtF
2	7.53	7.62	7.92	8.21	10.48
3	6.92	7.09	7.25	7.48	9.92
4	6.19	6.29	6.39	6.68	9.66
5	6.14	6.34	6.43	6.27	10.46
6	5.63	5.77	5.93	5.89	10.40
Max-Min	***1.90***	***1.85***	***1.98***	***2.32***	***0.82***

Parameter	Long-run average costs of the maintenance strategies				
γ	P-P	Im-Im	Im-P	TBM	RtF
0.5	5.62	5.66	5.71	5.66	8.19
1	6.41	6.61	6.72	6.67	10.27
1.5	6.65	6.82	7.05	7.05	9.64
2	6.68	6.82	7.14	7.26	11.18
2.5	7.04	7.20	7.31	7.91	11.78
Max-Min	***1.42***	***1.55***	***1.60***	***2.24***	***3.59***
Q	P-P	Im-Im	Im-P	TBM	RtF
0.1	6.14	6.35	6.47	6.59	9.75
0.3	6.28	6.40	6.50	6.73	9.78
0.5	6.69	6.82	6.97	7.05	10.84
0.7	6.79	6.92	7.23	7.24	9.85
0.9	6.51	6.62	6.75	6.93	10.70
Max-Min	***0.65***	***0.57***	***0.76***	***0.65***	***1.09***
P	P-P	Im-Im	Im-P	TBM	RtF
0.2	7.03	7.20	7.35	7.58	11.55
0.8	5.93	6.04	6.21	6.23	8.82
Max-Min	***1.10***	***1.15***	***1.14***	***1.36***	***2.73***

Table 31: Summary of level average analysis results from Table 30

	Maintenance Strategies					
Parameter	**P-P**	**Im-Im**	**Im-P**	**TBM**	**RtF**	**Average**
K/C	4.77	5.01	5.32	5.54	16.57	7.44
α	5.59	5.75	5.77	5.75	8.71	6.31
β	1.9	1.85	1.98	2.32	0.82	1.77
γ	1.42	1.55	1.6	2.24	3.59	2.08
Q	0.65	0.57	0.76	0.65	1.09	0.74
P	1.1	1.15	1.14	1.36	2.73	1.5

In Table 30 for every level of each parameter the mean long-run average costs from Taguchi Table is provided. From the Level Average analysis, it can be seen that the highest effect come from parameters α, ϒ, and β. Level Average analysis on three-state Taguchi shows similar results to two-state, i.e. the maximum effect also comes from parameters: α, β, and ϒ. (results can be found in appendix)

Following levels for each parameter have been considered in designing the new experiments:

Table 32 parameter levels of the new experiments

	α	β	ϒ
Level 1	2	2	0.5
Level 2	4	4	1
Level 3	6	6	2.5

All possible combinations of these parameters are considered, therefore the results are provided for $3^3 = 27$ experiments. The values of P matrix and Q matrix are fixed for all the combinations as below:

Two-state: $$P = \begin{bmatrix} 0.6 & 0.4 \\ 0 & 1 \end{bmatrix} \quad \& \quad Q = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$$

Three-state: $$P = \begin{bmatrix} 0.6 & 0.4 & 0.4 \\ 0 & 0.6 & 0.4 \\ 0 & 0 & 1 \end{bmatrix} \quad \& \quad Q = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$$

For the new experiments we do not consider K/C ≈ 0. The results are provided for K/C=0.2, 0.5, 1, and 2 for two-state and three-state equipment. In this section we only present the results for K/C = 2 and for two-state equipment, results for K/C=0.2, 0.5, and 1, can be found in appendix.

Long-run average costs from the new experiments for K/C = 2 are provided in Table 33:

Table 33: Long-run average costs from new experiment for K/C=2, C=10

	Two-State K/C=2				
α	**P-P**	**Im-Im**	**Im-P**	**TBM**	**RtF**
2	12.20	12.30	12.40	13.37	18.13
4	6.52	6.90	7.22	7.78	10.73
6	4.75	5.02	5.40	6.00	8.02
β	**P-P**	**Im-Im**	**Im-P**	**TBM**	**RtF**
2	9.80	10.15	10.62	11.88	13.16
4	7.26	7.50	7.74	8.36	12.19
6	6.41	6.57	6.66	6.93	11.52
γ	**P-P**	**Im-Im**	**Im-P**	**TBM**	**RtF**
0.5	7.73	7.63	7.85	7.52	10.88
1.5	7.83	8.13	8.33	9.02	12.60
2.5	7.90	8.45	8.84	10.62	13.39

The results from the Table 33 validate the results obtained from the experiments designed using Taguchi method. We can see that generally the lowest long-run average costs are obtained from the P-P strategy and the highest long-run average costs are obtained from the RtF strategy. The effect of parameters will be discussed in details in following sections.

