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Return Periods of Extreme Sea Levels From Short Records

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Extreme sea levels usually arise from a combination of the tides (assumed here to be deterministic) and storm surges (assumed stochastic). We show in this paper how tide and surge statistics derived from short (~1 year) records can be used to predict the occurrence of extremes with much longer return periods (~50 years). The method is based on an extension of the exceedance theory originally developed by Rice (1954) to study noise in electrical circuits. A comparison of predicted return periods with those obtained directly from a 50-year Markovian simulation of surge is used to validate the exceedance probability method. The method is next applied to the Canadian ports of Halifax and Victoria, which are dominated by semidiurnal and diurnal tides, respectively. To provide a stringent test of the method, just 1 year's data from each port are used to estimate the tide, surge statistics, and hence return periods. The predictions are found to compare well with the results of a conventional (Gumbel) extremal analysis based on more than 60 years of data provided allowance is made for (1) the anormality of the surge distribution and (2) seasonal changes of surge variance. The agreement suggests that the method may be successfully applied to other short sea level records or indeed to any partly deterministic process where return periods are of interest.

1. INTRODUCTION

The frequency of extremes in sea level, and its reciprocal, the return period, are of obvious practical importance to those coastal cities subject to flooding. Where many years of sea level data are available, the extremal analysis of *Gumbel* [1958] has proved to be of great use in deriving decadal scale return periods [e.g., *Graff*, 1981]. However, for many coastal regions the available sea level records are relatively short, and a conventional extremal analysis based on annual maxima is precluded.

Here it will be shown how return periods may be estimated from short records. The method is based on the probability of sea level exceedance in a given interval of time. The derivation of this exceedance probability for a normally distributed, stationary process was given originally by *Rice* [1954] in an analysis of noise in electrical circuits. This probability, and a related distribution of maxima, have appeared in a variety of oceanographic contexts. *Cartwright* [1958] has applied the probability of maxima to ocean wave studies, while *Garrett and Munk* [1972] have parameterized mixing due to internal wave breaking using an exceedance probability for wave shear. *Lumley* [1962] has also used an exceedance probability to relate Eulerian and Lagrangian fluid velocity statistics. In this study, the exceedance probability due to *Rice* [1954] will be extended to the nonstationary process of sea level where we assume the tide is deterministic and the surge is stochastic. We will refer to this exceedance probability method as the EPM.

While the focus of this paper is primarily on extreme sea levels, it will become clear that the EPM may be usefully applied to extreme currents, wave-tide action on sediment transport, or any other process that is in part deterministic and part stochastic. In this paper the deterministic component will be tides. However, it could equally well include a predictable trend, and we anticipate that the EPM may prove useful in the estimation of return periods for coastal sites subject to pronounced vertical crustal movement.

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In section 2 the EPM is outlined, and summaries are given of *Gumbel's* [1958] extremal analysis procedure and an alternative method for estimating return periods from short records due to *Pugh and Vassie* [1980]. The latter is based on the joint probability of tide and surge and is henceforth referred to as the JPM. A 50-year Markovian surge simulation is used to validate the EPM and compare return periods predicted from a Gumbel analysis and the JPM (section 3). The surge statistics and tidal component required for return period prediction at Halifax and Victoria are obtained in section 4 from 1 year of sea level data. Predictions are made in section 5 using the EPM and JPM and finally compared with benchmark extremal results obtained from a Gumbel analysis of more than 60 years of sea level data from each site.

2. THEORY

The purpose of this section is to outline the extension of the exceedance probability approach of *Rice* [1954] and review two alternative methods which are used later for comparison, i.e., the *Gumbel* [1958] type analysis of annual maxima and the joint probability approach of *Pugh and Vassie* [1980].

2.1. *Gumbel's Method*

Consider a record of length $N\Delta T$ where N is an integer (typically greater than 25) and ΔT is a subrecord length (typically 1 year) assumed to contain many independent extreme events. From each subrecord the largest extreme is extracted, and the resultant set ordered in an ascending sequence (η_m^*; $m = 1, N$). The probability that an (annual) extreme is greater than or equal to the mth exceedance is then estimated to be $\Phi(\eta_m^*) = (N + 1 - m)/(N + 1)$, with a return period (in years)

$$T_R = \Delta T/\Phi(\eta_m^*) \tag{1}$$

Now if sea level follows a type 1 distribution (see below), then *Gumbel* [1958] has shown that the extremal probability is given asymptotically by

$$\Phi(\eta^*) = 1 - \exp[-\exp(-y)] \tag{2}$$

where $y = (\eta^* - \alpha)/\beta$ is the "reduced variate" and α and 0.78β denote the mode and standard deviation of the extremes. (A type 1 distribution is defined to be any cumulative distribution (e.g., normal) which converges, with increasing η^*, toward unity at least as quickly as the exponential distribution,

$1 - \exp(-\eta^*)$.) Results for extremal return periods are then generally presented as a plot of η_m^* versus the reduced variate estimated from $y = -\ln[-\ln(1 - \Phi)]$. This often results in a straight line suitable for extrapolation; the return period (1) is associated with the abscissa.

Although Gumbel's method has proved very useful in the past, it is clear that a long sea level record is required if we are interested in (say) 50-year return periods; it is certainly inappropriate for records of 1 or 2 years duration. Another potential problem with this method, when applied to tidally dominated records, is in the common choice of 1 year for ΔT. The subrecord length should be long enough that the maxima can be considered a stationary process. Unfortunately, this is not the case when strong nodal modulations are present, and a better choice for ΔT would be 18.6 years.

2.2. *Joint Probability Method (JPM)*

For sea level records of only a few years duration, *Pugh and Vassie* [1980] have suggested that the extremal probability (2), and hence return period, may be estimated from the instantaneous probability that sea level exceeds η^*

$$Q_I(\eta^*) = \int_{\eta^*}^{\infty} P_{\eta_s}(\zeta)\, d\zeta \tag{3}$$

where P_{η_s} denotes the distribution of sea level η_s. Pugh and Vassie argue that at extreme levels the sea level process becomes independent over the sampling interval (taken to be 1 hour) and so the probability of the largest annual extreme exceeding η^* is given by

$$\Phi(\eta^*) = 1 - [1 - Q_I]^n \simeq nQ_I \tag{4}$$

where n is the number of samples in one year. (*Pugh and Vassie* [1980] assume a sampling interval of 1 hour and hence $n = 8766$.) Return periods in years are then recovered from (1).

The main advantage of the JPM is that the distribution of sea level used to estimate Q_I is obtained from

$$P_{\eta_s}(\zeta) = \int_{-\infty}^{\infty} P_\eta(\zeta - u) P_{\eta_T}(u)\, du \tag{5}$$

where P_η and P_{η_T} denote distributions of surge η and tide η_T; the convolution permits estimation of $P_{\eta_s}(\zeta)$ at sea level heights ζ that exceed those in the available data.

The main deficiency of the above approach involves the assumption of independence over the sampling interval. In fact the explicit dependence of return periods on the sampling interval is evident from (4), where halving the number of samples in a year, n, doubles T_R. This arbitrary feature of the JPM in part motivates the following exact treatment of return period prediction where the appropriate independence time will be obtained from the statistics of the sea level process.

