A DISTRIBUTION-FREE TEST FOR DEPRIVATION DOMINANCE

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Keywords and Phrases: deprivation profile; deprivation dominance; poverty; welfare; asymptotic distribution; statistical test

JEL Classification: C12, I32

ABSTRACT

The Rawlsian perspective on social policy pays particular attention to the least advantaged members of society, but how should "the least advantaged" be identified? The concept of deprivation dominance operationalizes in part the Rawlsian evaluation of the welfare of the least advantaged members of society, but a statistical procedure for testing deprivation dominance is needed. In this paper, we construct a new distribution-free test for deprivation dominance and apply it to Canadian income survey data.

1 Introduction

John Rawls has argued that the principle of justice as fairness implies that "social and economic inequalities are to satisfy two conditions: they must be (a) to the greatest benefit of the least advantaged members of society; and (b) attached to offices and positions open to all under conditions of fair equality of opportunity" (Rawls, 1982, p. 162). The ethical criterion that one should choose social policies which improve the well-being of "the least advantaged" has become very influential.

Empirical evidence on the actual distribution of income is therefore crucial, but in order to operationalize "the well-being of the least advantaged" as a criterion for choosing between social states and social policies, one requires: (a) some specification of who it is in society that are to be counted as "the least advantaged"; (b) some measure of the well-being of the least advantaged; and (c) given that the distribution of income inevitably must be estimated from sample data, some statistical test to ascertain whether a particular income distribution is really to be preferred to an alternative income distribution.

Corresponding to (a) and (b), the concept of *deprivation profile* has been recently studied under the similar but differently labelled setups [see, for example, Spencer and Fisher (1992), Shorrocks (1993), and Jenkins (1994)]. Shorrocks (1994,1995) has unified these setups and offered a general framework, and proposed several key concepts such as deprivation profile, deprivation dominance, and deprivation indices. Among these concepts, deprivation dominance has a particular importance because it reveals the partial ordering of two poverty profiles, and is helpful in evaluating the welfare impact of the changing intensity of poverty. This literature is a natural development from the poverty measurement literature initiated by Sen (1976) and extended by Foster, Greer and Thorbecke (1984). It is useful to note that deprivation dominance used here can also be viewed in principle as second-order stochastic dominance or generalized Lorenz dominance of a special kind between two distributions of poverty gaps (see Foster and Shorrocks, 1988).

This paper focuses on issue (c). Given the fact that deprivation dominance is, to date, a theoretical concept, we need a statistical test procedure for deprivation dominance. Because a deprivation profile is formulated from a truncation of an income distribution, such a test should be distribution-free. Along a somewhat different path, without using generalized Lorenz curves, Anderson (1996) proposed nonparametric tests of stochastic dominance between income distributions based on the Pearson goodness of fit test. Although Anderson (1996) did not deal with deprivation dominance explicitly, his framework is general and could be potentially applied to the comparison of deprivation profiles.[1] This paper, however, focuses on the deprivation dominance test based on an extension of Beach and Davidson (1983), and an application of Kodde and Palm (1986).

The main contributions of this paper are (1) the derivation of the asymptotic distribution for the vector of deprivation profile ordinates, and (2) the construction of the asymptotic test statistic for deprivation dominance. The proposed test is useful for testing the deprivation dominance relation between two deprivation profiles, and revealing the degrees of poverty intensity while not imposing any restriction on the form of distribution functions.

The remainder of the paper is organized as follows. In section 2, the deprivation profile and deprivation dominance are briefly reviewed, and the asymptotic distribution of the vector of deprivation profile ordinates and the test-statistic for deprivation dominance are developed. In section 3, we compare Canadian regional deprivation profiles based on the deprivation dominance test. Section 4 concludes.

2 Deprivation Profile and Deprivation Dominance Test

The measure of individual income, Y, is distributed across the population according to a distribution function F. For the same distribution function F, there exists a measure of individual deprivation, X. In the empirical study of poverty, a sample of n individual incomes, $\{y_i\}_{i=1}^{n}$, is considered. The sample

[1]We would like to thank one anonymous referee for bringing to our attention the potential use of Anderson (1996) in testing for deprivation dominance.

deprivation measure x_i of person i may be taken to be the absolute poverty gap $x_i(y_i, z) = \max\{z - y_i, 0\}$, which measures the absolute shortfall of the sample income, y_i, from the poverty line, $z > 0$. Let q be the number of the poor individuals in the sample whose incomes are less than or equal to the poverty line z.