Using the long-run average costs from Table 33, the average of cost savings for each strategy compared to RtF strategy for K/C=2 and C=10 is provided in Table 34:

Table 34: Average cost savings of maintenance strategies compared to RtF strategy from new experiments for K/C=2 for two-state equipment

α	**P-P**	**Im-Im**	**Im-P**	**TBM**
2	5.92	5.83	5.72	4.75
4	4.21	3.83	3.51	2.94
6	3.27	2.99	2.62	2.02
β	**P-P**	**Im-Im**	**Im-P**	**TBM**
2	3.37	3.01	2.54	1.29
4	4.93	4.69	4.45	3.84
6	5.1	4.95	4.86	4.59
Y	**P-P**	**Im-Im**	**Im-P**	**TBM**
0.5	3.14	3.25	3.03	3.36
1.5	4.77	4.47	4.27	3.58
2.5	5.49	4.94	4.55	2.77

From Table 34 it can be observed that as we move to the strategies on the left side, the long-run average cost savings increases (except when **ϒ=0.5)**. The reason is that parameter ϒ magnifies the effect of state covariates into the hazard rate function, therefore lower values of ϒ have lower effect on the hazard rate and consequently the savings of the CBM strategies are not notable. It can also be observed that the savings increase as the value of **β** or **ϒ** increases, and the savings decrease as the value of **α** increases. The cost savings presented as the percentage of the cost of the RtF strategy is provided in Table 35:

Table 35 Cost savings presented as the percentage of the cost of RtF strategy from new experiments for K/C = 2 for two-state equipment

α	P-P	Im-Im	Im-P	TBM
2	32.67%	32.16%	31.58%	26.21%
4	39.26%	35.71%	32.69%	27.44%
6	40.77%	37.34%	32.62%	25.13%
β	P-P	Im-Im	Im-P	TBM
2	25.57%	22.90%	19.27%	9.77%
4	40.44%	38.49%	36.52%	31.46%
6	44.33%	42.96%	42.17%	39.84%
γ	P-P	Im-Im	Im-P	TBM
0.5	28.90%	29.84%	27.82%	30.90%
1.5	37.83%	35.47%	33.86%	28.41%
2.5	41.00%	36.88%	34.00%	20.67%

The summary of the average cost savings compared to RtF strategy from the new experiments are provided in Table 36 for every value of K/C.

Table 36 Average cost savings of maintenance strategies compared to RtF strategy from new experiments for K/C = (0.2, 0.5, 1, 2) for two--state equipment

K/C	P-P	Im-Im	Im-P	TBM
0.2	0.15	0.03	0.03	0.02
0.5	0.64	0.38	0.34	0.21
1	1.72	1.41	1.30	0.91
2	4.47	4.22	3.95	3.24

Table 37 Cost savings from new experiments presented as the percentage of the cost of RtF strategy for each value of K/C for two-state equipment

K/C	P-P	Im-Im	Im-P	TBM
0.2	3.01%	0.72%	0.68%	0.45%
0.5	10.44%	6.31%	5.60%	3.44%
1	21.12%	17.37%	15.92%	11.09%
2	36.75%	34.64%	32.28%	26.65%

Table 38 Cost savings from new experiments presented as the percentage of the cost of a preventive replacement, C=10, for each value of K/C for two-state equipment

K/C	P-P	Im-Im	Im-P	TBM
0.2	1.50%	0.30%	0.30%	0.20%
0.5	6.40%	3.80%	3.40%	2.10%
1	17.20%	14.10%	13.00%	9.10%
2	44.70%	42.20%	39.50%	32.40%

The results from Tables 36, 37, and 38 validate the obtained results from the experiments designed using Taguchi experiment. We can see that the savings for K/C = 0.5, 1, and 2 are similar and as we move to the left of the Table the cost savings increase in either cases (Taguchi results and new experiments).

From the provided cost savings we can observe how much value each maintenance strategy can save per unit of time, given the failure parameters and/or the cost of a failure replacement and the cost of a preventive replacement. In order to better understand how the presented cost savings especially the cost savings illustrated in Table 26 to Table 29 and Table 36 to Table 39 can be used by the practitioner to choose the suitable strategy, an example is provided below:

Assume that a manufacturer wants to decide whether he should invest in condition monitoring technologies to collect state observation or actual state data, or no investment is necessary and a TBM or RtF strategy is cost effective. Consider that the present worth cost of implementing a CBM strategy (Imperfect-Perfect strategy) with monthly inspections for a 5 year period is $6,000, and a TBM strategy requires data analysis with a present worth cost of $2,000 for the same period. The manufacturer also has the opportunity to invest $7,000 to implement a CBM strategy assuming imperfect information (Imperfect-Imperfect Strategy). The failure parameters of the equipment are not known (due to lack of collected historical data or condition monitoring system), but a failure replacement costs $1500 (K+C) and a preventive replacement costs $500 (C).

Which strategy should the manufacturer choose?