2.3. *Exceedance Probability Method (EPM)*

The prediction of return periods here will involve determination of the probability that an exceedance occurs in a specific small interval $(t, t + dt)$. (In the context of the EPM we define an exceedance of η^* to occur when $\eta_s = \eta^*$ and $d\eta_s/dt > 0$.) This probability, originally discussed by *Rice* [1954], is denoted by

$$Q\, dt \tag{6}$$

and may be interpreted as the expected number of exceedances in $(t, t + dt)$. In our case the surge statistics may vary seasonally, and tides are assumed deterministic, and so the exceedance probability will be parametrically dependent on time; we thus write $Q = Q(t)$. A return period may be obtained from (6) by integrating forward in time until the expected number of exceedances

$$M = \int_t^{t+T} Q(\tau)\, d\tau \tag{7}$$

equals unity. The integration interval then defines the return period. Note that if the (sea level) process contains significant trends or periodicities in either the deterministic component or the statistics of the stochastic component then the return period will depend on the lower limit of integration. This is to be expected, and the ability to handle such nonstationarity is one of the advantages of the EPM.

In situations where the surge statistics, tide, and hence Q can be considered periodic in time, it is possible to obtain approximate return periods which avoid the long integrations required by the above definition. Taking the integration time T to equal the period of the tide-surge variability, then an approximate return period may be obtained from

$$T_R = T/M \tag{8}$$

with M given by (7). The approximate return period will generally differ from the exact ($M = 1$) definition by a small fraction of T and will be negligible for $T_R \gg T$. It will be shown in section 4 that for Halifax at least, the appropriate choice for T is 1 year and so the approximate return period definition (8) will be adequate for decadal scale return period predictions. We note that this approximate return period is more comparable with results from a Gumbel type analysis or the JPM, both of which assume a subrecord length that covers any significant periodicities in the statistics of the process. However, we stress that the exact return period from (7) should be used if there are significant trends or periodicities in the process that are comparable to the required return period.

The outstanding question is how to determine Q. The analysis of *Rice* [1954] shows that the exceedance probability of a purely stochastic process may be written as

$$Q = \int_0^{\infty} P_{\eta_s, \dot\eta_s}(\eta^*, v) v\, dv \tag{9}$$

where $P_{\eta_s, \dot\eta_s}$ denotes the joint distribution of sea level η_s with its derivative $\dot\eta_s = d\eta_s/dt$. Here, we regard surge as stochastic and tides as deterministic so (9) may be extended to

$$Q(t) = \int_0^{\infty} P_{\eta, \dot\eta}(\eta^* - \eta_T, v - \dot\eta_T) v\, dv \tag{10}$$

where $P_{\eta, \dot\eta}$ denotes the joint distribution of surge and its derivative. Assuming surge to have zero mean, the moments of $P_{\eta, \dot\eta}$ are defined by

$$\sigma^2 = \langle \eta^2 \rangle \tag{11}$$

$$\nu^2 = \langle \dot\eta^2 \rangle \tag{12}$$

$$\rho = \langle \eta\dot\eta \rangle (\sigma\nu)^{-1} = \nu^{-1}\, d\sigma/dt \tag{13}$$

where the angle brackets denote expectation. Anticipating a seasonal variation in the surge variance, it is assumed that

$$\sigma^2 = \sigma_0^2 (1 + \varepsilon \cos \Omega t) \tag{14}$$

where σ_0^2 denotes the yearly average of σ^2, $2\pi/\Omega$ is 1 year, and ε is a site-dependent parameter. A Taylor micro-time scale of surge variability may be defined by

$$\lambda = \sigma/\nu \tag{15}$$

and estimates (obtained in section 4) indicate λ to be reasonably constant throughout the year and of order 1 day. The correlation ρ is, in the context of the seasonal model (14), of order $\varepsilon\lambda\Omega$ and thus effectively zero. Indeed, from the definitions (13) and (15) the correlation ρ should in general be near zero as it will be of order $\lambda\Omega$ where $\Omega^{-1} \sim \sigma/(d\sigma/dt)$ is typically 1 year.

To proceed further, it will be assumed that the surge and its derivative are jointly normal in distribution. (The development of a more realistic model for $P_{\eta,\dot\eta}$ which includes skewness and kurtosis is deferred until section 5. The following normal model, however, is valuable in illuminating the relative importance of surge and tide in return period prediction.) Putting $\rho = 0$, it follows that η and $\dot\eta$ are statistically independent and the exceedance probability (10) is given by

$$Q(t) = [\lambda(2\pi)^{1/2}]^{-1}\sigma P_\eta(\eta^* - \eta_T)\{\exp(-\theta^2/2) + \theta(\pi/2)^{1/2}[1 + \operatorname{erf}(\theta/2^{1/2})]\} \tag{16}$$

where $\theta(t) = \dot\eta_T/\nu$ and

$$P_\eta(u) = [\sigma(2\pi)^{1/2}]^{-1} \exp(-u^2/2\sigma^2) \tag{17}$$

The expressions (16) and (17) represent a normal model from which return periods may be obtained for specified surge variance, microscale, and tide.

3. Discussion and Comparison of Methods

In order to focus our comparison of the three methods, 50 years of hourly surge data were first simulated as the second-order Markov process

$$\eta(t) = a\eta(t - \Delta) + b\eta(t - 2\Delta) + z(t)$$

where Δ is 1 hour and $z(t)$ was chosen at random from a normal $N(0, c)$ distribution. The parameters a, b, c were chosen [*Jenkins and Watts*, 1968] to yield a normally distributed surge time series that crudely corresponds to that found at Halifax in section 4 below; integral and microscales are near 40 and 10 hours, respectively, with a constant ($\varepsilon = 0$) surge variance of $\sigma_0{}^2 = 124.9$ cm^2. The three methods were then used to determine return periods when (1) the simulated surge dominates the tide (the strong surge limit) and (2) the tide dominates the simulated surge (the strong tide limit).

3.1. *Strong Surge Limit*

To test the methods in the strong surge limit, we have assumed for simplicity that $\eta_T = 0$, i.e., the sea level is just the simulated surge. Exceedances above a prescribed η^* ($\eta = \eta^*$, $\dot\eta > 0$) were then counted in the 50-year simulated surge record, and the frequency and hence return period of such events were determined. A plot of exceedance η^* versus return period is given in Figure 1. This provides a point of comparison for the three methods in the strong surge limit.

EPM. The parameter which determines the relative importance of tidal and surge variability in (16) is simply the amplitude of θ, the ratio of time derivatives $\dot\eta_T/\langle\dot\eta^2\rangle^{1/2}$. In a time-averaged sense, the amplitude of θ may be estimated by

$$\Gamma = \sigma_T\omega\lambda/\sigma \tag{18}$$

where σ_T and ω denote the rms tidal amplitude and frequency of the dominant tidal constituent.

For strong surge and weak tides, Γ is generally small, and the exceedance probability (16) asymptotes to that given by *Rice* [1954],

$$Q = \sigma[(2\pi)^{1/2}\lambda]^{-1}P_\eta(\eta^*) \tag{19}$$

If the surge variance is constant through time ($\varepsilon = 0$), (19) results in the return period

$$T_R = 2\pi\lambda \exp(\eta^{*2}/2\sigma^2) \tag{20}$$

for exceedance of η^* which clearly scales with the microscale λ. (Note that the return period of zero exceedances, $\eta^* = 0$, is simply $2\pi\lambda$, so that λ^{-1} may be interpreted as the dominant frequency of a narrow-band surge process. See also *Rice* [1954, p. 193] for further discussion.)