According to Shorrocks (1994, 1995), the deprivation profile $D(F, \cdot)$ for F is defined by

$$D(F, p) = \int_{F^{-1}(1-p)}^{\infty} x dF(x) = \int_{1-p}^{1} F^{-1}(w) dw, \quad p \in [0, 1]. \tag{1}$$

The deprivation profile is obtained by successively aggregating the deprivation values of the most disadvantaged members of society. The deprivation profile for F at p is the equally weighted sum for all the deprivation measures in the most deprived p percentile subgroup of the economic units being considered. $D(F, p)$ for $p \in [0, 1]$ has a continuous, concave graph; it begins at the origin and rises continuously to its maximum height which is the mean deprivation value $\mu(F) = \int_{-\infty}^{\infty} x dF(x) = \int_0^1 F^{-1}(w) dw$. Figure 1 demonstrates two different deprivation profiles with $D(F_2, \cdot)$ more deprived than $D(F_1, \cdot)$. The headcount ratio (or the poverty rate) can be defined as $H(F) = \frac{q}{n}$. The deprivation profile reaches its maximum height at $H(F) = \frac{q}{n}$. The point ξ in Figure 1 is an upper bound on individual deprivation measures.[2]

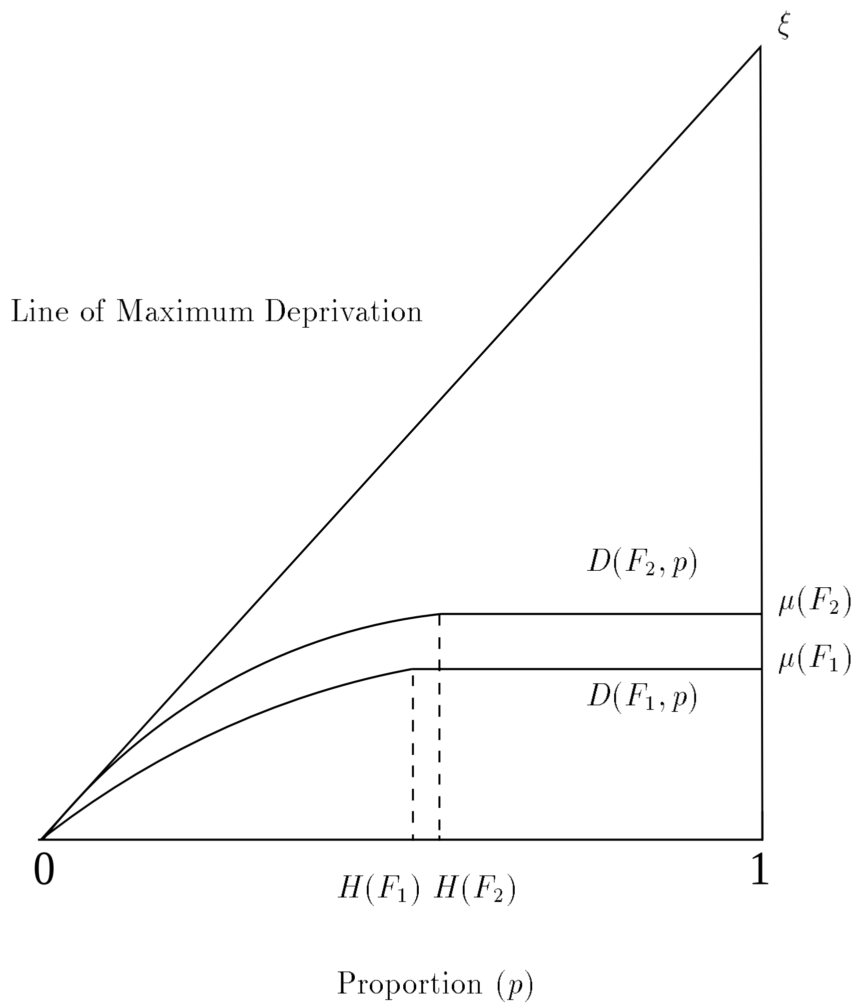
The deprivation profile $D(F, \cdot)$ for F is related to the generalized Lorenz curve $GL(F, \cdot)$ of the *deprivation measures* for F as follows: $D(F, p) = \mu(F) - GL(F, 1 - p)$, for $p \in [0, 1]$, where $GL(F, p) = \int_0^p F^{-1}(w) dw$. Given that lower deprivation measures are preferred, the distribution F_1 is said to *deprivation dominate* (denoted as D_d) the other distribution F_2 if $F_1 \neq F_2$ and $D(F_2, p) \geq D(F_1, p)$ for all $p \in [0, 1]$. Figure 1 shows that the "lower de-