Since we do not have the failure parameters we can use the results from Table 28 to estimate the average long-run savings of implementing each strategy. The ratio of extra cost of failure replacement (K) over a preventive replacement (C) is equal to K/C=$1000/$500=2. Table 28 Cost savings presented as the percentage of the cost of a preventive replacement, C=10, for each value of K/C for two-state equipment provides values of the maintenance strategies for a preventive replacement cost of $10. The saving percentage of a preventive replacement cost as illustrated in Table 38 for K/C =2 is as below:

Table 39 savings presented as the percentage of the cost of a preventive replacement, C=10, from Taguchi experiments for K/C=2, two-state equipment

K/C	**P-P**	**Im-Im**	**Im-P**	**TBM**
2	46.55%	43.69%	41.32%	40.37%

Therefore for a preventive replacement cost of C=$500, at each monthly inspection point the savings obtained from the maintenance strategies are as provided in Table 40:

Table 40 monthly savings for C=$500, and K/C = 2

K/C	**P-P**	**Im-Im**	**Im-P**	**TBM**
2	$232.75	$218.45	$206.6	$201.85

What comes later enables the practitioner to incorporate the current value of the saving and the amount of investment required, to choose the right strategy while the time value of money is considered. Table 41 shows the summary of the cost of each strategy for the manufacturer for the five year period, coming from the example:

Table 41 cost of implementing the maintenance strategies

Im-Im	Im-P	TBM
$7,000	$6,000	$2,000

The present value of these savings considering monthly interest rate of i, and monthly savings of A, can be calculated as below [90]:

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)}\right]$$

The present value of savings for each strategy compared to RtF strategy for an interest rate of i=0.01 and for 10 year period is provided below (n is equal to 10×12=120):

Table 42 present value of savings for the maintenance strategies for i=0.01 and 10 year period

i	P-P	Im-Im	Im-P	TBM
0.01	$53,011	$49,754	$47,055	$45,973

From Table 41, the cost of TBM strategy is $2,000, while the savings of TBM compared to RtF is $45,973. Therefore, by using TBM instead of RtF, the manufacturer can gain a present net worth value of profit of $43,973. In Table 43 we can see the costs, savings, and profit of the maintenance strategies for the above example:

Table 43 Costs, Savings, and Profits of each maintenance strategy compared to RtF strategy

Present Worth	Im-Im	Im-P	TBM
Costs	$7,000	$6,000	$2,000
Savings	$49,754	$47,055	$45,973
Profit	$42,754	$41,055	$43,973

Therefore, the manufacturer may decide to use the TBM strategy. The TBM provides net present worth profit of $43,973 compared to RtF, which is relatively higher than the profit of implementing Im-P strategy or Im-Im strategy.

Now, consider that another option is available to the practitioner; perfect information from equipment can be available at present worth cost of $8,000 over the 5 year period (Perfect-Perfect Strategy).

Should the manufacturer invest in obtaining perfect information from equipment?

From Table 42, we have the present net worth of implementing P-P strategy. The profit of implementing P-P strategy can be found in Table below:

Table 44 Cost, Saving, and Profit of implementing P-P strategy

	P-P
Costs	$8,000
Savings	$53,011
Profit	$45,011

Therefore, the P-P strategy has the highest profit amongst the maintenance strategies and it is recommended that the practitioner invest $8,000 into implementing the CBM strategy and to collect perfect information from the equipment.

The example above illustrates how the average cost savings can be used by the practitioners to choose the right maintenance strategy when limited information is available on failure behaviour of the system. This decision making is based on the expected behaviour of the system. If the failure parameters are known, depending on the ratio of K/C, the practitioner may use the Table 34 to Table 38 for K/C = 2 or the similar Tables provided in appendix for

K/C = 0.2, 0.5, and 1, to choose the best strategy based on the average cost savings provided.

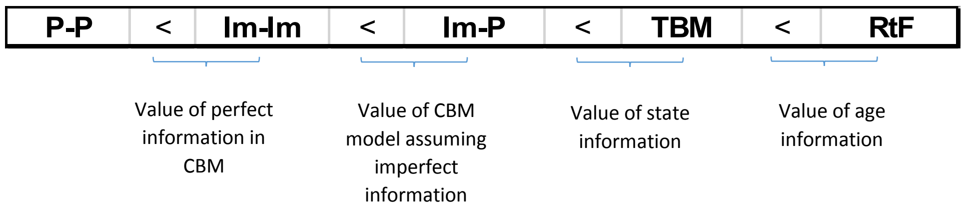
3.4 Value of Information in Maintenance

In sections 3.1 and 3.2 we provided the results obtained from the experiments designed using Taguchi method. Section 3.3 introduced new experiments designed to validate Taguchi results and provided more detailed cost savings information for the practitioner. An example was illustrated in section 3.3 to provide a better understanding on how the cost savings can be used by the practitioner. The cost savings were obtained by comparing the long-run average costs of the maintenance strategies to the RtF strategy. However, an important part of this work is to identify value of information in maintenance. We know from chapter 2 and earlier in this chapter that the value of information is obtained by relative comparison of the long-run average costs of the maintenance strategies. Value of information such as age, observation, and perfect state information from equipment as well as the value of implementing the CBM strategy assuming imperfect state information from equipment (Im-Im strategy).