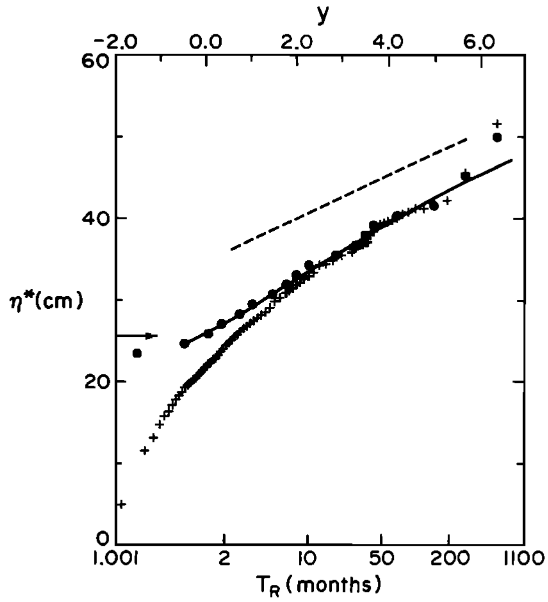

Fig. 1. Critical level η^* against return period T_R for the 50-year simulated surge process (i.e., $\eta_T = 0$). The circles denote the mean time between η^* exceedances calculated directly from the 50 years of hourly surge data. The solid and dashed curves show predictions from the EPM, (20), and the JPM, (21). The crosses denote return periods associated with monthly maxima which were estimated using the Gumbel approach, (1). The horizontal arrow indicates α, the most likely (or modal) monthly extreme.

Return periods have been predicted using Rice's formula (20) and the appropriate values of σ and λ for the simulated Markovian surge. The results are generally in very good agreement with the direct estimates described above (Figure 1), thus confirming (20). This is perhaps not surprising, as both sets of return periods are based on a normally distributed surge with identical statistics. (The increased scatter at large η^* is presumably due to the few exceedances recorded at such levels, leading to poor direct estimates of T_R.) The more interesting question is how successful are the other two methods?

Gumbel. Six hundred monthly extremes were extracted from the 50-year simulated data and ordered to permit estimation of Φ and return periods (see (1)). At high η^* the return periods of the monthly extrema agree well with the exceedance return periods. At η^* below the most likely monthly extreme (α), the return period of the exceedances is shorter than that of the monthly extremes. This is to be expected, as while exceedances of low η^* may be quite frequent, an extreme at low η^* must by definition be the highest level in the given time ΔT and possibly a rare event. We note that in certain practical applications (e.g., overtopping of sea defences) this difference could be important and the return period of exceedances more relevant.

JPM. Return periods may also be obtained from the JPM, although strictly the JPM was developed for the strong tide limit. The instantaneous exceedance probability (4) may

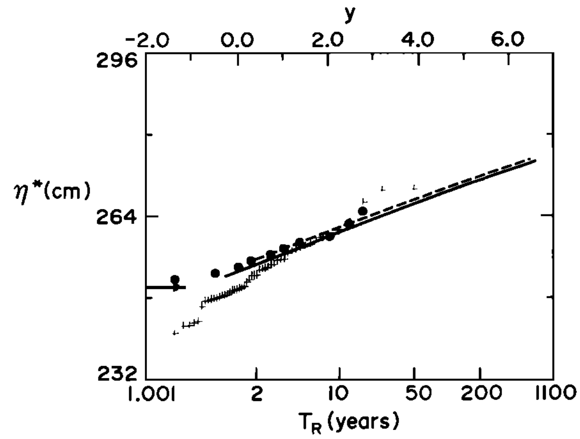

Fig. 2. Critical level η^* against return period T_R for the simulated sea level process at Halifax. (The 50-year simulated surge record was added to the 1970 predicted tide at Halifax, which was assumed to repeat over each of the 50 years.) The circles denote the mean time between η^* exceedances calculated directly from the 50 years of simulated Halifax sea level. The solid and dashed curves show predictions from the EPM, (22), and the JPM, (3)–(5). (The former were based on an integration time T (see (8)) of 1 year.) The crosses denote return periods associated with annual maxima which were estimated using the Gumbel approach, (1). The horizontal arrow indicates α, the most likely (or modal) annual extreme.

be integrated analytically for a normal surge distribution to give

$$T_R = 2\Delta t/[\text{erfc}\,(\eta^*/\sigma(2)^{1/2}] \tag{21}$$

where Δt is an "independence time" over which extremes decorrelate and is taken by *Pugh and Vassie* [1980] to be the sampling interval of 1 hour. Results from (21) seriously underestimate the return periods if we take $\Delta t = 1$ hour (Figure 1). By equating (20) and (21) the correct independence time is given by $\Delta t \simeq \sigma\lambda(2\pi)^{1/2}/\eta^*$, which is nearer to 6 hours for $\eta^*/\sigma = 4$ at Halifax. The dependence of Δt on $\sigma\lambda/\eta^*$ shows that the JPM is not applicable in the strong surge limit.

3.2. *Strong Tide Limit*

To test the methods in the strong tide limit, the 50 years of simulated surge data were added to the Halifax predicted tide, assumed periodic over 1 year. (The details of the predicted tide are not important at this stage although we note it is predominantly semidiurnal with an rms amplitude of $\sigma_T = 46$ cm. Given the rms surge is $\sigma_0 = 11$ cm, the strong tide limit pertains at Halifax.) Again the frequency of exceedance of a prescribed η^* was noted, and return periods were estimated. A plot of η^* versus return period is shown in Figure 2.

EPM. In the strong tide limit, tidal variability dominates that due to surge, and over a tidal cycle Γ, the magnitude of $\theta(t)$, may be made sufficiently large such that the exceedance probability (16) asymptotes to

$$Q(t) = \tfrac{1}{2}\{\dot{\eta}_T + |\dot{\eta}_T|\}P_\eta(\eta^* - \eta_T) \tag{22}$$

The return period ($T_R \sim Q^{-1}$) is now independent of λ and scales with the dominant tidal period, $2\pi/\omega$. Note that exceedances will tend to occur at rising tide where, by (22), Q is nonzero.

Return periods were estimated from the general expression (16) and the strong tide limit (22). (In both cases an integration time T of 1 year was used, which includes all significant periodicities in the Halifax record. The integrations were achieved by interpolating the tide to half hour intervals and estimating $\dot{\eta}_T$ with a centered second-order finite difference scheme. The interpolation was necessary to ensure sufficient accuracy of the trapezoidal rule integration as Q varies substantially for small changes in tidal height. Return periods were then obtained from (7) and (8).) The return periods from (16) and (22) were found to be indistinguishable, thus justifying (22) as a strong tide limit approximation. The exceedance return periods are presented in Figure 2, and again the agreement with those estimated directly from the simulated data is very good.

Gumbel. Annual maxima of sea level (tide plus simulated surge) were again extracted, ordered in an ascending sequence, and plotted (Figure 2). For high η^* the return periods of the annual maxima agree well with those of the exceedances. As in the strong surge limit, exceedances occur more frequently than annual maxima at low η^*, as expected.

JPM. Return periods were obtained from (3)–(5) by assuming the surge distribution to be normal and estimating the distribution of the tide directly from a histogram of the 1 year of Halifax predicted tide. The results presented in Figure 2 slightly underestimate the extremal return periods although the agreement is very good.