[2]As Shorrocks (1994) points out, the deprivation is best understood in the following way: Assume that F corresponds to an equally weighted sample of n individual deprivation values $\{x_i\}_{i=1}^n$ arranged in *decreasing* order, the deprivation profile can be characterized by

$$D(F, \frac{k}{n}) = \frac{1}{n} \sum_{i=1}^{k} x_i,$$

for $k = 1, 2, \ldots, n$.

Figure 1: Derivation Profiles



privation" profile deprivation dominates the "higher deprivation" profile (or $D(F_1, \cdot)\ D_d\ D(F_2, \cdot)$). The distribution F_1 deprivation dominates the other distribution F_2 if and only if F_1 can be obtained from F_2 by a sequence of decrements and/or equalizations.

Now, we turn to the development of the asymptotic distribution of the deprivation profile $D(F, \cdot)$. Consider K points on the generalized Lorenz curve are chosen such that $0 < p_1 < p_2 < \cdots < p_{K-1} < p_K = 1$. The K population quantiles corresponding to these p_i's are denoted by $\xi_{p_i} = F^{-1}(p_i)$, $i = 1, 2, \ldots, K$. The conditional mean and variance of income less than or equal to ξ_{p_i} are denoted as $\gamma_i = E(X|X \leq \xi_{p_i})$ and $\lambda_i^2 = E[(X - \gamma_i)^2|X \leq \xi_{p_i}]$, respectively, for $i = 1, 2, \ldots K$. γ_K and λ_K^2 are the overall or unconditional mean and variance, respectively. The $K \times 1$ vector of generalized Lorenz ordinates at $p_1, p_2, \cdots, p_K$ is given by $\theta = [p_1\gamma_1, p_2\gamma_2, \cdots, p_K\gamma_K]'$. The population quantities can be estimated consistently by their sample analogs which are denoted by the same symbols with a hat "$\widehat{\ }$". Let the sample observations $\{x_i\}_{i=1}^n$ be ordered in *increasing order* so that $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$. Let the sample quantile be $\widehat{\xi}_p = x_{(r)}$, where $x_{(r)}$ is the r-th order statistic with $r = [np]$ denoting the greatest integer less than or equal to np. $\widehat{\gamma}_i = \frac{1}{r_i}\sum_{j=1}^{r_i} x_{(j)}$ is the sample counterpart of $\gamma_i = E(X|X \leq \xi_{p_i})$ and $\widehat{\lambda}_i^2 = \frac{1}{r_i}\sum_{j=1}^{r_i}(x_{(j)} - \widehat{\gamma}_i)^2$ the sample counterpart of $\lambda_i^2 = E[(X - \gamma_i)^2|X \leq \xi_{p_i}]$ with $i = 1, 2, \ldots, K$.

Beach and Davidson (1983) show that under the conditions that the population has finite mean and variance and that the distribution function F is strictly monotonic and twice differentiable, the K vector of generalized Lorenz ordinates $\widehat{\theta} = [p_1\widehat{\gamma}_1, p_2\widehat{\gamma}_2, \cdots, p_K\widehat{\gamma}_K]'$ for $GL(F, \cdot)$ is asymptotically normal in that $\sqrt{n}(\widehat{\theta} - \theta) \to \mathcal{N}(0, \Sigma)$ where, for $i \leq j$, the ij-th element of the $K \times K$ variance-covariance matrix Σ is given by

$$\sigma_{i,j} = p_i[\lambda_i^2 + (1 - p_j)(\xi_i - \gamma_i)(\xi_j - \gamma_j) + (\xi_i - \gamma_i)(\gamma_j - \gamma_i)]. \qquad (2)$$

For $i \leq j$, $\sigma_{i,j}$ (Σ) can be consistently estimated by

$$\widehat{\sigma}_{i,j} = p_i[\widehat{\lambda}_i^2 + (1 - p_j)(x_{(r_i)} - \widehat{\gamma}_i)(x_{(r_j)} - \widehat{\gamma}_j) + (x_{(r_i)} - \widehat{\gamma}_i)(\widehat{\gamma}_j - \widehat{\gamma}_i)] \qquad (3)$$

$(\widehat{\Sigma})$.