Value of age information is obtained by comparing the long-run average costs of the TBM strategy and the RtF strategy (which for convenience it is symbolized as RtF/TBM), value of observation information (imperfect state information) by comparing the Im-P strategy and the TBM strategy (which for convenience it is symbolized as TBM/Im-P), value of CBM model assuming imperfect information is obtained by comparing the Im-Im strategy with Im-P strategy (which for convenience it is symbolized as Im-P/Im-Im), and the value of perfect information is obtained by comparing the P-P strategy with the Im-Im strategy (which for convenience it is symbolized as Im-Im/P-P).

Using the results obtained from the experiments designed using the Taguchi method and additional new experiments, we showed in sections 3.1, and 3.2 that the long-run average costs of the maintenance strategies follow the order presented in Figure 10:

Figure 10 maintenance strategies ordered by the mean long-run average costs



As it can be observed from Figure 10, feeding more information into the decision making leads to higher long-run cost savings. In this section we investigate the present net worth value of information in maintenance strategies, and we analyze the effect of failure parameters on the value of information.

We present the results as the present net worth of the cost savings in order to highlight the importance of the cost savings. In the following Tables the present net worth values of information for the maintenance strategies, considering planning horizons of 2, 5, & 10 years, and monthly interest rates of 0.01, 0.02, and 0.03. The results are shown for different parameter values when K/C=2, and assuming the cost of a preventive replacement to be equal to C=$10.

Table 45 Present net worth of information for K/C=2, i=0.01, and C=$10

	Two-State K/C=2, i=0.01											
	Im-Im/P-P			Im-P/Im-Im			TBM/Im-P			RtF/TBM		
α	2 year	5 year	10 year	2 year	5 year	10 year	2 year	5 year	10 year	2 year	5 year	10 year
2	$2.60	$8.08	$22.70	$2.67	$8.08	$22.77	$25.90	$78.43	220..92	$127.12	$384.89	$1,084.14
4	$10.10	$30.72	$86.54	$8.54	$25.87	$72.88	$14.95	$45.28	$127.54	$78.78	$238.54	$671.89
6	$7.21	$21.80	$61.49	$10.14	$30.72	$86.54	$16.00	$48.51	$136.65	$53.94	$163.33	$460.07
β	2 year	5 year	10 year	2 year	5 year	10 year	2 year	5 year	10 year	2 year	5 year	10 year
2	$9.34	$28.30	$79.70	$12.55	$38.05	$107.04	$33.65	$101.80	$286.97	$34.18	$103.50	$291.53
4	$6.41	$19.40	$54.66	$6.41	$19.40	$54.60	$16.50	$50.13	$141.21	$102.28	$309.69	$872.32
6	$4.27	$12.93	$36.44	$2.40	$7.27	$20.49	$7.21	$21.83	$61.49	$122.58	$371.15	$1,045.42
γ	2 year	5 year	10 year	2 year	5 year	10 year	2 year	5 year	10 year	2 year	5 year	10 year
0.5	-$2.67	-$8.08	-$22.77	$5.87	$17.78	$50.10	-$8.81	-$26.68	-$75.60	$89.73	$271.69	$765.27
1.5	$8.01	$24.25	$68.28	$5.34	$16.17	$45.55	$18.42	$55.79	$157.15	$95.60	$289.48	$815.38
2.5	$14.68	$44.47	$125.26	$10.41	$31.53	$88.82	$47.53	$143.93	$405.41	$73.97	$223.98	$630.09

From Table 45 we can see savings as high as $1,084 for 10 year planning horizon with monthly inspections, while K/C=2, cost of preventive replacement is equal to $10, and the interest rate is 0.01. This amount of saving is 1,084/10=108.4 times the cost of a preventive replacement (C=$10). Note that $10 is a relatively small cost for a preventive replacement. Consider a preventive replacements with cost of $1000, in which case, the savings can be as high as 108.4×$1000=$108,400.

Following we investigate the present net worth values of Table 45 first by type of information to obtain the value of age, observation, CBM model assuming imperfect information, and value of perfect information in CBM; and later we investigate the present net worth values by system parameters to understand the effect of system parameters on value of information.

Net worth value of each type of information is investigated for K/C=0.2, 0.5, 1, 2.

3.4.1 Value of Age Information

The value of age information can be derived by comparing the TBM strategy with the RtF strategy. The results below show the present net worth value of implementing a TBM strategy, in other words, the value of age information.

Table 46 Value of age information by K/C, Cost of a preventive replacement is equal to $10

	RtF/TBM (Value of age information)								
	i=0.01			i=0.02			i=0.03		
K/C	2 year	5 year	10 year	2 year	5 year	10 year	2 year	5 year	10 year
0.2	$0.53	$1.62	$4.56	$0.60	$2.24	$9.57	$0.67	$3.17	$21.82
0.5	$5.61	$16.98	$47.83	$6.26	$23.48	$100.52	$7.02	$33.24	$229.10
1	$24.30	$73.58	$207.26	$27.14	$101.75	$435.60	$30.42	$144.06	$992.78
2	$86.53	$261.99	$737.95	$96.63	$362.28	$1,550.94	$108.29	$512.91	$3,534.74

We can see savings as low as $0.53 and as high as $3,534. The practitioner may choose not to invest in obtaining age information and use the RtF strategy instead, if the ratio of K/C is 0.2 or lower. Depending on the cost of applying the TBM strategy, the interest rate, and the ratio of K/C, a proper decision can be made.