The agreement is in part fortuitous. This is most easily demonstrated by assuming the tide to be of the form $\eta_T = a \sin \omega t$, i.e., to have just one constituent. In this case the EPM, through use of (22), predicts an exceedance return period of

$$T_R \simeq 4\pi/\{\omega \text{ erfc } [(\eta^* - a)/\sigma(2)^{1/2}]\} \tag{23}$$

assuming $\eta^* \geq a$. The return period from the JPM may also be obtained from the distribution of the single tidal constituent, i.e.,

$$\begin{aligned} P_{\eta T} &= \pi^{-1}(a^2 - \xi^2)^{-1/2} & |\xi| \leq a \\ P_{\eta T} &= 0 & \text{other} \end{aligned} \tag{24}$$

and the normal surge distribution, which together imply

$$Q_I \simeq \tfrac{1}{4} \text{ erfc } [(\eta^* - a)/\sigma(2)^{1/2}] - \frac{1}{\pi}\int_0^a P_\eta(\eta^* - \xi) \sin^{-1}(\xi/a)\, d\xi \tag{25}$$

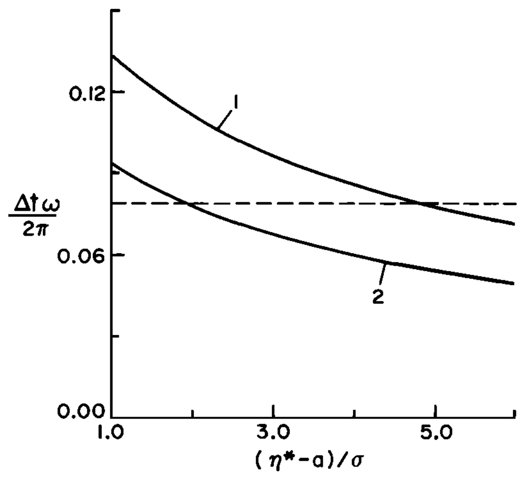

Fig. 3. The "correct" independence time Δt for the JPM as a function of critical level η^*, tidal amplitude a, and surge standard deviation σ. The independence time is scaled by the period of the single constituent tide ($2\pi/\omega$). Curves 1 and 2 correspond to $a/\sigma = 5$ and $a/\sigma = 10$, respectively. The dashed line shows $\Delta t = 1$ hour for a semidiurnal tide.

To obtain a return period, it is necessary to multiply Q_I^{-1} by some independence time scale, Δt, which *Pugh and Vassie* [1980] take to be the sampling interval of 1 hour. However, by equating the JPM return period of $\Delta t/Q_I$ with T_R from (23) it is clear that the correct independence time should be

$$\Delta t = Q_I T_R \tag{26}$$

which will depend on ω, σ, a, and η^*. This dependence is illustrated in Figure 3, where the independence time, scaled by the tidal period, is plotted against $(\eta^* - a)/\sigma$ for the a/σ ratios of 5 and 10 that crudely correspond to those found below for Halifax and Victoria. The dashed horizontal line represents the 1-hour independence time for a semidiurnal tide.

These results show why the JPM and exceedance return periods in Figure 2 are in reasonable agreement. For exceedance levels that are several σ above the tidal amplitude, the correct independence time for Halifax is of order 1 hour. However, for exceedance levels below or above about $5\sigma + a$, the correct independence time will be larger or smaller than 1 hour, and so the JPM will underestimate and overestimate T_R. Further, for a diurnal tide the correct independence time and return period will double and hence differ from those of the JPM by an additional factor of 2.

To summarize the results of the Markovian simulations, the EPM gives accurate estimates of exceedance return periods in both strong surge and strong tide limits. Further, for large η^* the EPM can also provide good estimates of the return period of annual extremes (which approach those of exceedances as η^* increases). The JPM can provide good results in the strong tide limit if Δt is chosen correctly; the JPM is inappropriate in the strong surge limit.

4. ANALYSIS OF THE HALIFAX AND VICTORIA RECORDS

The simulations of section 3 were based on idealized surge statistics, e.g., normal distribution, constant variance ($\varepsilon = 0$), no trend, etc. The real world is obviously more complicated, and so, as a prelude to using the EPM on observations, we will now describe the Halifax and Victoria sea level variability.

Hourly estimates of sea level were obtained at Victoria and Halifax for the years 1911–1982 and 1920–1982, respectively, from the Marine Environmental Data Service (Ottawa, Canada). The monthly extremes for Halifax (Figure 4) exhibit a clear linear trend of 0.375 cm yr^{-1} which is identical to that found for the monthly means. These trends are thought due to

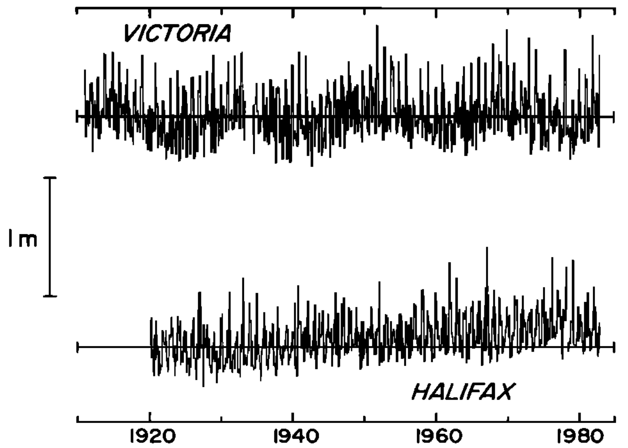

Fig. 4. Time series plot of monthly maxima based on all available hourly observations for Halifax and Victoria. Each series has zero mean.

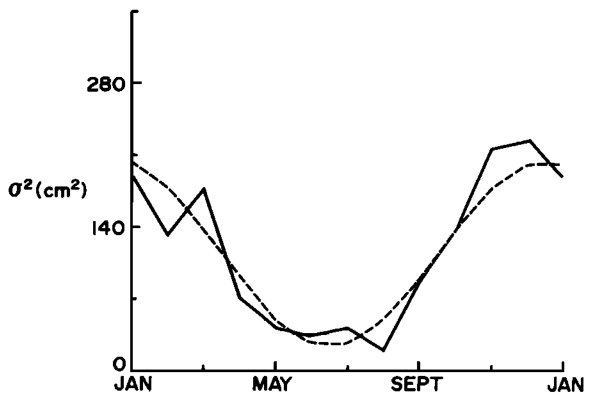

Fig. 5. Estimates of the surge variance at Victoria for each month of 1970 (solid line) and the model $\sigma^2 = \sigma_0^2(1 + \varepsilon \cos \Omega t)$ where $\sigma_0 =$ 10.7 cm and $\varepsilon = 0.8$ (dashed line).

vertical crustal movement and eustatic changes [e.g., *Vanicek*, 1976]. Over the 63 years of data the trend is responsible for a change of 23 cm that is comparable with the changes in extremes ($\sim$60 cm). Apart from this trend and a clear seasonal modulation the monthly extremes appear reasonably stationary with no evidence of significant nodal variation. At Victoria the seasonal modulation is again present although the trend is much reduced (0.032 cm yr^{-1}). However, there is now a clear nodal modulation which is responsible for an underlying change of about 24 cm in the extremes over an 18.6-year period.