The first main result of this paper is the asymptotic distribution for a set of deprivation profile ordinates. From $D(F,p) = \mu(F) - GL(F, 1-p)$ for $p \in [0,1]$, the deprivation profile can be derived from the generalized Lorenz curve. That is, the $K \times 1$ vector of deprivation profile ordinates,

$$\widehat{\phi} = [(\widehat{\gamma}_K - p_{K-1}\widehat{\gamma}_{K-1}), (\widehat{\gamma}_K - p_{K-2}\widehat{\gamma}_{K-2}), \cdots, (\widehat{\gamma}_K - p_1\widehat{\gamma}_1), \widehat{\gamma}_K]', \qquad (4)$$

corresponding to $[p_1, p_2, \cdots, p_K]'$ for $D(F, \cdot)$ can be expressed as a linear combination of the $K \times 1$ vector of generalized Lorenz ordinates, $\widehat{\theta}$. Note $p_K \widehat{\gamma}_K = \widehat{\gamma}_K$, since $p_K = 1$, and $\widehat{\gamma}_K$ is an estimator for $\mu(F)$. $GL(F, \cdot)$ corresponding to p_i, $i = 1, 2, \ldots, K$, are estimated by $p_i \widehat{\gamma}_i$, $i = 1, 2, \ldots, K$. To facilitate the linear transformation, a special $K \times K$ matrix R should be defined as:

$$R = \begin{bmatrix} 0 & \cdots & 0 & -1 & 1 \\ 0 & \cdots & -1 & 0 & 1 \\ 0 & \cdots & 0 & 0 & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ -1 & \cdots & 0 & 0 & 1 \\ 0 & \cdots & 0 & 0 & 1 \end{bmatrix}. \qquad (5)$$

Then $\widehat{\phi}$ and ϕ may be expressed respectively as

$$\widehat{\phi} = R\widehat{\theta}, \qquad \text{and} \qquad \phi = \mathrm{R}\theta. \qquad (6)$$

Based on Beach and Davidson (1983), $\sqrt{n}(\widehat{\theta} - \theta) \rightarrow \mathcal{N}(0, \Sigma)$, we have $\sqrt{n}(R\widehat{\theta} - R\theta) \rightarrow \mathcal{N}(0, R\Sigma R')$. Thus, $\sqrt{n}(\widehat{\phi} - \phi) \rightarrow \mathcal{N}(0, \Omega)$ where $\widehat{\phi} = R\widehat{\theta}$, $\phi = R\theta$, and $\Omega = R\Sigma R'$. The $K \times K$ variance-covariance matrix Ω is given by

$$\Omega = \begin{bmatrix} \mathbf{C} & \mathbf{b} \\ \mathbf{b}' & \sigma_{K,K} \end{bmatrix}, \qquad (7)$$

where the $K \times 1$ row vector $\mathbf{b}$ is given by

$$\mathbf{b} = [(\sigma_{K,K} - \sigma_{K-1,K}), (\sigma_{K,K} - \sigma_{K-2,K}), \ldots, (\sigma_{K,K} - \sigma_{1,K})]', \tag{8}$$

and the ij-th element $c_{i,j}$ of the $K \times K$ matrix $\mathbf{C}$ is given by

$$c_{i,j}=\sigma_{K-i,K-j} + \sigma_{K,K} - \sigma_{K-i,K} - \sigma_{K,K-j}, \tag{9}$$

for $i \leq j$. The variance-covariance matrix of a deprivation profile, Ω, can also be consistently estimated by using $\widehat{\sigma}_{i,j}$ to replace $\sigma_{i,j}$.