3.4.2. Value of Observation Information

The Im-P strategy considers both the age and observation information of equipment. Comparing the Im-P strategy with the TBM strategy provides the value of observation information. Collecting observation information and implementing the CBM strategy can be very costly. Therefore the present net value of observation information should be high enough in order to be selected as the right maintenance strategy. From Table 47, we can conclude that unless for a long planning horizon and high interest rate, it is possibly not justifiable to implement a CBM strategy. Especially when the ratio of K/C is lower than 0.5. However, the savings for higher ratios of K/C and longer planning horizon are quite significant.

Table 47 Value of state information by K/C, Cost of a preventive replacement is equal to $10

	TBM/Im-P (Value of state information)								
	i=0.01			i=0.02			i=0.03		
K/C	2 year	5 year	10 year	2 year	5 year	10 year	2 year	5 year	10 year
0.2	$0.27	$0.81	$2.28	$0.30	$1.12	$4.79	$0.33	$1.58	$10.91
0.5	$3.47	$10.51	$29.61	$3.88	$14.54	$62.23	$4.35	$20.58	$141.83
1	$10.42	$31.54	$88.83	$11.63	$43.61	$186.69	$13.04	$61.74	$425.48
2	$18.96	$57.41	$161.71	$21.18	$79.39	$339.87	$23.73	$112.40	$774.59

3.4.3. Value of CBM assuming imperfect information

The CBM strategy assuming imperfect information (Im-Im) and the CBM strategy assuming perfect information (Im-P) both consider the observations from equipment instead of actual state of equipment for decision making, but the former translates the observations into a stochastic distribution of actual state of equipment and the later considers the observations directly as the exact state of equipment. From Table 48 it can be concluded that Im-Im contributes to huge net present worth of savings. Since the Im-Im strategy and Im-P strategy have similar assumptions and are only different in mathematical modeling, implementing the Im-Im strategy is not much more costly compared to Im-P strategy. Therefore, it is reasonable to always implement the CBM assuming imperfect information instead of CBM assuming perfect information. However, for lower ratios of K/C as it can be seen in the Table below, the present net worth of savings are not noticeable.

Table 48 Net present value of CBM assuming imperfect information by K/C, Cost of a preventive replacement is equal to $10

	Im-P/Im-Im (Value of CBM assuming imperfect information)								
	i=0.01			i=0.02			i=0.03		
K/C	2 year	5 year	10 year	2 year	5 year	10 year	2 year	5 year	10 year
0.2	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
0.5	$1.07	$3.23	$9.11	$1.19	$4.47	$19.15	$1.34	$6.33	$43.64
1	$2.94	$8.89	$25.05	$3.28	$12.30	$52.66	$3.68	$17.41	$120.01
2	$7.21	$21.83	$61.50	$8.05	$30.19	$129.24	$9.02	$42.74	$294.56

3.4.4. Value of perfect information in CBM

Table 49 shows the value of having perfect information from equipment in CBM. The perfect information can be provided by investing in more condition monitoring technologies comparing to imperfect information. Depending on the equipment (the possibility of collecting perfect information from the equipment), and the costs of a failure replacement and a preventive replacement; the present value of perfect information may justify investment into obtaining perfect information from equipment.

Table 49 Value of perfect information by K/C, Cost of a preventive replacement is equal to $100

	Im-Im/P-P (Value of perfect information in CBM)								
	i=0.01			i=0.02			i=0.03		
K/C	2 year	5 year	10 year	2 year	5 year	10 year	2 year	5 year	10 year
0.2	$3.20	$9.70	$27.33	$3.58	$13.42	$57.44	$4.01	$19.00	$130.92
0.5	$6.94	$21.02	$59.22	$7.75	$29.07	$124.46	$8.69	$41.16	$283.65
1	$8.28	$25.07	$70.61	$9.25	$34.66	$148.39	$10.36	$49.07	$338.20
2	$6.68	$20.22	$56.94	$7.46	$27.95	$119.67	$8.36	$39.58	$272.74

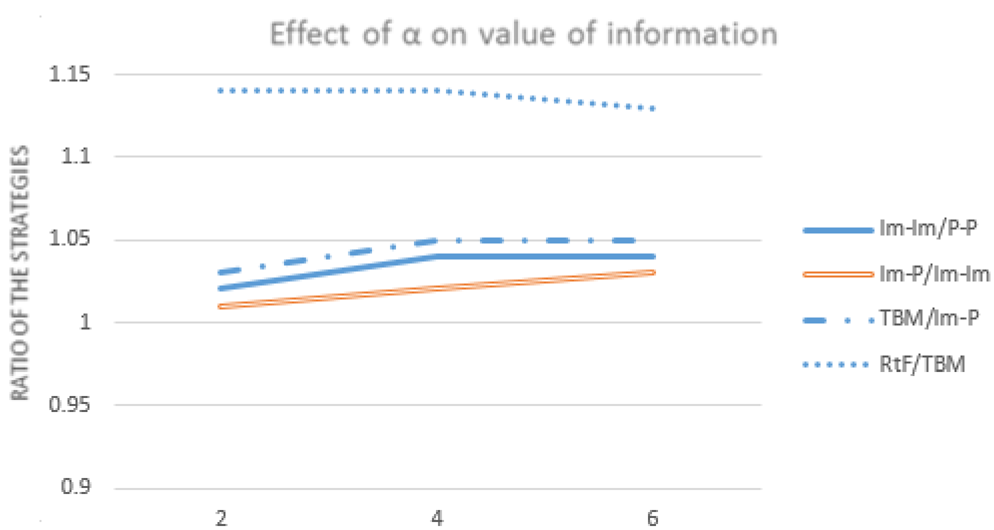
3.5. Effect of Parameters on Value of Information