The purpose of this paper is to present a method for determining return periods from short records. Therefore, as a severe test of the EPM, only 1 year of data will be analyzed to estimate the tidal signal and surge statistics necessary for return period prediction. The year 1970 was chosen arbitrarily, and the tidal package of *Foreman* [1977] used to determine the tidal constituents, predicted tide and "surge" (observed sea level minus predicted tide). At Halifax, apparent timing errors resulted in some 23% of the surge variance being due to residual tide. However, rather than choose another analysis year, we have persisted with the 1970 data so as to show how such errors affect predicted return periods. Indeed, where only short data records are available, timing errors may be unavoidable. Estimates of surge variance were obtained for each month of 1970 at Victoria and Halifax (Figures 5 and 6). A strong seasonal dependence is apparent at both sites. At Victoria the cycle is well represented by the model, $\sigma^2 = \sigma_0^2(1 + \varepsilon \cos \Omega t)$, where the yearly averaged variance (σ_0^2) is 113.6 cm and the parameter ε is 0.8. For Halifax the model parameters $\sigma_0^2 = 124.9$ cm and $\varepsilon = 0.8$ lead to an underestimate of the December spike in surge variance; elsewhere the fit is reasonable. The spike in September is due to residual tide presumably unremoved due to timing errors.

To determine the surge microscale λ, the effect of residual tide must be considered because, in analogy with enstrophy, λ is determined principally by the high-frequency components of the power spectrum of surge. This may be shown by first defining the normalized power spectrum

$$E(\omega) = 2\sigma^{-2} \int_0^\infty \langle \eta(\tau)\eta(t + \tau)\rangle \exp(-2\pi i\omega\tau)\, d\tau \tag{27}$$

The microscale due to energy below some cutoff ω may then be written as

$$\lambda(\omega) = \left\{8\pi^2 \int_0^\omega \zeta^2 E(\zeta)\, d\zeta\right\}^{-1/2} \tag{28}$$

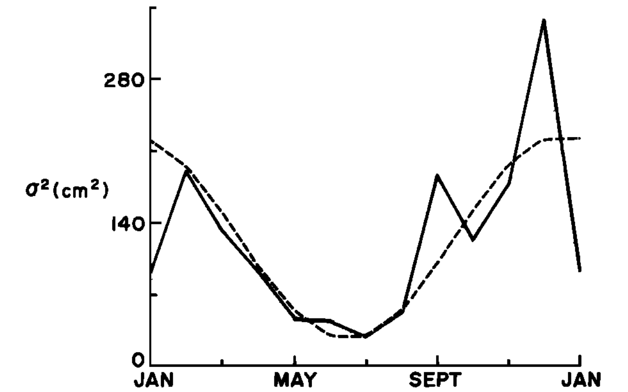

Fig. 6. Estimates of the surge variance at Halifax for each month of 1970 (solid line) and the model $\sigma^2 = \sigma_0{}^2(1 + \varepsilon \cos \Omega t)$ where $\sigma_0 = 11.2$ cm and $\varepsilon = 0.8$ (dashed line).

Formally, both (27) and (28) are nonstationary (dependent on season), and so estimates of $E(\omega)$, $\lambda(\omega)$, and the normalized surge variance,

$$\sigma_N{}^2(\omega) = 2 \int_0^\omega E(\zeta)\, d\zeta \tag{29}$$

were obtained for Victoria for "summer" (April–September 1970) and "winter" (remainder of 1970).

The seasonal estimates of $\lambda(\omega)$ and $\sigma_N{}^2(\omega)$ are similar (Figure 7), indicating that the nonstationarity of the surge process is primarily contained in the variance. The seasonal estimates of $E(\omega)$ were also similar but, for clarity, only the winter spectrum is presented (Figure 7). It is clear that while most (85%) of the reduced residual energy lies below the diurnal peak, the tidal energy reduces λ from near 17 hours to 4 hours. We shall therefore take λ to be 17 hours and note that this estimate, as against that of 4 hours, is more compatible with the passage time of storms responsible for surges. The

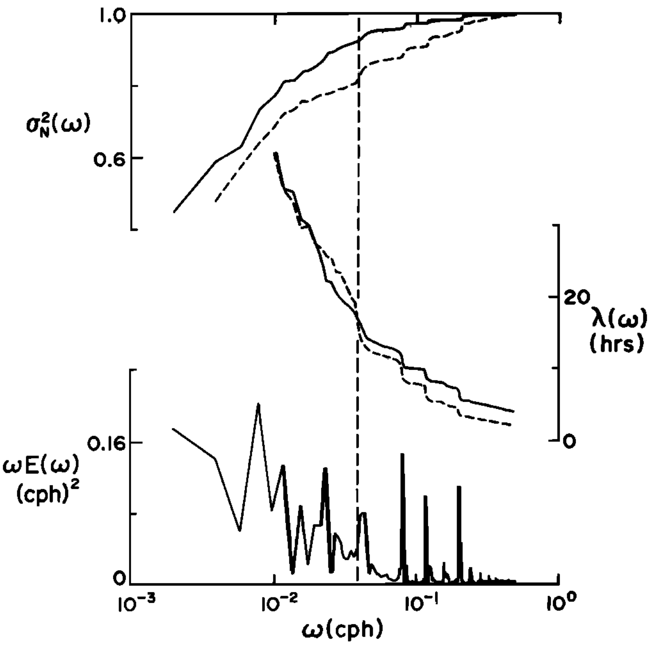

Fig. 7. Summer (dashed curve) and winter (solid curve) estimates of normalized variance ($\sigma_N{}^2(\omega)$, (29)), microscale ($\lambda(\omega)$, (28)) and autospectrum ($E(\omega)$, (27)) for Victoria, 1970. The summer $E(\omega)$ is not presented for clarity. The vertical dashed line marks the cutoff frequency chosen to lie just below the diurnal peak due to residual tide.

TABLE 1. Summary of the Surge and Tide Statistics for Halifax and Victoria Based on Hourly Data for 1970

Statistic	Halifax	Victoria
$\sigma_0{}^2$, cm^2	124.9	113.6
$f\sigma_0{}^2$, cm^2	96.2	96.6
$\sigma_T{}^2$, cm^2	2080	4550
ε	0.8	0.8
λ, hours	10	17
Γ	21	28
S	0.3	0.1
$\dot{S}$	0.0	0.0
K	3.8	3.3
$\dot{K}$	3.8	3.3

significant amount of residual tidal energy also suggests that the observed surge variance should be reduced by some fraction f. Assuming all energy above, and including, the diurnal peak in $E(\omega)$ to be tidal, we find f to be 0.85 at Victoria and the reduced variance, $f\sigma_0{}^2$, to be 96.6 cm^2. This estimate corresponds to the lower 80% confidence bound for $\sigma_0{}^2$ if we assume the independence time to be twice the surge integral time scale of 40 hours. We shall therefore make predictions of return period using both $\sigma_0{}^2$ and the reduced value, $f\sigma_0{}^2$. Note that for the predictions it will not be necessary to amplify the tidal signal to account for the reduced residual tidal energy because the 1970 tidal variance, $\sigma_T{}^2 = 4550$ cm^2, is much greater than $(1 - f)\sigma^2$. Inspection of the exceedance probability, (23), shows that it is the 15% change in σ^2 rather than the 0.3% change in tidal energy which is most important in return period prediction. These variance estimates also imply that the estimated microscale for Victoria need not be exact as the parameter $\Gamma = \sigma_T\omega\lambda/\sigma \simeq 28$ is much larger than unity and the strong tide limit should pertain.