As in Figure 1, the headcount ratio in F_1, $H(F_1)$, is lower than that $(H(F_2))$ in F_2. The deprivation profile $D(F_1, \cdot)$ reaches it maximum height $\mu(F_1)$ at $H(F_1)$ while the deprivation profile $D(F_2, \cdot)$ reaches it maximum height $\mu(F_2)$ at $H(F_2)$. The two deprivation profiles from $H(F_2) > H(F_1)$ to 1 are parallel horizontal lines. Thus, it is only necessary to have the test implemented on $D(F_1, p)$ and $D(F_2, p)$ over the interval of $p \in [0, \max(H(F_1), H(F_2))]$. Because of this, $\widehat{\phi}_1$ and $\widehat{\phi}_2$ should be truncated. Both truncated $\widehat{\phi}_1$ and $\widehat{\phi}_2$ should have the same dimension $k < K$, and represent $D(F_1, p)$ and $D(F_2, p)$ for $p \in [0, \max(H(F_1), H(F_2))]$.

There are generally two approaches with which one can evaluate the deprivation dominance between two deprivation profiles. One is to implement the Studentized Maximum Modulus (SMM) procedure to the pair-wise comparison of the differences at each of the selected percentiles following Beach and Richmond (1985). This literature is well-known and widely used. In this case, the inference must be made on the basis of pair-wise comparisons, and no direct test of a deprivation dominance relationship is available. Alternatively, following Xu (1998), one can combine the pair-wise comparisons into one test and test the deprivation dominance relationship directly. In this case, the deprivation dominance of one deprivation profile over another must be clearly specified under the null hypothesis.

To test for deprivation dominance between two deprivation profiles, define a $k \times k$ variance-covariance matrix for the difference between the truncated $\phi_2 - \phi_1$ as $\frac{1}{n}(\Omega_1 + \Omega_2)$. The new test-statistic T for deprivation dominance, the second main result of this paper, is based on Kodde and Palm (1986). To test $H_0: \ \phi_2 - \phi_1 \geq 0$ against H_a: $\phi_2 - \phi_1 \ngeq 0$, the test-statistic T for deprivation dominance is given by:

$$T = \Delta'[\frac{1}{n}(\Omega_1 + \Omega_2)]^{-1}\Delta, \tag{10}$$

where $\Delta = [(\widehat{\phi}_2 - \widehat{\phi}_1) - (\widetilde{\phi}_2 - \widetilde{\phi}_1)]$; $\widehat{\phi}_1$and $\widehat{\phi}_2$ are the unrestricted estimates while $\widetilde{\phi}_1$ and $\widetilde{\phi}_2$ are the restricted estimates minimizing

$$[(\widehat{\phi}_2 - \widehat{\phi}_1) - (\phi_2 - \phi_1)]'[\frac{1}{n}(\Omega_1 + \Omega_2)]^{-1}[(\widehat{\phi}_2 - \widehat{\phi}_1) - (\phi_2 - \phi_1)] \tag{11}$$

$$s.t. \ \ (\phi_2 - \phi_1) \geq 0.$$

If the least favorable values in the vector $\phi_2 - \phi_1$ satisfy the null hypothesis with equality, the test-statistic T is asymptotically distributed as a weighted sum of χ^2 random variables with different degrees of freedom:

$$sup_{\phi_2 - \phi_1 \geq 0} Pr(T \geq q | \frac{1}{n}(\Omega_1 + \Omega_2)) = \sum_{i=0}^{k} Pr[\chi^2(k-i) \geq q] W(k, i, \frac{1}{n}(\Omega_1 + \Omega_2)).$$

The upper- and lower-bounds for the critical values for testing inequality restrictions are provided by Kodde and Palm (1986).[3] For example, when k is 5 and the significance level α is set to 5%, the upper-and lower-bounds are 2.706 and 10.371, respectively. When k is 11 and the significance level α is set to 5%, the upper-and lower-bounds are 2.706 and 19.045, respectively. Decision rules based on the test-statistic T are: if the test-statistic T exceeds the upper-bound value, reject H_0; and if the test-statistic T is smaller than the lower-bound