In this section the effect of system parameters on value of information in the maintenance strategies are investigated. This analysis is performed to find possible trends in value of information for different values of system parameters. Understanding these trends can be useful for the practitioners to predict if the cost savings of each maintenance strategy will increase or decrease as a result of different system parameters. This analysis is only performed as an illustration of the trends in long-run average costs savings. However, the practitioner may use the developed application which will be presented later in this chapter, in order to obtain more accurate estimates on cost savings of each maintenance strategy.

3.5.1. Effect of parameter α

The value of age information decreases as the parameter α increases. This can be observed from Table 50. However, it appears that the value of observation information, value of CBM model assuming imperfect information, and value of perfect information in CBM generally increases as the value of parameter α increases. Increasing the value of parameter α, decreases the hazard rate (refer to equation 7). Therefore, we can conclude that the observation information, CBM model assuming imperfect information, and perfect information in CBM provide more value when the failures occur on a low frequency.

Table 50 Effect of α on value of information from new experiments

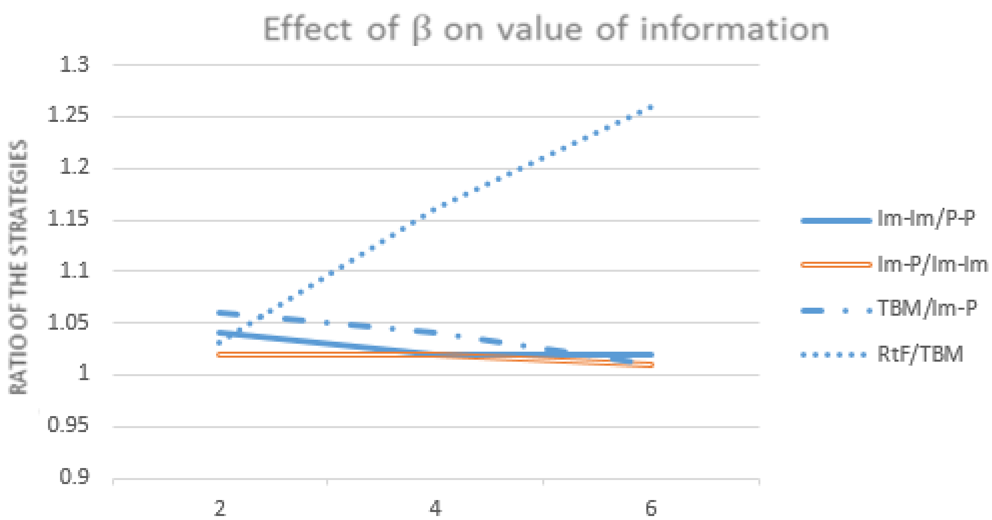


alpha	Im-Im/P-P	m-P/Im-Im	TBM/Im-P	RtF/TBM
2	1.02	1.01	1.03	1.14
4	1.04	1.02	1.05	1.14
6	1.04	1.03	1.05	1.13

3.5.2. Effect of Parameter β

Parameter β seems to have the reverse effect of parameter α. It can be observed that the increase in parameter β leads to the increase of value of age information, decrease of value observation information, decrease of value of CBM assuming imperfect information, and decrease of value of perfect information. The reason is that unlike the parameter α, increasing the value of parameter β increases the hazard rate (refer to equation 7). This effect in noticeable in Table 51.