Estimates of $E(\omega)$, $\lambda(\omega)$, and $\sigma_N{}^2(\omega)$ were also obtained for Halifax. Apparent timing errors caused about 23% of the surge variance to be due to residual tide. The resulting factor of $f = 0.77$ implies a reduced surge variance of $f\sigma_0{}^2 = 96.2$ cm^2. A microscale of 10 hours was estimated for energy below the dominant semidiurnal peak. Again the strong tide limit ($\Gamma \gg 1$) should pertain at Halifax because $\sigma_T{}^2 = 2080$ cm^2 and Γ is near 21 for the dominant semidiurnal tide.

The 1970 surge statistics for Halifax and Victoria are summarized in Table 1 along with estimates of skewness and kurtosis for η (S and K) and $\dot{\eta}$ ($\dot{S}$ and $\dot{K}$). The skewness and kurtosis suggest that the surge process at Halifax may be far from normal.

5. RETURN PERIODS FOR HALIFAX AND VICTORIA

The simulations demonstrated that accurate predictions of return periods may be obtained for a normally distributed surge process using the EPM. However, as noted above, the surge processes at Halifax and Victoria are not normal, exhibiting both skewness and kurtosis. The influence of such anormality is explored below after predictions based on the normal model for 1970 surge statistics are found to be in poor agreement with those from a Gumbel type analysis of all available annual extrema from Halifax and Victoria. A contaminated normal model for surge is subsequently developed to allow for skewness and kurtosis.

5.1. *Normal Surge Model*

In order to compare predicted and extremal return periods at Halifax, the strong trend in sea level was first removed by adjusting all annual extremes to the mean level of the 1970

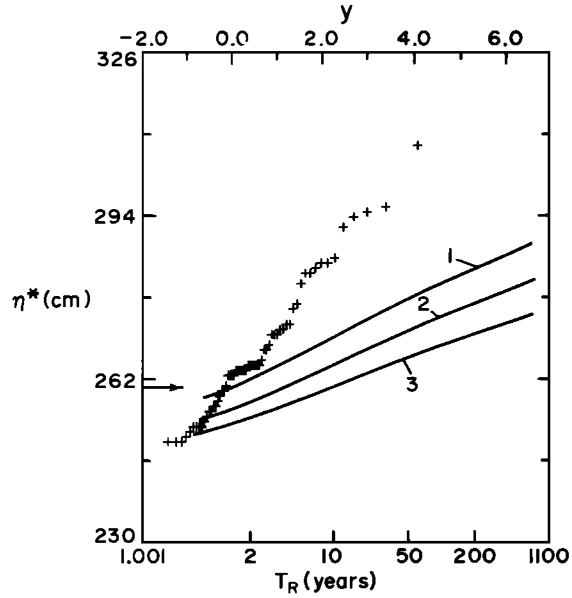

Fig. 8. The crosses denote extremal return periods based on all annual extrema for Halifax. (The extremes were adjusted for the linear trend.) The horizontal arrow denotes the most likely (or modal) annual extreme. Curves marked 1 and 2 were obtained using the EPM with the normal surge model (16). Curve 1 was based on the observed surge variance σ_0^2, and curve 2 was based on the reduced surge variance $f\sigma_0^2$ (Table 1). Both curves were based on $\varepsilon = 0.8$. Curve 3 was based on a constant surge variance throughout the year ($\varepsilon = 0$) of σ_0^2.

analysis year. Extremal return periods were then obtained following Gumbel and are plotted in Figure 8. (Each annual extreme was extracted from a year beginning in July so as to ensure that each winter, when most surge events occur, would contribute only one annual extreme.) It is noted that the adjustment for trend resulted in a significant decrease in the slope of the extremal $\eta^* - T_R$ curve with the largest recorded extremes near 294 cm essentially unaltered, while those at return periods of 2 years increased by about 8 cm.

Return periods of exceedance were also predicted by integrating (16) using the 1970 predicted Halifax tide and the normal surge parameters determined in section 4: $\sigma_0^2 = 124.9$ cm^2, $\varepsilon = 0.8$, and $\lambda = 10$ hours. These results, plotted in Figure 8, were indistinguishable from those obtained by integrating the strong tide limit probability (22). Note, however, that the EPM seriously overestimates the extremal return periods for η^* above the extremal mode, $\alpha = 261$ cm.

Return periods were also obtained with the tidally reduced variance, $f\sigma_0^2 = 96.2$ cm^2 (Figure 8). The reduction results in a nearly parallel shift of the predicted η^*-T_R curve and does not lead to the steeper slope evident in the extremal results. The importance of modeling the seasonal nature of the surge is also illustrated in Figure 8 by the further increase in return periods when seasonality is excluded ($\varepsilon = 0$).

A similar set of calculations was made for Victoria. Extremal return periods from the EPM were obtained by integrating the strong tide approximation (22) over 19 years of predicted tide to allow for a full nodal modulation. The return periods of the annual maxima (defined over a surge year, July–June) were estimated using a Gumbel analysis. The results from the EPM again seriously overestimate the return periods from the Gumbel analysis (Figure 9).

Given the agreement found with the simulations of normally distributed surge, the above discrepancy was thought to be due to the positive skewness and kurtosis of surge, which would make large exceedances more frequent at the expense of intermediate exceedances. To account for such anormality, a contaminated normal model for surge was developed for inclusion in the EPM. (While other distributions might be chosen to allow for skewness or kurtosis, the contaminated normal model developed below involves only a trivial extension of the EPM.)

5.2. *A Contaminated Normal Model for Q*

Our contaminated normal model for the joint distribution of surge and its derivative is defined as the sum of two bivariate normals, i.e.,

$$P_{\eta\dot{\eta}} = \sum_{i=1}^{2} a_i P_{\eta_i\dot{\eta}_i} \tag{30}$$

where $a_1 + a_2 = 1$, and the surge and its derivative are again assumed to be statistically independent. The moments of the two bivariate normals $\{\mu_i, \sigma_i, \nu_i / i = 1, 2\}$ are related to those of the surge by

$$\mu = \sum_{i=1}^{2} a_i \mu_i \tag{31}$$

$$\sigma^2 = \sum_{i=1}^{2} a_i(\mu_i^2 + \sigma_i^2) - \mu^2 \tag{32}$$

$$\nu^2 = \sum_{i=1}^{2} a_i \nu_i^2 \tag{33}$$

where we have introduced the surge mean $\mu = \langle\eta\rangle$, which is here assumed nonzero. (Although the surge data will generally have zero mean, the error accompanying the least squares fit of the contaminated normal model to data can lead to a probability density function with nonzero mean. We could of course constrain the least squares fitting procedure to ensure $\mu = 0$, but this was not considered necessary; the real point of the contaminated normal model is to fit the observed surge distribution in the positive tail.)