[3]The reason for computing the upper- and lower-bonds for the critical value is that computing the weights W can be nontrivial. The computing of weights involves evaluation of k-multiple integrals, and closed forms are only available for a small integer k. Kodde and Palm (1986) provide a partial solution to this problem by computing the upper- and lower-bound critical values that do not require computation of the weights. These bounds are given by $\alpha_l = \frac{1}{2}Pr(\chi_1^2 \geq q_l)$, and $\alpha_u = \frac{1}{2}Pr(\chi_{k-1}^2 \geq q_u) + \frac{1}{2}Pr(\chi_k^2 \geq q_u)$, where q_l and q_u are the lower- and upper-bounds, respectively, for the critical values of the test-statistic. A lower-bound for the critical value is obtained by choosing a significance level α and setting degrees of freedom (df) equal to one. An upper-bound for the critical value is obtained by choosing a significance level α and setting df equal to k.

value, do not reject H_0. If the test-statistic T is in the inconclusive region, then the weights W in the distribution can be determined by a numerical simulation, and the simulated p-value of the test-statistic T can be compared with a chosen significance level α to reach a decision [see Wolak (1989), p.215].[4]

Regarding the new test, we have some additional observations: (1) An issue of considerable practical importance is the possible ambiguity of deprivation ordering of income distributions. The alternative hypothesis H_a:$\phi_2 - \phi_1 \ngeq 0$ is not equivalent to $\phi_2 - \phi_1 < 0$. The former is weaker than the latter. While $\phi_2 - \phi_1 \geq 0$ represents a weak form of deprivation dominance, $\phi_2 - \phi_1 \ngeq 0$ represents any violation of $\phi_2 - \phi_1 \geq 0$. (2) The variance-covariance matrix $\frac{1}{n}(\Omega_1 + \Omega_2)$ should be replaced by $(\frac{\Omega_1}{n_1} + \frac{\Omega_2}{n_2})$ when the two sample sizes are n_1 and n_2, respectively, and $n_1 \neq n_2$. (3) Since the sample size of the poverty data is usually very large, this asymptotic test is clearly suitable to this kind of data. (4) As Wolak (1991) points out, the upper- and lower-bounds are *slack* bounds and can be used to draw asymptotically valid inferences.

The proposed test fills the void of the literature and satisfies the need for a statistical test procedure for deprivation dominance. The new test is distribution-free in the sense that there is no need to specify or assume any functional form of distribution functions for an underlying data set.

3 A Regional Comparison of Deprivation in Canadian Society

It is well-known that regional disparity in poverty experience exists in Canada but the disparity of poverty experience has been often studied in terms of headcount ratios. In this section, we conduct a regional comparison of deprivation profiles using income data for four regions in Canada: Western Canada (Manitoba, Saskatchewan, Alberta, and British Columbia), Ontario, Quebec, and Atlantic Canada (New Brunswick, Prince Edward Island, Nova Scotia,

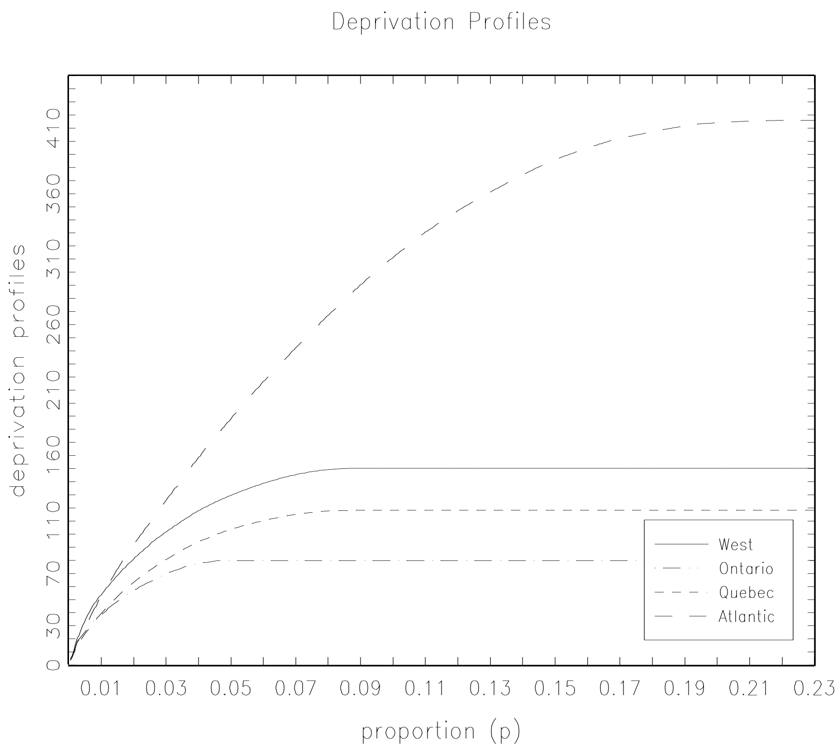
[4]Fisher, Willson and Xu (1998) provide an example of the application of the Wolak procedure. For applied economists, the bounds are relatively easy to implement for many situations.