Table 51 Effect of β on value of information from sample experiments

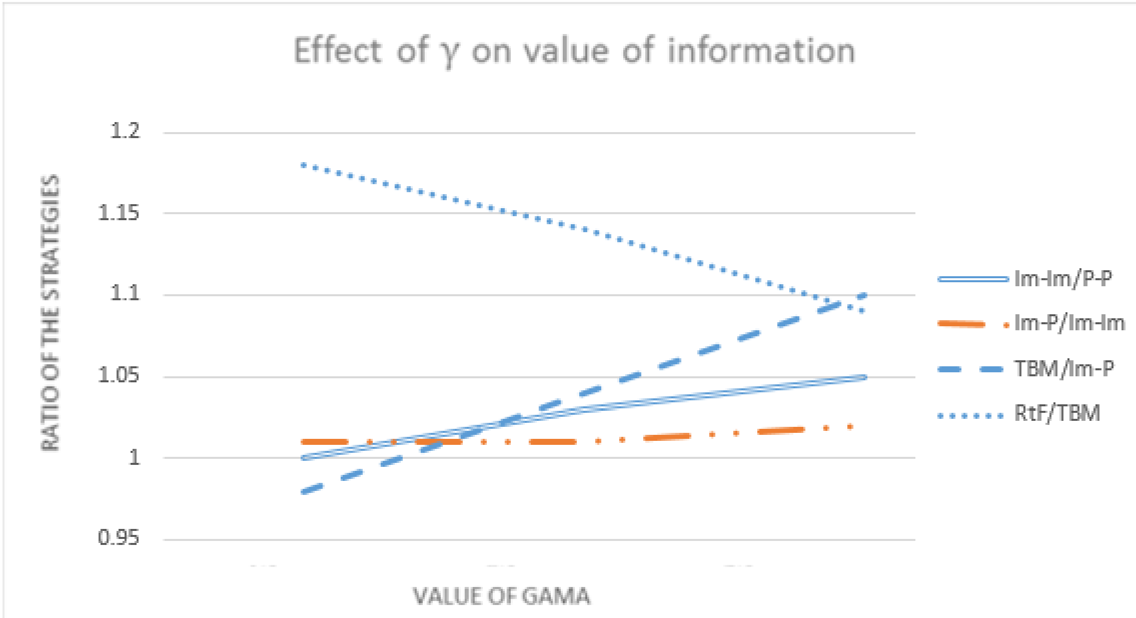


beta	Im-Im/P-P	Im-P/Im-Im	TBM/Im-P	RtF/TBM
2	1.04	1.02	1.06	1.03
4	1.02	1.02	1.04	1.16
6	1.02	1.01	1.01	1.26

3.4.3. Effect of Parameter ϒ

Parameter ϒ seems to significantly affect value of age information and value of observation information. As the value of parameter ϒ increases, the value of age information decreases and the value of state information increases. It is quite expected that higher values of parameter ϒ increase the value of state information. The reason is that higher values of ϒ exponentially (since the state dependent function is exponential, refer to equation 18) magnify the effect of health state values in the hazard function, therefore, magnifying the effect of state information.

Table 52 Effect of Υ on value of information from sample experiments



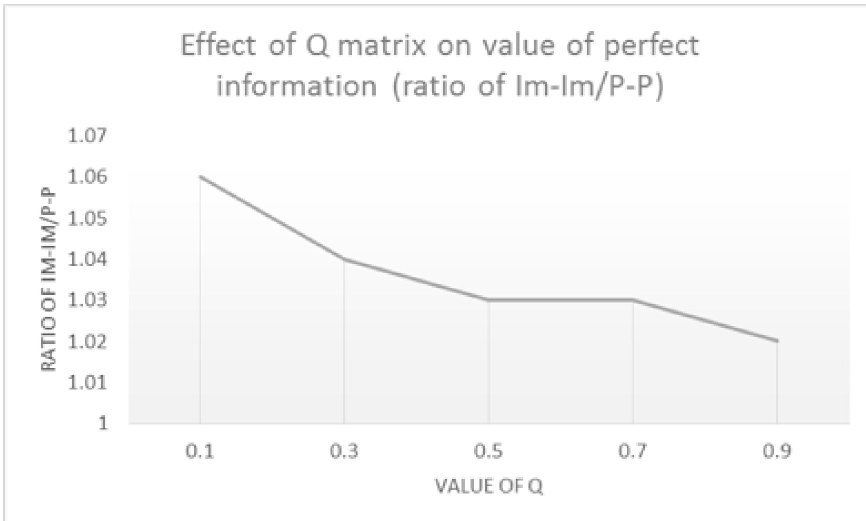
gamma	Im-Im/P-P	Im-P/Im-Im	TBM/Im-P	RtF/TBM
0.5	1	1.01	0.98	1.18
1.5	1.03	1.01	1.04	1.14
2.5	1.05	1.02	1.1	1.09

3.4.4. Effect of Parameter q

Higher values of parameter q mean higher accuracy in translating the observed state information into actual state of equipment. Perfect information can be obtained if parameter q is close to 1. From the results from Table 53 we can see that the closer the value of parameter q is to 1, the closer the value of Im-Im strategy to P-P strategy. This value also can be interpreted as the value of proper model usage with imperfect information. The higher the imperfection of data the higher the value of usage of the proper model.

Table 53 Effect of Q on long-run average costs and value of perfect information, from new experiments

Q	P-P	Im-Im	Im-Im/P-P
0.1	6.4	6.76	1.06
0.3	6.57	6.81	1.04
0.5	6.88	7.12	1.03
0.7	6.97	7.16	1.03
0.9	6.94	7.05	1.02



The effect of q illustrates that the CBM assuming imperfect information can provide the best results by estimating the actual state of equipment. Since the cost of implementing the CBM assuming imperfect information may be just slightly higher than the cost of implementing the CBM strategy assuming perfect information, it is recommended that the practitioner uses the CBM strategy assuming imperfect information instead of the CBM assuming perfect information.

3.4.5. Effect of P matrix

The P matrix does not show any considerable effect on the value of information.