It is possible to substitute the contaminated normal distribution (30) into a suitably generalized form of (16), integrate,

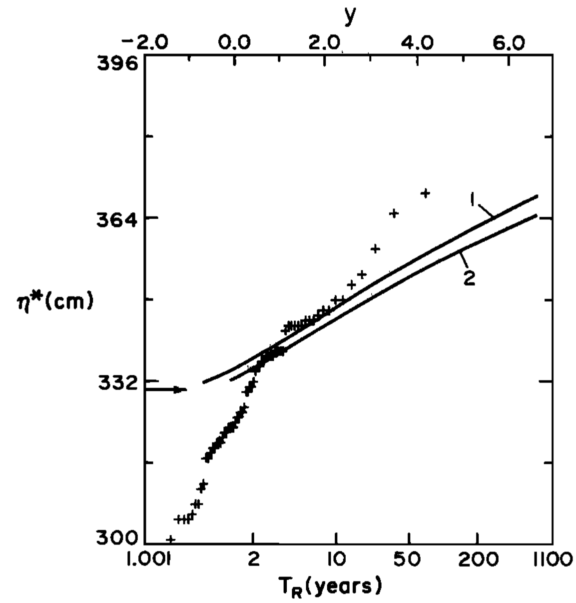

Fig. 9. As for Figure 8 but for Victoria and without correction for a linear trend. The predictions were based on 19 years of predicted tide, so a full nodal modulation was included.

TABLE 2. Fitted and Tidally Reduced ($f^{1/2}$) Surge Parameters for the Contaminated Normal Model of Surge Distribution

	Halifax		Victoria	
Statistic	Fitted	Reduced	Fitted	Reduced
a_1	0.873	0.873	0.710	0.710
μ_0, cm	0.48	0.42	0.78	0.72
μ_{01}, cm	−0.81	−0.71	−1.15	−1.06
μ_{02}, cm	9.33	8.19	5.50	5.07
σ_0^2, cm^2	122.1	94.1	107.3	91.2
σ_{01}, cm	9.44	8.29	7.65	7.05
σ_{02}, cm	16.10	14.14	13.98	12.89

Statistics were derived from hourly surge data from Halifax and Victoria for 1970.

and thus find the return periods directly. However, it is fairly straightforward to show that at both Halifax and Victoria the strong tide limit applies to each of the bivariate normals in (30), i.e.,

$$\sigma_T\omega/v_i \gg 1 \qquad i = 1, 2 \tag{34}$$

It then follows that in the strong tide limit, the exceedance probability will again be given by

$$Q(t) = \tfrac{1}{2}\{\dot{\eta}_T + |\dot{\eta}_T|\}P_\eta(\eta^* - \eta_T) \tag{35}$$

but with P_η now given by

$$P_\eta(\eta^* - \eta_T) = \sum_{i=1}^{2} a_i[(2\pi)^{1/2}\sigma_i]^{-1} \cdot \exp\,[-(\eta^* - \eta_T - \mu_i)^2/2\sigma_i^2] \tag{36}$$

Note that the precise estimation of v_i (or equivalently λ_i) is not required in the strong tide limit.

To model the seasonal variation in surge variance, each of the μ_i and σ_i^2 should seasonally modulate as follows:

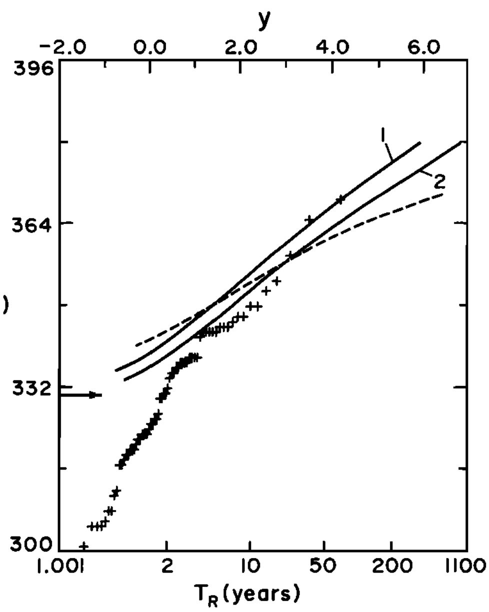

Fig. 10. The crosses denote extremal return periods based on all annual maxima for Victoria. Curves marked 1 and 2 were obtained from the EPM with the contaminated normal model for surge, i.e., (35) and (36). Curve 1 was based on the observed surge distribution, and curve 2 was based on the reduced ($f^{1/2}$) parameters (Table 2). Both curves were based on $\varepsilon = 0.8$. The dashed curve shows predictions from the JPM using the observed surge distribution for 1970. All three predictions were based on 19 years of predicted tide, so a full nodal modulation was included.

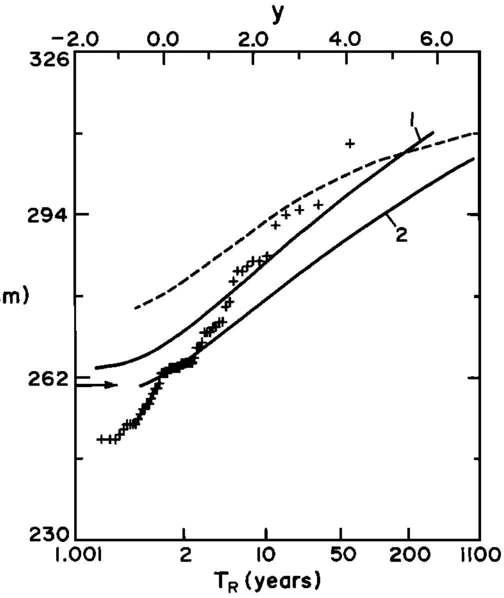

Fig. 11. As in Figure 10 but for Halifax. (The annual extrema were adjusted for the linear trend, and the predictions were based on 1 year of predicted tide.)

$$\mu_i = \mu_{0i}(1 + \varepsilon \cos \Omega t)^{1/2} \tag{37}$$

$$\sigma_i^2 = \sigma_{0i}^2(1 + \varepsilon \cos \Omega t) \tag{38}$$

Thus the contaminated normal model (36)–(38) involves seven parameters that are to be determined from the 1970 surge data: $\{a_i, \mu_{0i}, \sigma_{0i}/i = 1, 2\}$ and ε. The value of ε was set at 0.8 for both Halifax and Victoria (Table 1). The remaining six parameters were estimated by fitting a sum of two normal distributions to the histogram of 1970 surge data at Halifax and Victoria. (We used the nonlinear least squares program, NONLIN, of the SPSS suite of programs with the constraint that $a_1 + a_2 = 1$. The positive tail was weighted most heavily in the fitting procedure because it is the extreme positive surges which cause the exceedances of interest. The bin size of the histogram of 1970 surge data was 5 cm.) The least squares estimates $\{a_i, \mu_{0i}, \sigma_{0i}/i = 1, 2\}$ for Halifax and Victoria are given in Table 2; together with ε they define the seasonal, anormal models for surge distribution.

5.3. *Contaminated Normal Results*

Return periods were predicted using the contaminated normal model (35)–(38), $\varepsilon = 0.8$, and two sets of parameters: $\{a_i, \mu_{0i}, \sigma_{0i}/i = 1, 2\}$ and $\{a_i, f^{1/2}\mu_{0i}, f^{1/2}\sigma_{0i}/i = 1, 2\}$. The latter are simply the least squares estimates after correction for residual tide, and both sets of parameters are given in Table 2.