and Newfoundland) from the 1992 Canadian Survey of Family Expenditures. We adopt the ethical perspective that although households may contain different numbers of individuals and have different household total incomes, a deprivation profile should be used to assess the deprivation of *individuals*. We therefore compute the equivalent income of each individual in each household using the equivalence scale proposed by the OECD.[5] We convert the family income to individual income using the equivalence scale adopted by OECD (1982). The low income cutoff point is computed as the half of median equivalent income [see, among others, Sharif and Phipps (1994)]. The poverty line is chosen to be the half median of the national individual incomes. The deprivation measures are computed for Western Canada , Ontario, Quebec, and Atlantic Canada.

Figure 2 presents the deprivation profiles for Western Canada, Ontario, Quebec, and Atlantic Canada. The figure shows that Atlantic Canada was deprivation-dominated by Western Canada, Quebec, and Ontario. The vertical distance from the deprivation profile of Atlantic Canada to the one of Western Canada was so great that Atlantic Canada was likely to be deprivation-dominated by Western Canada. Although there were differences among the deprivation profiles among Western Canada, Quebec, and Ontario, these deprivation profiles were fairly closed to one another. It was not clear whether or not the differences were statistically significant because the deprivation profile ordinates were computed based on sample data and sampling uncertainty existed surrounding the estimates of deprivation profile ordinates. The estimates and standard errors of deprivation profile ordinates are provided in Table 1. Two observations are in order. First, indeed the point estimates of a deprivation profile are subject to the influence of sampling variation. One cannot use a simple "eyeball test" to judge if one region deprivation dominates another. Second, although the four deprivation profiles reach their maximum heights at different proportions, their estimates and standard errors are all constant

[5]The OECD equivalence scale is: single adult (1.0), second and additional adults (0.7), and each child (0.5). We recognize that we are implicitly assuming an equal division of resources within each household, but since our focus in this paper is the demonstration of our test-statistic, this paper will neglect the important issue of intra-household inequality.

Figure 2: Derivation Profiles for Western Canada, Ontario, Quebec, and Atlantic Canada



Notes: The deprivation profiles for Western Canada, Ontario, Quebec, and Atlantic Canada are computed based on the 1992 Canadian Survey of Family Expenditures.

at the proportions between 0.22 and 1. The deprivation dominance test can be implemented meaningfully for all the proportions at which at least one deprivation profile does not reach its maximum height.

In order to provide a firmer basis for poverty comparison, we apply the proposed test procedure to each and every pair of deprivation profiles in two different directions. The test results are presented in Table 2. In Table 2, the first column shows the deprivation dominance relation to be tested. The second column lists the number of the proportions at which at least one deprivation profile does not reach its maximum height. The third column provides the test statistic. In the fourth column, the simulated p-value is provided only if the statistic falls into an inconclusive region. The last column shows the decision based on the 5% significance level. The results show that Western Canada, Ontario, and Quebec deprivation dominated Atlantic Canada but not the other way around. Thus, all other three regions deprivation dominated Atlantic Canada. Ontario deprivation dominated Western Canada but not the other way around. The ambiguity might come from the comparison between Western Canada and Quebec, and between Ontario and Quebec. This ambiguity can be examined by using the proposed test. While the tests in one direction indicate that Quebec deprivation dominated Western Canada, and that Ontario deprivation dominated Quebec, the tests in the opposite direction reject that Western Canada deprivation dominated Quebec, and that Quebec deprivation dominated Ontario if the significance level is chosen to be 5%. The differences among them were in fact statistically significant at the 5% level. Thus these deprivation profiles can be unambiguously ranked.[6]

4 Concluding Remarks

The concepts of deprivation profile and dominance have been recently proposed in welfare economics and poverty studies. However, there is no specialized test procedure available for applied research. This paper fills the void by

[6]Osberg and Xu (1998) discuss trends within Canadian provinces in poverty intensity over the period of 1984-1995.