Table 54 Effect of P on value of information for two-state equipment from Taguchi experiments

P	Im-Im/PP	Im-P/ImIm	TBM/ImP	RtF/TBM
0.2	1.02	1.02	1.03	1.52
0.8	1.02	1.03	1.00	1.42

Table 55 Effect of P on value of information for three-state equipment from Taguchi experiments

P	Im-Im/PP	Im-P/ImIm	TBM/ImP	RtF/TBM
0.2	1.08	1.03	1.00	1.43
0.8	1.06	1.04	0.99	1.40

The summary of effect of parameters on the value of information is provided in Table 56. We can see how increasing the value of each parameter effects the value of information.

Table 56 Summary of the effect of increasing the value of parameters on value of information

Type of information	alpha	beta	gama	Q
Value of age information (RtF/TBM)	Decrease	Increase	Decrease	
Value of state information (TBM/Im-P)	Increase	Decrease	Increase	
Value of CBM assuming imperfect information (Im-P/Im-Im)	Increase	Decrease	Increase	
Value of perfect state information (Im-Im/P-P)	Increase	Decrease	Increase	Decrease

The results provided earlier show the value of information in maintenance, and the effect of parameters on value of information. They provide the practitioner with some information to select the proper maintenance strategy. The results also highlight the importance of information in maintenance decision making. From the results the practitioner can estimate the expected cost savings obtainable from each maintenance strategy. However, more accurate value of information cannot be obtained unless failure parameters are estimated and long-run average costs of each maintenance strategy are calculated for the estimated parameters. In the next section we provide a demo of the application which is developed to obtain more accurate information regarding the cost savings and value of information.

3.6. A Demo of the Application Developed to Select the Suitable Strategy

We developed an application that uses history of the failure data of equipment, and estimates the PHM parameters as well as Time Based and Run-to failure parameters. Using the estimated parameters, the application calculates long-run average costs of the maintenance strategies and provides the estimation of expected present net worth value of each strategy.

The application can read the failure data from an excel file. Each failure data set should be entered in the format below:

k	Observed value
0	1
1	2
2	2
Failure Time:	
2.3	

The observations from equipment should be entered for each observation point of time k, and the time that the equipment fails should be inputted. This application can perform calculations for two-state and three-state equipment. Using this information, the failure parameters are estimated and by inserting the cost of failure replacement (C+K) and preventive replacement (C), long-run average costs are calculated. Other set of inputs include the period of inspections, if it's monthly, weekly or yearly, and the length of planning horizon. The interest rate should also be entered to enable the application to calculate the present net worth value of the maintenance strategies.

The output includes the long-run average costs, and the cost saving of applying each strategy. The output is illustrated in Figure 11:

```
Cost of a preventive replacement is $100
Cost of a failure replacement is    $200
--------------------------------------------
Long-Run Average Costs:
------------------------
P-P:    65.2051
Im-Im:  69.0625
Im-P:   72.2862
TBM:    77.8013
RtF:    107.3627
------------------------
Value of Each maintenance strategy

Savings for   5   year period:
Im-Im/P-P     Im-P/Im-Im     TBM/Im-P     RtF/TBM
------------------------------------------------------
  431.6          360.04        617.22       3305.25
```

Figure 11 snapshot of the output of the developed application to calculate value of information

The example above shows the value of each maintenance strategy for a 5 year planning period with an interest rate of 0.02, considering monthly inspections. The practitioner can compare the results with the amount of investment and operational costs required to implement each maintenance strategy and make the right decision.

Chapter 4: Conclusion

The application of monitoring equipment health condition is becoming increasingly essential especially as Condition-based maintenance (CBM) strategies are being more commonly used in maintenance decision making. Condition monitoring techniques are evolving to a degree where they can perfectly observe and report health condition of equipment. In this study the application of proportional hazards model (PHM) in CBM has been investigated by considering two assumptions: the condition of equipment is perfectly observable (perfect information), and equipment condition is not perfectly observable but stochastically related to the actual condition (imperfect information). Long-run average costs were calculated for the CBM strategies as well as for a time-based maintenance approach and a run to failure maintenance approach. The cost analysis shows the effectiveness and the profit CBM can provide for a business and at the same time the value of having age information, observation information, and perfect state information will be recognized. The effect of hazard function parameters, and state and observation transition matrices were also investigated in order to provide the practitioner with an understanding on existing trends in long-run average cost savings of the maintenance strategies. In the end an application was introduced which can provide the practitioner with more accurate cost savings of the maintenance strategies.

The main contribution of this study is to introduce and investigate the monetary value of perfect information and the monetary value of implementing the CBM strategy assuming imperfect information. A general framework is constructed accordingly, and an application is developed to illustrate value of information by calculating long-run average costs of the maintenance strategies. The significance of this study is that it shows the value of various sorts of information from equipment, which can be compared to the cost of obtaining those information to magnify the financial profit of choosing the proper strategy. To the authors knowledge no such analysis has been done in the literature. For future work it is suggested to perform the calculations of the long-run average costs for a wider range of parameters and for greater number of parameter levels. Inclusion of partial repairs is another potential future work to extend this work.

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