For Victoria the contaminated normal model results in a marked steepening of the η^*-T_R curve over those obtained assuming normality (Figure 10). The best fit to the Gumbel analysis above the mode ($\alpha = 331$ cm) lies somewhere between those obtained from the fitted and reduced ($f^{1/2}$) parameters. It is encouraging to note, however, that both curves, based on just 1 year of data, can reasonably reproduce the extremal return periods based on about 60 years of data.

Predicted return periods were also obtained for Halifax using the EPM, and the results are presented in Figure 11. Predictions from the fitted surge parameters again reasonably represent the extremal results although they are not as accurate as predictions for Victoria. This discrepancy may be due to the inadequate modeling of the December spike in the surge variance at Halifax (Figure 6). Assuming this spike to be typical of the December surge variance, then perhaps a more elaborate model than $\sigma^2 = \sigma_0{}^2(1 + \varepsilon \cos \Omega t)$ would lead to the required shortening of return periods. The strong sensitivity of the return periods to the yearly averaged surge variance is also illustrated for Halifax, where reducing $\sigma_0{}^2$ by 23% to allow for residual tide leads to a twofold increase in the predicted return period for $\eta^* = 278$ cm. This reduced surge variance also corresponds to the lower 80% confidence bound for $\sigma_0{}^2$, and so the change in return period indicates the inherent prediction error associated with an analysis based on just 1 year of data. If longer time series of sea level were used, better estimates of the surge distribution, skewness, kurtosis, and variance would be obtained, presumably leading to improved predictions.

Finally, return period estimates were obtained from the JPM using the 1970 observed surge distribution (unadjusted for residual tide) and the predicted tide at Halifax and Victoria. (Nineteen years of predicted tide were used to estimate the tidal distribution at Victoria.) The results for each site are presented in Figures 10 and 11. The predicted return periods near 10 years are in reasonable agreement with those obtained from the extremal analysis. At longer or shorter return periods, however, the JPM overestimates or underestimates the extremal results more seriously than those predicted from the contaminated normal model. Indeed, such behavior of the JPM return period predictions was demonstrated for a single constituent tide in section 3.2.

6. SUMMARY AND DISCUSSION

By assuming tides to be deterministic and surges to be stochastic, the analysis of *Rice* [1954] was extended to allow prediction of return periods for sea level exceedances. In the strong surge limit the general expression for exceedance probability (16) was shown to asymptote to that given by *Rice* [1954], and the return periods were shown to scale with the surge microscale λ, a time scale of surge variability. Of more interest, perhaps, is the case where tidal variability dominates the sea level process and for which the probability of exceedance is approximately given by

$$Q(t) = \tfrac{1}{2}\{\dot{\eta}_T + |\dot{\eta}_T|\}P_\eta(\eta^* - \eta_T) \qquad (39)$$

so that return periods now scale with the dominant tidal period and are independent of λ.

This extension of the Rice method (which we referred to as the exceedance probability method, or EPM) was validated in both strong surge and strong tide limits using a 50-year simulation of hourly surge data. In addition, this analysis showed that return periods of exceedances and extremes are almost identical for heights above the most likely extreme. Below the modal extreme, exceedances occur more frequently because extremes, by definition, must be the largest event in a chosen time interval.

For Halifax and Victoria more than 60 years of sea level data were analyzed following Gumbel in order to obtain a benchmark of extremal return periods. For these sites the strong tide limit was found to pertain, and return periods were predicted from (39) using a normal and contaminated normal model fitted to just 1 year of surge data. The results indicated that reasonable return periods may be predicted if allowance is made for (1) the positive kurtosis and skewness of the surge, which acts to make extreme exceedances more probable, and (2) the seasonal variation in the surge variance, which results in an effective variance which is larger than the annual average, $\sigma_0{}^2$.

More accurate return periods may well be obtained by using several years of data and through the development and fitting of more elaborate models for the surge statistics. Here, however, we have confined ourselves to relatively simple models, i.e., a contaminated normal and $\sigma^2 = \sigma_0{}^2(1 + \varepsilon \cos \Omega t)$. Even though many years of sea level data were available to us, only 1 year was chosen in order to provide a stringent test of the EPM. The reasonable results suggest that the EPM may be usefully employed in those regions where only short records are available and a Gumbel type of extremal analysis is precluded. Such predictions might well be made using the strong-tide exceedance probability (39) which will pertain provided the rms tide, σ_T, is much greater than the surge variance. (We expect that in general, $\omega\lambda$ will be of order 1 or greater so that $\sigma_T/\sigma \gg 1$ implies $\Gamma = \omega\lambda\sigma_T/\sigma \gg 1$.) At sites where the tide and surge variance are of comparable magnitude, the general exceedance probability (16) might also be used to predict return periods. However, a contaminated normal model would most likely be required in order to account for possible skewness and kurtosis of the surge distribution.

The EPM was also used to examine the prediction method suggested by *Pugh and Vassie* [1980]. In the strong tide limit, the independence time Δt required to make their method (JPM) exact was shown to differ significantly from 1 hour because of its dependence on the surge variance, tidal period and amplitude, and exceedance level. It was also shown that the JPM is totally inappropriate in the strong surge limit (although we recognize that the method was not designed to be employed in such circumstances). This will presumably inhibit the use of the JPM in extreme current studies where the nontidal variance is often a significant proportion of the observed current variance.

One of the advantages of the EPM is that the return periods can be estimated with respect to a given time origin (usually the present) if the process under study exhibits significant nonstationarity. For example, the influence of trends in sea level, due to climate changes or vertical crustal movement, may be evaluated with a simple integration forward in time. Changes in storm severity on the incidence of flooding may also be evaluated by postulating an annual increase in surge variance. The exceedance probability might also be used in predicting return periods of extreme ocean currents or indeed of any geophysical process made up of a stochastic and a deterministic component.

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REFERENCES

Cartwright, D. E., On estimating the mean energy of sea waves from the highest waves in a record. *Proc. R. Soc. London, Ser. A, 247,* 22–48, 1958.

Foreman, M. G. G., Manual for tidal heights, analysis and prediction, *Pac. Mar. Serv. Rep. 77-10,* 101 pp., Inst. of Ocean Sci., Patricia Bay, B. C., 1977.

Garrett, C., and W. Munk, Oceanic mixing by breaking internal waves, *Deep Sea Res., 19,* 823–832, 1972.

Graff, J., An investigation of the frequency distribution of annual sea level maxima at ports around Great Britain, *Estuarine Coastal Shelf Sci., 12,* 389–449, 1981.

Gumbel, E. J., *Statistics of Extremes,* 375 pp., Columbia University Press, New York, 1958.

Jenkins, G. M., and D. G. Watts, *Spectral Analysis and Its Applications*, 525 pp., Holden-Day, San Francisco, Calif., 1968.

Lumley, J. L., The mathematical nature of the problem of relating Lagrangian and Eulerian statistical functions in turbulence, in *The Mechanics of Turbulence*, pp. 17–26, Gordon and Breach, New York, 1962.

Pugh, D. T., and J. M. Vassie, Applications of the joint probability method for extreme sea level computations, *Proc. Inst. Civ. Eng., Part 2*, *69*, 959–975, 1980.

Rice, S. O., The mathematical analysis of random noise, in *Noise and Stochastic Processes*, edited by N. Wax, pp. 133–294, Dover, New York, 1954.

Vanicek, P., Vertical crustal movements pattern in Maritime Canada, *Can. J. Earth Sci.*, *13*(5), 661–667, 1976.

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