Table 1: Deprivation Profile Ordinates: Estimates and Standard Errors

	West Canada		Ontario	
p	Estimate	St. Error	Estimate	St. Error
.02	81.11	6.07	57.42	6.56
.04	117.96	7.83	77.44	8.96
.06	138.87	9.41	79.90	10.47
.08	149.28	11.25	79.90	10.47
.10	150.18	11.99	79.90	10.47
.12	150.18	11.99	79.90	10.47
.14	150.18	11.99	79.90	10.47
.16	150.18	11.99	79.90	10.47
.18	150.18	11.99	79.90	10.47
.20	150.18	11.99	79.90	10.47
.22	150.18	11.99	79.90	10.47
⋮	⋮	⋮	⋮	⋮
1.00	150.18	11.99	79.90	10.47
	Quebec		Atlantic	
p	Estimate	St. Error	Estimate	St. Error
.02	63.19	5.58	91.91	3.32
.04	93.94	7.76	158.59	4.05
.06	110.28	9.68	216.62	5.36
.08	117.60	11.57	267.59	6.12
.10	118.19	12.13	311.84	7.77
.12	118.19	12.13	347.44	9.68
.14	118.19	12.13	374.39	11.00
.16	118.19	12.13	394.96	13.44
.18	118.19	12.13	407.29	15.03
.20	118.19	12.13	414.17	16.77
.22	118.19	12.13	415.89	17.70
⋮	⋮	⋮	⋮	⋮
1.00	118.19	12.13	415.89	17.70

Notes: The computations are based on the 1992 Canadian Survey of Family Expenditures.

Table 2: Deprivation Dominance Tests

Deprivation Dominance (D_d)			k	Test	p-value	Decision at $\alpha = .05$
Atlantic	D_d	West	11	189.970	—	Reject
West	D_d	Atlantic	11	.000	—	Do not reject
Atlantic	D_d	Quebec	11	245.010	—	Reject
Quebec	D_d	Atlantic	11	.000	—	Do not reject
Atlantic	D_d	Ontario	11	376.330	—	Reject
Ontario	D_d	Atlantic	11	.000	—	Do not reject
West	D_d	Ontario	5	20.379	—	Reject
Ontario	D_d	West	5	.000	—	Do not reject
West	D_d	Quebec	5	4.854	.049802	Reject
Quebec	D_d	West	5	.000	—	Do not reject
Quebec	D_d	Ontario	5	5.839	.031334	Reject
Ontario	D_d	Quebec	5	.000	—	Do not reject

Notes: The deprivation dominance test is performed between each and every pair of deprivation profiles in two directions. k is the number of the deprivation profile ordinates used for the test. p-value is the simulated p-value for the test statistic. The upper and lower bounds for the critical value corresponding to $k = 5$, at the 5% significance level, are 2.706 and 10.371, respectively. The upper and lower bounds for the critical value corresponding to $k = 11$, at the 5% significance level, are 2.706 and 19.045, respectively.

proposing a new distribution-free test for deprivation dominance. To the best of our knowledge, this test is the first specialized test of this kind following the Beach and Davidson (1983) tradition.

This paper uses our test procedure to evaluate the regional disparity of poverty experience in Canada, using the 1992 Survey of Family Expenditures. Poverty intensity was unambiguously higher in Atlantic Canada than in any other regions, and the difference was statistically and economically significant. Although the poverty intensity in Quebec was similar to that of Western Canada and Ontario, the differences in deprivation between Western Canada and Quebec, and between Quebec and Ontario were indeed statistically significant at the 5% significance level. Hence these deprivation profiles can be unambiguously ranked.

ACKNOWLEDGEMENTS

We wish to thank Tony Shorrocks for pointing out the issue and providing an earlier version of his paper on deprivation profiles and deprivation indices, and Essie Maasoumi and an anonymous referee for their helpful comments and suggestions. We also wish to thank Lynn Lethbridge for research assistance. Research support from Social Sciences and Humanities Research Council of Canada (SSHRC) and Dalhousie University Research Fund are gratefully acknowledged.

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